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Title: Speed Limits: From Thermodynamics to Annealing

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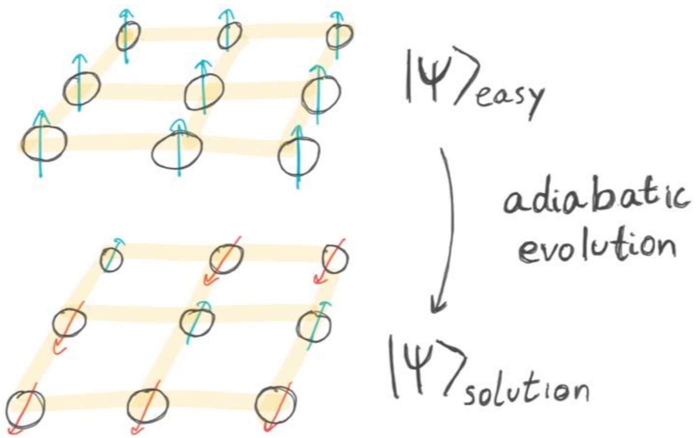
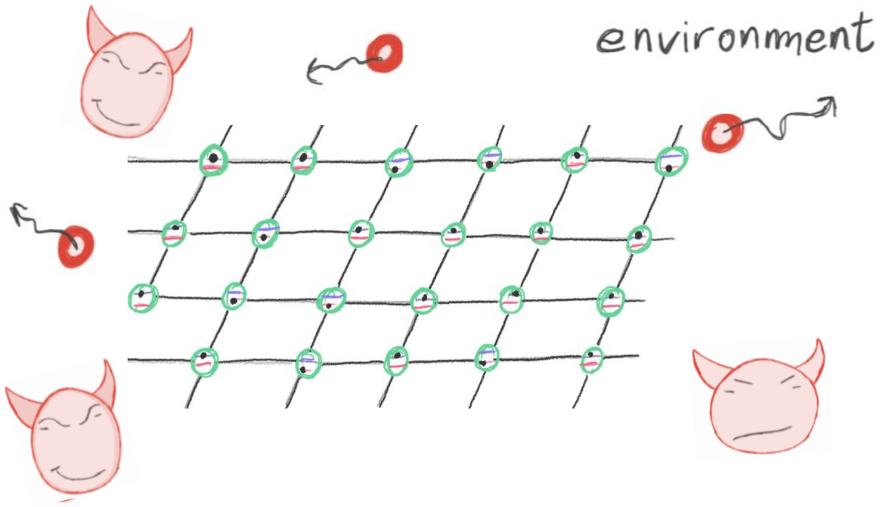
Speed Limits: From Thermodynamics to Annealing

Luis Pedro García-Pintos
Los Alamos National Laboratory

Dynamics of a complex quantum system: *How do we usually solve it?*





~~Simulations~~, ~~master equations~~

memory constraints

$$\frac{d\rho_t}{dt} \approx -i[H,\rho_t] + \sum_n \gamma_n [L_n[L_n,\rho_t]]$$

approximations
(weak coupling, Markov)

Today – alternative first-principles analysis:

general traits of dynamics from minimal assumptions

$$\frac{d\langle A\rangle}{dt} = ?$$

I. Introduction: Quantum Speed Limits

II. Speed Limits on Observables

III. An application: Quantum Annealing

I. **Introduction: Quantum Speed Limits**

II. Speed Limits on Observables

III. An application: Quantum Annealing

Quantum Speed Limits

Bounds to the speed of
evolution of a quantum system

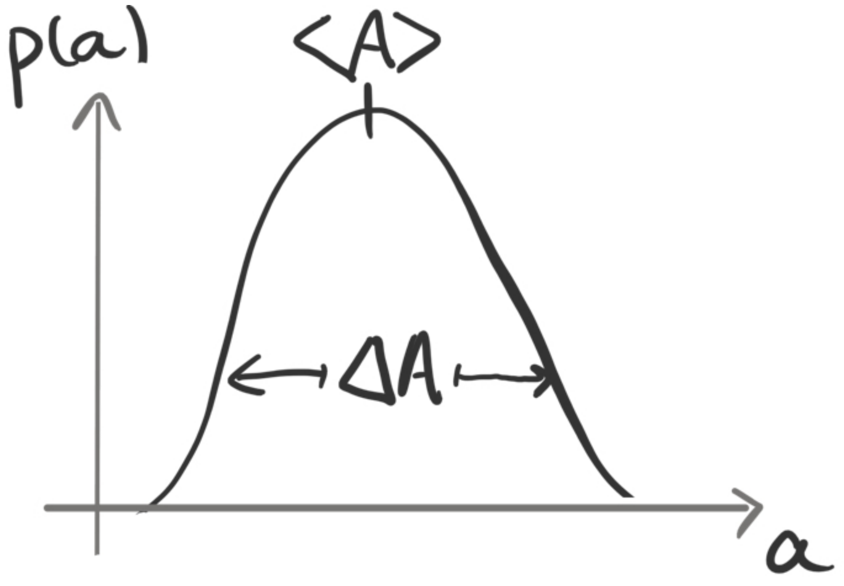
J. Phys. USSR 9, 249–254 (1945)

The Uncertainty Relation Between Energy and Time in Non-relativistic Quantum Mechanics

By *L. Mandelstam*[1] and *Ig. Tamm*

For isolated systems:

$$\left|\frac{d\langle A\rangle}{dt}\right| \leq 2\,\Delta A\,\Delta H$$



expectation value ⟹ $\langle A\rangle = \mathrm{Tr}\left[A\rho_t\right]$

standard deviations ⟹ $\begin{cases} \Delta A = \sqrt{\langle A^2\rangle - \langle A\rangle^2} \\ \Delta H = \sqrt{\langle H^2\rangle - \langle H\rangle^2} \end{cases}$

Mandelstam-Tamm's Uncertainty Relation:
limits to the speed of any physical process on isolated **quantum** systems

Quantum Speed Limits

For isolated systems:

$$\left|\frac{d\langle A\rangle}{dt}\right| \leq 2\,\Delta A\,\Delta H$$

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The Uncertainty Relation Between Energy and Time in Non-relativistic Quantum Mechanics

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Lots of posterior work:
speed limits on the state
of an open quantum system

$$\dot{\rho}_t$$

TOPICAL REVIEW

Quantum speed limits: from Heisenberg's uncertainty principle to optimal quantum control

Sebastian Deffner[1] and Steve Campbell[2]

Published 10 October 2017 •

Shortcoming: speed of observables
can be very different!

$$|\psi_0\rangle = |\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\rangle$$

$$|\psi_t\rangle = |\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\rangle$$

First aim: **speed limits on observables for open quantum systems**

I. Introduction: Quantum Speed Limits

II. Speed Limits on Observables $\frac{d\langle A\rangle}{dt} \leq ?$

III. An application : Quantum Annealing

Classical Stochastic Systems

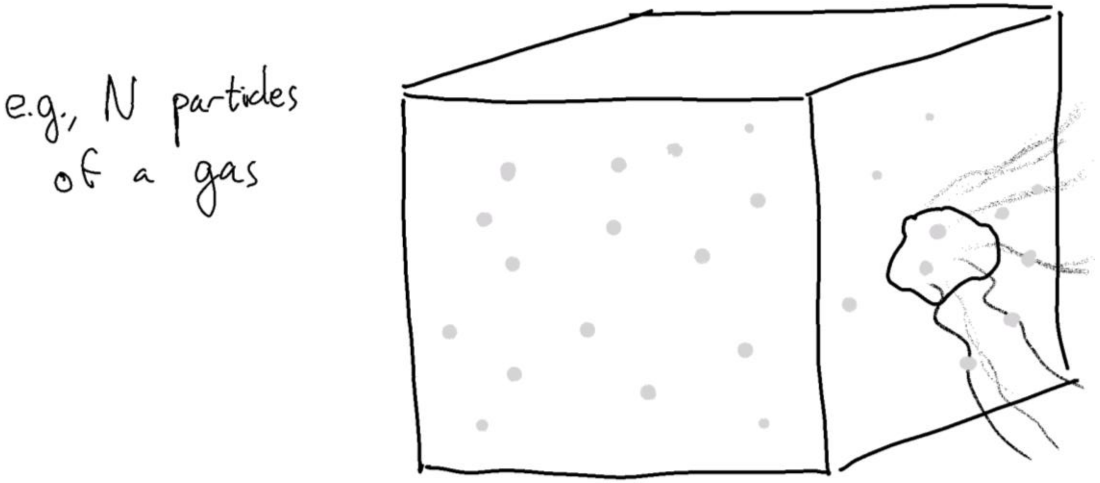

Article | Published: 21 September 2020

Time–information uncertainty relations in thermodynamics

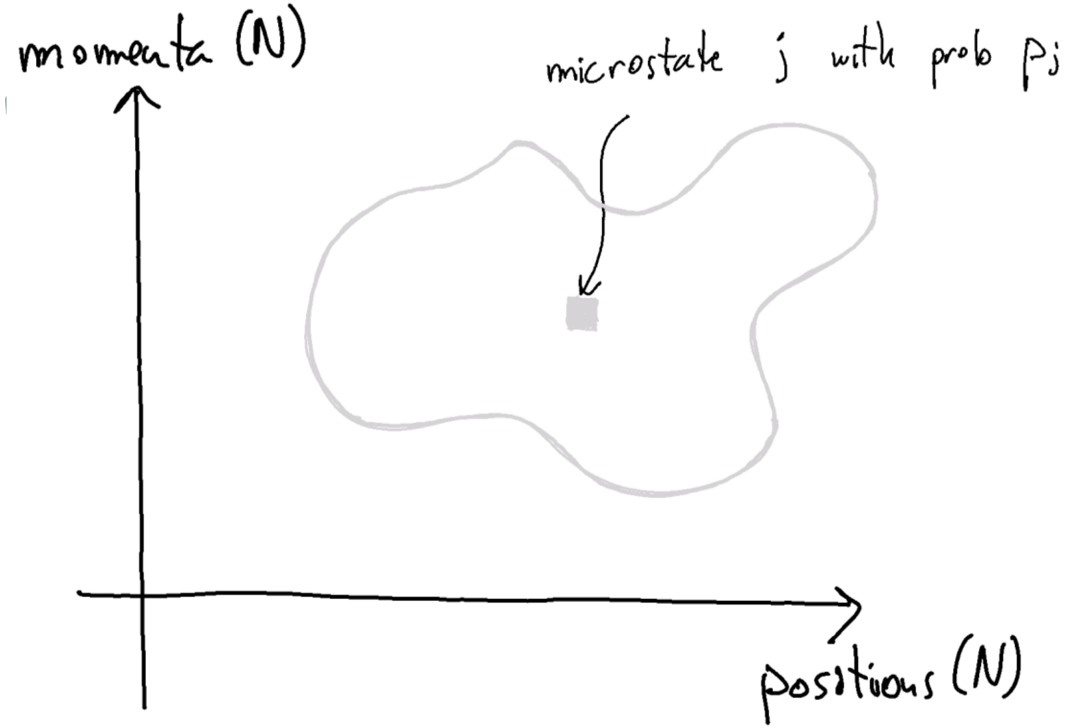
Schuyler B. Nicholson, Luis Pedro García-Pintos, Adolfo del Campo & Jason R. Green

Nature Physics **16**, 1211–1215(2020) | Cite this article

Classical system with 'states' j that occur with probabilities p_j



(e.g., j denotes a point in phase space)



The distribution $\mathbf{p} = \{p_{x_1}, p_{x_2}, \ldots\}$ represents the state of the system

Dynamics: $\mathbf{p}(t) = \{p_1(t), p_2(t), \ldots\}$

Classical observable A takes values a_j for state j (e.g., particle density, energy)

$$\langle A \rangle = \sum_j p_j(t) a_j$$

Classical Speed Limits

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Classical speed limit on observables

$$\text{surprisal} \longleftarrow I_j := \ln \frac{1}{p_j} \qquad \text{cov}(A,B) := \langle AB \rangle - \langle A \rangle \langle B \rangle \qquad \text{assumptions: } p_j \neq 0 \quad \& \quad \exists\, \dot{p}_j$$

$$\left| \frac{d\langle A \rangle}{dt} \right| = \left| \text{cov}(A, \dot{I}) \right| \leq \Delta A \; \Delta \dot{I}$$

speed

classical uncertainty

dynamics

(Classical) Fisher Information (~ measure of speed of p_j)

$$\mathcal{I}_F := (\Delta \dot{I})^2 = \sum_j p_j \left(\tfrac{d}{dt} \ln p_j \right)^2$$

Uncertainty relation bounds the speed of classical stochastic processes

Compare: quantum bound $\left| \frac{d\langle \hat{A} \rangle}{dt} \right| \leq 2\, \Delta \hat{A}\, \Delta \hat{H}$

Applications to stochastic thermodynamics:

heat flow $|\dot{Q}|$

entropy rates $|\dot{S}|$

Dynamics of Open Quantum Systems

PHYSICAL REVIEW X **12**, 011038 (2022)

Unifying Quantum and Classical Speed Limits on Observables

Luis Pedro García-Pintos[1,*] Schuyler B. Nicholson,[2] Jason R. Green,[3,4,5] Adolfo del Campo,[6,7,5] and Alexey V. Gorshkov[1]

Any open quantum system evolves following

$$\frac{d}{dt}\rho_t = -i[H,\rho_t] + \mathcal{D}(\rho_t)$$

coherent "quantum"

incoherent "classical" (*changes in probabilities*)

~ bound similarly to Mandelstam-Tamm 1945

~ bound similarly to Nat Phys 2020 (classical speed limits)

$$\rho_t = U_t \sum_j p_j(t)\, |j\rangle_0 \langle j|\, U_t^\dagger; \quad H_t \equiv i\dot{U}_t U_t^\dagger$$

$\rho_t = \sum_j p_j(t)\, |j\rangle_t \langle j|$ ⟹ density matrix

$-i[H,\rho_t]$ ⟹ coherent evolution

$\mathcal{D}(\rho_t)$ ⟹ incoherent evolution (dephasing, errors, environment)

If $\mathcal{D}(\rho_t) = 0 \Longrightarrow$ 'Ideal' isolated quantum dynamics

If $H = 0 \Longrightarrow$ *Classical* stochastic dynamics

Speed Limits for Open Systems

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$$\frac{d}{dt}\rho_t = -i[H,\rho_t] + \mathcal{D}(\rho_t)$$

coherent "quantum" → incoherent "classical"

$$\left|\frac{d\langle A\rangle}{dt}\right| = \left|\mathrm{cov}\left(A_C, L_C\right) + \mathrm{cov}\left(A_I, L_I\right)\right|$$

$$\leq \; \Delta A_C \, \Delta L_C \; + \; \Delta A_I \, \Delta L_I$$

$$\rho_t = \sum_j p_j(t)\, |j\rangle\langle j|$$

$$A_C = \sum_{j\neq k} A_{jk}\, |j\rangle\langle k|$$

$$A_I = \sum_j A_{jj}\, |j\rangle\langle j|$$

Coherent Fisher Information $\mathcal{I}_F^C := \left(\Delta L_C\right)^2$

Incoherent Fisher Information $\mathcal{I}_F^I := \left(\Delta L_I\right)^2$

$$L_C := -2i \sum_{j\neq k} \frac{\langle j|\,[H_t,\rho_t]\,|k\rangle}{(p_j + p_k)}\, |j\rangle\langle k|$$

$$L_I := \sum_j \frac{d\ln p_j}{dt}\, |j\rangle\langle j|$$

Speed Limits for Open Systems

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coherent "quantum"

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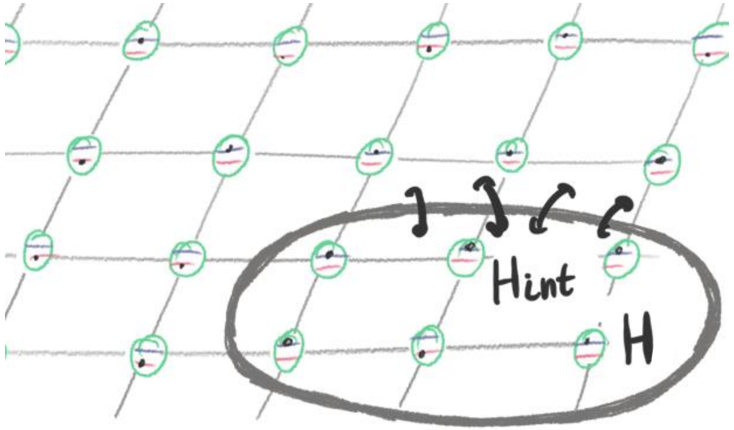
$$\leq \Delta A_C\,\Delta L_C + \Delta A_I\,\Delta L_I$$

$$\rho_t = \sum_j p_j(t)\,|j\rangle\langle j|$$

$$A_C = \sum_{j\neq k} A_{jk}\,|j\rangle\langle k|$$

$$A_I = \sum_j A_{jj}\,|j\rangle\langle j|$$

Incoherent Fisher Information $\mathcal{I}_F^I \leq 4(\Delta H_{\text{int}})^2$



Speed Limits for Open Systems

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$$\rho_t = \sum_j p_j(t)\,|j\rangle\langle j|$$

$$\left|\frac{d\langle A\rangle}{dt}\right| = \left|\mathrm{cov}\left(A_C, L_C\right) + \mathrm{cov}\left(A_I, L_I\right)\right|$$

$$\leq \Delta A_C\,\Delta L_C + \Delta A_I\,\Delta L_I$$

$$A_C = \sum_{j\neq k} A_{jk}\,|j\rangle\langle k|$$

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J. Phys. USSR 9, 249–254 (1945)

The Uncertainty Relation Between Energy and Time in Non-relativistic Quantum Mechanics

By *L. Mandelstam*[1] and *Ig. Tamm*

Lebedev Physical Institute, Academy of Sciences of the USSR

Article | Published: 21 September 2020

Time–information uncertainty relations in thermodynamics

Schuyler B. Nicholson, Luis Pedro García-Pintos, Adolfo del Campo & Jason R. Green

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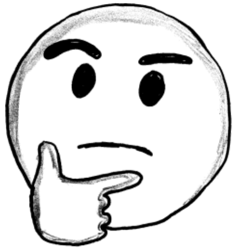
*So, how are these **general** bounds useful anyways?*



$$\frac{d\langle A\rangle}{dt} \leqslant ?$$

Can they accurately reflect timescales of evolution?

Second aim: **characterize regimes where speed limits are saturated**

Classical and quantum thermodynamics

Article | Published: 21 September 2020

Time–information uncertainty relations in thermodynamics

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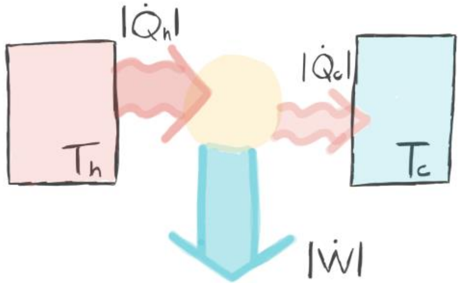
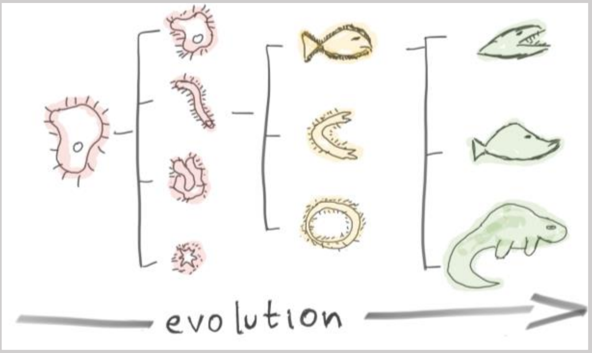


Evolutionary biology*

Diversity and Fitness Uncertainty Allow for Faster Evolutionary Rates

Luis Pedro García-Pintos[1,*]

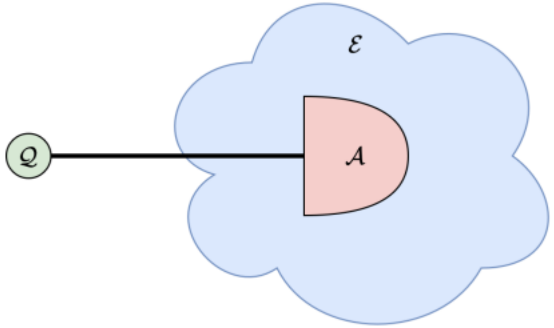


$$\frac{d\langle A\rangle}{dt} \lessgtr ?$$

Quantum-to-classical transition

Bounding the Minimum Time of a Quantum Measurement

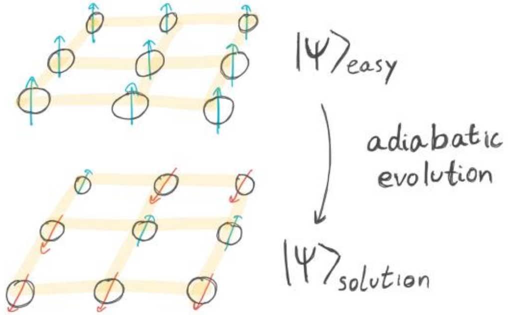
Nathan Shettell,[1] Federico Centrone,[2] and Luis Pedro García-Pintos[3]



Quantum annealing*

Lower Bounds on Quantum Annealing Times

Luis Pedro García-Pintos,[1,2,*] Lucas T. Brady,[3,4] Jacob Bringewatt,[1,2] and Yi-Kai Liu[1,5]



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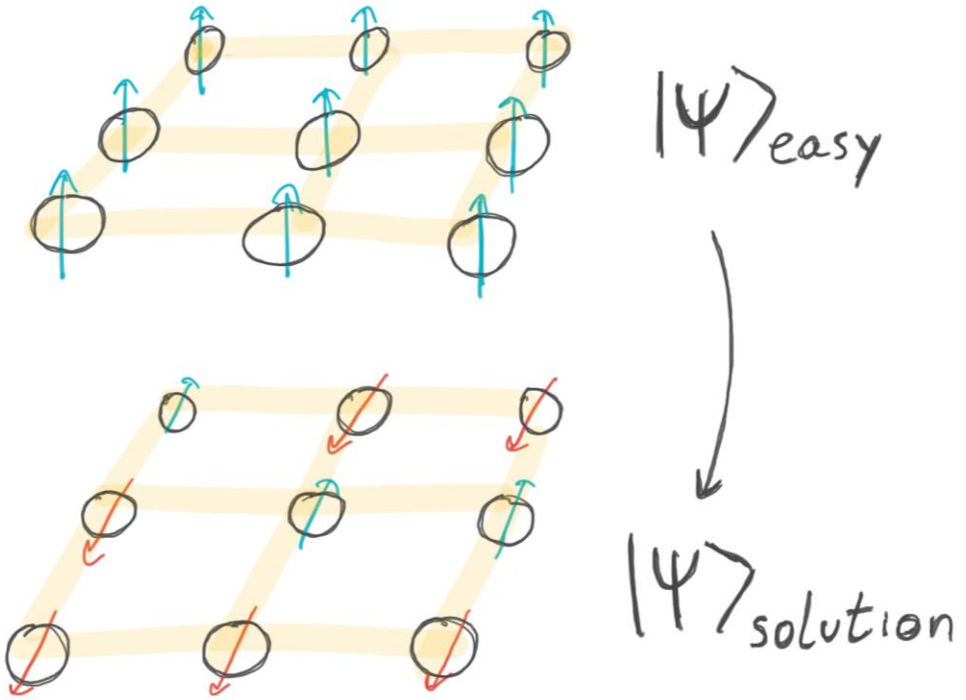
Quantum annealing

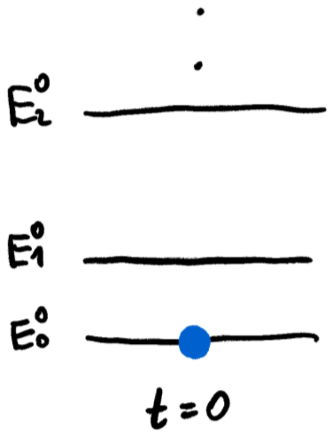
$$H_t = (1 - g_t) H_0 + g_t H_1 \qquad g_0 = 0; \quad g_{t_f} = 1$$

$$|\psi\rangle(0) = |E_0^{t=0}\rangle$$

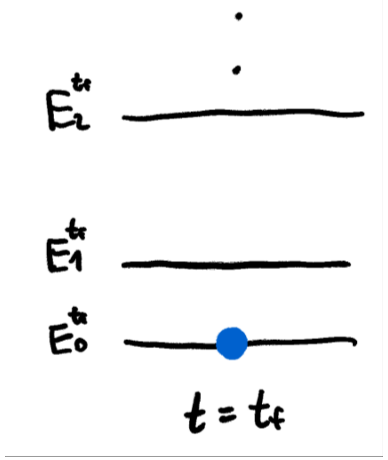
aim: $|\psi\rangle(t_f) = |E_0^{t_f}\rangle$



$$H_t = \sum_{j=0}^{d} E_j^t \, |E_j^t\rangle\langle E_j^t|$$



$\langle H_0 \rangle_0 = 0$



aim: $\langle H_1 \rangle_{t_f} = 0$

$$\langle H_i \rangle_t = \langle \psi | \, H_i \, | \psi \rangle \, (t)$$

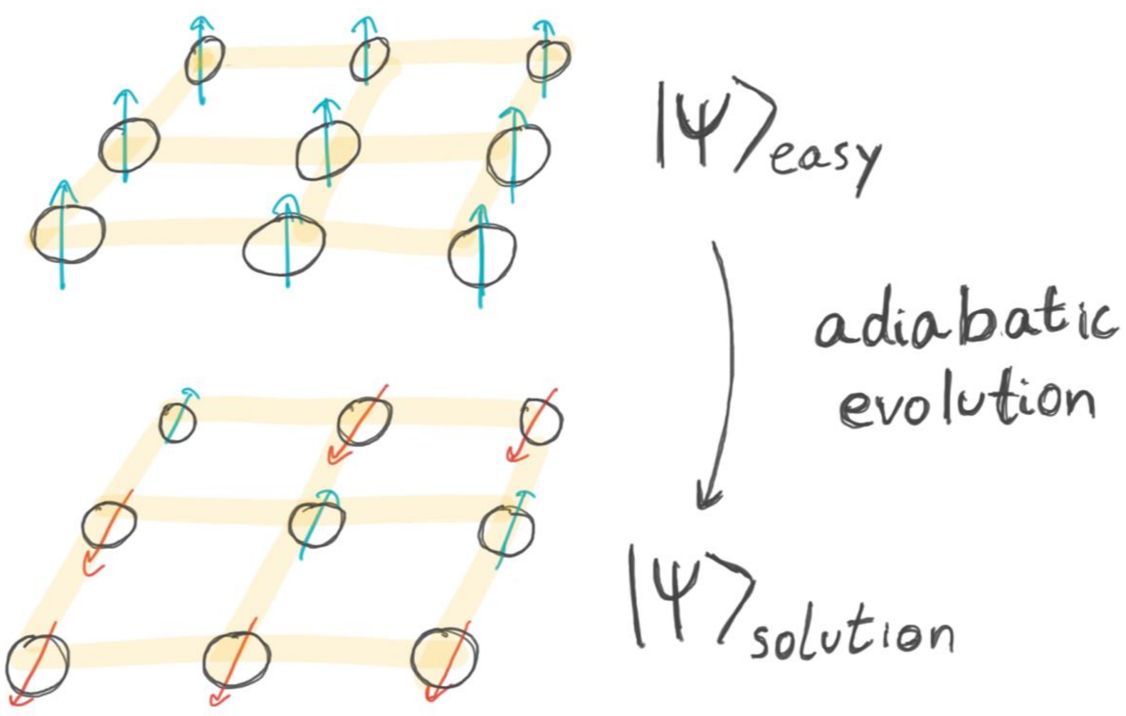
Quantum annealing

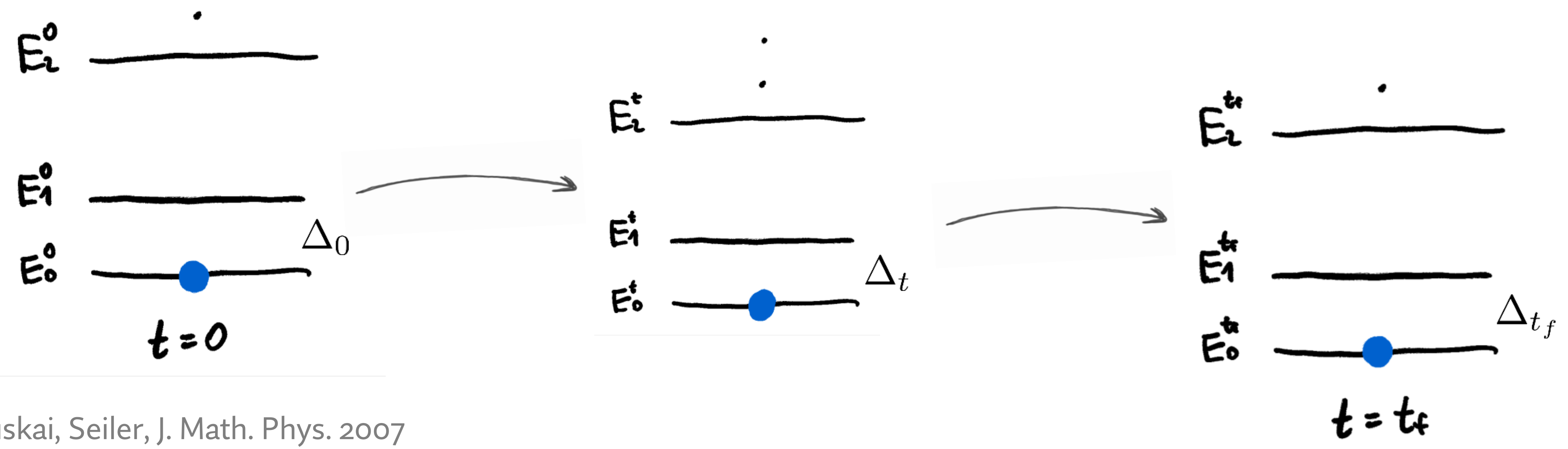
$$H_t = (1 - g_t)H_0 + g_t H_1$$

$$|\psi\rangle(0) = |E_0^{t=0}\rangle \qquad \text{aim: } \quad |\psi\rangle(t_f) = |E_0^{t_f}\rangle$$

Adiabatic theorem – sufficient condition:

system ends close to the target state if the transition is slow enough





Jansen, Ruskai, Seiler, J. Math. Phys. 2007

Quantum annealing

$$H_t = (1 - g_t) H_0 + g_t H_1$$

$|\psi\rangle(0) = |E_0^{t=0}\rangle$ aim: $|\psi\rangle(t_f) = |E_0^{t_f}\rangle$

Adiabatic theorem – sufficient condition:
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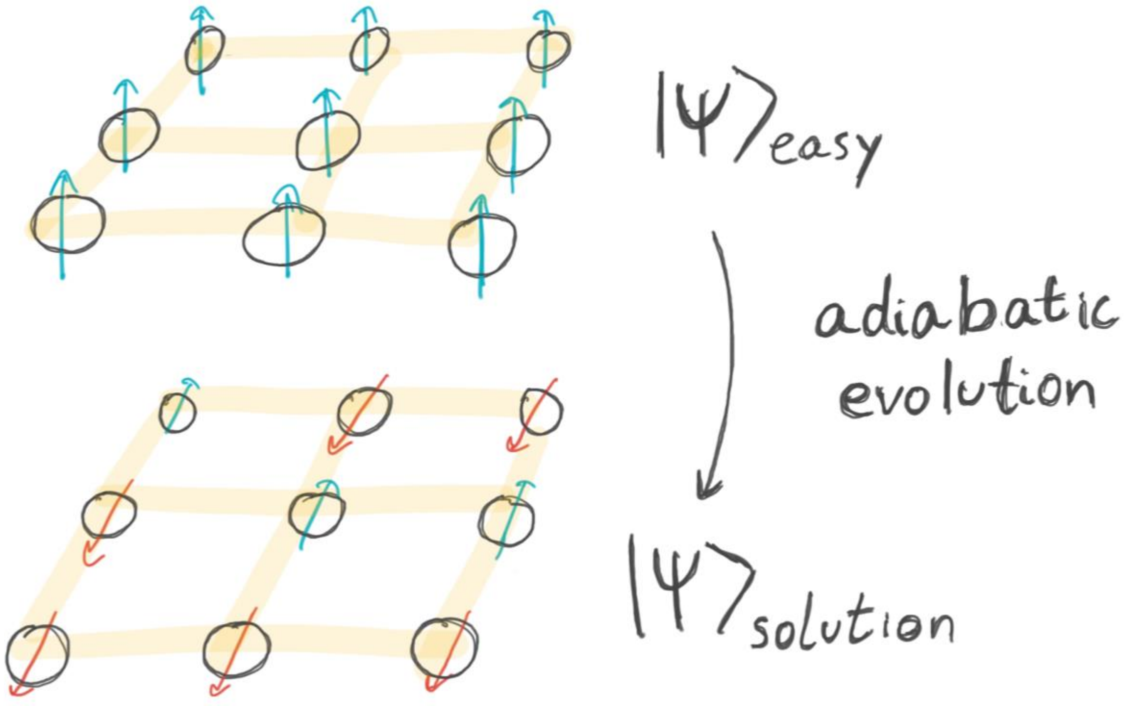


However, *annealing need not be adiabatic*!

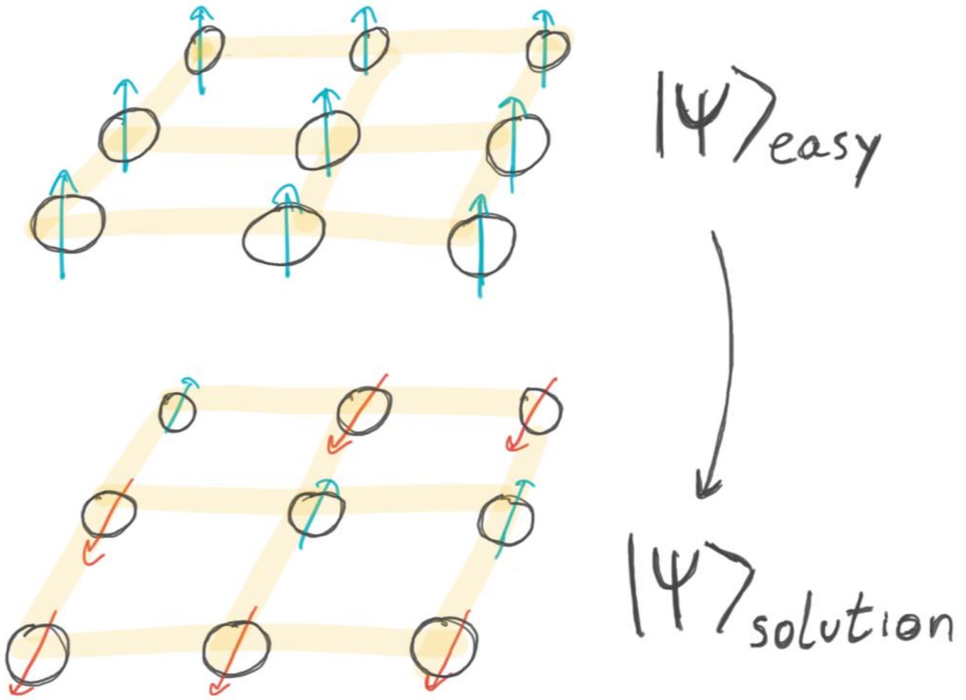
How fast can it be?

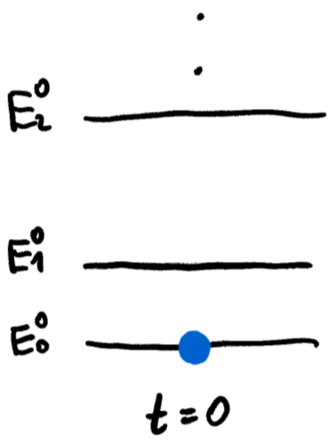
Jansen, Ruskai, Seiler, J. Math. Phys. 2007

Quantum annealing

$$H_t = (1 - g_t)H_0 + g_t H_1$$

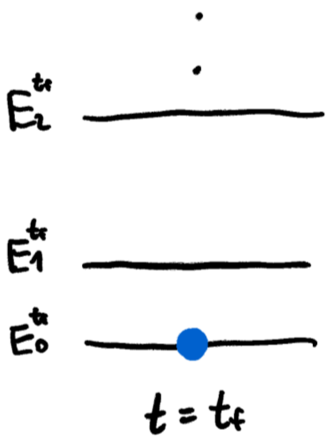
$|\psi\rangle(0) = |E_0^{t=0}\rangle$ aim: $|\psi\rangle(t_f) = |E_0^{t_f}\rangle$





$\langle H_0\rangle_0 = 0$

$t_f = ?$



aim: $\langle H_1\rangle_{t_f} = 0$

$\langle H_0\rangle_0 - \langle H_1\rangle_0 < 0$

$\langle H_0\rangle_{t_f} - \langle H_1\rangle_{t_f} > 0$

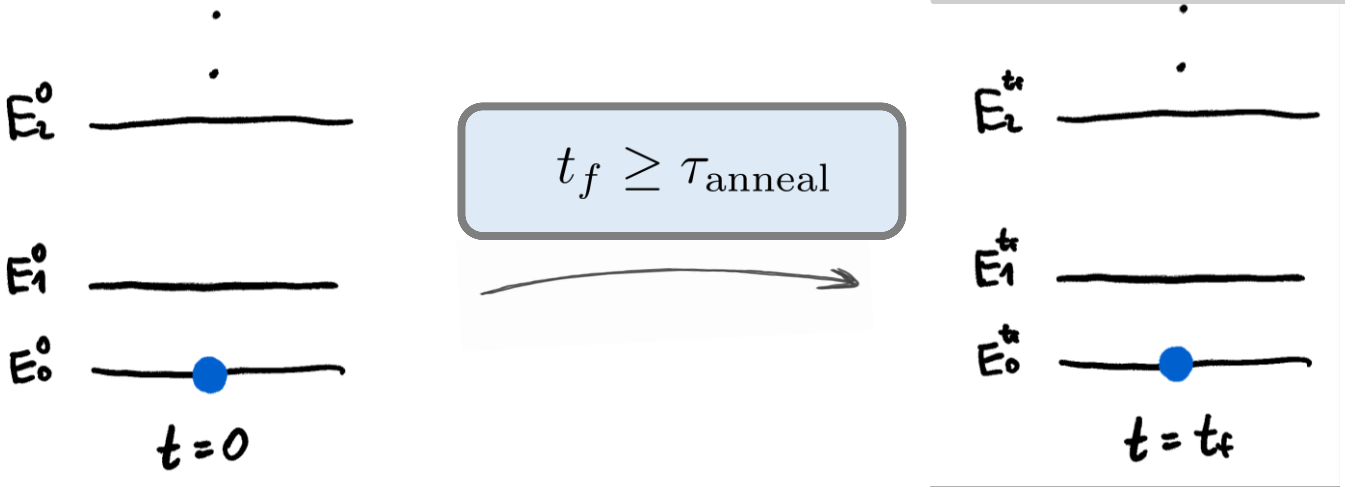
$\langle H_i\rangle_t = \langle\psi|\, H_i\, |\psi\rangle\, (t)$

Lower bounds on annealing times

How fast can annealing be?

Lower Bounds on Quantum Annealing Times

Luis Pedro García-Pintos, Lucas T. Brady, Jacob Bringewatt, and Yi-Kai Liu
Phys. Rev. Lett. **130**, 140601 – Published 5 April 2023



$\langle H_0 \rangle_0 = 0$

aim: $\langle H_1 \rangle_{t_f} = 0$

$||A|| = \max \mathrm{eigs}(A)$

$\langle H_i \rangle_t = \langle \psi | H_i | \psi \rangle (t)$

error term

$\langle H_1 \rangle_{t_f} = \langle \psi | H_1 | \psi \rangle (t_f)$

***Necessary** condition for annealing:* $t_f \geq \tau_{\mathrm{anneal}}$

$$\tau_{\mathrm{anneal}} \equiv \frac{\langle H_0 \rangle_{t_f} + \langle H_0 \rangle_0 - \langle H_1 \rangle_{t_f}}{\left|\left|[H_1, H_0]\right|\right|}$$

Lower bounds on annealing times

How fast can annealing be?

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Phys. Rev. Lett. **130**, 140601 – Published 5 April 2023

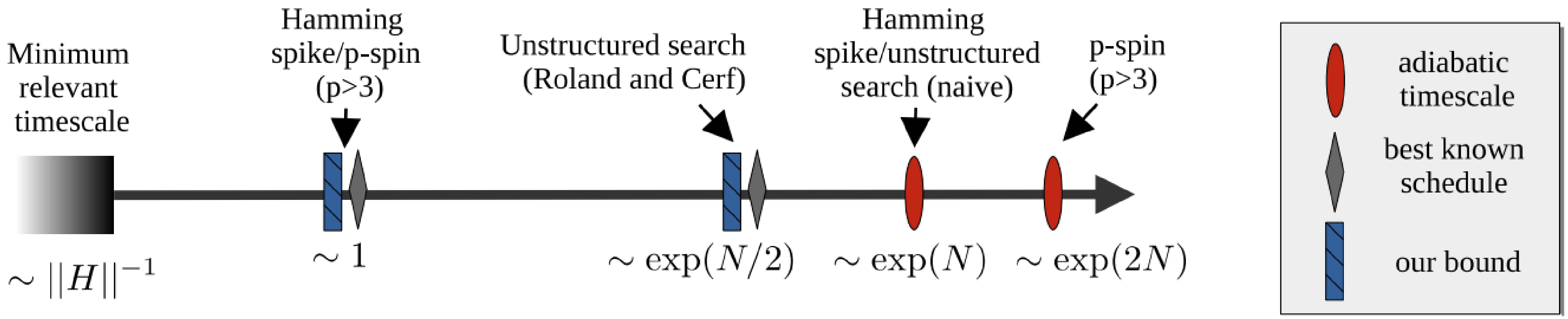

FIG. 1. **Annealing timescales.** An illustration of the range of possible timescales in annealing problems and how our bounds and the adiabatic timescales fit in the picture.

***Necessary** condition for annealing:* $t_f \geq \tau_{\text{anneal}}$

$$\tau_{\text{anneal}} \equiv \frac{\langle H_0 \rangle_{t_f} + \langle H_0 \rangle_0 - \langle H_1 \rangle_{t_f}}{\big\| [H_1, H_0] \big\|}$$

Lower bounds on annealing times

$$\|A\|_1 = \mathrm{Tr}\left[\sqrt{AA^\dagger}\right] \qquad \|A\| = \max \mathrm{eigs}(A)$$

How fast can annealing be?

Lower Bounds on Quantum Annealing Times

Luis Pedro García-Pintos, Lucas T. Brady, Jacob Bringewatt, and Yi-Kai Liu
Phys. Rev. Lett. **130**, 140601 – Published 5 April 2023

Defining $\langle H_1\rangle_0 = \langle\psi_0| H_1 |\psi_0\rangle \qquad \langle H_0\rangle_{t_f} = \langle\psi_{t_f}| H_0 |\psi_{t_f}\rangle \qquad \langle H_1\rangle_{t_f} = \langle\psi_{t_f}| H_1 |\psi_{t_f}\rangle$

$$C_1(|\psi_t\rangle) = \min_{\sigma_t:[\sigma_t, H_t]=0} \left\| |\psi_t\rangle\langle\psi_t| - \sigma_t \right\|_1$$

measure of coherence in eigenbasis of H_t

Quantifying Coherence

T. Baumgratz, M. Cramer, and M. B. Plenio
Phys. Rev. Lett. **113**, 140401 – Published 29 September 2014

***Necessary** conditions for annealing:* $t_f \geq \tau_1 \geq \tau_2$

$$\tau_1 \equiv 2\frac{\langle H_0\rangle_{t_f} + \langle H_0\rangle_0 - \langle H_1\rangle_{t_f}}{\left\|[H_1, H_0]\right\| \frac{1}{t_f}\int_0^{t_f} C_1(|\psi_t\rangle)dt} \qquad \tau_2 \equiv \frac{\langle H_0\rangle_{t_f} + \langle H_0\rangle_0 - \langle H_1\rangle_{t_f}}{\left\|[H_1, H_0]\right\|}$$

Summary

I. General framework to derive bounds on $\frac{d\langle A\rangle}{dt}$ and characterize saturation

II. Speed limits reflect actual dynamics in a range of problems

III. Saturable lower bounds on annealing times

IV. Coherence in energy influences annealing times

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Thank you!