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# RAPID, APPROXIMATE MULTI-AXIS VIBRATION TESTING

Ethan Cramer

March 23<sup>rd</sup>, 2023

# RAPID APPROXIMATE MULTI-AXIS VIBRATION TESTING

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01

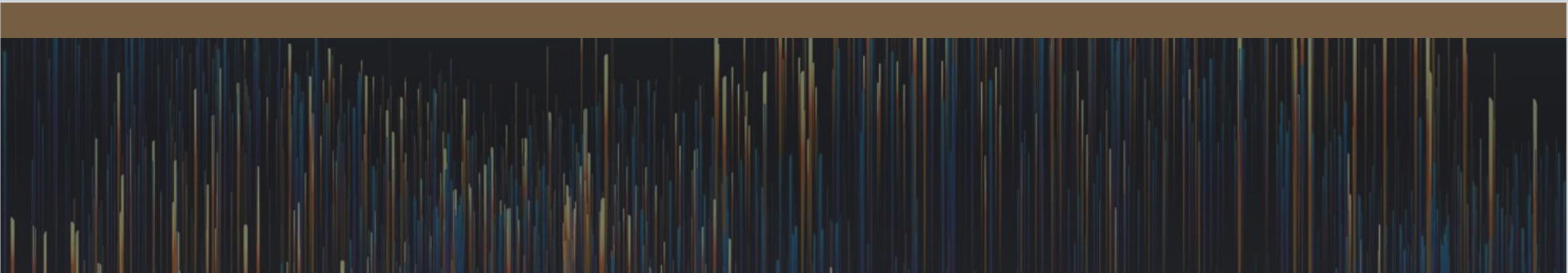
INTRODUCTION

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LITERATURE REVIEW

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METHODOLOGY



04

RESULTS

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DISCUSSION

06

CONCLUSIONS

# INTRO

## Background

- What is the context of this thesis?

## Objectives

- What are the desired outcomes?

# BACKGROUND

- The aerospace industry uses vibration shaker tables to perform component durability testing.

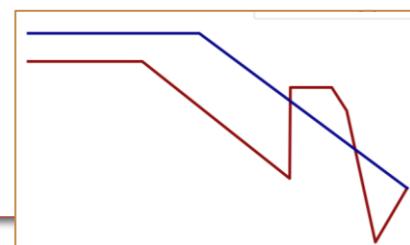
## Measure Field Responses

Acceleration measurements are taken during a field test.



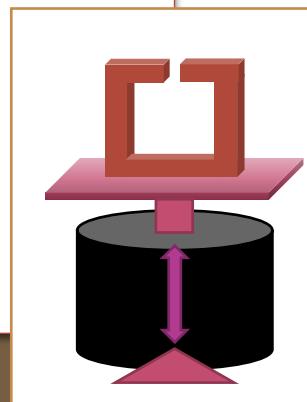
## Design Service Environment Specification

A specification is derived from the measured accelerations.



## Run Test

A laboratory test attempts to hit the derived specification.

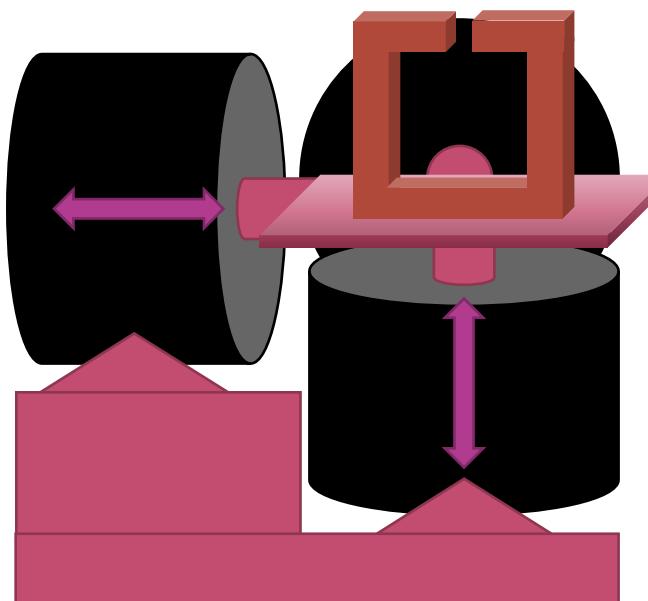


# BACKGROUND

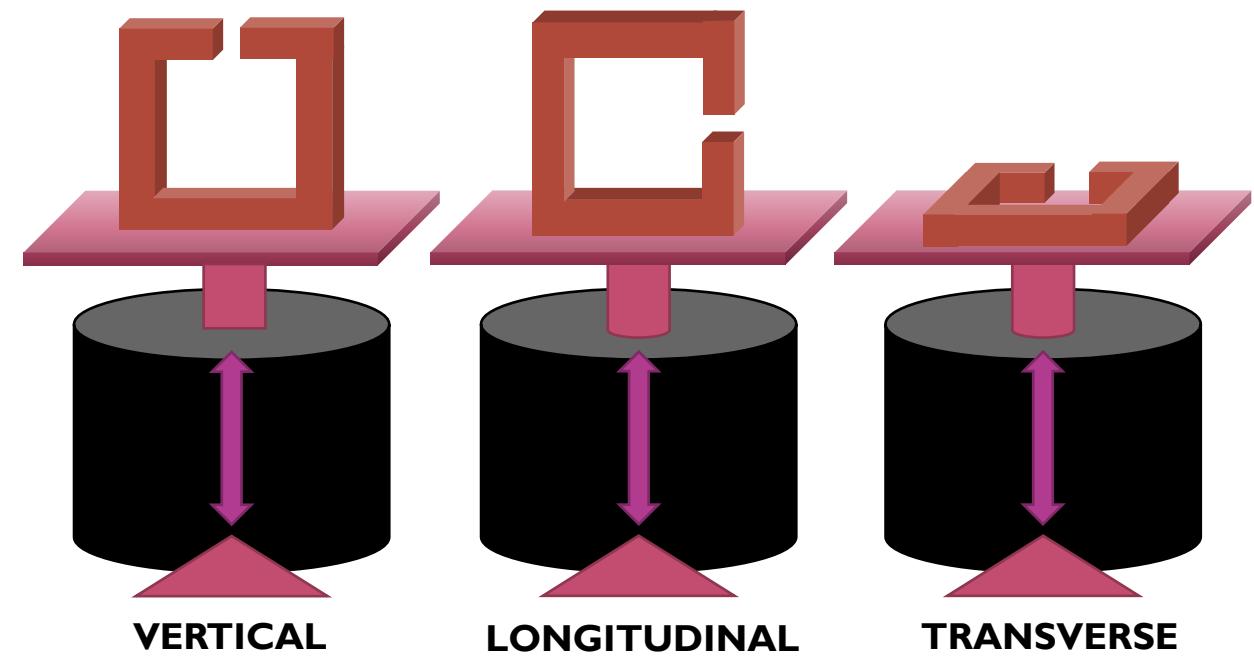
- Two categories of shaker tables: **single-axis** and **multi-axis**.
  - They are differentiated by their number of independent degrees of freedom.
- Multi-axis shakers can run more realistic environments but are not yet common in aerospace due to cost and complexity.
- **Sequential single-axis testing** attempts to replace a multi-axis shaker test with three sequential single-axis tests.

# BACKGROUND

## Multi-Axis Testing



## Sequential Single-Axis Testing



# BACKGROUND

Unfortunately, sequential single-axis tests **do not produce realistic environments.**

Sequential tests assume zero response in the off-axes.  
Usually, this is far from true.

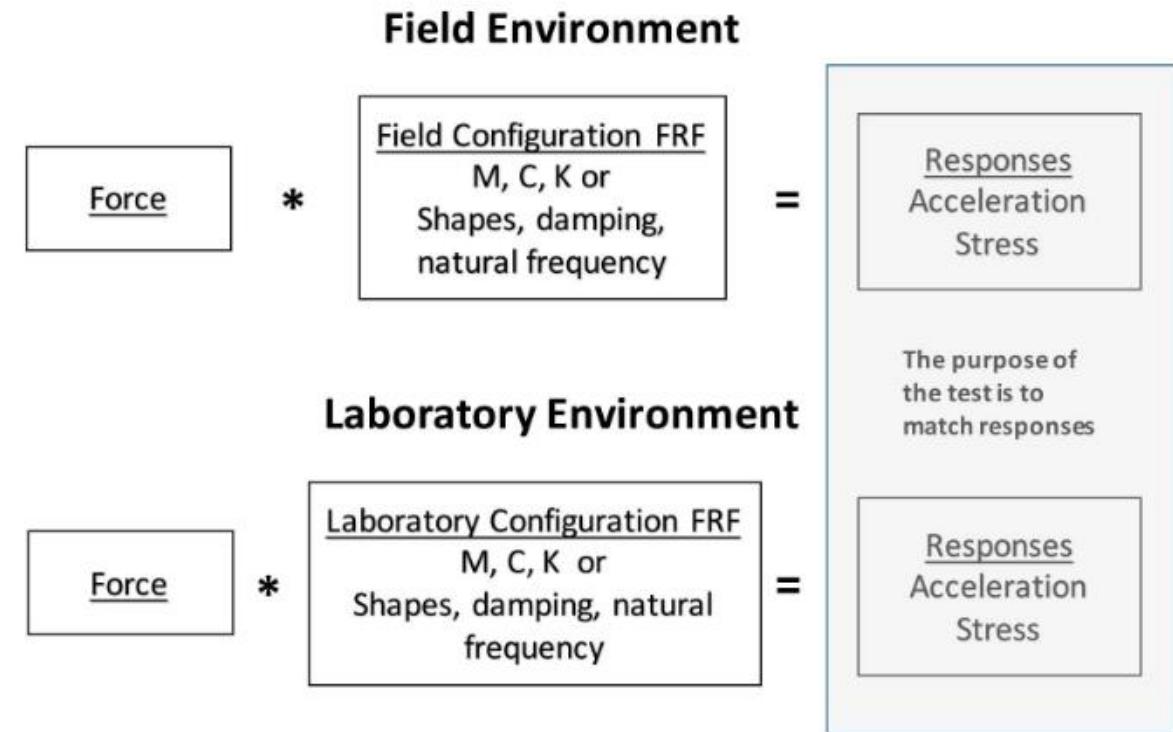
If cross-axis responses are significant, then three sequential tests cause each axis to experience full duration response three times.

This over-testing leads to unnecessary costs in strengthening the component design.

# BACKGROUND

## A poorly designed test fixture can also lead to unrealistic tests.

- A test's goal is to match the field environment in a laboratory test.
- The test fixture defines the test's boundary condition.
- For various practical and historic reasons, tests usually use rigid fixtures.



Schoenherr et al., 2018

# OBJECTIVES

## **The problem:**

Sequential single-axis testing is unrealistic but multi-axis shaker tables are not readily available.

## **A proposed solution:**

Approximate a multi-axis test using a single-axis shaker table and an optimized test fixture.

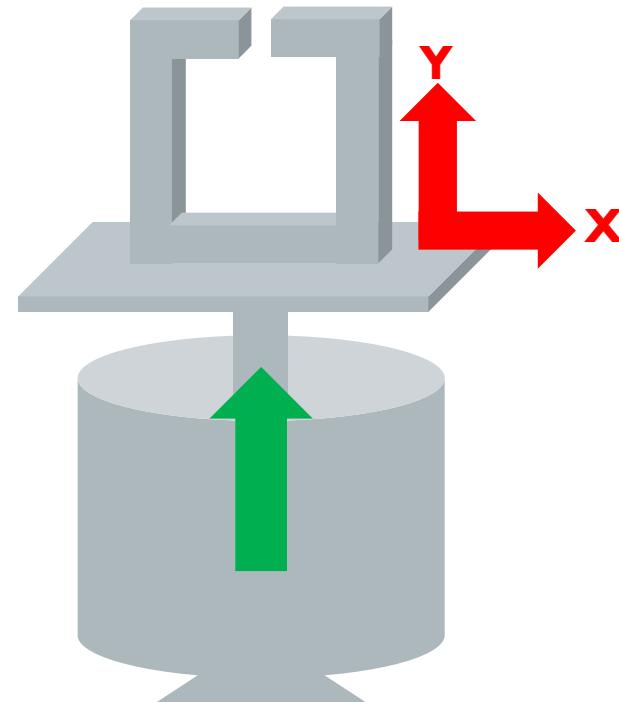
# OBJECTIVES

Three strategies to approximate a multi-axis test:

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Three strategies to approximate a multi-axis test:

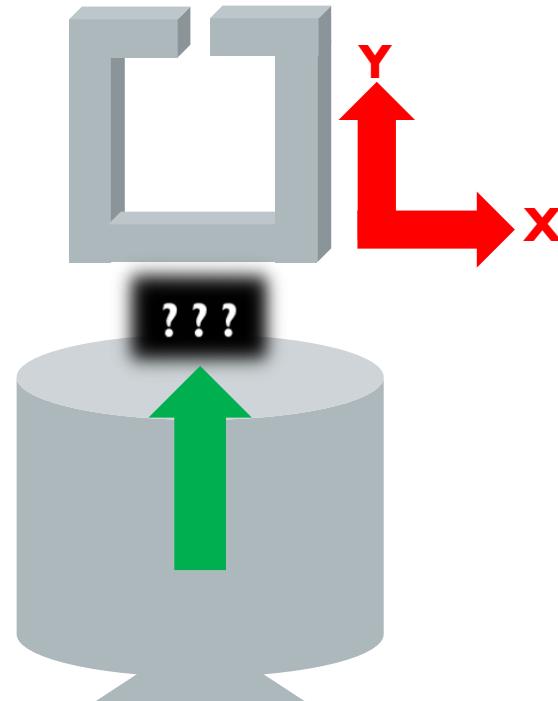
- I. A **single-input, multiple output** (SIMO) test strategy



# OBJECTIVES

Three strategies to approximate a multi-axis test:

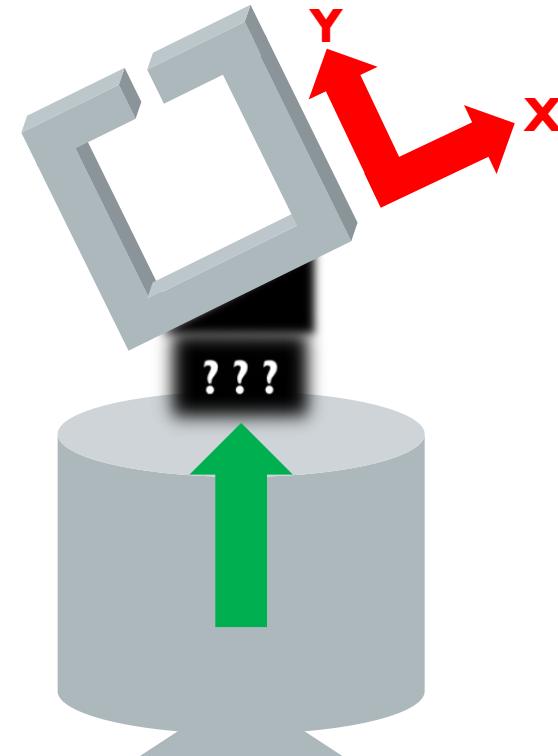
1. A **single-input, multiple output** (SIMO) test strategy
2. Test fixture optimization



# OBJECTIVES

Three strategies to approximate a multi-axis test:

1. A **single-input, multiple output** (SIMO) test strategy
2. Test fixture optimization
3. Angle optimization
  - I. This helps a single input achieve multiple targets



# OBJECTIVES

The objectives of the research are as follows:

- i. To assess the quality of the proposed method.
- ii. To understand how much test quality improvement is possible with a well-designed test fixture.
- iii. To determine the effect of increasing the number of test fixture optimization parameters.

# LITERATURE REVIEW

Overview of vibration testing literature

Single-axis versus multi-axis testing

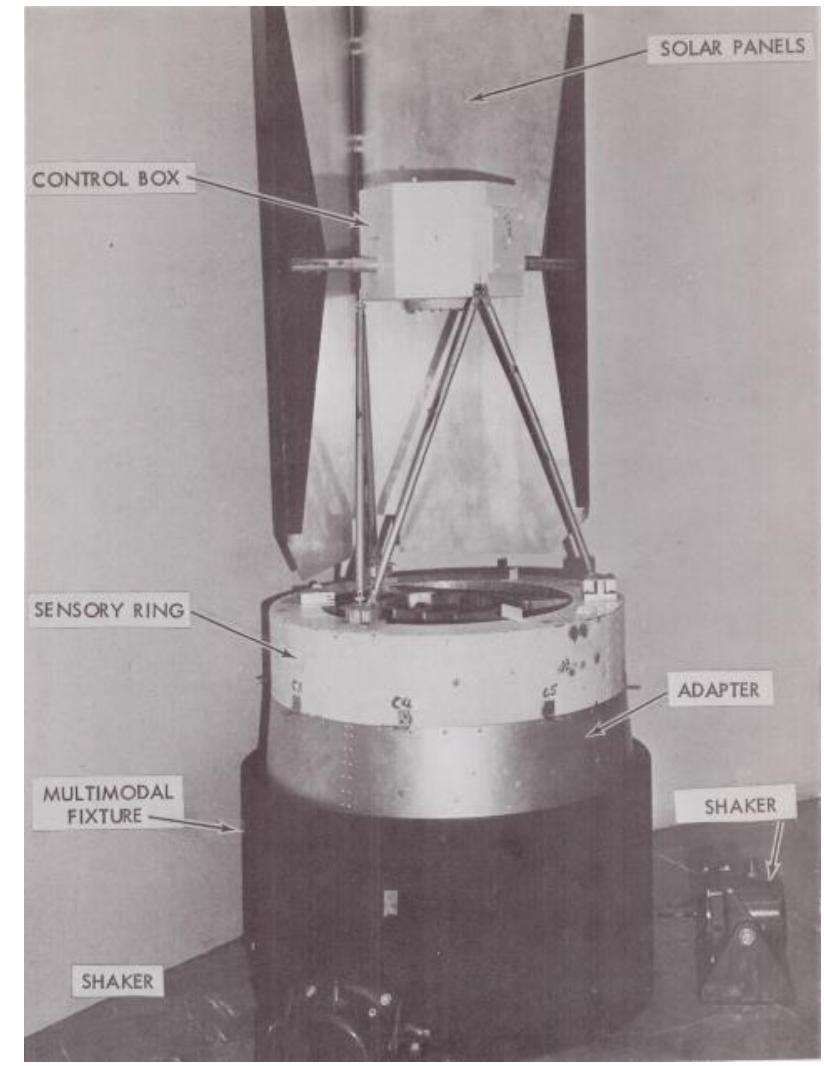
Impedance modification approaches

Input control approaches

Structural optimization

# OVERVIEW OF VIBRATION TESTING

- Attempts to improve vibration tests via test fixture design began in the Apollo era.
- A component's real boundary condition can be difficult to reproduce in the lab.
- NASA researchers made early efforts at quantifying the mechanical impedance of boundary conditions (On, 1967) and matching the field test's boundary condition in the lab (Scharton, 1969).



Multimodal test fixture, (Scharton, 1969)

- The field and lab environment's mismatch is exacerbated in sequential single-axis testing.

Compared to a 6DOF test, sequential single-axis testing produces different...

Failure times

Failure distributions

Maximum Von Mises stress

Location of maximum Von Mises stress

Modal participations

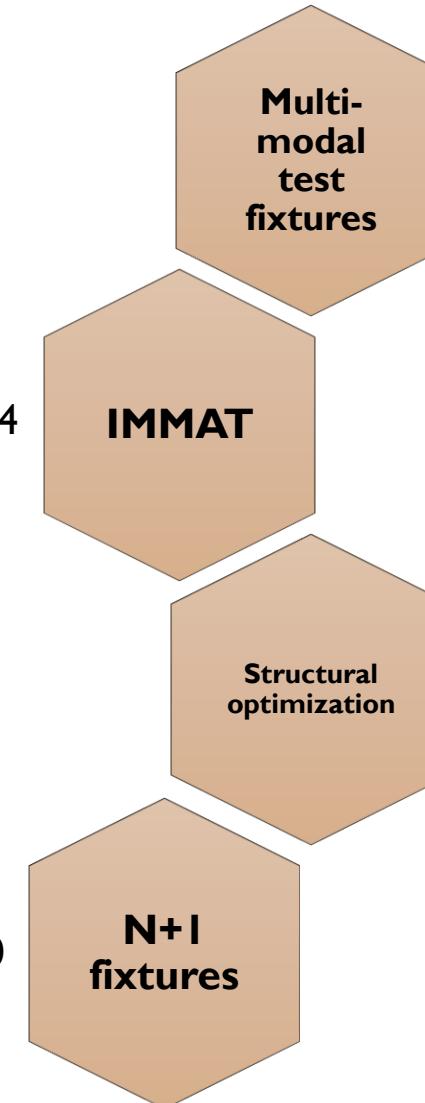
French et al., 2006

Gregory et al., 2009

# IMPEDANCE MODIFICATION APPROACHES

- Two categories of approaches are trying to rectify the mismatch between field and lab environments:
  1. **Impedance modification**
  2. **Input control**

Daborn et al., 2014



Scharton, 1969

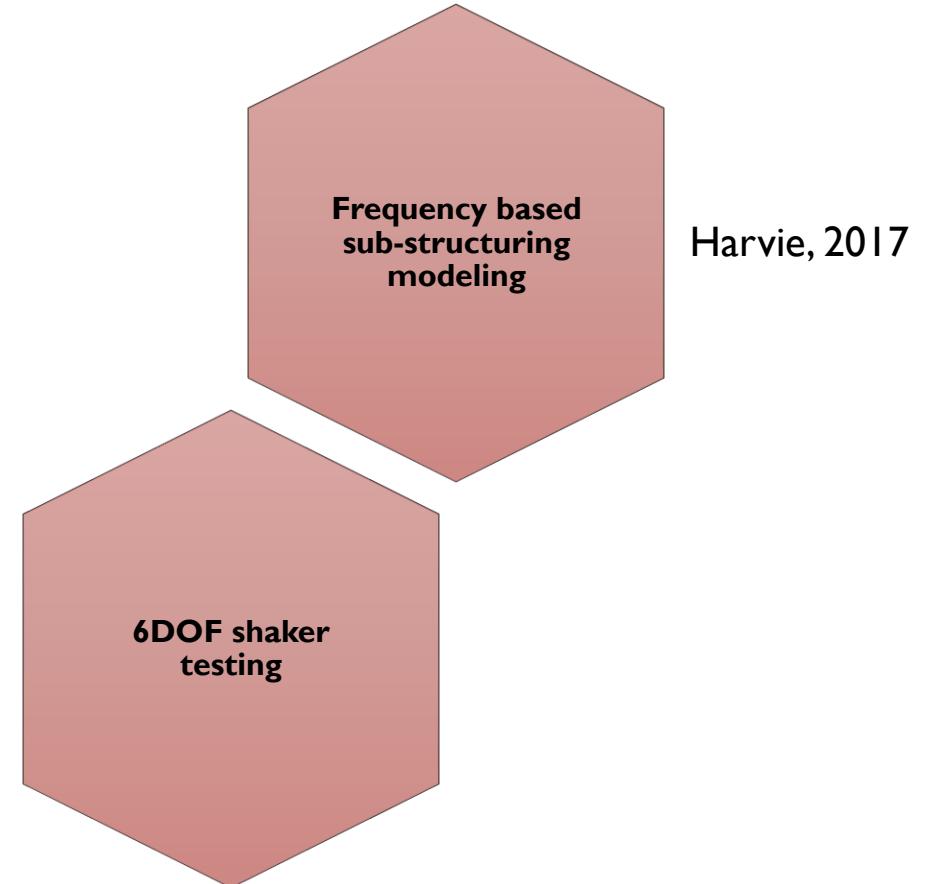
Schoenherr, 2018

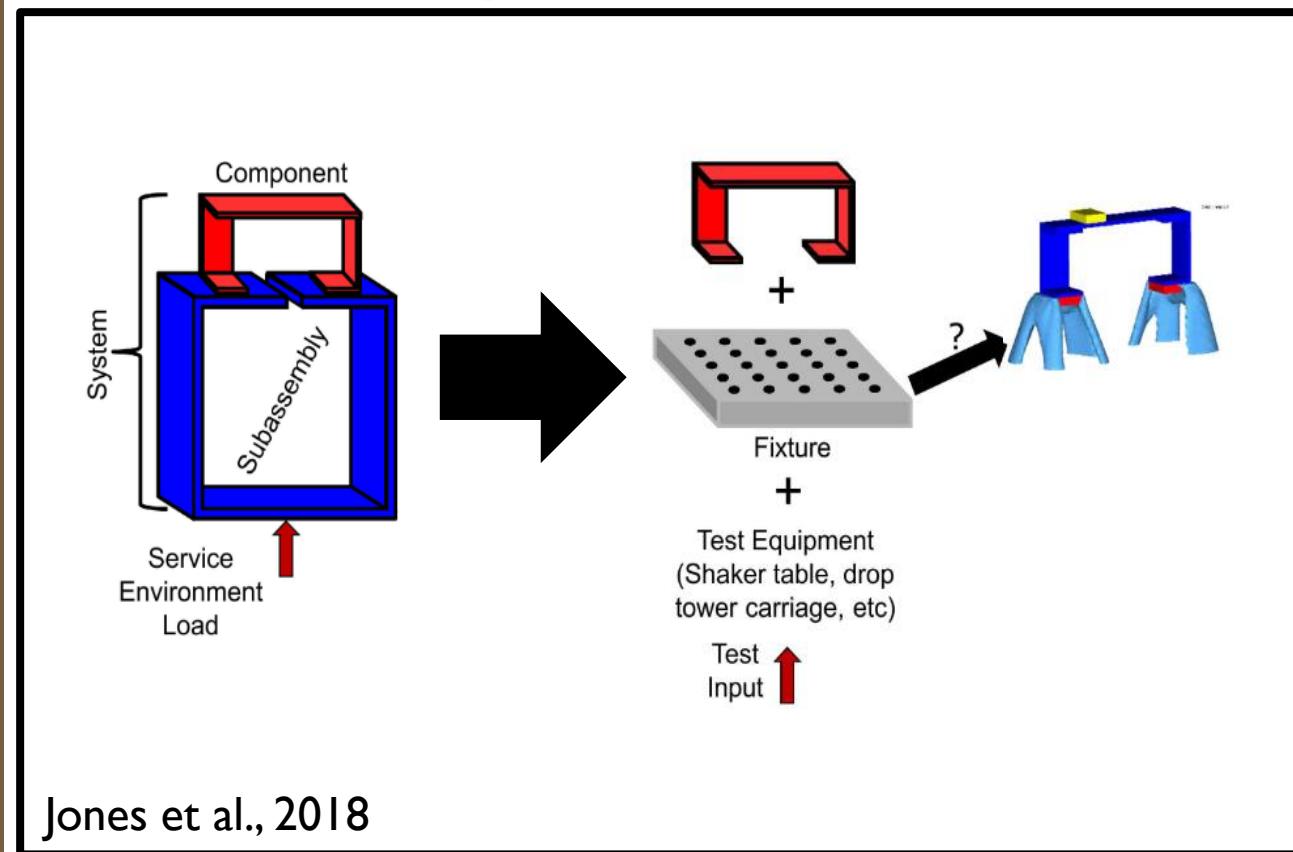
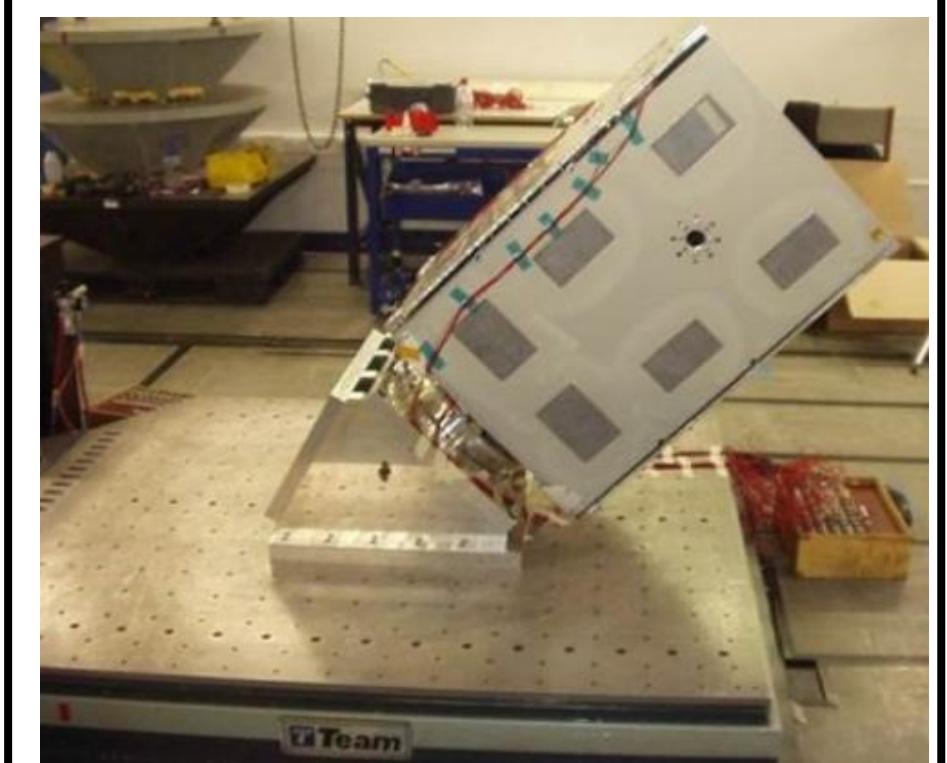
Hall, 2020

# INPUT CONTROL APPROACHES

- Two categories of approaches are trying to rectify the mismatch between field and lab environments:
  1. Impedance modification
  2. **Input control**

Schoenherr et al., 2019



STRUCTURAL  
OPTIMIZATIONTopological  
OptimizationAngle  
Optimization

Jones et al., 2018

Knight et al., 2018

# METHOD

Case studies

Finite element model

Service environment

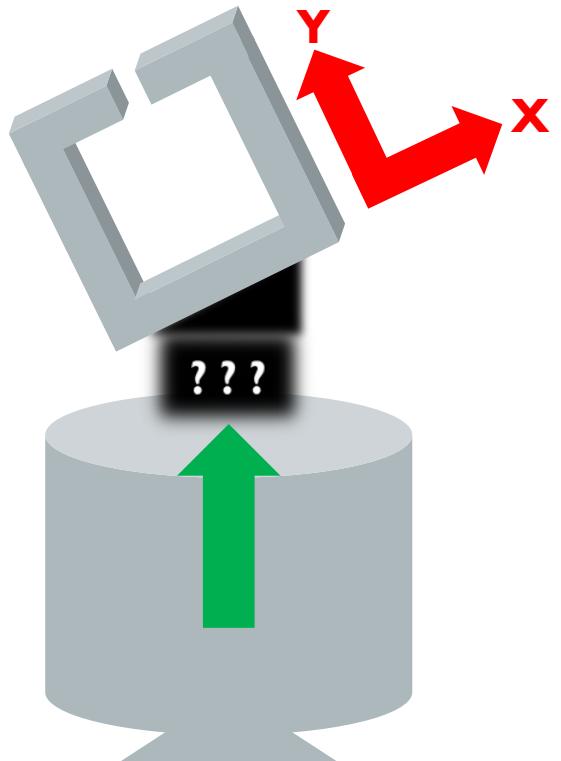
Boundary conditions

Test quality metrics

Simulation approach

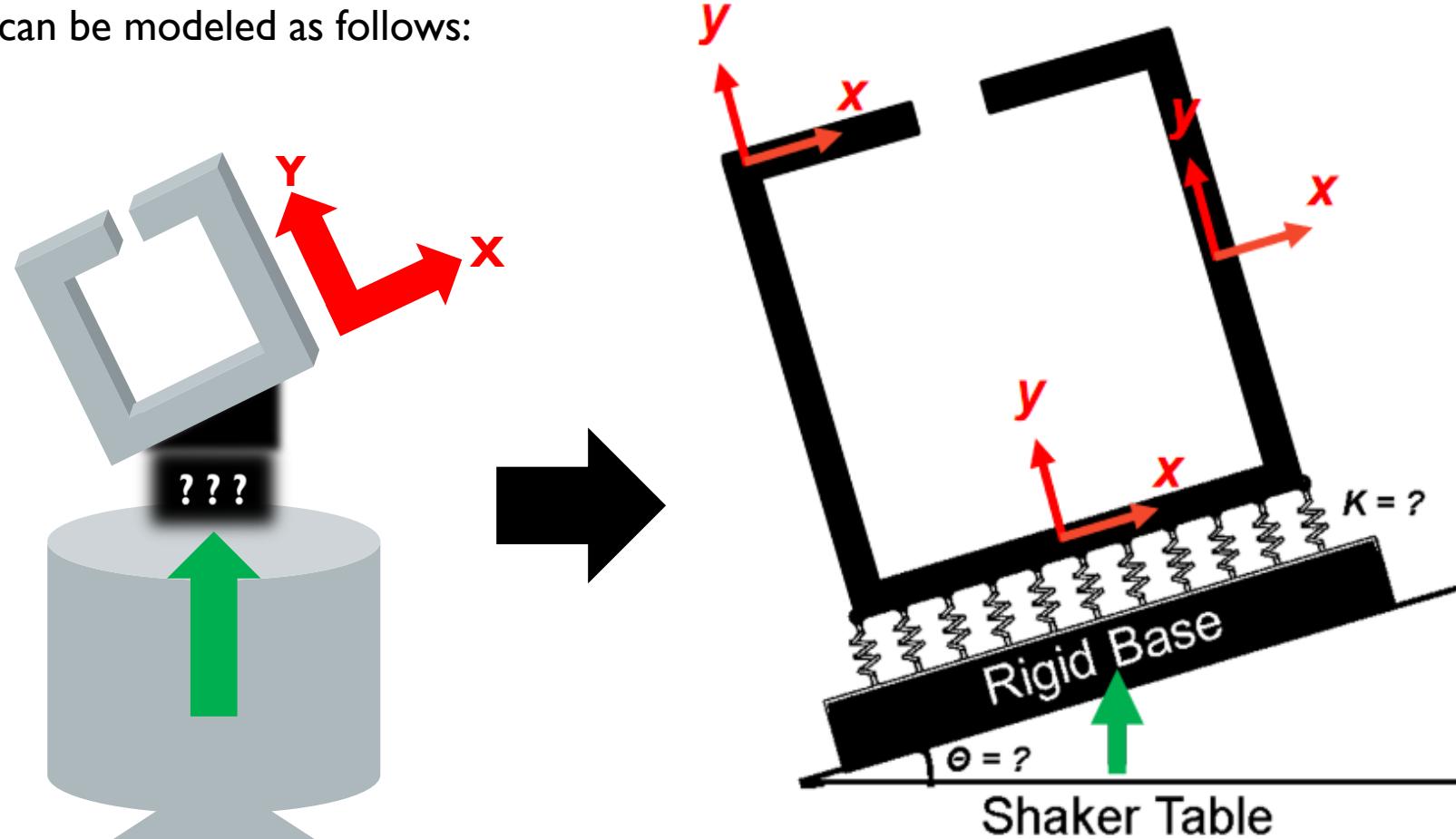
# CASE STUDIES

The proposed method can be modeled as follows:



# CASE STUDIES

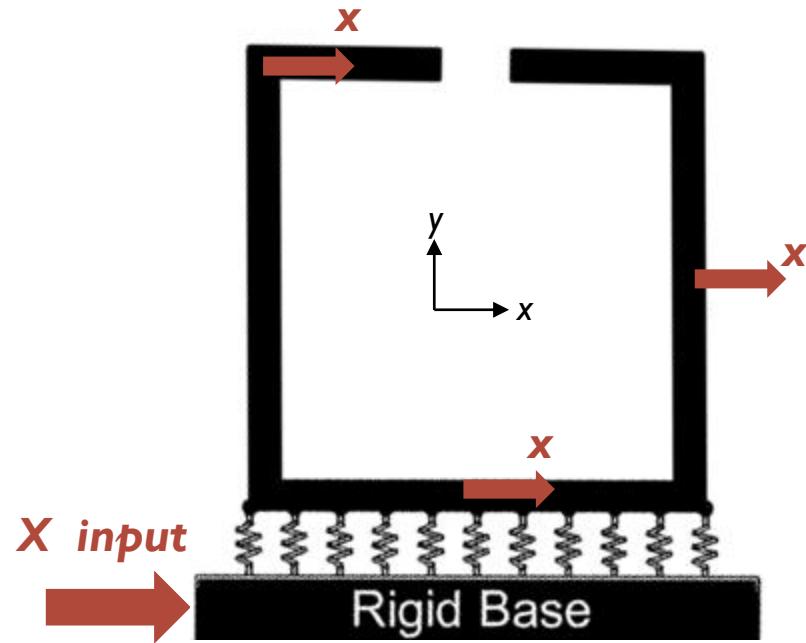
The proposed method can be modeled as follows:



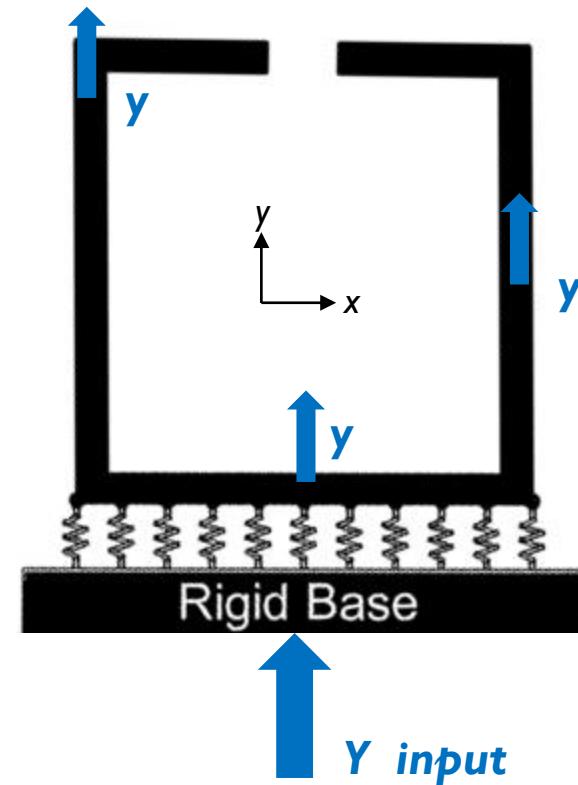
# CASE STUDIES

And a sequential test can be similarly represented:

## X-axis test



## Y-axis test



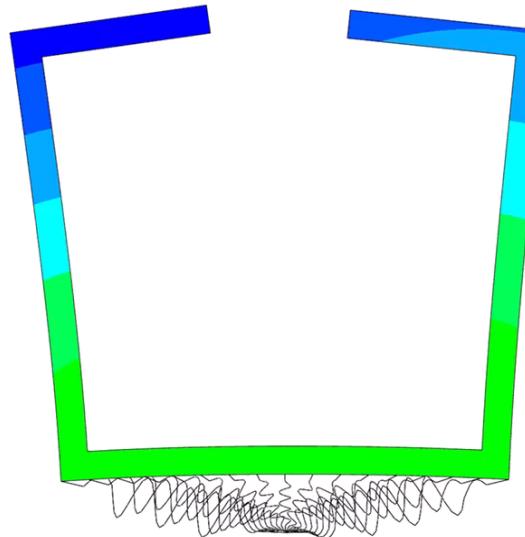
# CASE STUDIES

- Four case studies are presented to compare sequential testing to the proposed method:

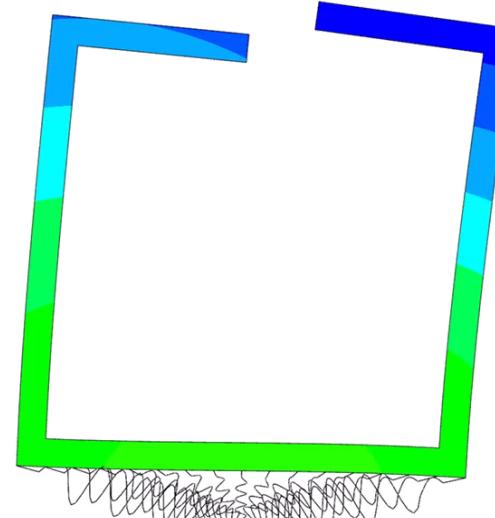
Case	Test Fixture Design	Comparisons
1	Rigid	<ol style="list-style-type: none"><li>Sequential single-axis (single control location)</li><li>Sequential single-axis (all control locations)</li><li>SIMO multi-axis (all control locations)</li></ol>
2	Optimized (2 parameters)	<ol style="list-style-type: none"><li>Sequential single-axis</li><li>SIMO multi-axis</li></ol>
3	Optimized (4 parameters)	<ol style="list-style-type: none"><li>Sequential single-axis</li><li>SIMO multi-axis</li></ol>
4	Optimized (8 parameters)	<ol style="list-style-type: none"><li>Sequential single-axis</li><li>SIMO multi-axis</li></ol>

# FEA MODEL

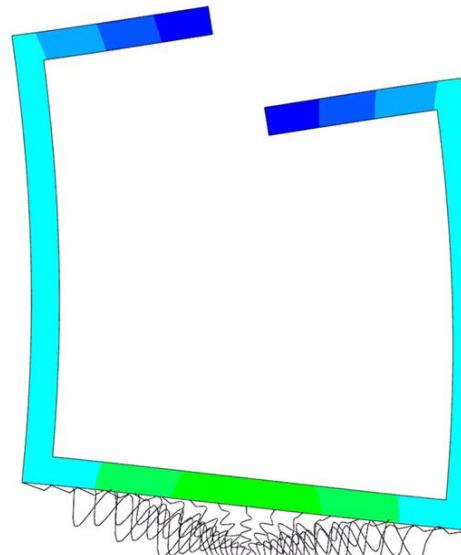
- Simulations were carried out using an Abaqus FEA model consisting of:
  1. 2-D BARC without the removable component
  2. Test fixture, idealized by sixteen springs
  3. Rigid base, representing a vertically-oriented, single-axis shaker
- The test fixture springs have an independent X-axis and Y-axis stiffness.



**Mode 1: 135.5 Hz**



**Mode 2: 136.4 Hz**



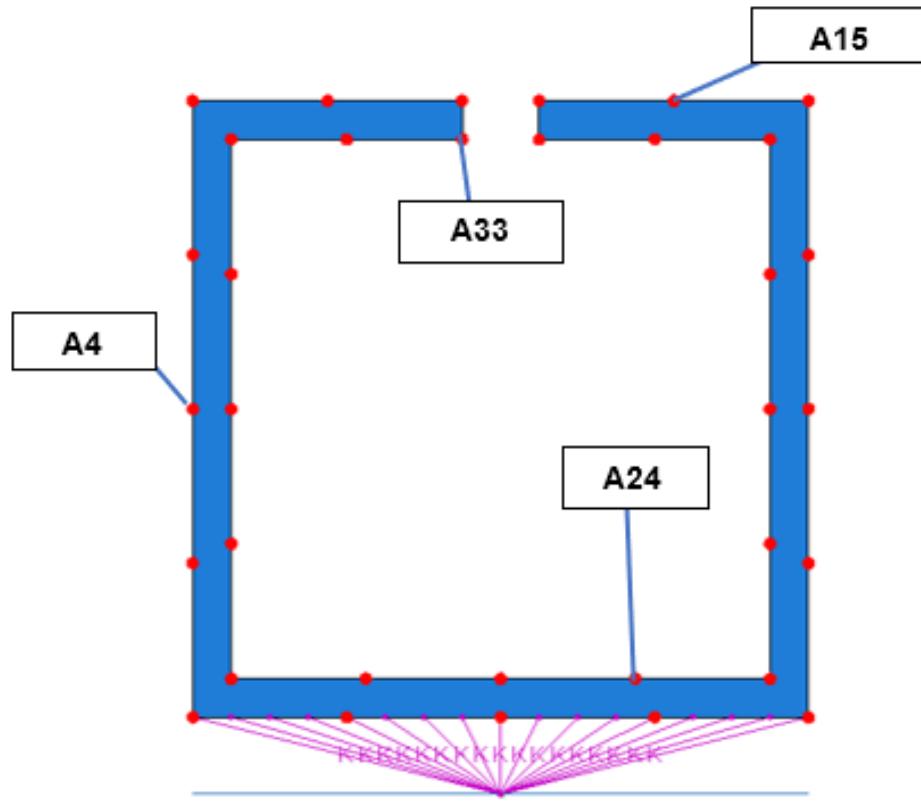
**Mode 3: 437.3 Hz**

# FEA MODEL

- The shaker input is derived to achieve the lowest possible error between a target response and measured response.
- The locations where measured response is used to derive a shaker input are known to as **control locations**.

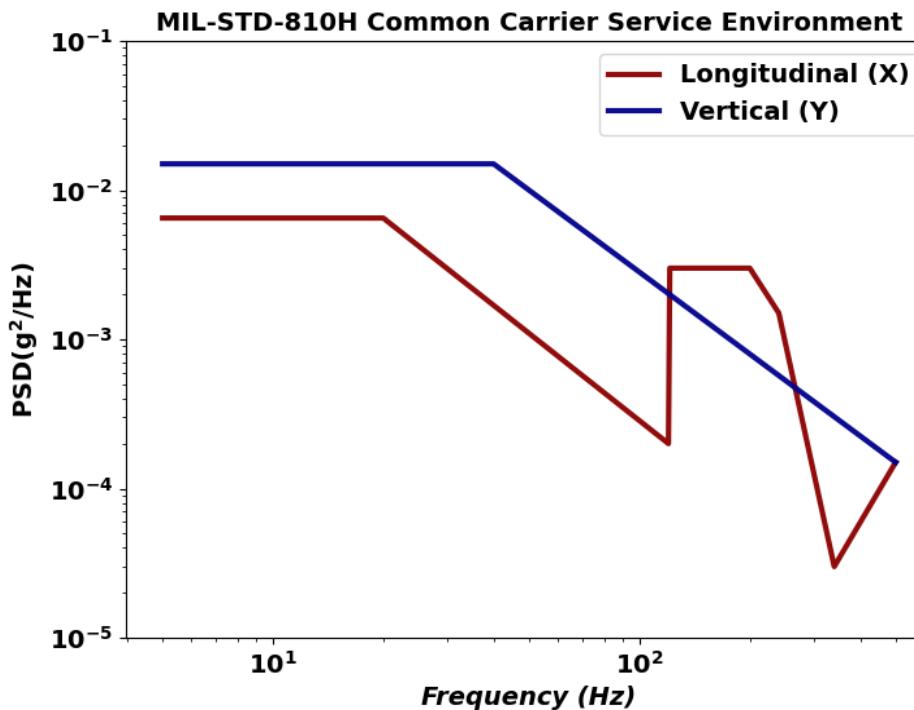


## Measurement Locations



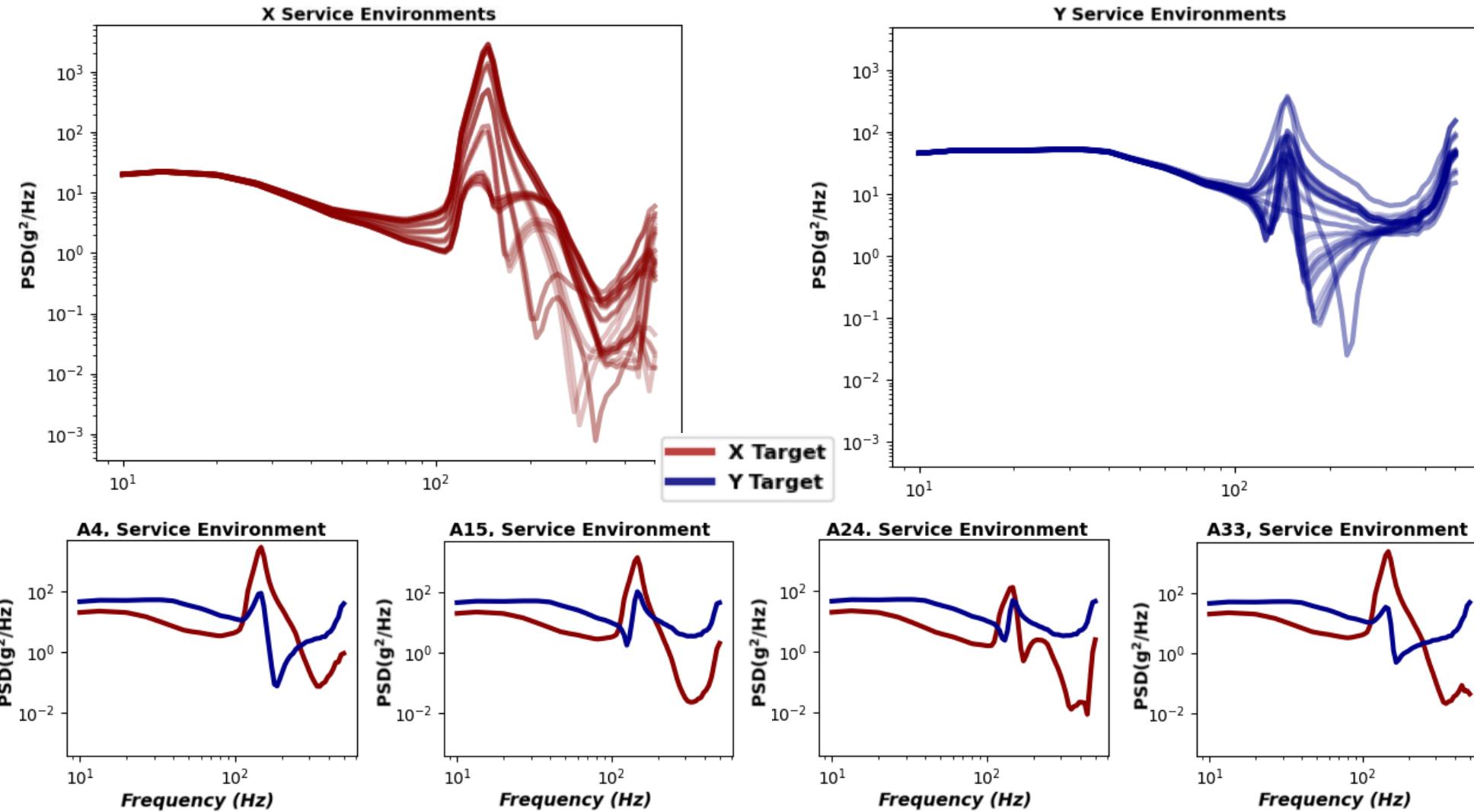
# SERVICE ENVIRONMENT

- A **service environment** is the real environment the test article is expected to experience in its lifetime.



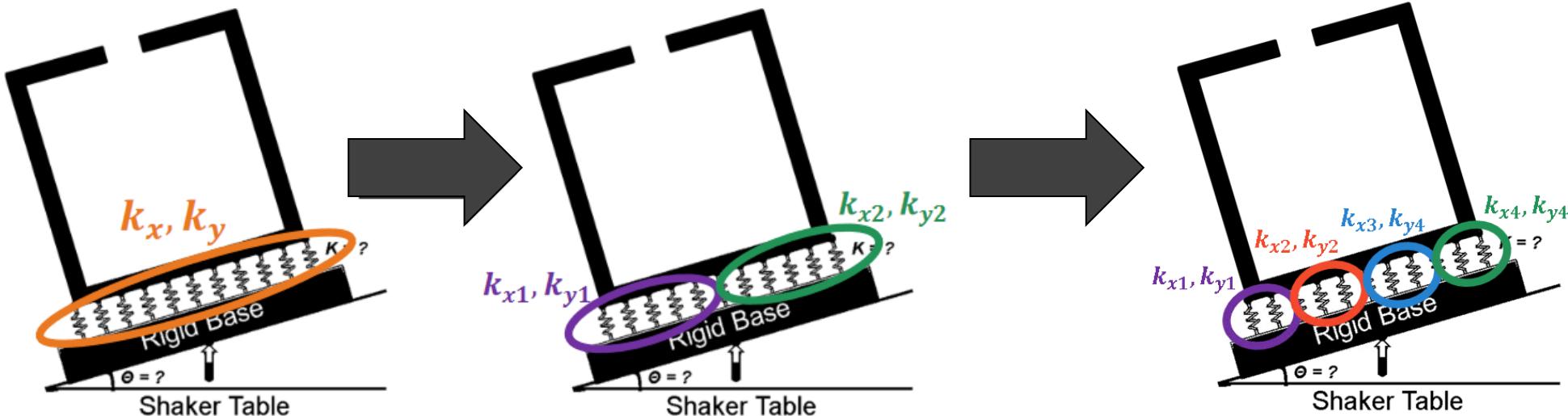
- This is a common transportation environment base excitation
- To generate a set of targets from this base excitation...
  1. Apply X and Y excitations simultaneously to the base of the model
  2. Measure acceleration responses on the BARC
- Targets were generated using a flexible boundary condition ( $k_x$  and  $k_y = 10^6 \text{ N/m}$ )

# SERVICE ENVIRONMENT



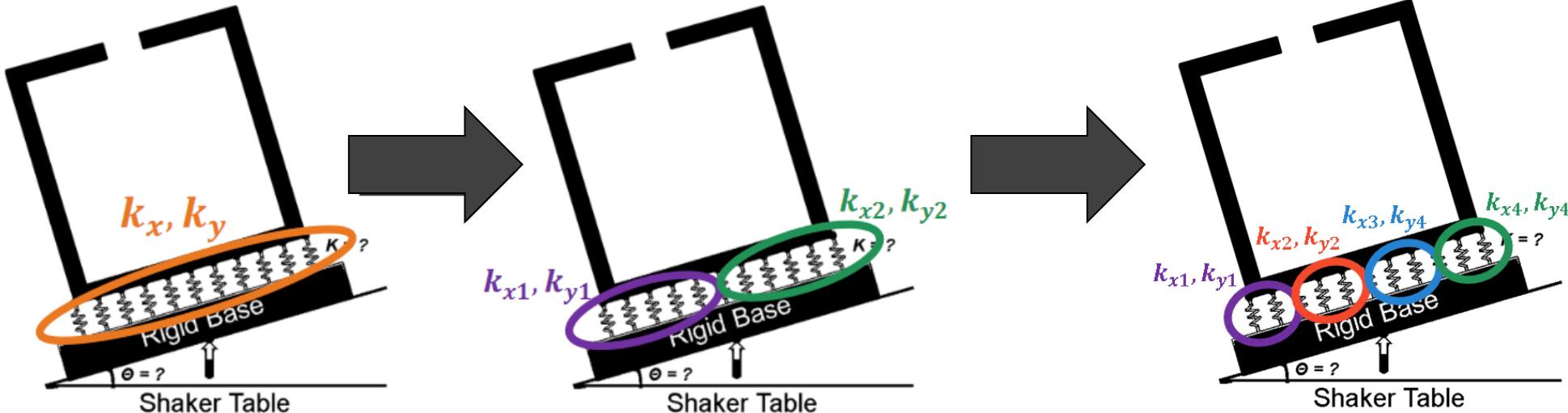
# BOUNDARY CONDITIONS

- The boundary conditions are determined by the test fixture.
- In our model, the boundary condition is determined by the stiffness of the springs.
- Lower number of optimization parameters → more tractable search
- In Case 1, the boundary condition is rigid.
- In Cases 2, 3, and 4, the boundary condition is optimized using an increasing number of parameters to determine how much test fixture complexity contributes to improving the possible solutions.



# BOUNDARY CONDITIONS

Case	Test Fixture Design	Optimization Parameters	
		Sequential Single-Axis	SIMO Multi-Axis
1	Rigid	-	$\theta$
2	Optimized (2 parameters)	$k_x, k_y$	$k_x, k_y, \theta$
3	Optimized (4 parameters)	$k_{x1}, k_{x2}, k_{y1}, k_{y2}$	$k_{x1}, k_{x2}, k_{y1}, k_{y2}, \theta$
4	Optimized (8 parameters)	$k_{x1}, k_{x2}, k_{x3}, k_{x4},$ $k_{y1}, k_{y2}, k_{y3}, k_{y4}$	$k_{x1}, k_{x2}, k_{x3}, k_{x4}$ $k_{y1}, k_{y2}, k_{y3}, k_{y4}, \theta$



# TEST QUALITY METRICS

## RMS dB Error (RDBE):

RDBE is a scalar metric that represents the average mismatch between a response PSD and a target PSD across all frequencies.

$$RDBE = \sqrt{\frac{\sum_{i=1}^n (10 \log_{10} \frac{\hat{y}_i}{y_i})^2}{n}}$$

In both equations,  $\hat{y}_i$  is the value of a response PSD at the  $i^{th}$  frequency line, and  $y_i$  is the value of the target PSD at the  $i^{th}$  frequency line. There are  $n$  total frequency lines.

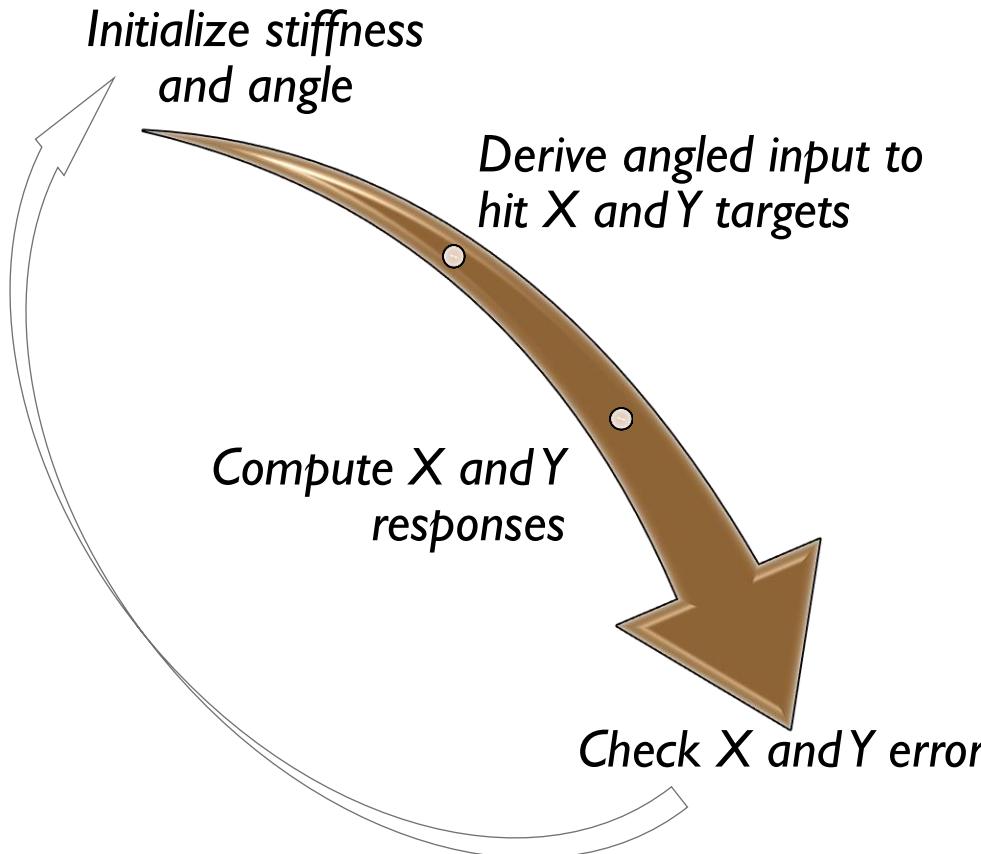
## Percentage of Frequency Lines within a 3 dB Tolerance (FTOL):

FTOL is a scalar metric that counts the percentage of frequency lines where the response PSD differs from the target PSD by less than 3 dB.

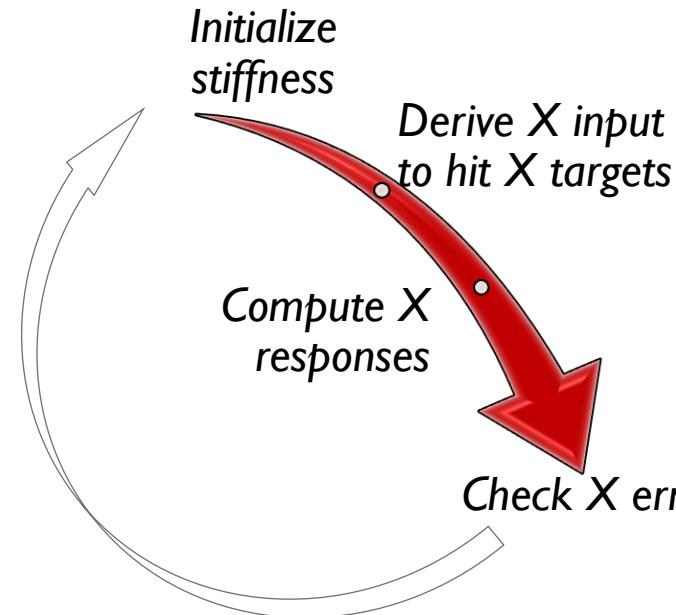
$$\% FL = 100 * \frac{1}{n} \sum_{i=1}^n \begin{cases} 1 & \text{if } \left| 10 \log_{10} \frac{\hat{y}_i}{y_i} \right| \leq 3 \text{ dB} \\ 0 & \text{if } \left| 10 \log_{10} \frac{\hat{y}_i}{y_i} \right| > 3 \text{ dB} \end{cases}$$

# SIMULATION APPROACH

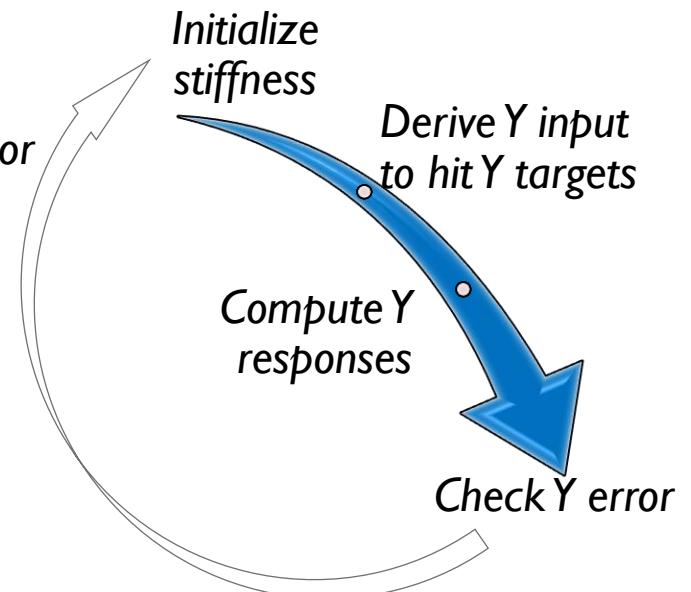
## **SIMO Multi-Axis Test**



## **X-Axis Sequential Test**



## **Y-Axis Sequential Test**



# RESULTS

Case study 1

Case study 2

Case study 3

Case study 4

# CASE STUDY 1

- For case 1, all tests used a rigid fixture ( $k_x$  and  $k_y = 10^9 \text{ N/m}$ )

Case	Test Fixture Design	Comparisons
1	Rigid	<ol style="list-style-type: none"><li>Sequential single-axis (single control location)</li><li>Sequential single-axis (all control locations)</li><li><b>SIMO multi-axis (all control locations)</b></li></ol>
2	Optimized (2 parameters)	<ol style="list-style-type: none"><li>Sequential single-axis</li><li>SIMO multi-axis</li></ol>
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4	Optimized (8 parameters)	<ol style="list-style-type: none"><li>Sequential single-axis</li><li>SIMO multi-axis</li></ol>

# CASE STUDY 1

- For the single control location sequential tests, choosing the best location improves the RDBE by 2.2 dB and the FTOL by 3.9% over the worst location.

Sequential Single-Axis (Worst performing control location)				Sequential Single-Axis (Best performing control location)			
	X-Error (avg)	Y-Error (avg)	<u>Mean Error</u>		X-Error (avg)	Y-Error (avg)	<u>Mean Error</u>
<b>RDBE</b>	9.9 dB	3.2 dB	<b><u>6.6 dB</u></b>	<b>RDBE</b>	5.6 dB	3.2 dB	<b><u>4.4 dB</u></b>
<b>FTOL</b>	64.6%	83.9%	<b><u>74.3%</u></b>	<b>FTOL</b>	71.6%	84.7%	<b><u>78.2%</u></b>

# CASE STUDY 1

- When the test controls to all locations, the RDBE further decreases to **3.8 dB**.
- It's interesting that the FTOL decreases slightly.

## Sequential Single-Axis (All control locations)

	X-Error (avg)	Y-Error (avg)	<u>Mean Error</u>
<b>RDBE</b>	5.1 dB	2.6 dB	<b><u>3.8 dB</u></b>
<b>FTOL</b>	70.3%	85.2%	<b><u>77.8%</u></b>

## Sequential Single-Axis (Worst performing control location)

## Sequential Single-Axis (Best performing control location)

	X-Error (avg)	Y-Error (avg)	<u>Mean Error</u>		X-Error (avg)	Y-Error (avg)	<u>Mean Error</u>
<b>RDBE</b>	9.9 dB	3.2 dB	<b><u>6.6 dB</u></b>	<b>RDBE</b>	5.6 dB	3.2 dB	<b><u>4.4 dB</u></b>
<b>FTOL</b>	64.6%	83.9%	<b><u>74.3%</u></b>	<b>FTOL</b>	71.6%	84.7%	<b><u>78.2%</u></b>

# CASE STUDY 1

- The SIMO multi-axis test's **RDBE** increases by 1.0 dB while the **FTOL** falls by 3.8%.

Sequential Single-Axis (All control locations)				SIMO Multi-Axis (All control locations)			
	X-Error (avg)	Y-Error (avg)	<u>Mean Error</u>		X-Error (avg)	Y-Error (avg)	<u>Mean Error</u>
<b>RDBE</b>	5.1 dB	2.6 dB	<b><u>3.8 dB</u></b>	<b>RDBE</b>	5.7 dB	3.9 dB	<b><u>4.8 dB</u></b>
<b>FTOL</b>	70.3%	85.2%	<b><u>77.8%</u></b>	<b>FTOL</b>	69.2%	78.7%	<b><u>74.0%</u></b>

This decrease in measured test quality metrics may be justified because the SIMO multi-axis test eliminates cross-axis responses that are unavoidable in the sequential test.

# CASE STUDY 2

Optimizing the test fixture stiffness with two parameters:

1. Improved the sequential test by 0.1 dB and 2.2%.
2. Improved the SIMO test by 0.3 dB and 1.6%.

	Mean Error	
	Sequential Single-Axis	SIMO Multi-Axis
RDBE	3.7 dB	4.5 dB
FTOL	80.0%	75.6%

# CASE STUDY 2

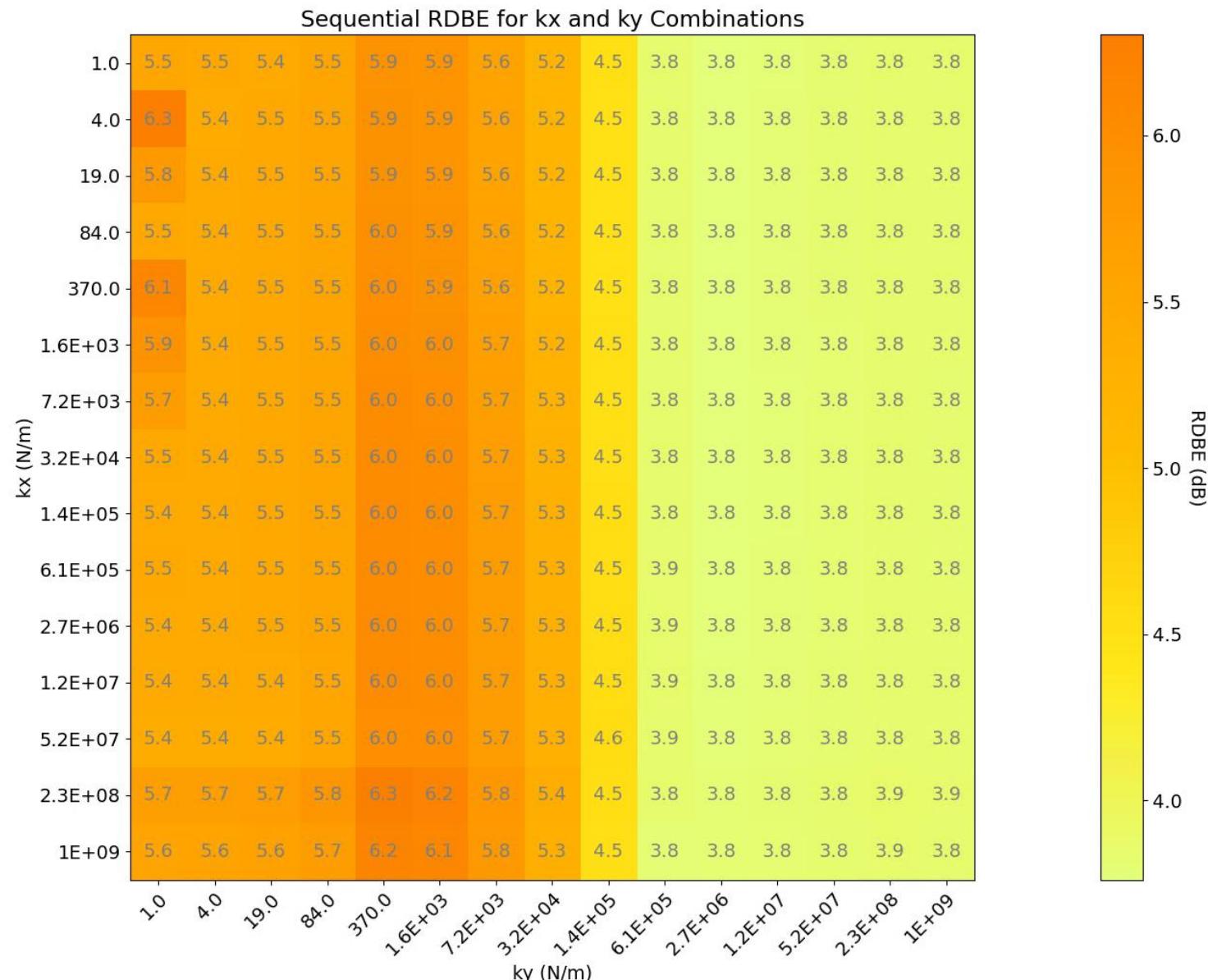
The solution found for each test was:

1. For the sequential test,  $k_x = 3.2 * 10^4$  and  $k_y = 2.7 * 10^6$  N/m.
2. For the SIMO test,  $k_x = 1.2 * 10^7$ ,  $k_y = 2.7 * 10^6$  N/m, and  $\theta = 27^\circ$ .

	Mean Error	
	Sequential Single-Axis	SIMO Multi-Axis
RDBE	3.7 dB	4.5 dB
FTOL	80.0%	75.6%

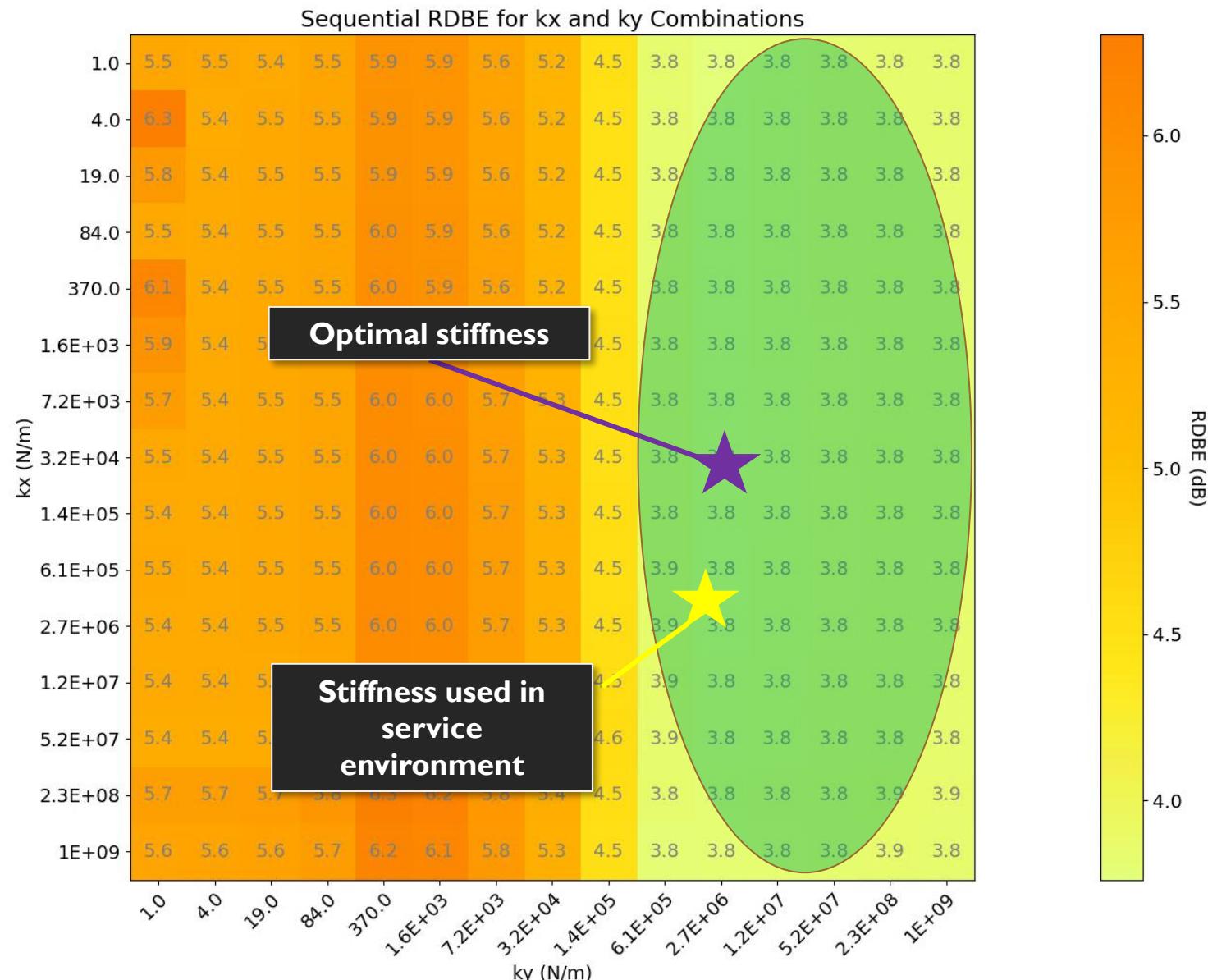
# CASE STUDY 2

- The sequential test's stiffness optimization found the X-stiffness had little impact while the Y-stiffness needed to be above  $1.4 * 10^5 \text{ N/m}$  to produce good error.



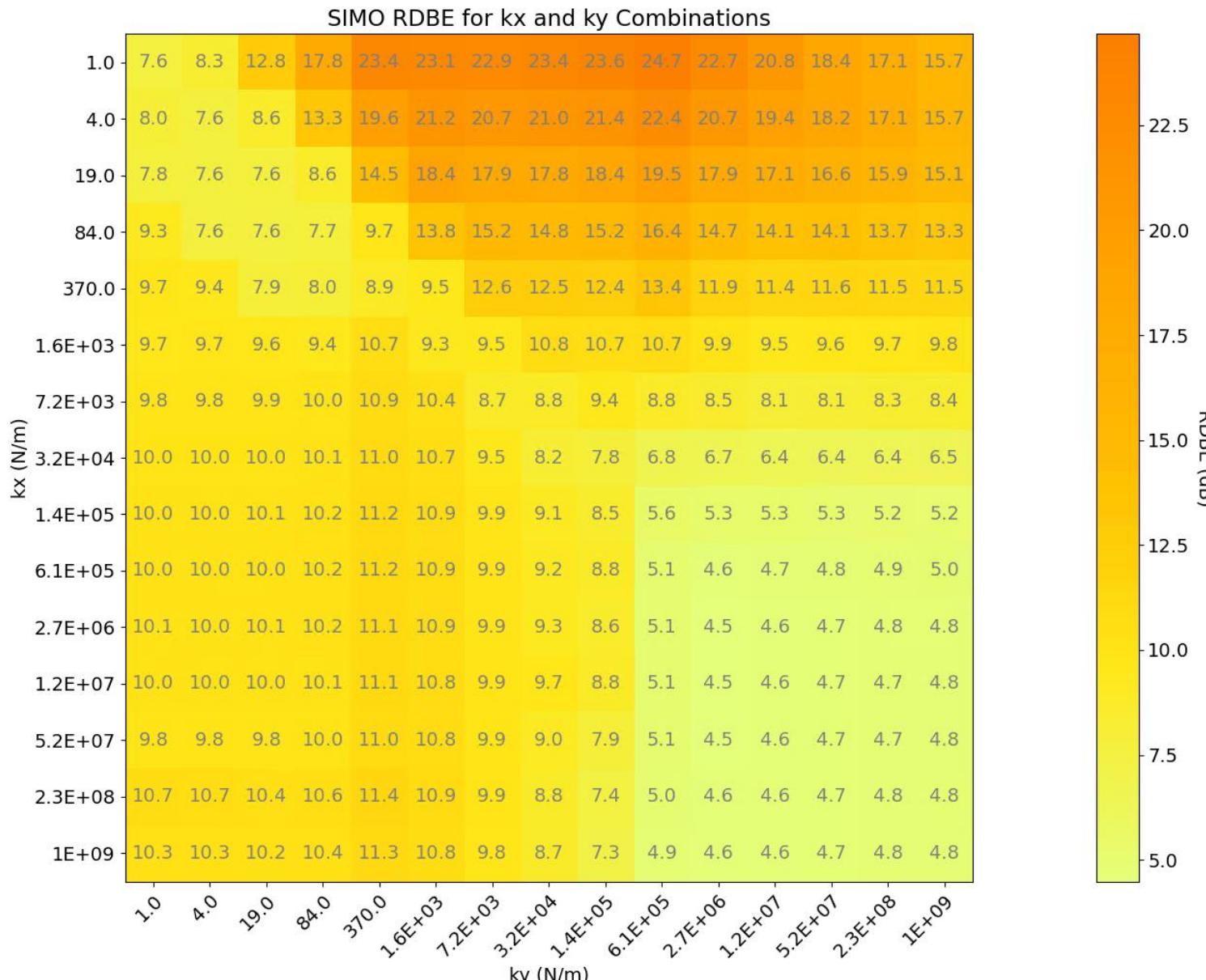
# CASE STUDY 2

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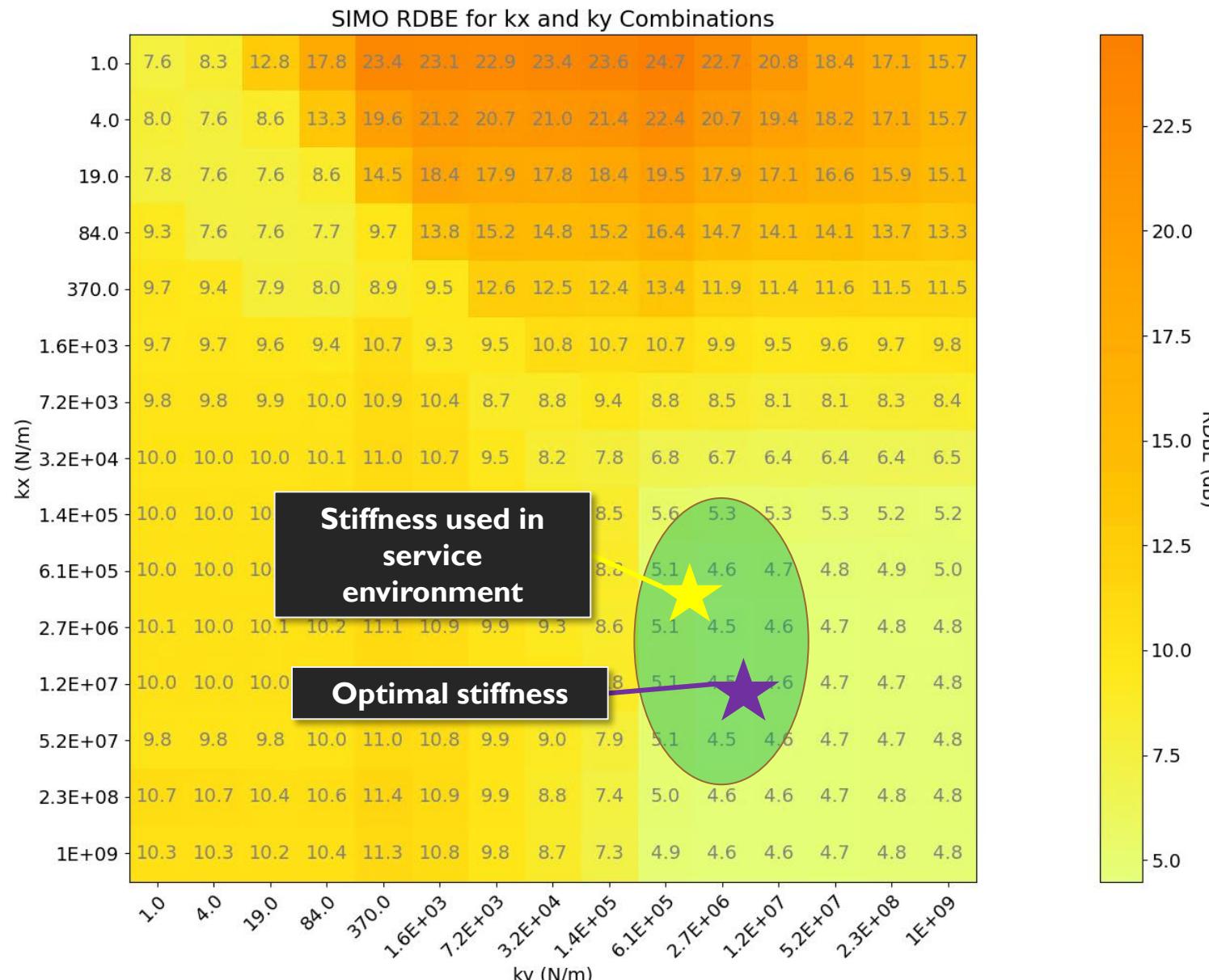
# CASE STUDY 2

- The SIMO solution space was smooth with a clear minimum region, but not fully convex.
- The minimum region provides good values to start a local optimization.

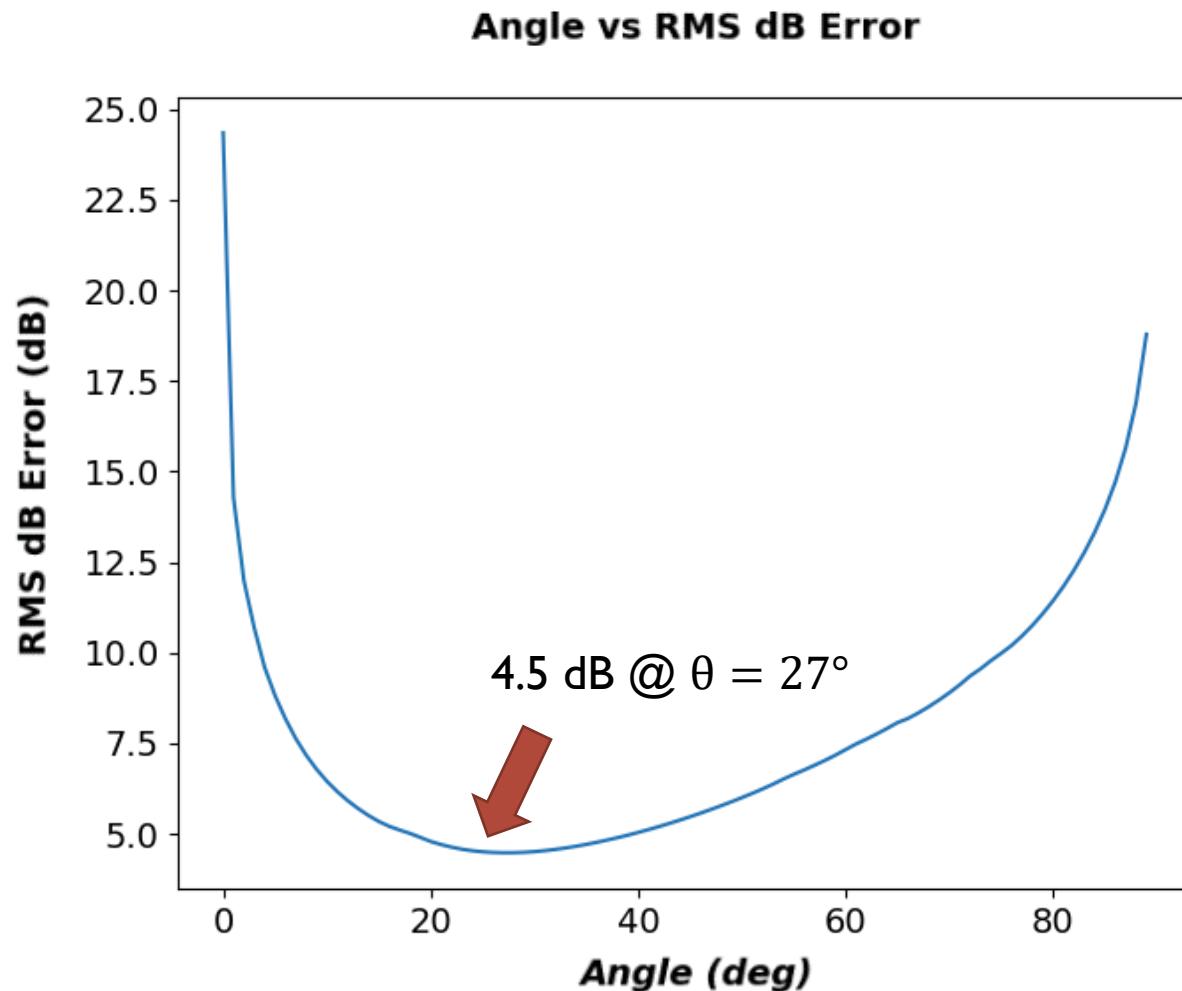


# CASE STUDY 2

- The SIMO solution space was smooth with a clear minimum region.
- The minimum region provides good values to start a local optimization.



# CASE STUDY 2



- The SIMO test's angle optimization has a clear minimum.

# CASE STUDY 3

The solution found for each test was:

1. For the sequential test,  $k_{x1} = 3.2 * 10^4$ ,  $k_{x2} = 1$ ,  $k_{y1} = 2.7 * 10^6$ ,  $k_{y2} = 2.7 * 10^6$  N/m.
2. For the SIMO test,  $k_{x1} = 19$ ,  $k_{x2} = 2.3 * 10^8$ ,  $k_{y1} = 2.7 * 10^6$ ,  $k_{y2} = 2.7 * 10^6$  N/m,  $\theta = 27^\circ$ .

	Mean Error	
	Sequential Single-Axis	SIMO Multi-Axis
RDBE	3.7 dB	4.5 dB
FTOL	80.0%	75.6%

# CASE STUDY 4

The solution found for each test was:

1. For the sequential test,

$$k_{x1} = 1, k_{x2} = 1, k_{x3} = 1, k_{x4} = 1, k_{y1} = 1, k_{y2} = 10^9, k_{y3} = 10^9, k_{y4} = 1 \text{ N/m.}$$

2. For the SIMO test,

$$k_{x1} = 10^9, k_{x2} = 3.2 * 10^4, k_{x3} = 2.7 * 10^6, k_{x4} = 2.7 * 10^6, k_{y1} = 2.7 * 10^6, k_{y2} = 2.7 * 10^6, \\ k_{y3} = 10^9, k_{y4} = 2.7 * 10^6 \text{ N/m, } \theta = 27^\circ.$$

	Mean Error	
	Sequential Single-Axis	SIMO Multi-Axis
RDBE	3.7 dB	4.5 dB
FTOL	79.6%	75.3%

# DISCUSSION

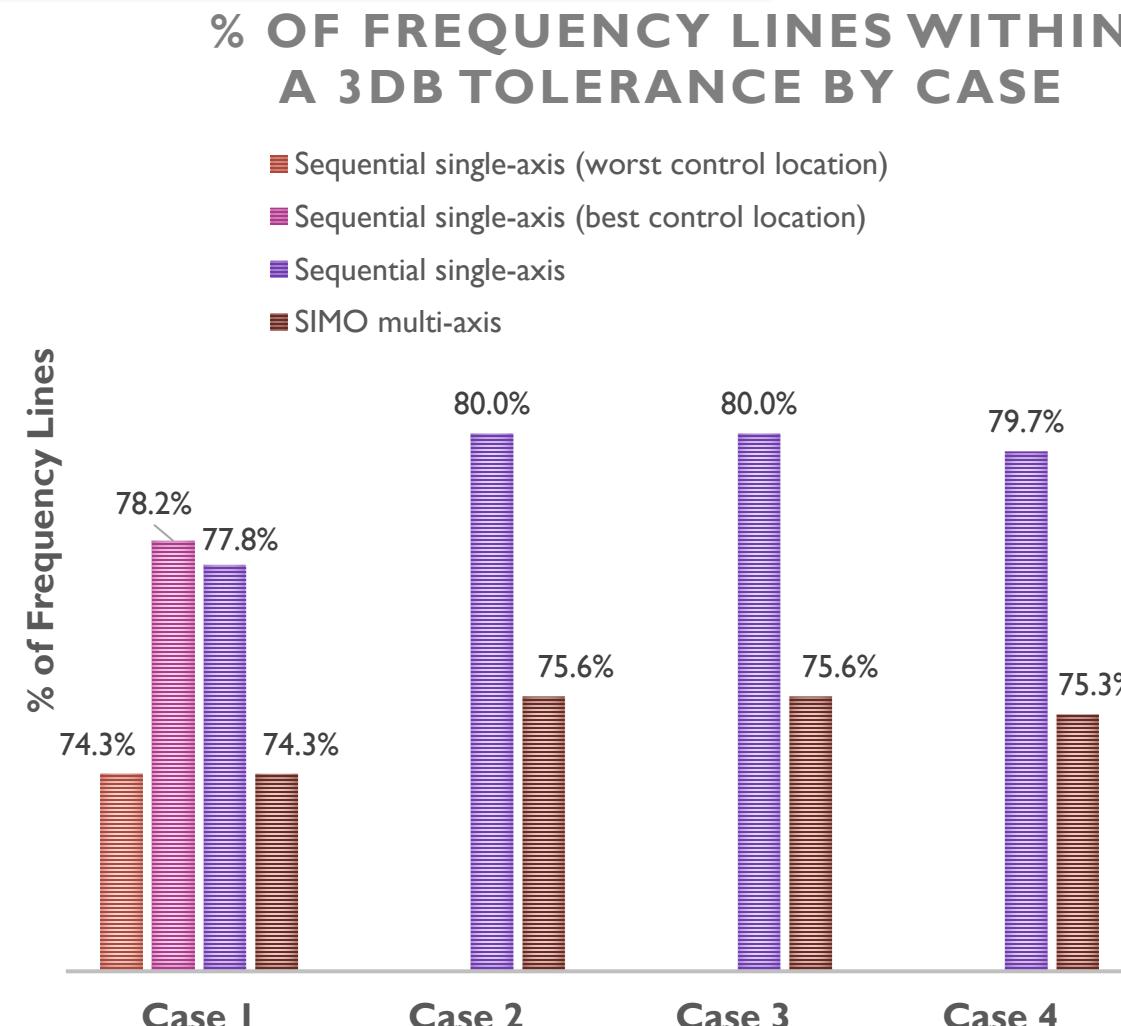
## Key findings

- What did we learn from the results of each case study?

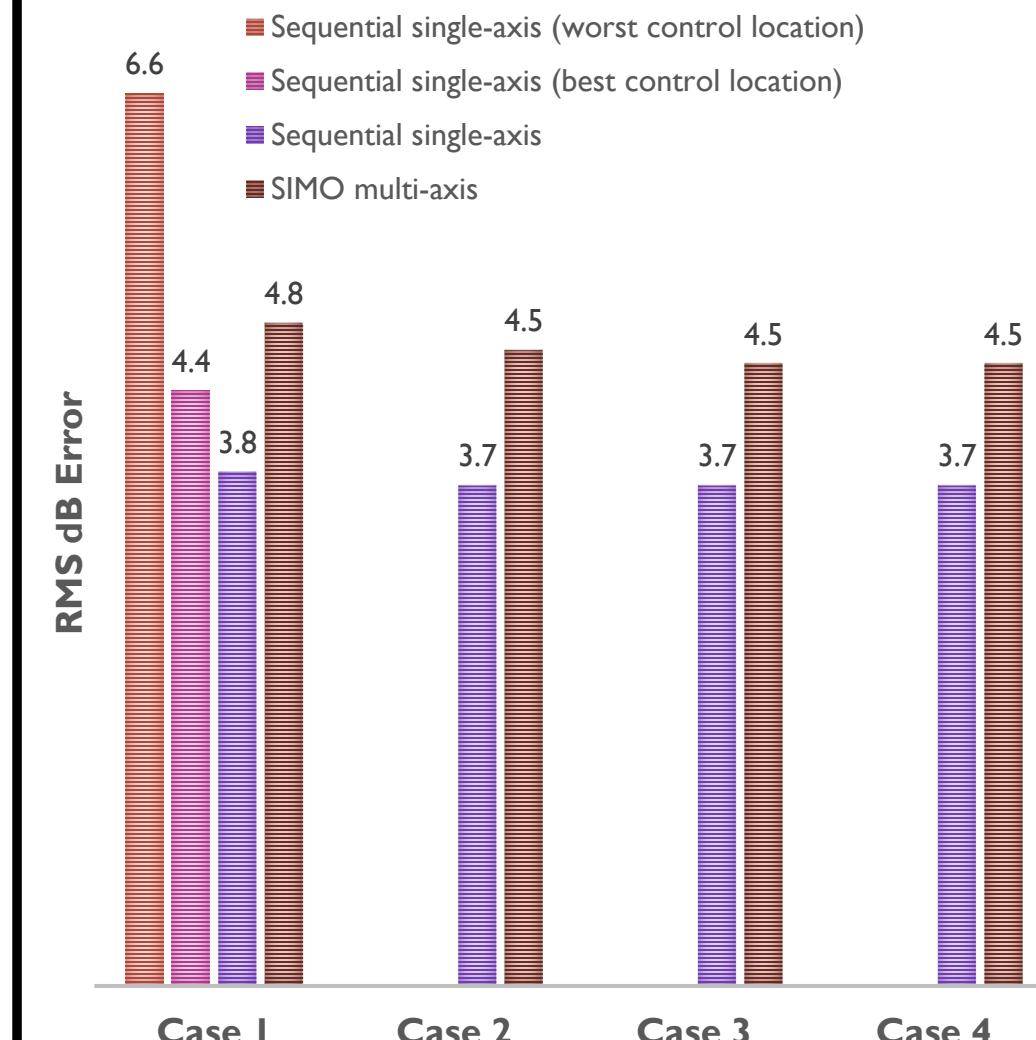
## Objectives

- Were the originally stated outcomes achieved?

# KEY FINDINGS

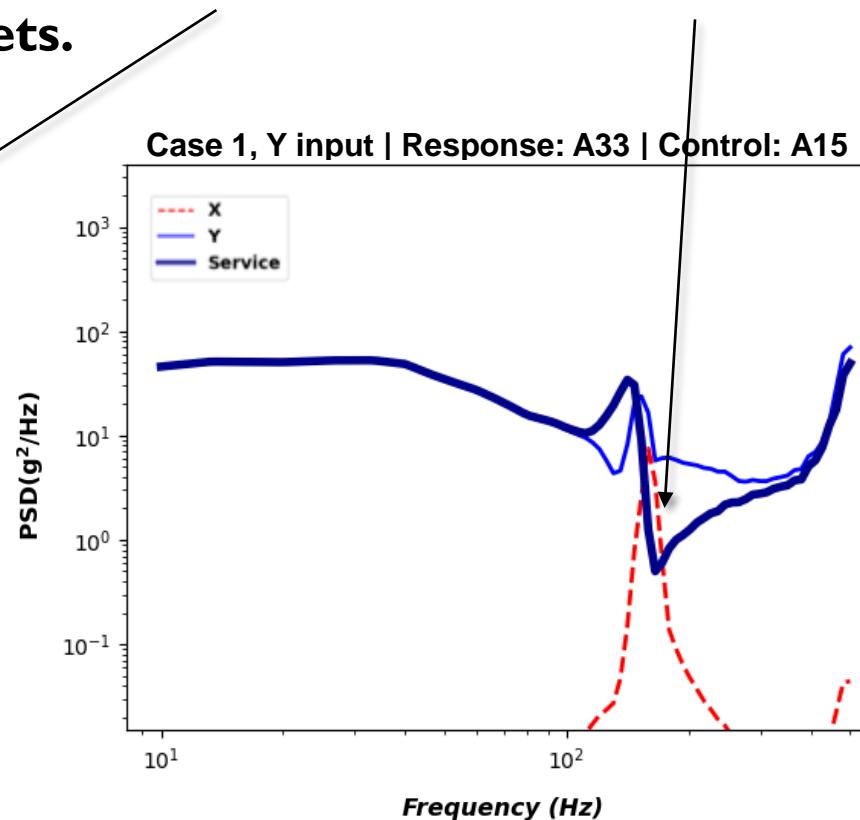
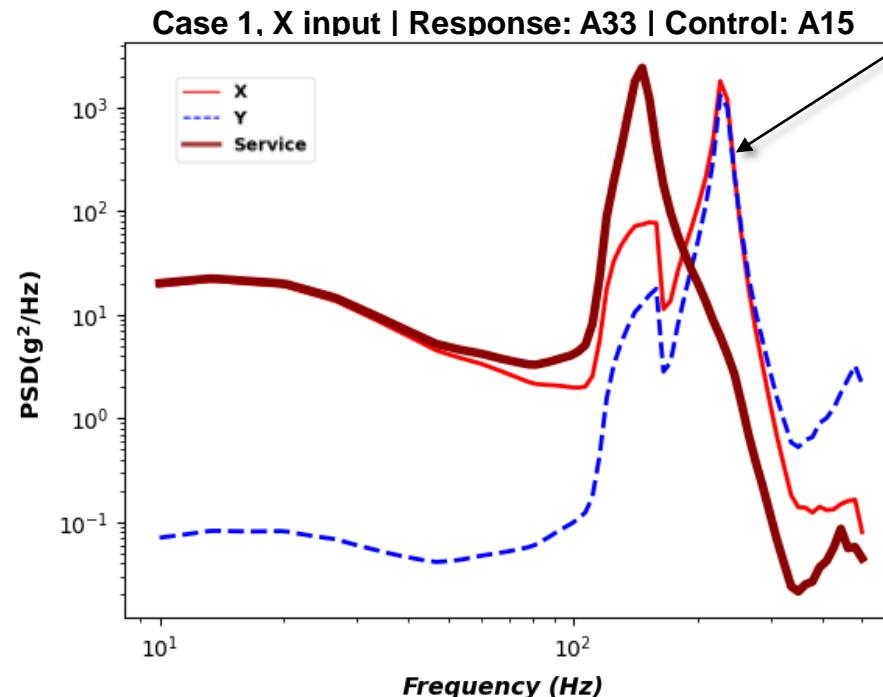


## RMS DB ERROR BY CASE



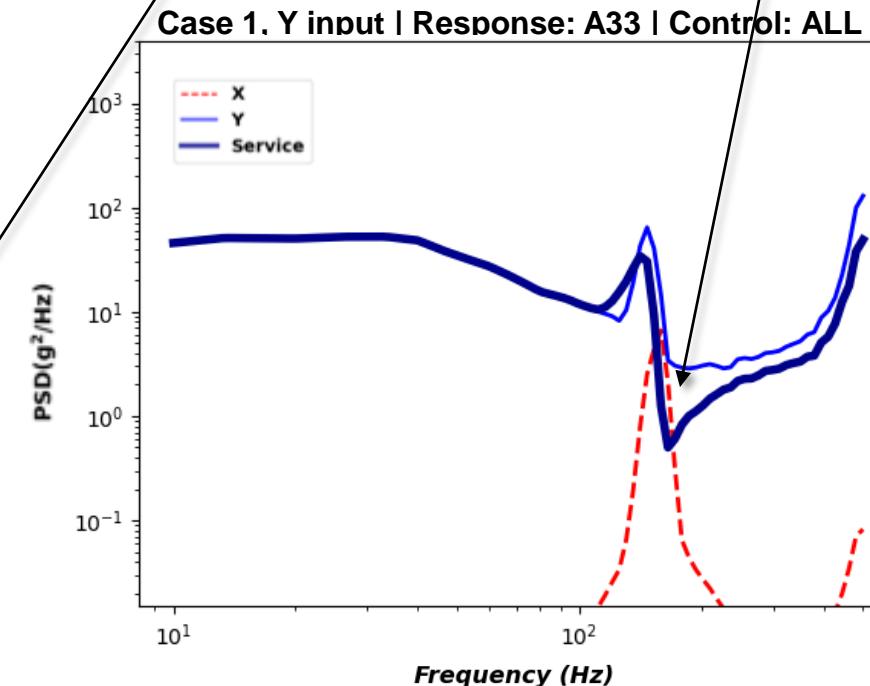
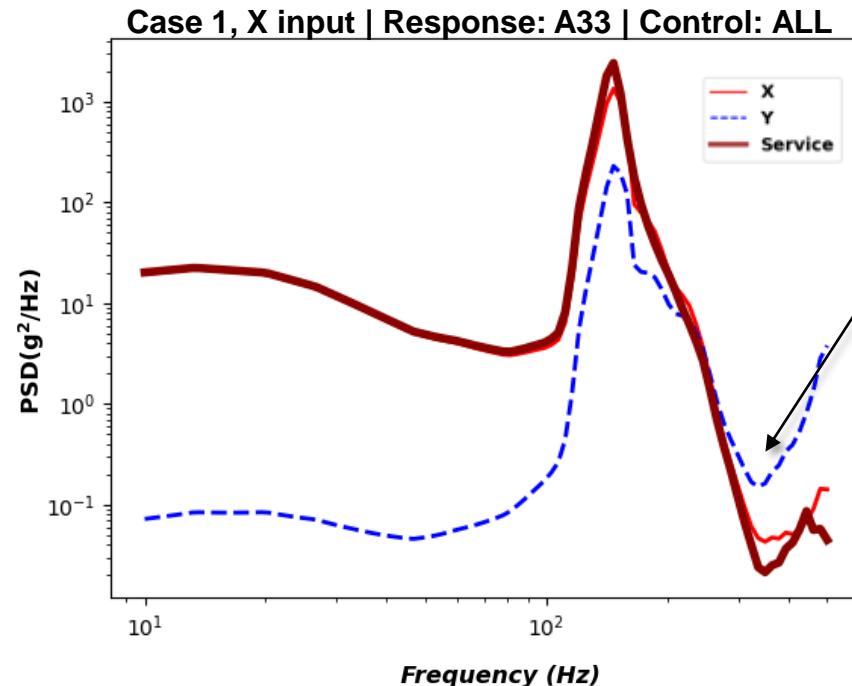
## KEY FINDINGS

**Off-axis response occasionally exceeds both the X and Y targets.**

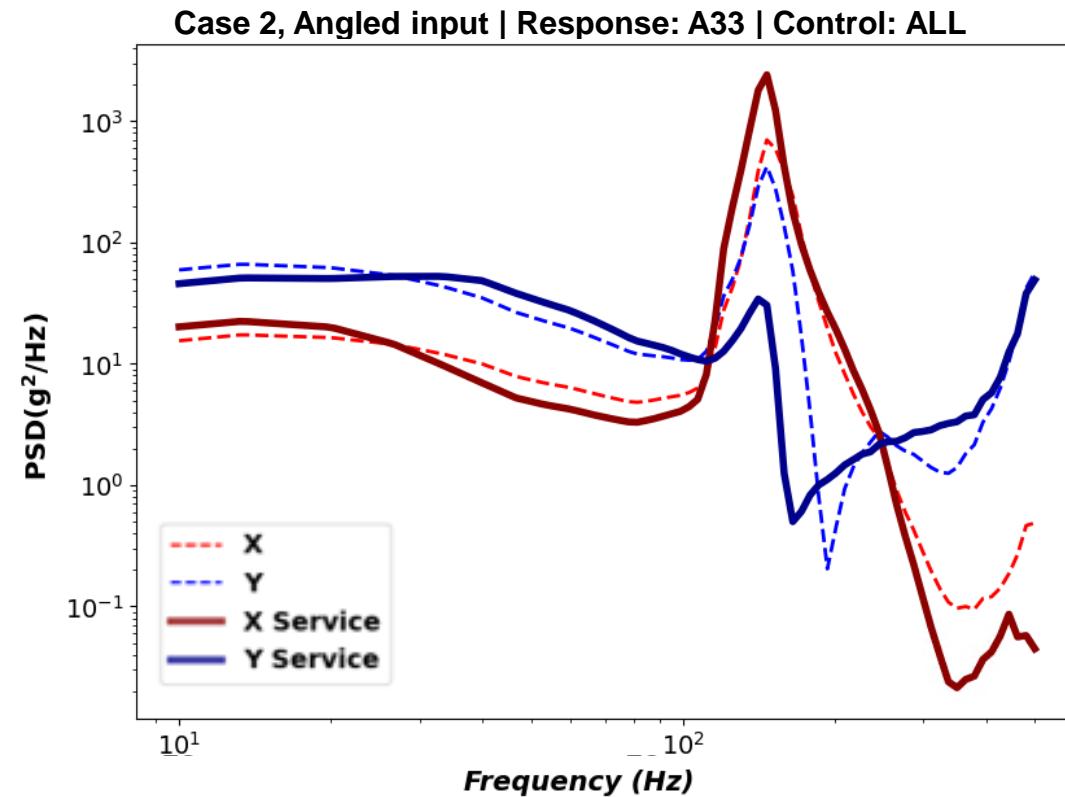


## KEY FINDINGS

**Off-axis response occasionally exceeds both the X and Y targets.**



## KEY FINDINGS



# OBJECTIVES

The objectives of the research are as follows:

- i. To assess the quality of the proposed method.
  - The proposed method allowed a single test to approximate a multi-axis test with on-axis errors only slightly worse than sequential testing.
- ii. To understand how much test quality improvement is possible with a well-designed test fixture.
  - A dynamically optimized test fixture improved both the sequential test and SIMO test.
- iii. To determine the effect of increasing the number of test fixture optimization parameters.
  - For this specific model, target, and optimization strategy, more stiffness parameters showed no improvement beyond the initial optimization.

# CONCLUSIONS

**Summary**

**Future work**

## SUMMARY

Over-testing was avoided for a small on-axis error penalty.

Both stiffness and angle optimization were necessary to produce the best-case SIMO test.

The stiffness solution space was tractable.

Increased test fixture complexity did not lead to better results.

- A fully validated method would enable wide access to rapid, approximate multi-axis vibration testing and eliminate the over-conservatism of sequential single-axis testing.

## FUTURE WORK

- The proposed method produced promising results, but more work is needed to fully validate the method...

Robust global optimization + local optimization.

Assessment of larger target populations.

Combine proposed method with input modification strategies.

Design and validation work to operationalize the approach.

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