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RAPID, APPROXIMATE MULTI-AXIS VIBRATION TESTING

Ethan Cramer

March 23rd, 2023

RAPID APPROXIMATE MULTI-AXIS VIBRATION TESTING

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Mechanical and Energy Engineering

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INTRODUCTION

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06

CONCLUSIONS

INTRO

Background

- What is the context of this thesis?

Objectives

- What are the desired outcomes?

BACKGROUND

- The aerospace industry uses vibration shaker tables to perform component durability testing.

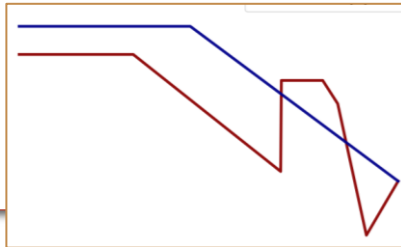
Measure Field Responses

Acceleration measurements are taken during a field test.



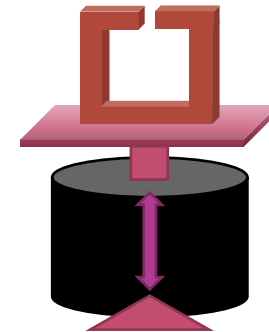
Design Service Environment Specification

A specification is derived from the measured accelerations.



Run Test

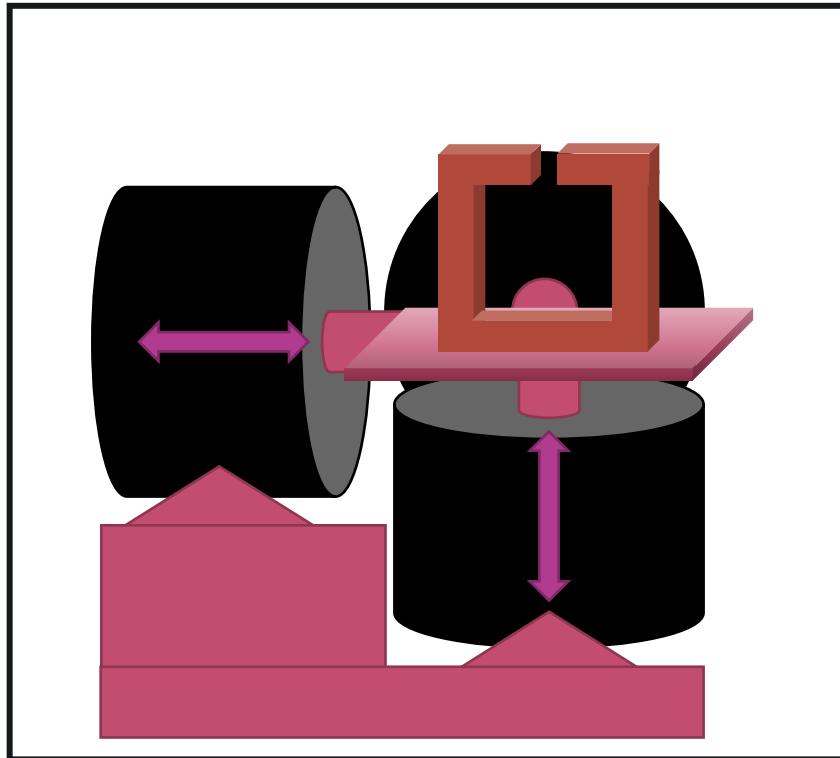
A laboratory test attempts to hit the derived specification.



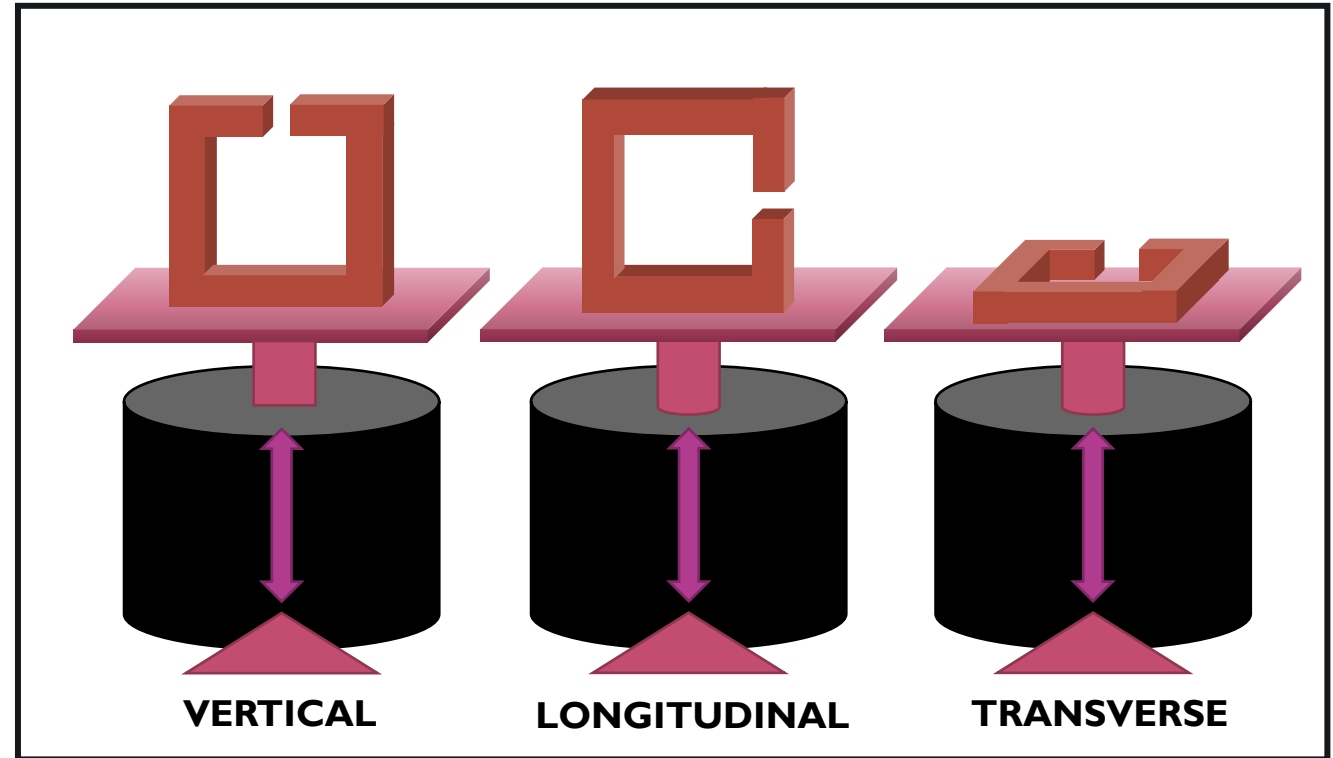
BACKGROUND

- Two categories of shaker tables: **single-axis** and **multi-axis**.
 - They are differentiated by their number of independent degrees of freedom.
- Multi-axis shakers can run more realistic environments but are not yet common in aerospace due to cost and complexity.
- **Sequential single-axis testing** attempts to replace a multi-axis shaker test with three sequential single-axis tests.

Multi-Axis Testing




Sequential Single-Axis Testing




Unfortunately, sequential single-axis tests **do not produce realistic environments.**

Sequential tests assume zero response in the off-axes.
Usually, this is far from true.



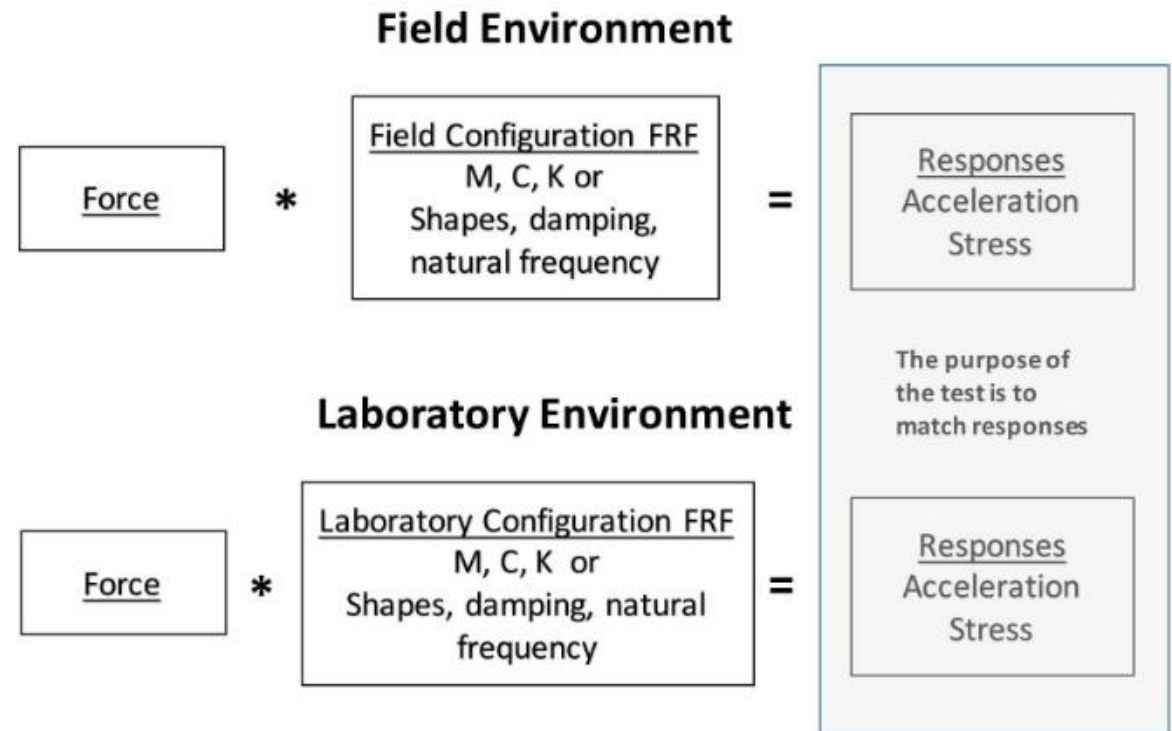
If cross-axis responses are significant, then three sequential tests cause each axis to experience full duration response three times.



This over-testing leads to unnecessary costs in strengthening the component design.

A poorly designed test fixture can also lead to unrealistic tests.

- A test's goal is to match the field environment in a laboratory test.
- The test fixture defines the test's boundary condition.
- For various practical and historic reasons, tests usually use rigid fixtures.



Schoenherr et al., 2018

OBJECTIVES

The problem:

Sequential single-axis testing is unrealistic but multi-axis shaker tables are not readily available.

A proposed solution:

Approximate a multi-axis test using a single-axis shaker table and an optimized test fixture.

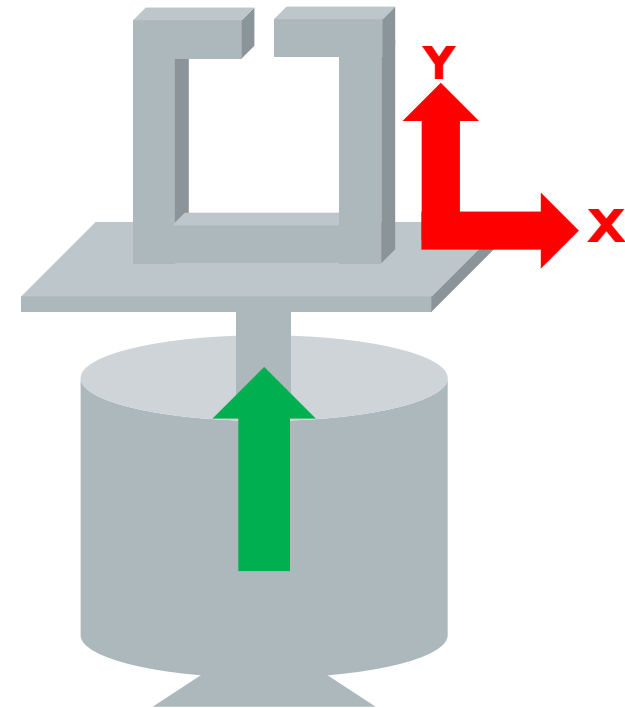
OBJECTIVES

Three strategies to approximate a multi-axis test:

OBJECTIVES

Three strategies to approximate a multi-axis test:

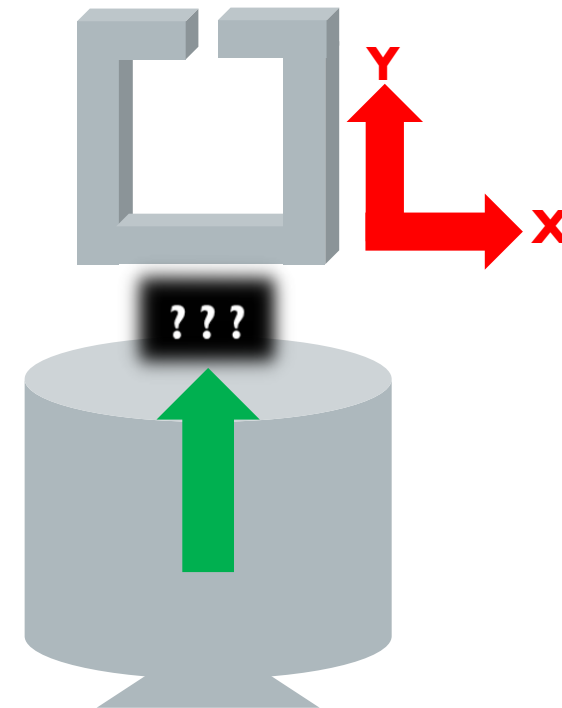
- I. A single-input, multiple output (SIMO) test strategy



OBJECTIVES

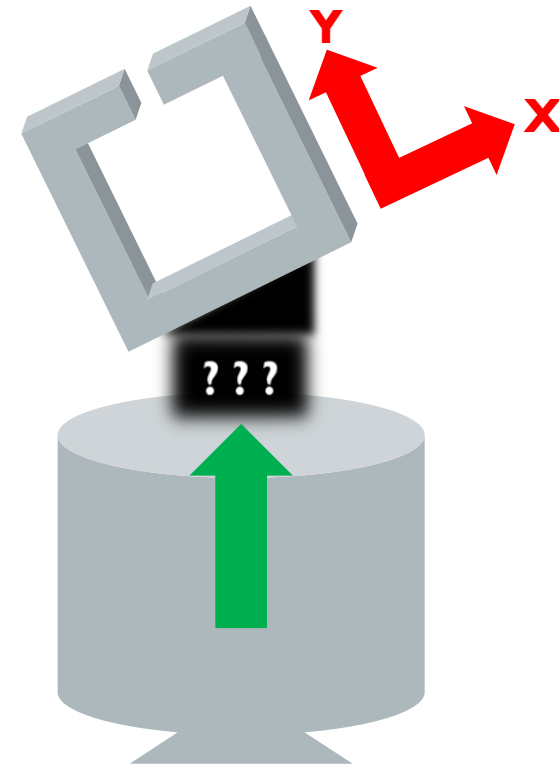
Three strategies to approximate a multi-axis test:

1. A **single-input, multiple output** (SIMO) test strategy
2. Test fixture optimization



Three strategies to approximate a multi-axis test:

1. A **single-input, multiple output** (SIMO) test strategy
2. Test fixture optimization
3. Angle optimization
 1. This helps a single input achieve multiple targets



OBJECTIVES

The objectives of the research are as follows:

- i. To assess the quality of the proposed method.
- ii. To understand how much test quality improvement is possible with a well-designed test fixture.
- iii. To determine the effect of increasing the number of test fixture optimization parameters.

LITERATURE REVIEW

Overview of vibration testing literature

Single-axis versus multi-axis testing

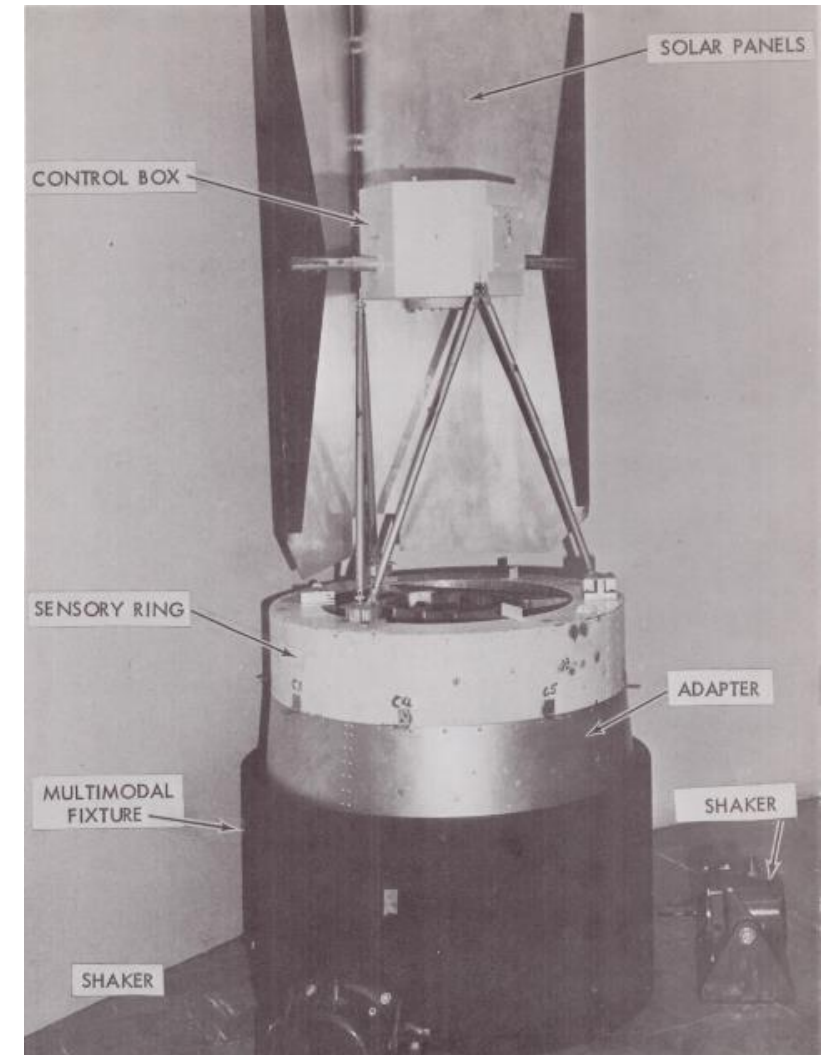
Impedance modification approaches

Input control approaches

Structural optimization

OVERVIEW OF VIBRATION TESTING

- Attempts to improve vibration tests via test fixture design began in the Apollo era.
- A component's real boundary condition can be difficult to reproduce in the lab.
- NASA researchers made early efforts at quantifying the mechanical impedance of boundary conditions (On, 1967) and matching the field test's boundary condition in the lab (Scharton, 1969).



Multimodal test fixture, (Scharton, 1969)

- The field and lab environment's mismatch is exacerbated in sequential single-axis testing.

Compared to a 6DOF test, sequential single-axis testing produces different...

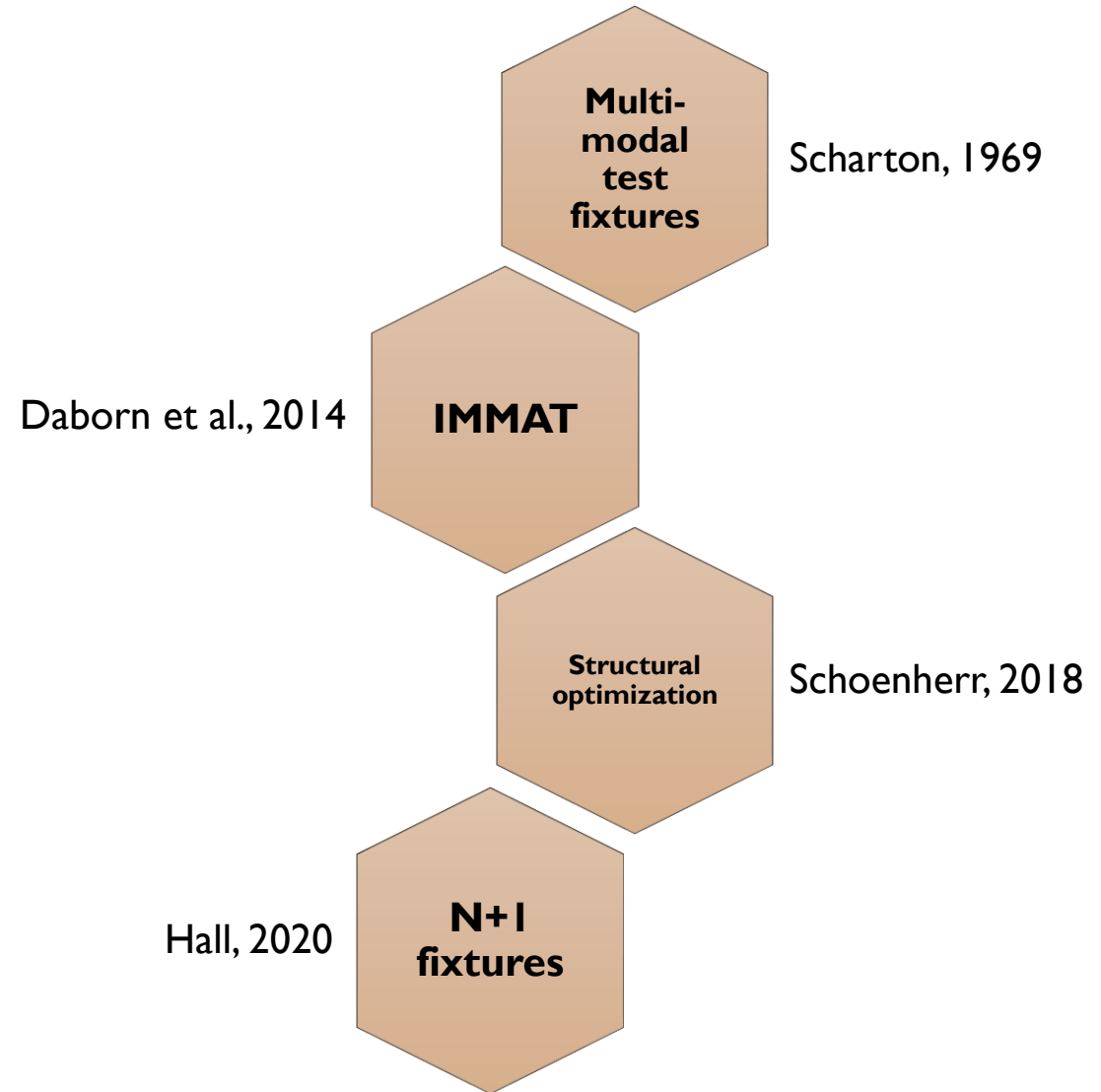
Failure times	Failure distributions	Maximum Von Mises stress	Location of maximum Von Mises stress	Modal participations
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French et al., 2006

Gregory et al., 2009

IMPEDANCE MODIFICATION APPROACHES

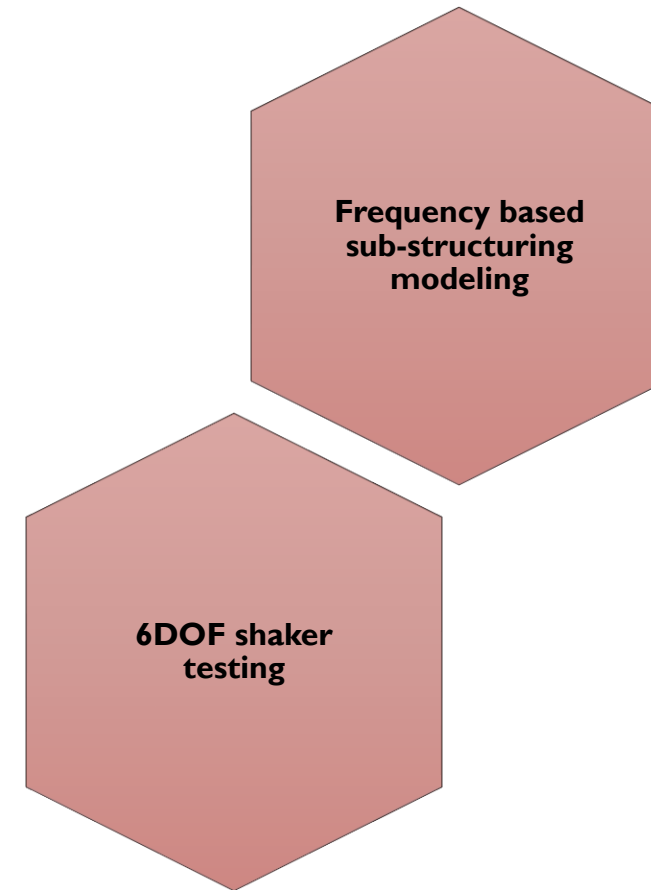
- Two categories of approaches are trying to rectify the mismatch between field and lab environments:
 1. **Impedance modification**
 2. **Input control**



INPUT CONTROL APPROACHES

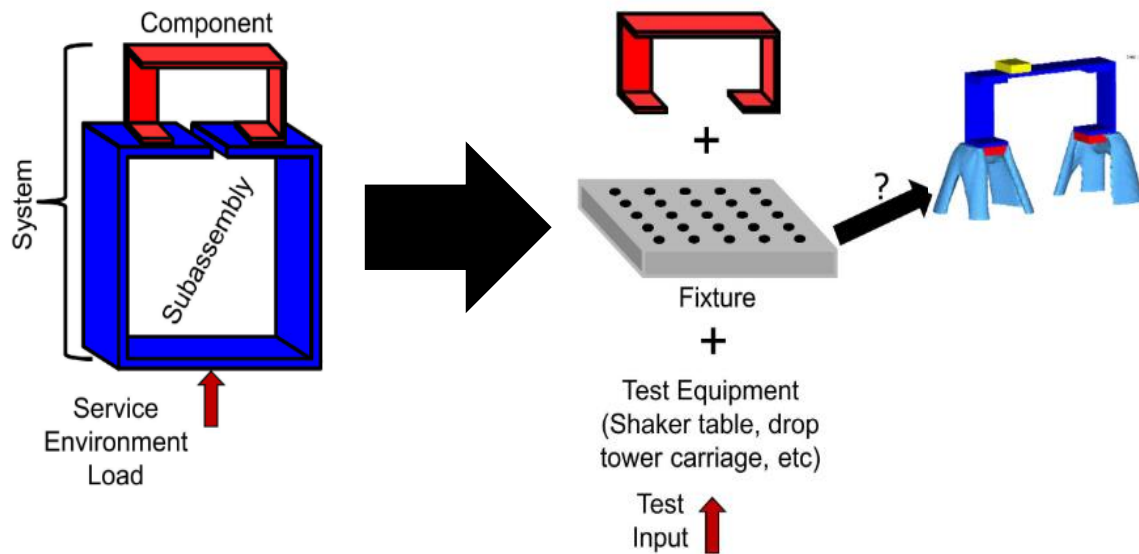
- Two categories of approaches are trying to rectify the mismatch between field and lab environments:
 1. Impedance modification
 2. **Input control**

Schoenherr et al., 2019



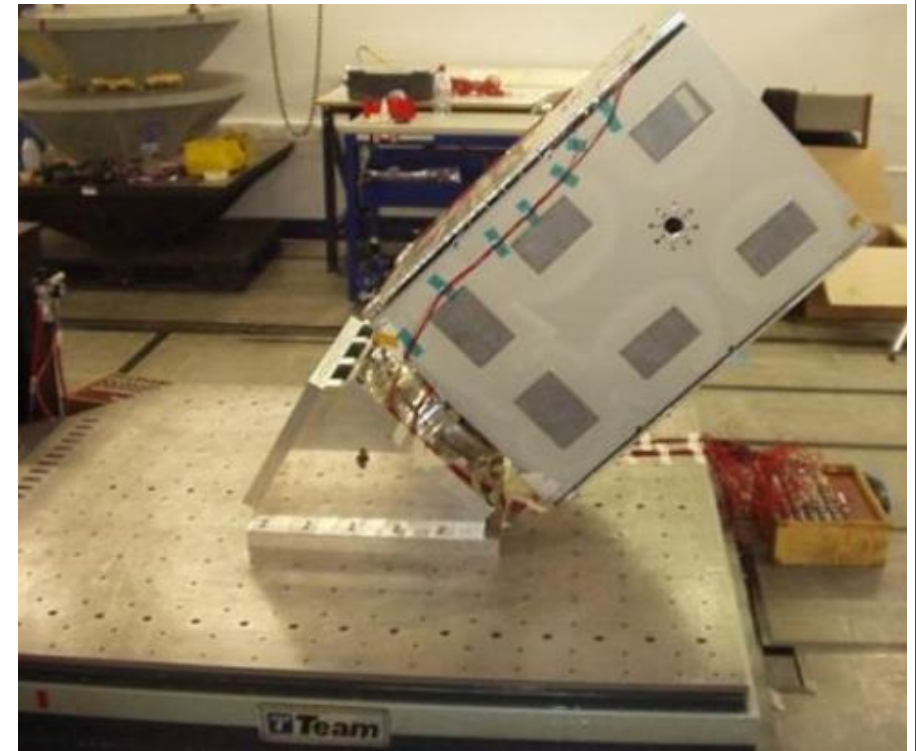
Harvie, 2017

Topological Optimization



Jones et al., 2018

Angle Optimization



Knight et al., 2018

METHOD

Case studies

Finite element model

Service environment

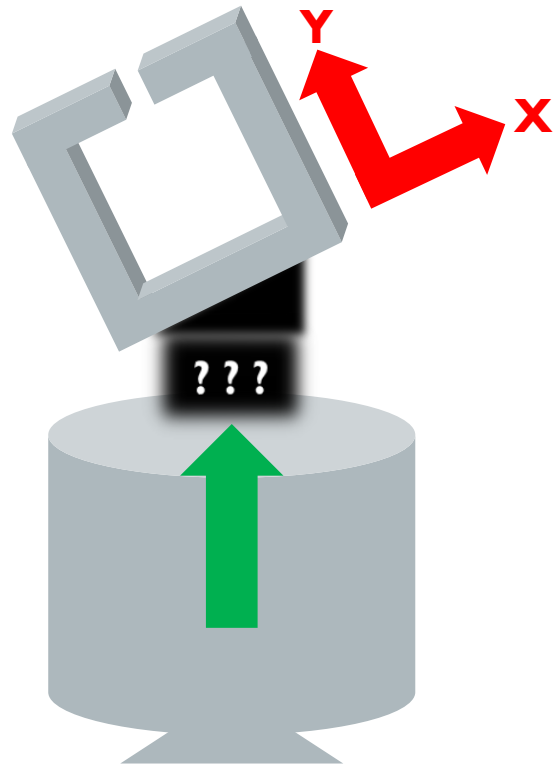
Boundary conditions

Test quality metrics

Simulation approach

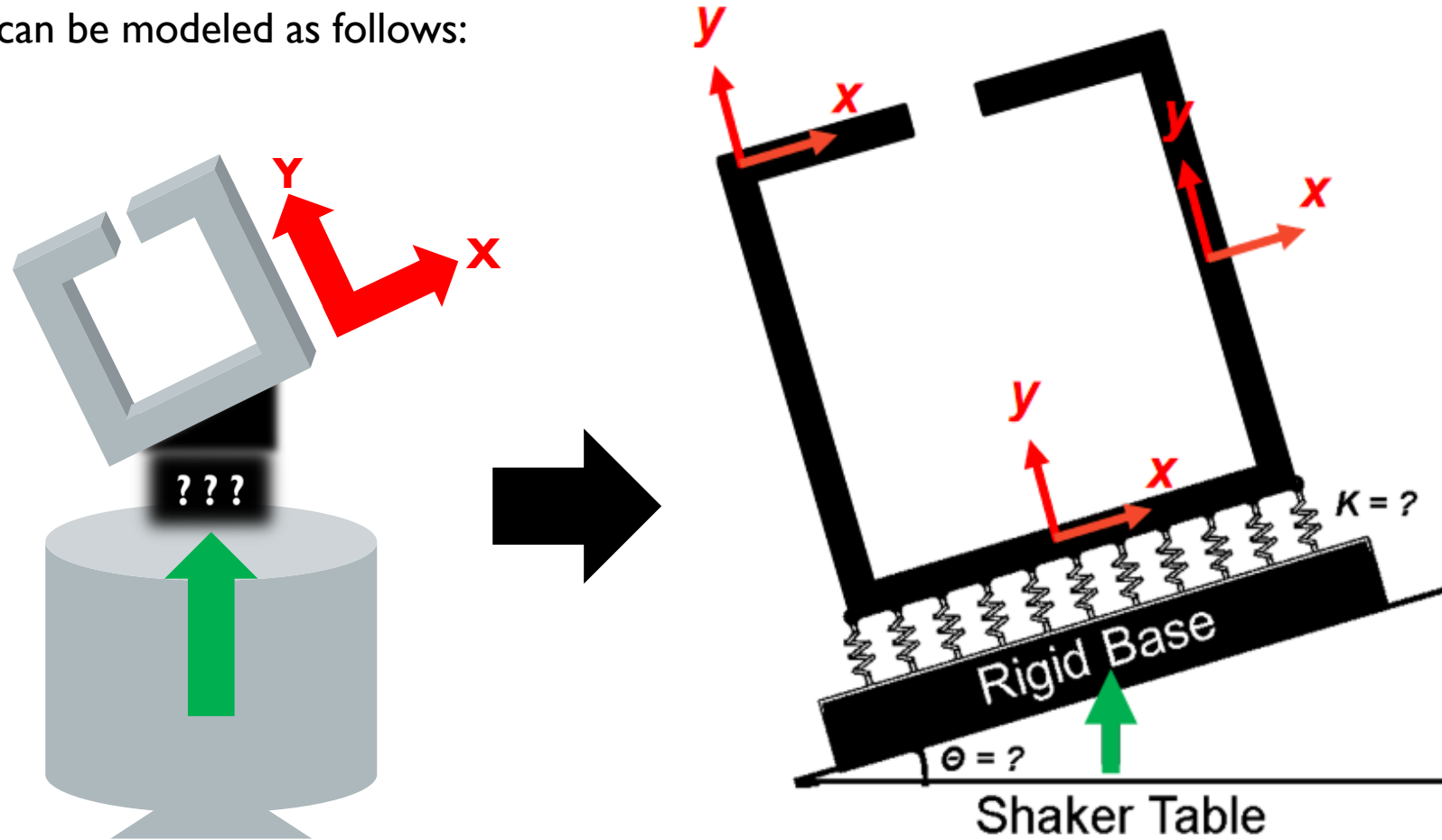
CASE STUDIES

The proposed method can be modeled as follows:



CASE STUDIES

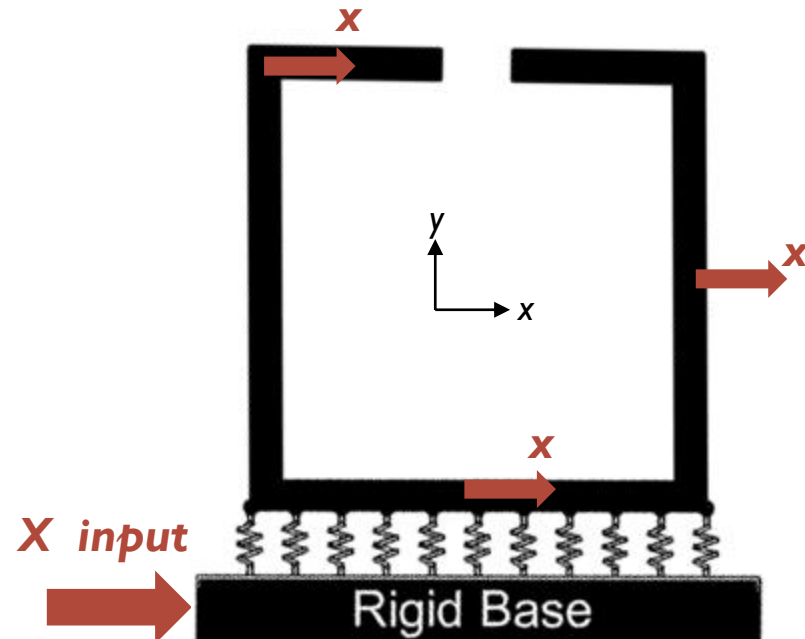
The proposed method can be modeled as follows:



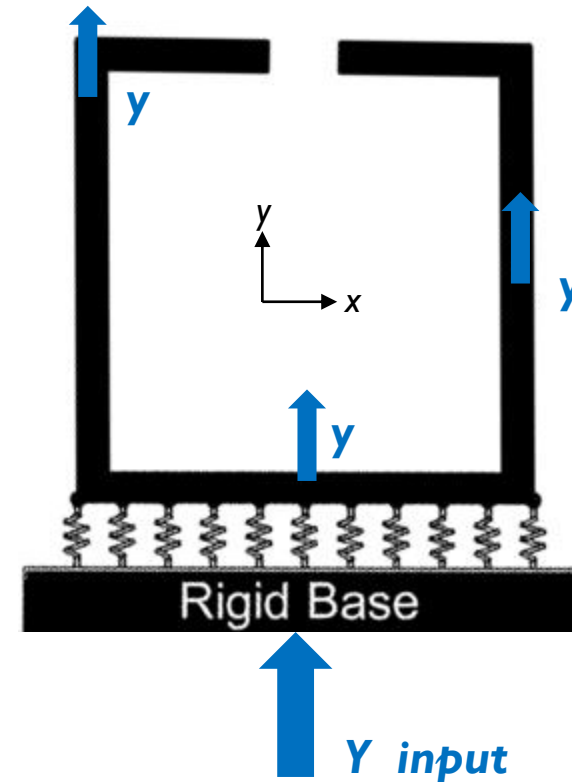
CASE STUDIES

And a sequential test can be similarly represented:

X-axis test



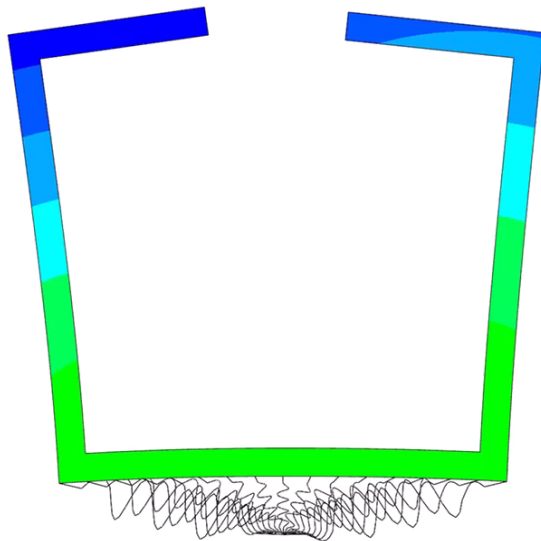
Y-axis test



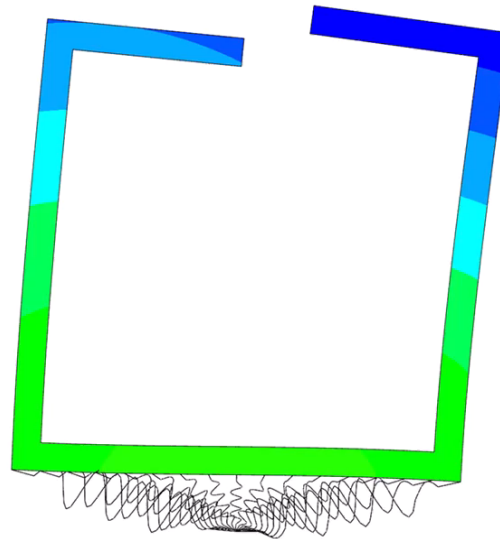
- Four case studies are presented to compare sequential testing to the proposed method:

Case	Test Fixture Design	Comparisons
1	Rigid	<ol style="list-style-type: none">1. Sequential single-axis (single control location)2. Sequential single-axis (all control locations)3. SIMO multi-axis (all control locations)
2	Optimized (2 parameters)	<ol style="list-style-type: none">1. Sequential single-axis2. SIMO multi-axis
3	Optimized (4 parameters)	<ol style="list-style-type: none">1. Sequential single-axis2. SIMO multi-axis
4	Optimized (8 parameters)	<ol style="list-style-type: none">1. Sequential single-axis2. SIMO multi-axis

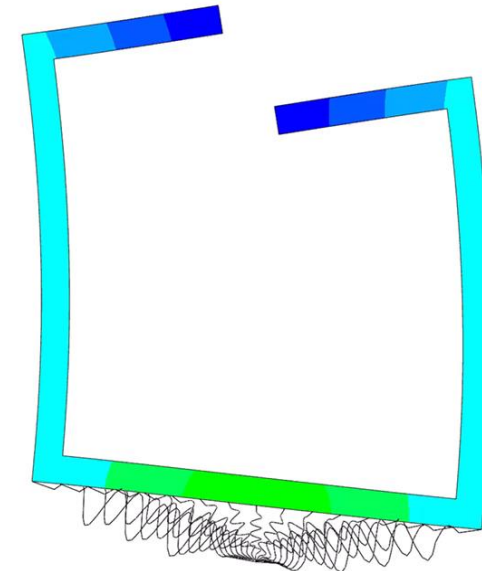
- Simulations were carried out using an Abaqus FEA model consisting of:
 1. 2-D BARC without the removable component
 2. Test fixture, idealized by sixteen springs
 3. Rigid base, representing a vertically-oriented, single-axis shaker
- The test fixture springs have an independent X-axis and Y-axis stiffness.



Mode 1: 135.5 Hz



Mode 2: 136.4 Hz

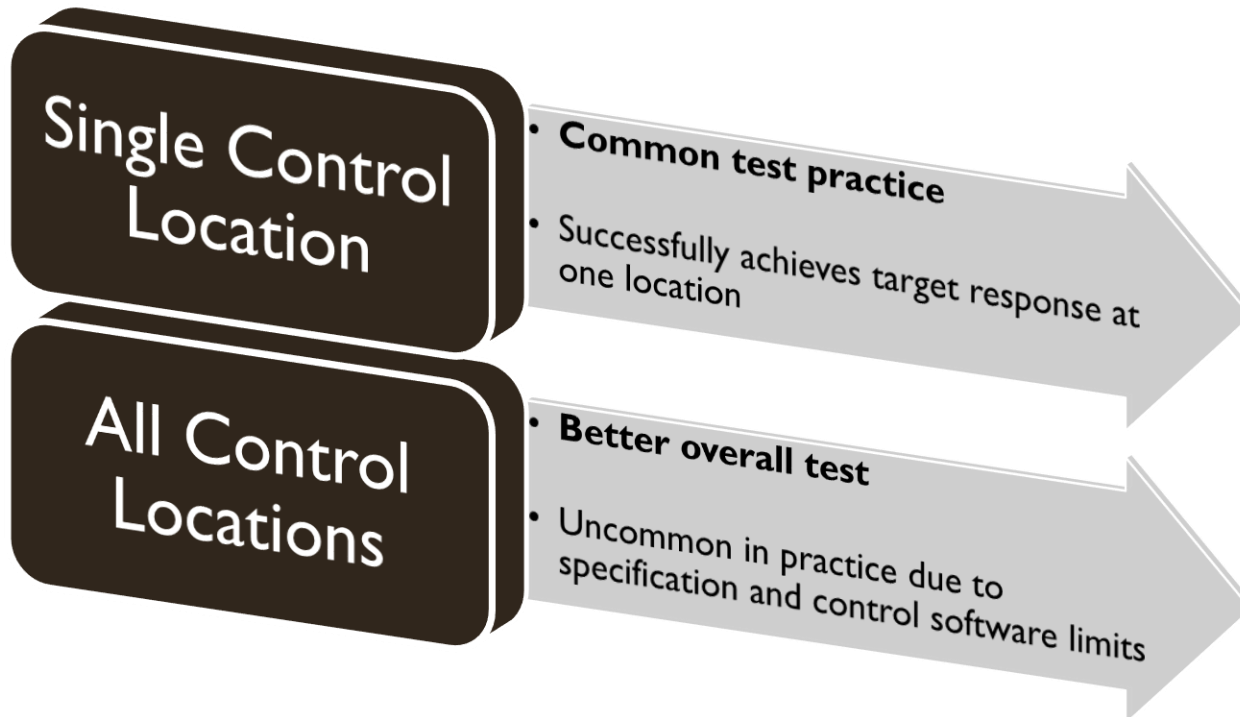
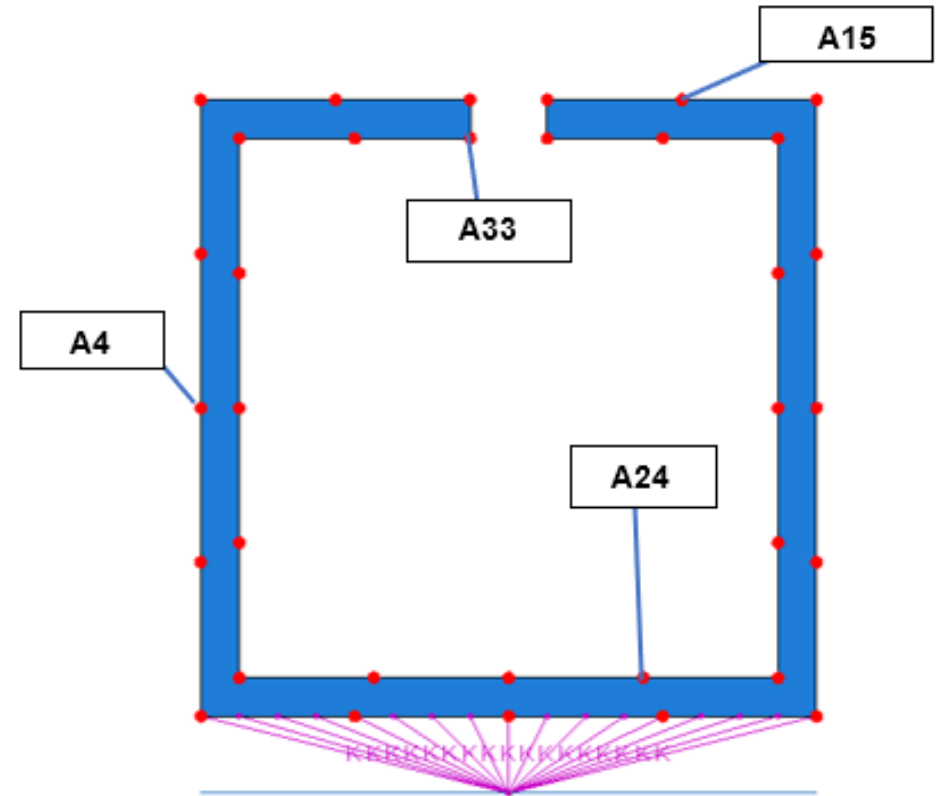


Mode 3: 437.3 Hz

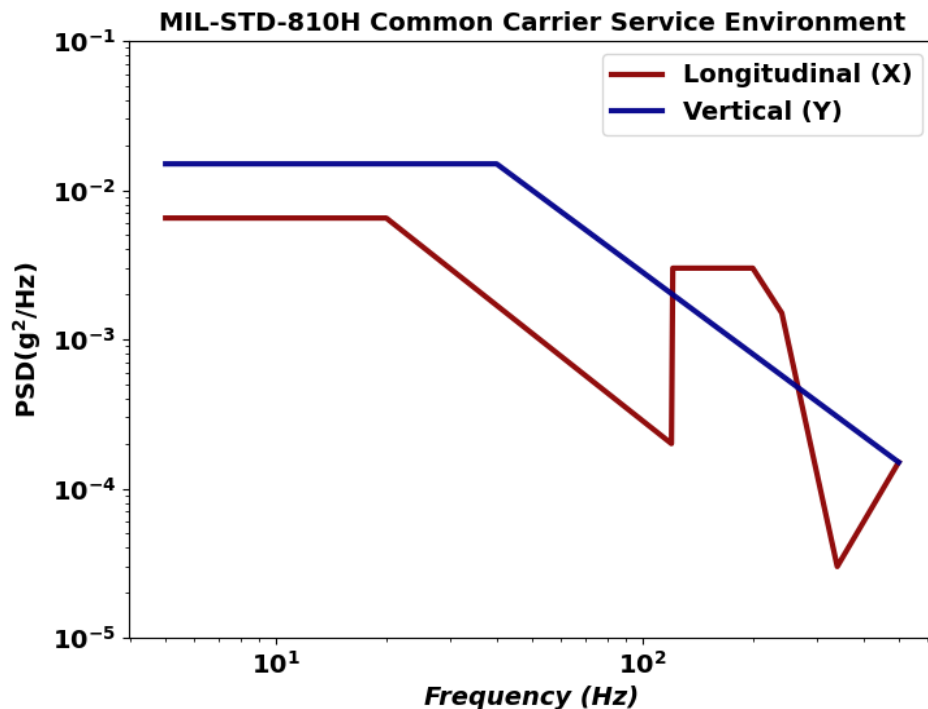
FEA MODEL

- The shaker input is derived to achieve the lowest possible error between a target response and measured response.
- The locations where measured response is used to derive a shaker input are known to as **control locations**.

Measurement Locations



- A **service environment** is the real environment the test article is expected to experience in its lifetime.

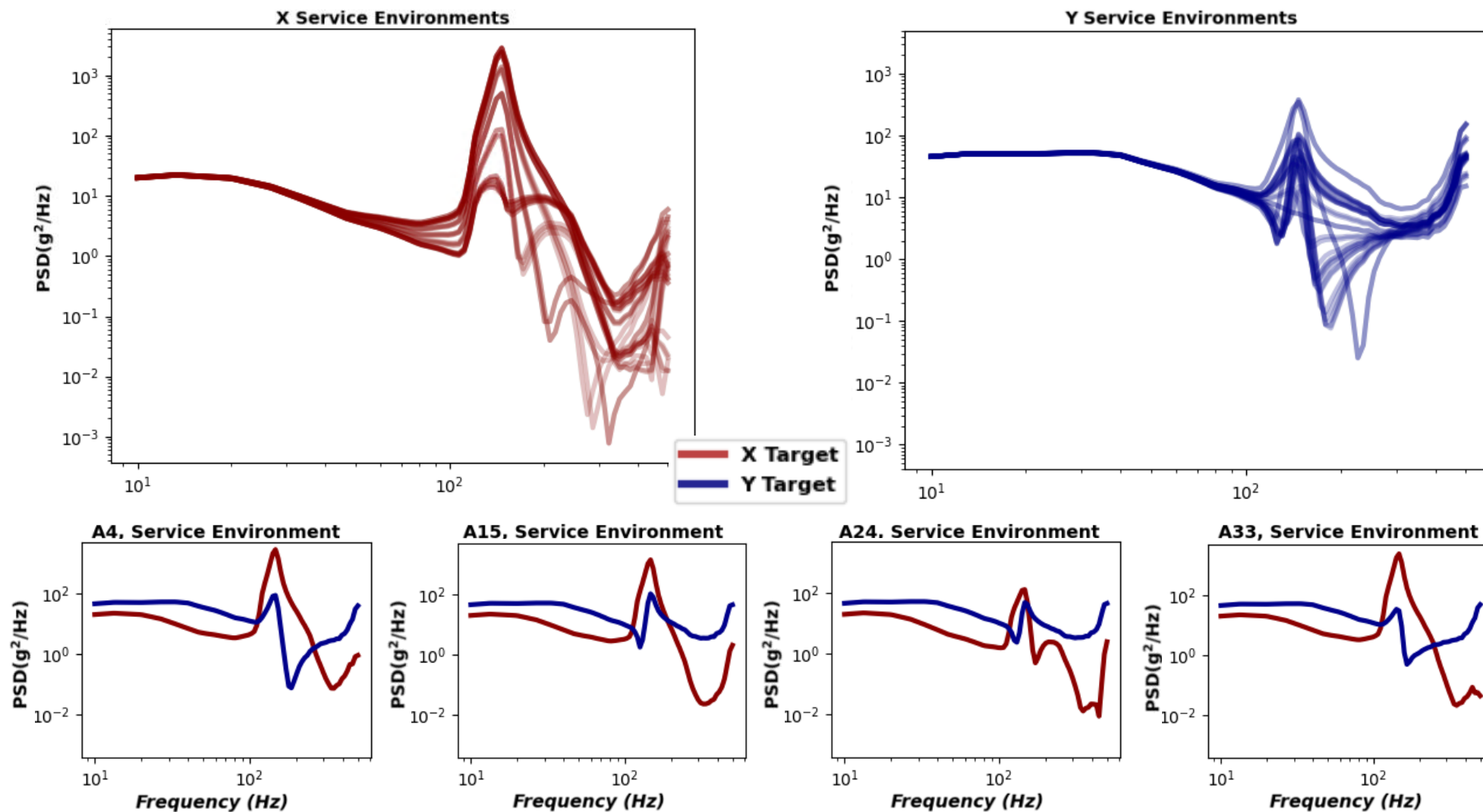


Defense Logistics Agency, 2019

- This a common transportation environment base excitation
- To generate a set of targets from this base excitation...
 1. Apply X and Y excitations simultaneously to the base of the model
 2. Measure acceleration responses on the BARC
- Targets were generated using a flexible boundary condition (k_x and $k_y = 10^6 \text{ N/m}$)

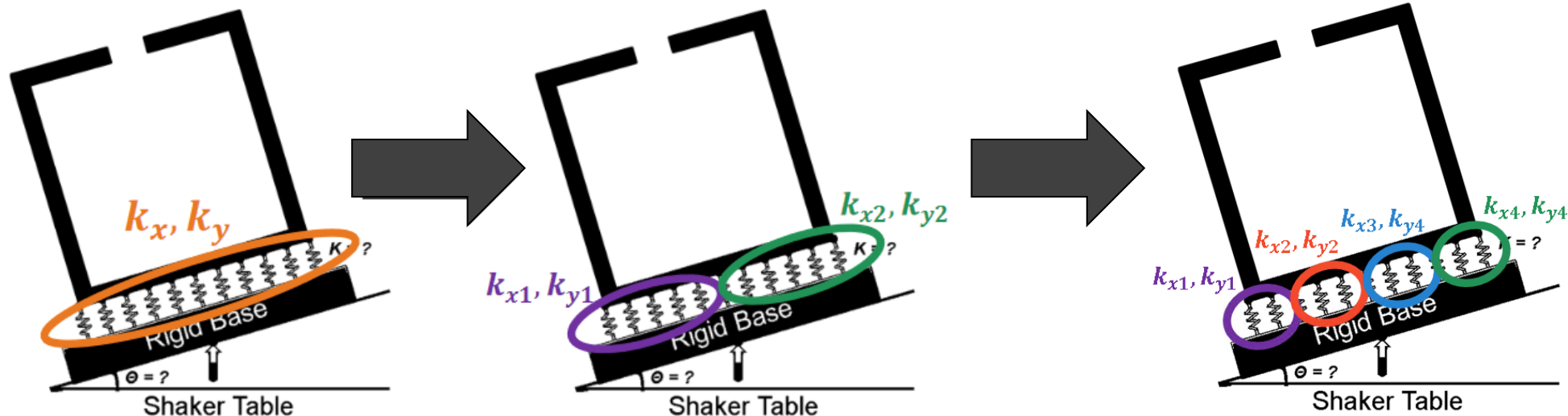
SERVICE ENVIRONMENT

Method
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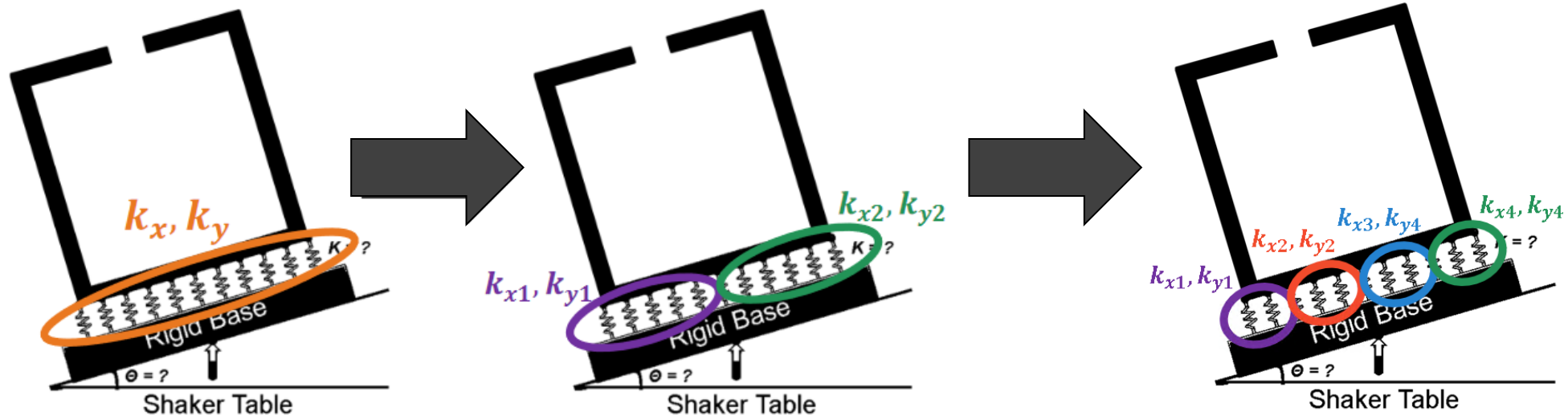
BOUNDARY CONDITIONS

- The boundary conditions are determined by the test fixture.
- In our model, the boundary condition is determined by the stiffness of the springs.
 - Lower number of optimization parameters \rightarrow more tractable search
- In Case 1, the boundary condition is rigid.
- In Cases 2, 3, and 4, the boundary condition is optimized using an increasing number of parameters to determine how much test fixture complexity contributes to improving the possible solutions.



BOUNDARY CONDITIONS

Case	Test Fixture Design	Optimization Parameters	
		Sequential Single-Axis	SIMO Multi-Axis
1	Rigid	-	θ
2	Optimized (2 parameters)	k_x, k_y	k_x, k_y, θ
3	Optimized (4 parameters)	$k_{x1}, k_{x2}, k_{y1}, k_{y2}$	$k_{x1}, k_{x2}, k_{y1}, k_{y2}, \theta$
4	Optimized (8 parameters)	$k_{x1}, k_{x2}, k_{x3}, k_{x4},$ $k_{y1}, k_{y2}, k_{y3}, k_{y4}$	$k_{x1}, k_{x2}, k_{x3}, k_{x4},$ $k_{y1}, k_{y2}, k_{y3}, k_{y4}, \theta$



RMS dB Error (RDBE):

RDBE is a scalar metric that represents the average mismatch between a response PSD and a target PSD across all frequencies.

$$RDBE = \sqrt{\frac{\sum_{i=1}^n (10 \log_{10} \frac{\hat{y}_i}{y_i})^2}{n}}$$

Percentage of Frequency Lines within a 3 dB Tolerance (FTOL):

FTOL is a scalar metric that counts the percentage of frequency lines where the response PSD differs from the target PSD by less than 3 dB.

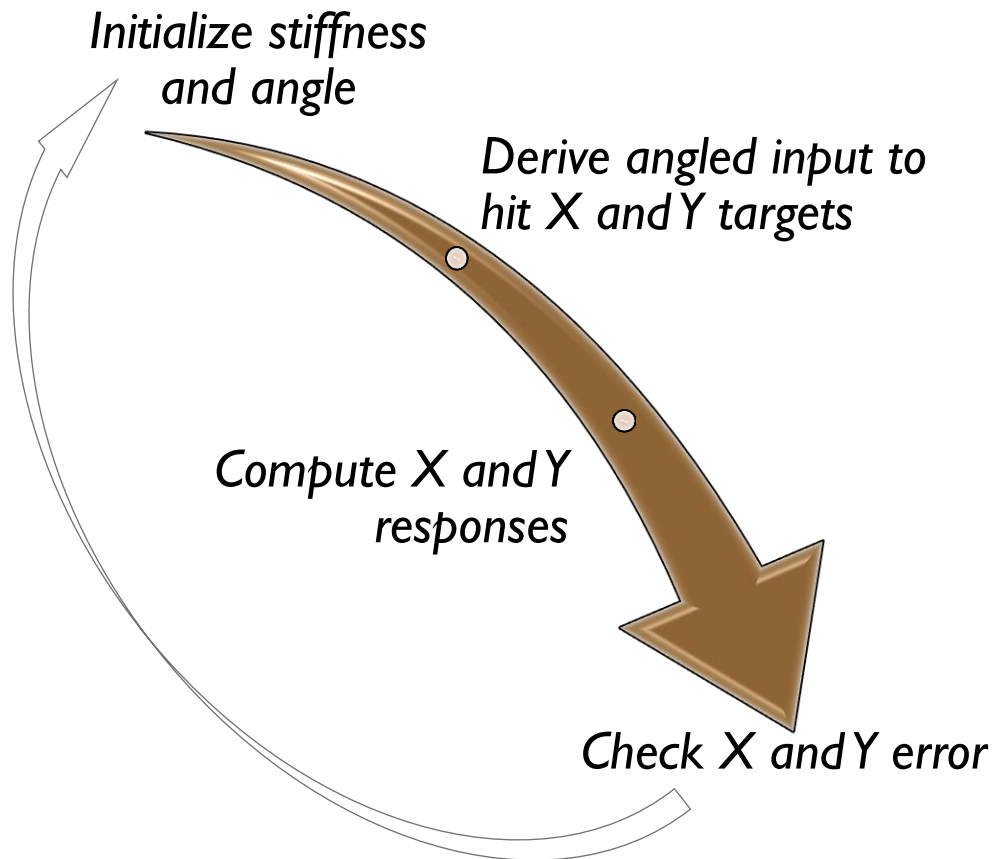
$$\% FL = 100 * \frac{1}{n} \sum_{i=1}^n \begin{cases} 1 & \text{if } \left| 10 \log_{10} \frac{\hat{y}_i}{y_i} \right| \leq 3 \text{ dB} \\ 0 & \text{if } \left| 10 \log_{10} \frac{\hat{y}_i}{y_i} \right| > 3 \text{ dB} \end{cases}$$

In both equations, \hat{y}_i is the value of a response PSD at the i^{th} frequency line, and y_i is the value of the target PSD at the i^{th} frequency line. There are n total frequency lines.

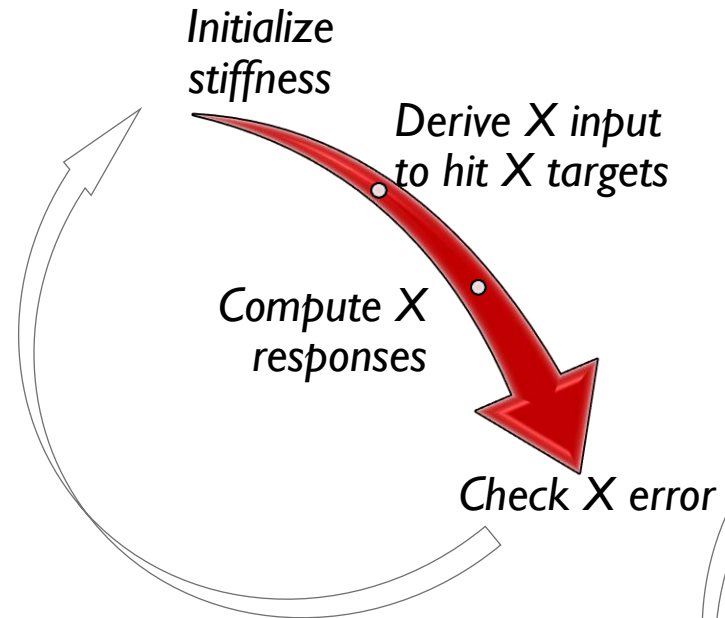
SIMULATION APPROACH

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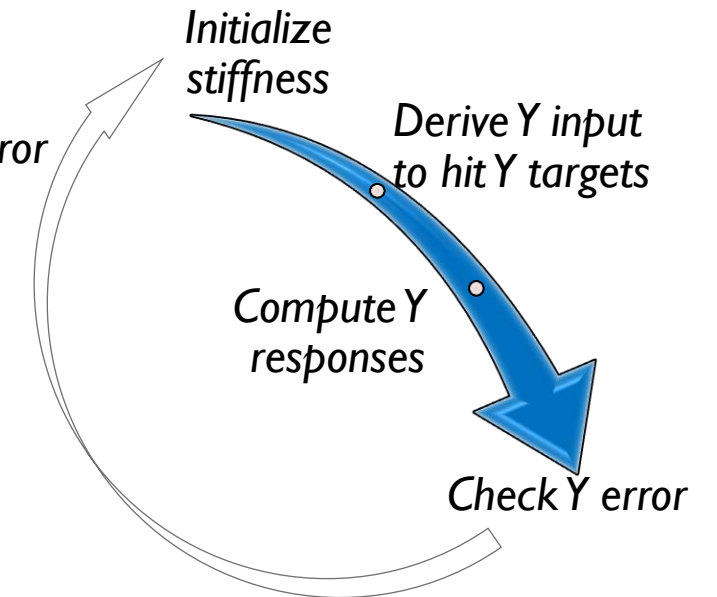
SIMO Multi-Axis Test



X-Axis Sequential Test



Y-Axis Sequential Test



RESULTS

Case study 1

Case study 2

Case study 3

Case study 4

- For case I, all tests used a rigid fixture (k_x and $k_y = 10^9 \text{ N/m}$)

Case	Test Fixture Design	Comparisons
I	Rigid	<ol style="list-style-type: none"> 1. Sequential single-axis (single control location) 2. Sequential single-axis (all control locations) 3. SIMO multi-axis (all control locations)
2	Optimized (2 parameters)	<ol style="list-style-type: none"> 1. Sequential single-axis 2. SIMO multi-axis
3	Optimized (4 parameters)	<ol style="list-style-type: none"> 1. Sequential single-axis 2. SIMO multi-axis
4	Optimized (8 parameters)	<ol style="list-style-type: none"> 1. Sequential single-axis 2. SIMO multi-axis

CASE STUDY 1

- For the single control location sequential tests, choosing the best location improves the RDBE by 2.2 dB and the FTOL by 3.9% over the worst location.

Sequential Single-Axis (Worst performing control location)				Sequential Single-Axis (Best performing control location)			
	X-Error (avg)	Y-Error (avg)	<u>Mean Error</u>		X-Error (avg)	Y-Error (avg)	<u>Mean Error</u>
RDBE	9.9 dB	3.2 dB	<u>6.6 dB</u>	RDBE	5.6 dB	3.2 dB	<u>4.4 dB</u>
FTOL	64.6%	83.9%	<u>74.3%</u>	FTOL	71.6%	84.7%	<u>78.2%</u>

CASE STUDY 1

- When the test controls to all locations, the RDBE further decreases to **3.8 dB**.
- It's interesting that the FTOL decreases slightly.

Sequential Single-Axis (All control locations)

	X-Error (avg)	Y-Error (avg)	<u>Mean Error</u>
RDBE	5.1 dB	2.6 dB	<u>3.8 dB</u>
FTOL	70.3%	85.2%	<u>77.8%</u>

Sequential Single-Axis (Worst performing control location)

	X-Error (avg)	Y-Error (avg)	<u>Mean Error</u>
RDBE	9.9 dB	3.2 dB	<u>6.6 dB</u>
FTOL	64.6%	83.9%	<u>74.3%</u>

Sequential Single-Axis (Best performing control location)

	X-Error (avg)	Y-Error (avg)	<u>Mean Error</u>
RDBE	5.6 dB	3.2 dB	<u>4.4 dB</u>
FTOL	71.6%	84.7%	<u>78.2%</u>

CASE STUDY 1

- The SIMO multi-axis test's **RDBE** increases by **1.0 dB** while the **FTOL** falls by **3.8%**.

Sequential Single-Axis (All control locations)				SIMO Multi-Axis (All control locations)			
	X-Error (avg)	Y-Error (avg)	<u>Mean Error</u>		X-Error (avg)	Y-Error (avg)	<u>Mean Error</u>
RDBE	5.1 dB	2.6 dB	<u>3.8 dB</u>	RDBE	5.7 dB	3.9 dB	<u>4.8 dB</u>
FTOL	70.3%	85.2%	<u>77.8%</u>	FTOL	69.2%	78.7%	<u>74.0%</u>

This decrease in measured test quality metrics may be justified because the SIMO multi-axis test eliminates cross-axis responses that are unavoidable in the sequential test.

Optimizing the test fixture stiffness with two parameters:

1. Improved the sequential test by 0.1 dB and 2.2%.
2. Improved the SIMO test by 0.3 dB and 1.6%.

	Mean Error	
	Sequential Single-Axis	SIMO Multi-Axis
RDBE	3.7 dB	4.5 dB
FTOL	80.0%	75.6%

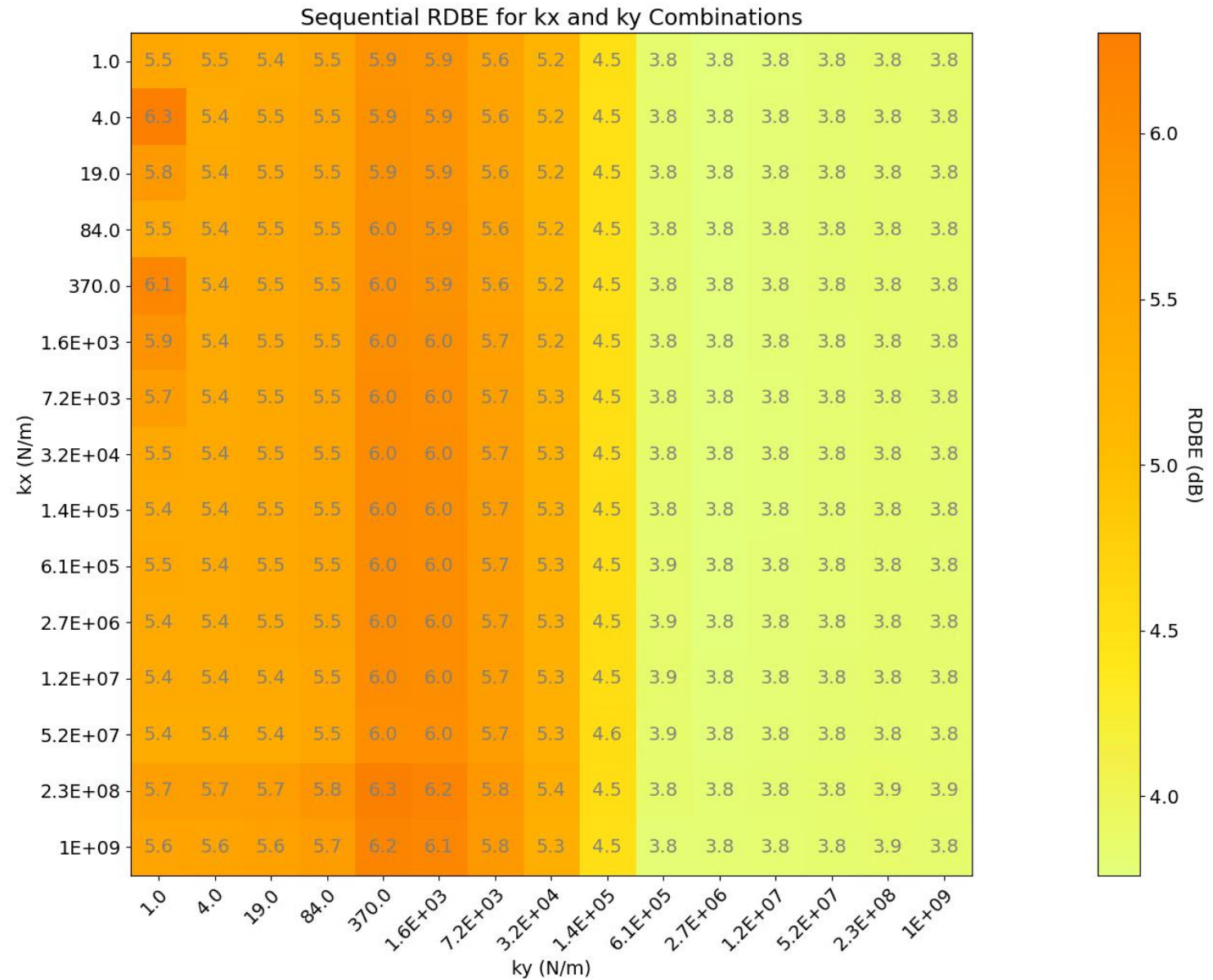
The solution found for each test was:

1. For the sequential test, $k_x = 3.2 * 10^4$ and $k_y = 2.7 * 10^6$ N/m.
2. For the SIMO test, $k_x = 1.2 * 10^7$, $k_y = 2.7 * 10^6$ N/m, and $\theta = 27^\circ$.

	Mean Error	
	Sequential Single-Axis	SIMO Multi-Axis
RDBE	3.7 dB	4.5 dB
FTOL	80.0%	75.6%

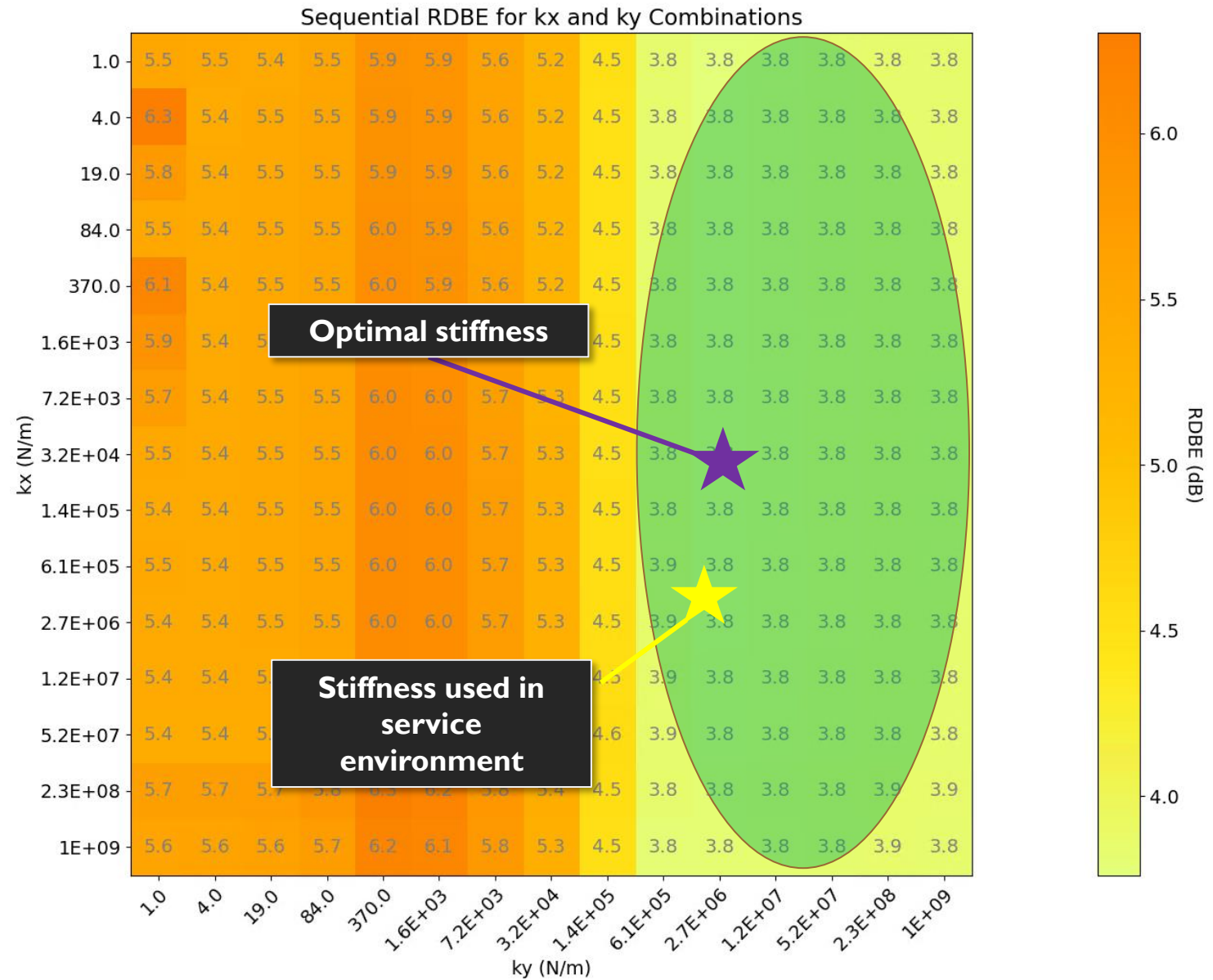
CASE STUDY 2

- The sequential test's stiffness optimization found the X-stiffness had little impact while the Y-stiffness needed to be above $1.4 \times 10^5 \text{ N/m}$ to produce good error.



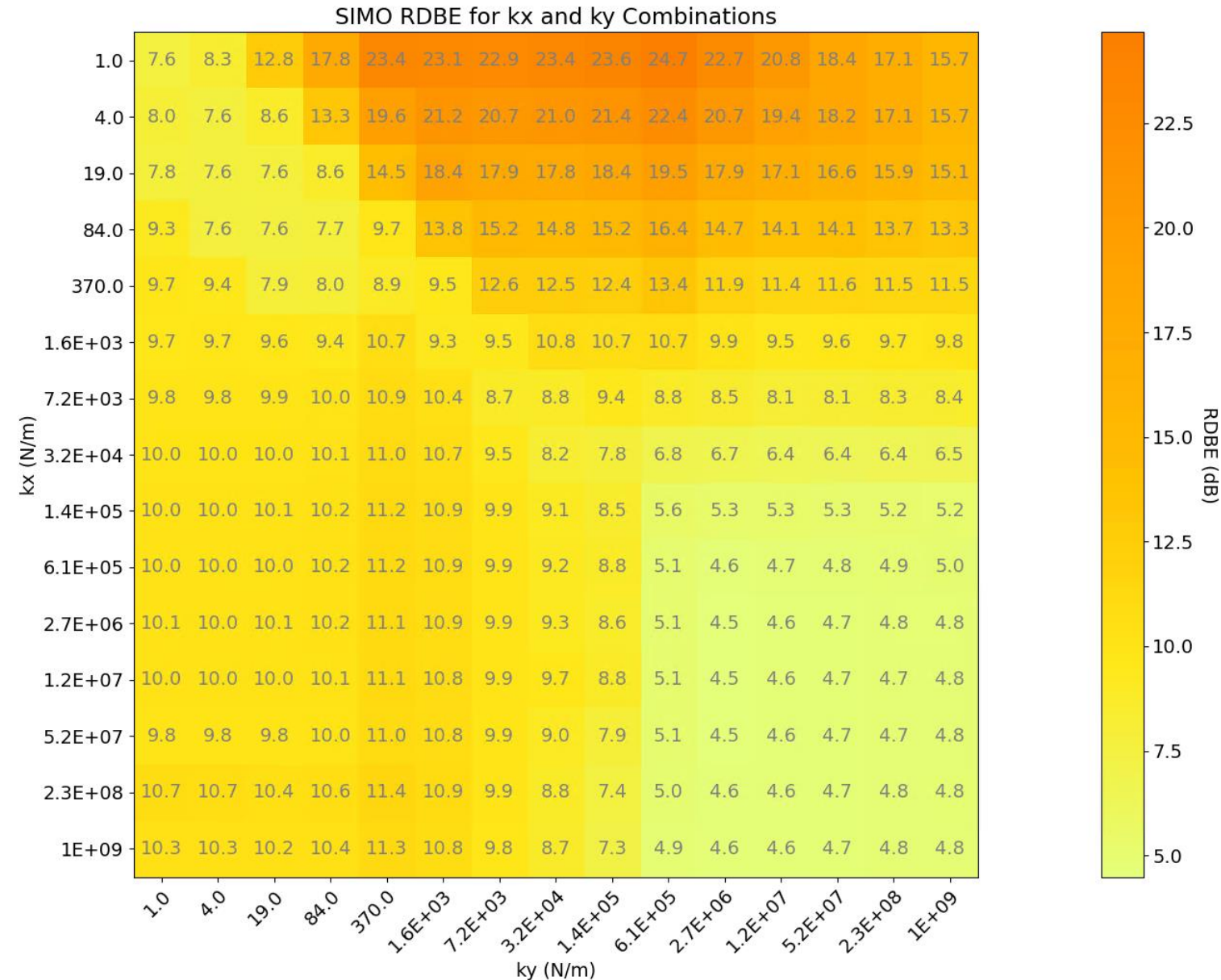
CASE STUDY 2

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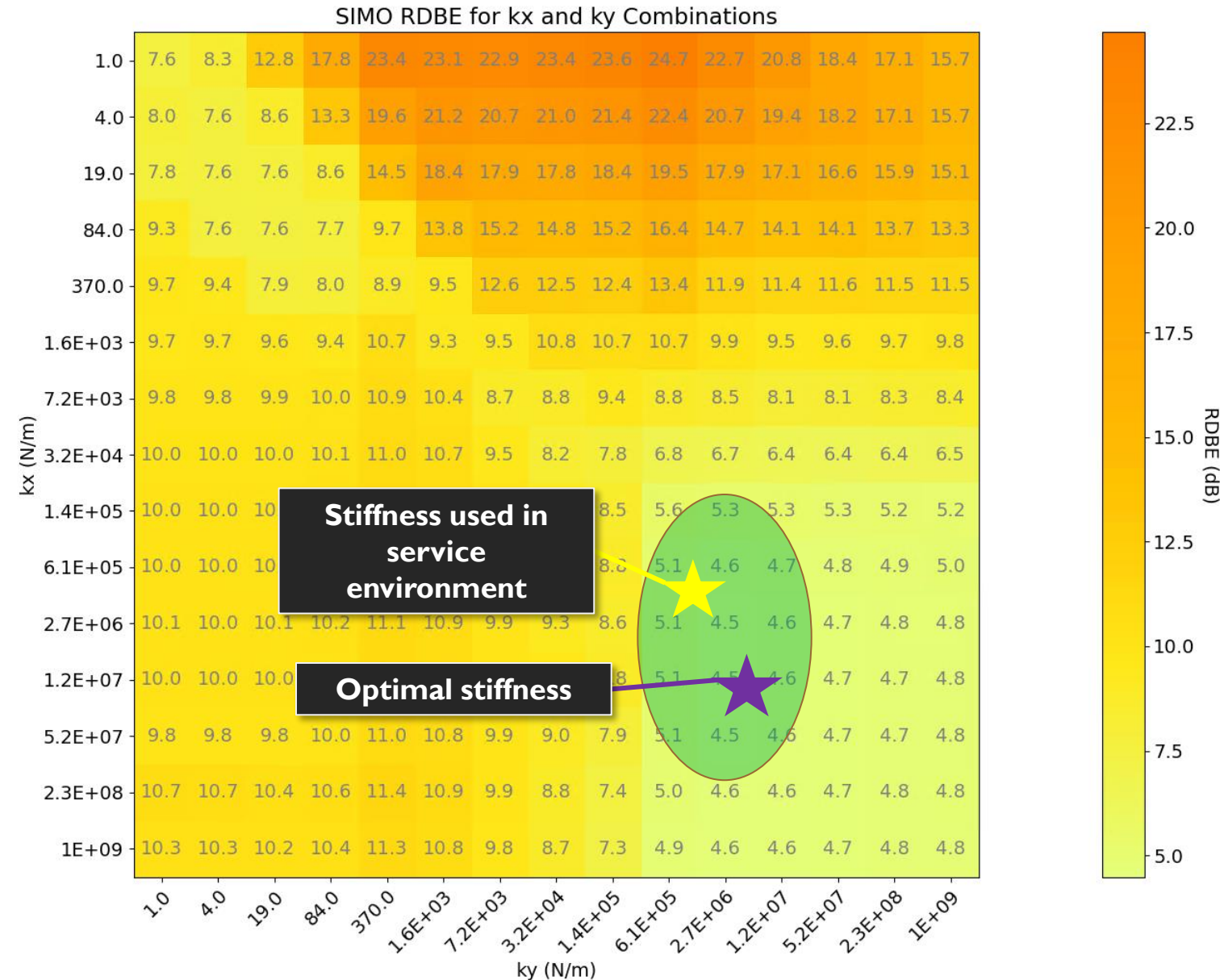
CASE STUDY 2

- The SIMO solution space was smooth with a clear minimum region, but not fully convex.
- The minimum region provides good values to start a local optimization.



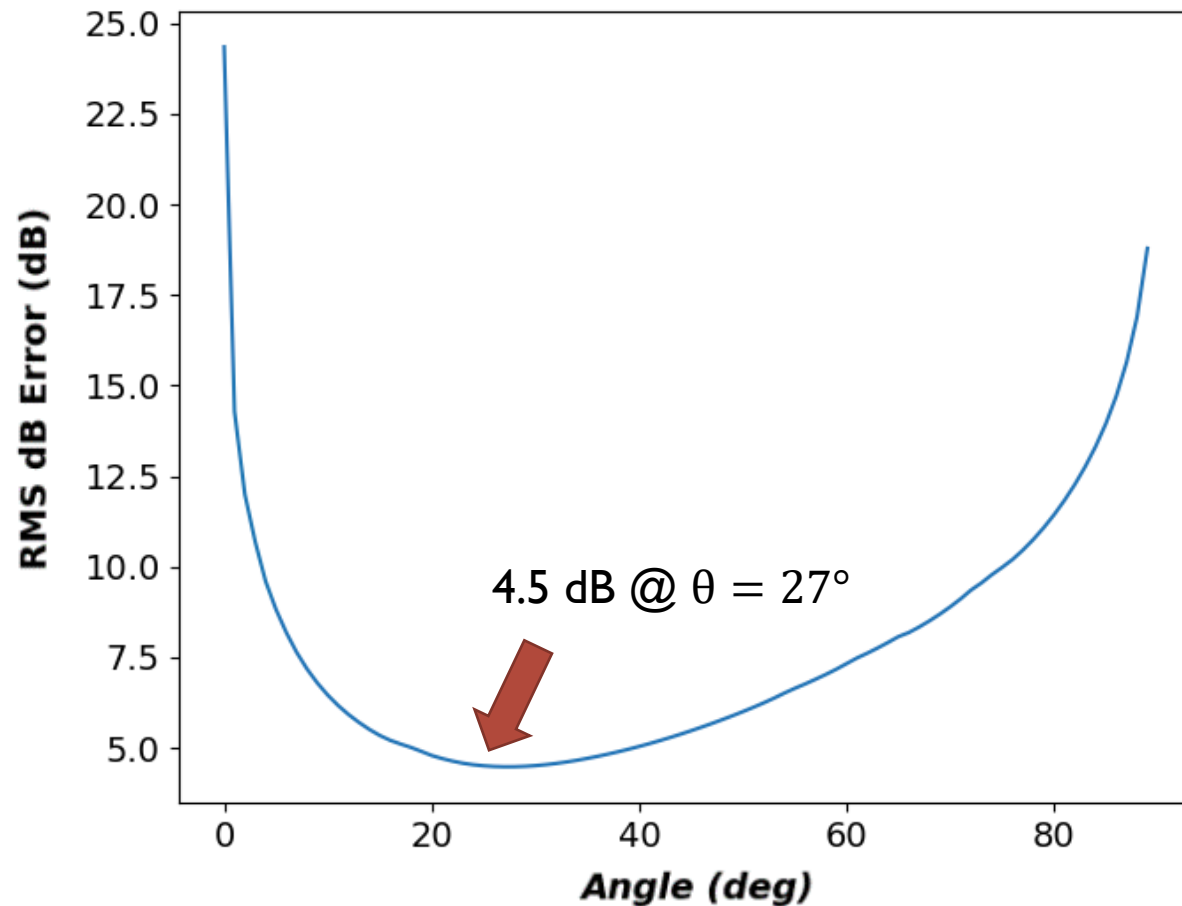
CASE STUDY 2

- The SIMO solution space was smooth with a clear minimum region.
- The minimum region provides good values to start a local optimization.



CASE STUDY 2

Angle vs RMS dB Error



- The SIMO test's angle optimization has a clear minimum.

The solution found for each test was:

1. For the sequential test, $k_{x1} = 3.2 * 10^4, k_{x2} = 1, k_{y1} = 2.7 * 10^6, k_{y2} = 2.7 * 10^6$ N/m.

2. For the SIMO test, $k_{x1} = 19, k_{x2} = 2.3 * 10^8, k_{y1} = 2.7 * 10^6, k_{y2} = 2.7 * 10^6$ N/m, $\theta = 27^\circ$.

	Mean Error	
	Sequential Single-Axis	SIMO Multi-Axis
RDBE	3.7 dB	4.5 dB
FTOL	80.0%	75.6%

The solution found for each test was:

1. For the sequential test, $k_{x1} = 1, k_{x2} = 1, k_{x3} = 1, k_{x4} = 1, k_{y1} = 1, k_{y2} = 10^9, k_{y3} = 10^9, k_{y4} = 1 \text{ N/m}.$

2. For the SIMO test, $k_{x1} = 10^9, k_{x2} = 3.2 * 10^4, k_{x3} = 2.7 * 10^6, k_{x4} = 2.7 * 10^6, k_{y1} = 2.7 * 10^6, k_{y2} = 2.7 * 10^6, k_{y3} = 10^9, k_{y4} = 2.7 * 10^6 \text{ N/m}, \theta = 27^\circ.$

	Mean Error	
	Sequential Single-Axis	SIMO Multi-Axis
RDBE	3.7 dB	4.5 dB
FTOL	79.6%	75.3%

DISCUSSION

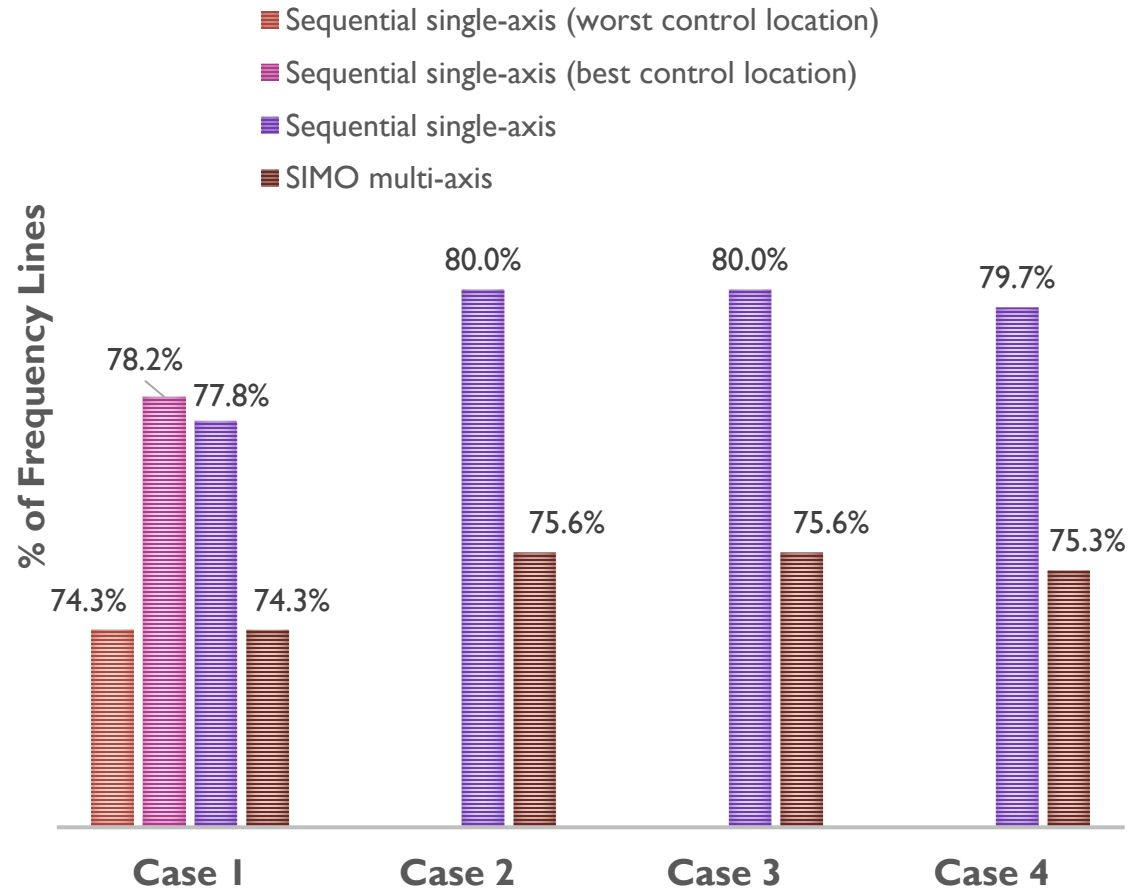
Key findings

- What did we learn from the results of each case study?

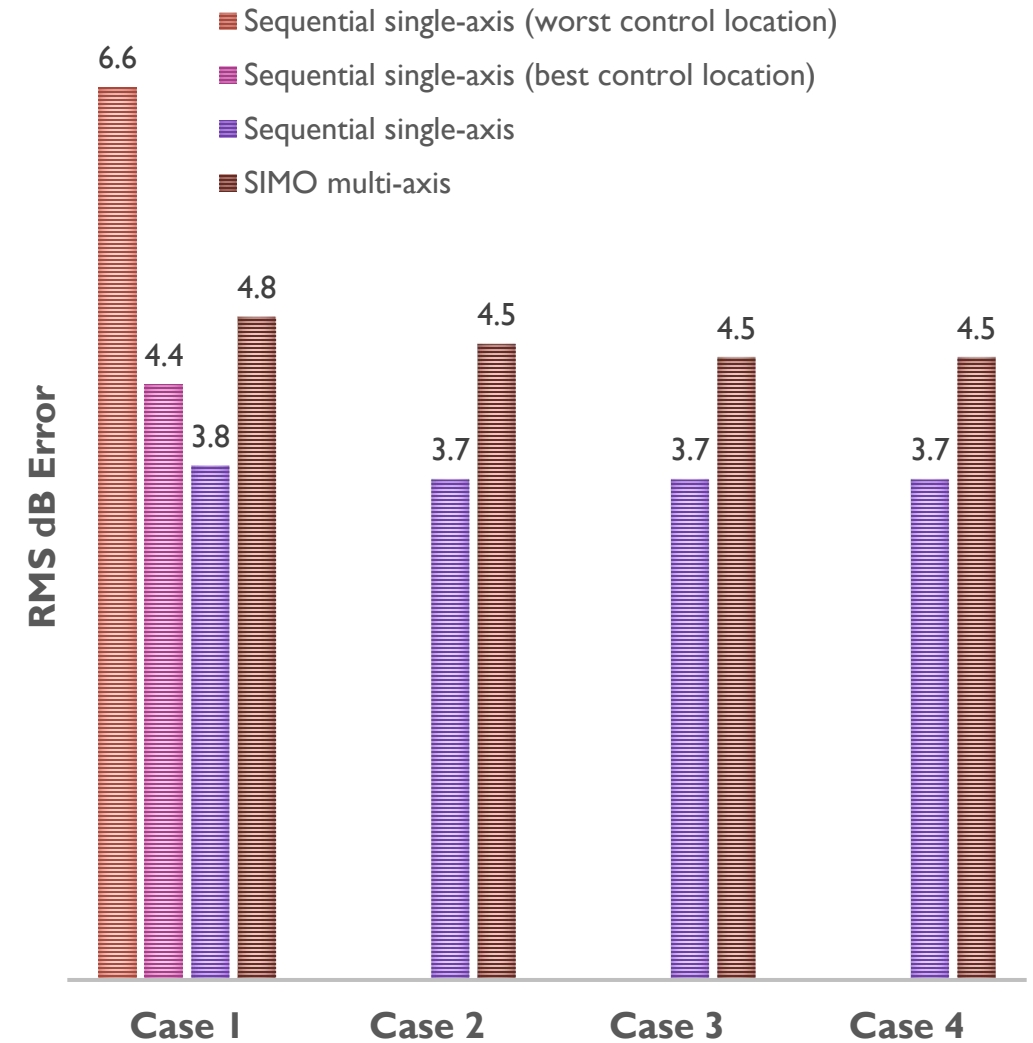
Objectives

- Were the originally stated outcomes achieved?

KEY FINDINGS

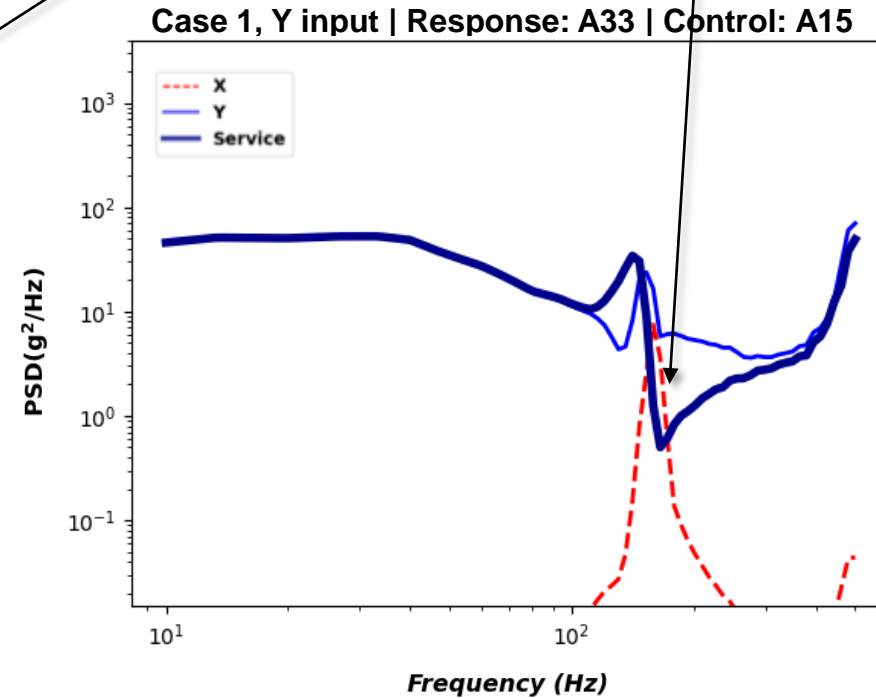
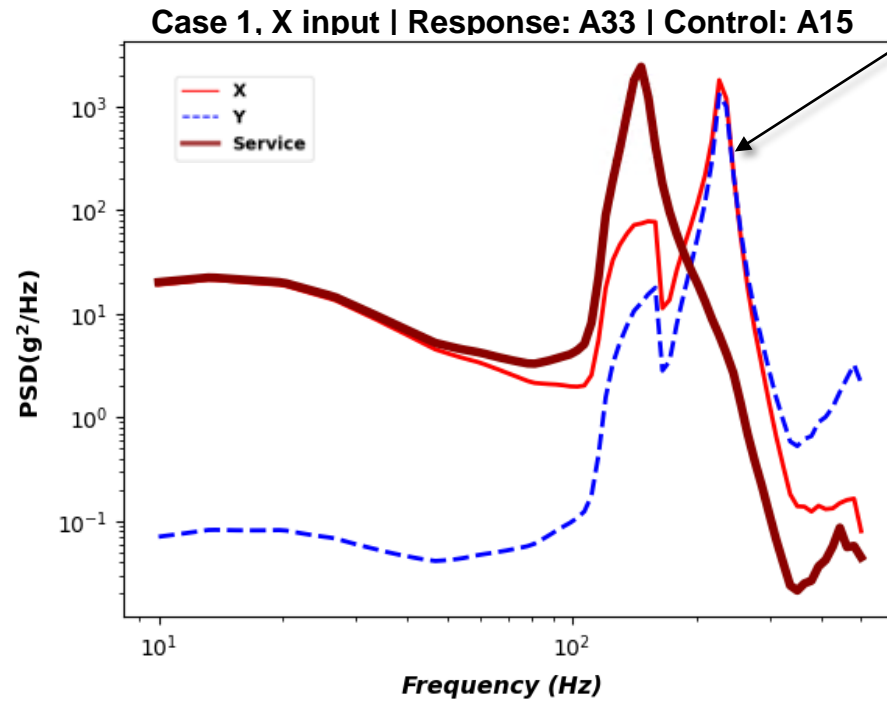
% OF FREQUENCY LINES WITHIN
A 3DB TOLERANCE BY CASE

RMS DB ERROR BY CASE



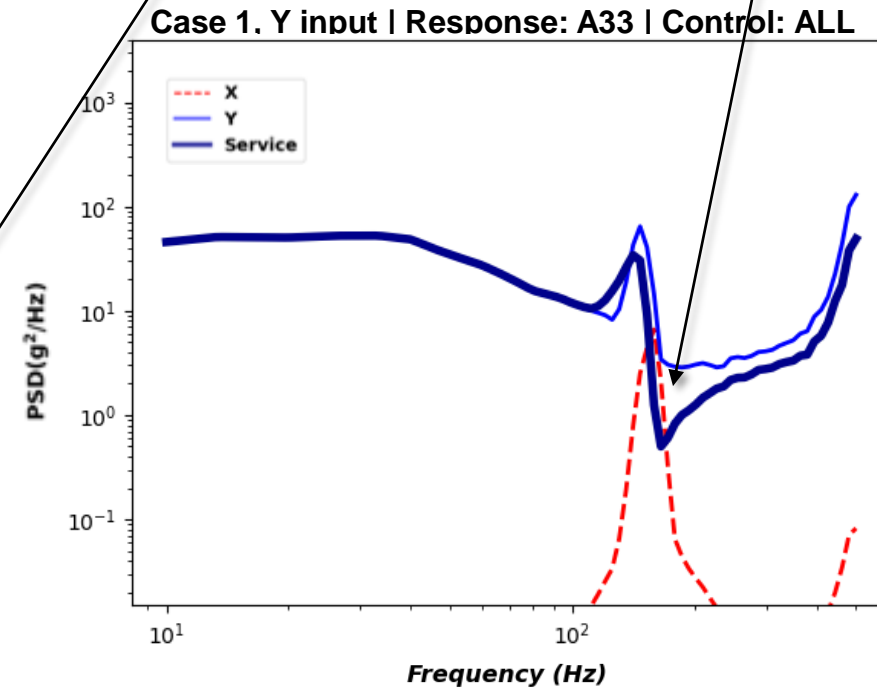
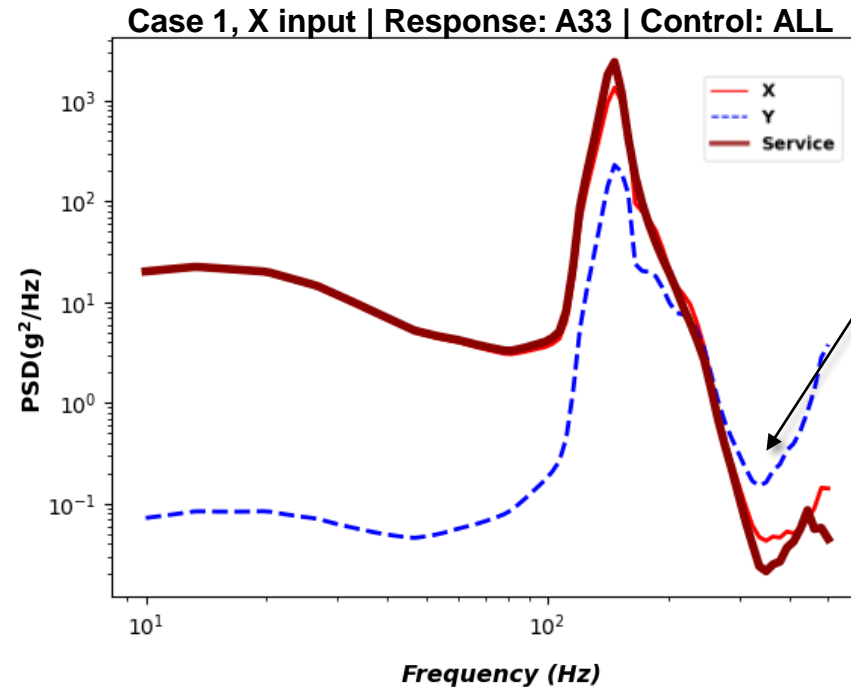
KEY FINDINGS

Off-axis response occasionally exceeds both the X and Y targets.

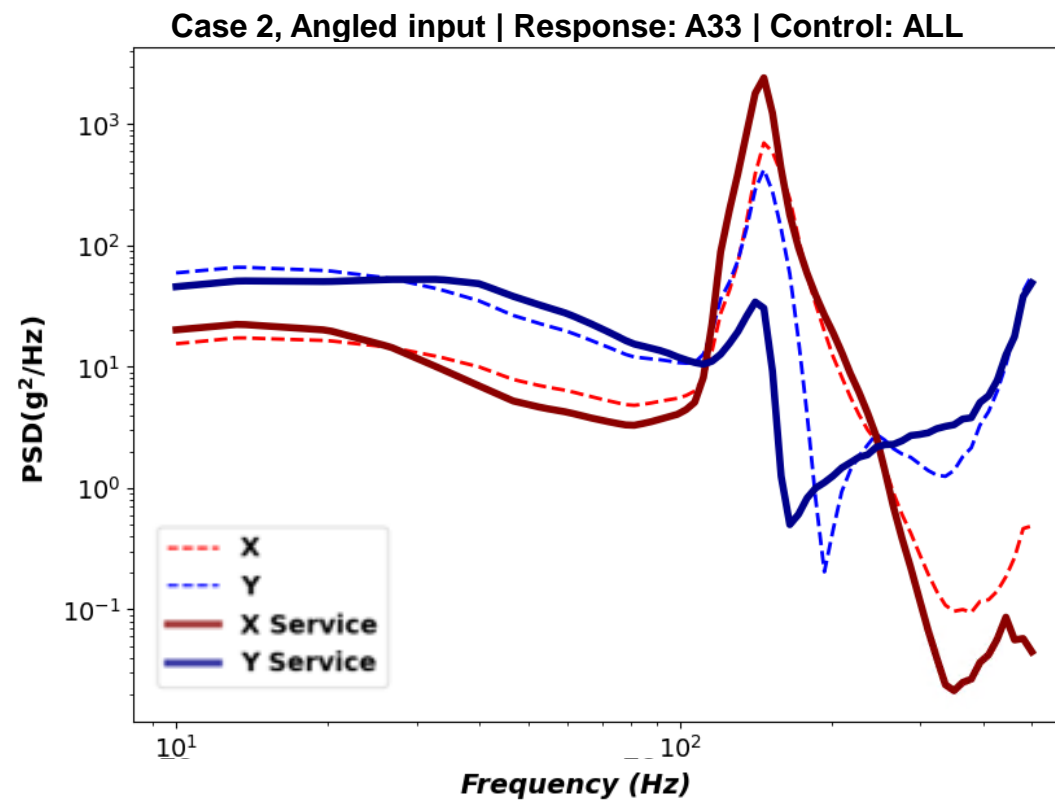


KEY FINDINGS

Off-axis response occasionally exceeds both the X and Y targets.



KEY FINDINGS



OBJECTIVES

The objectives of the research are as follows:

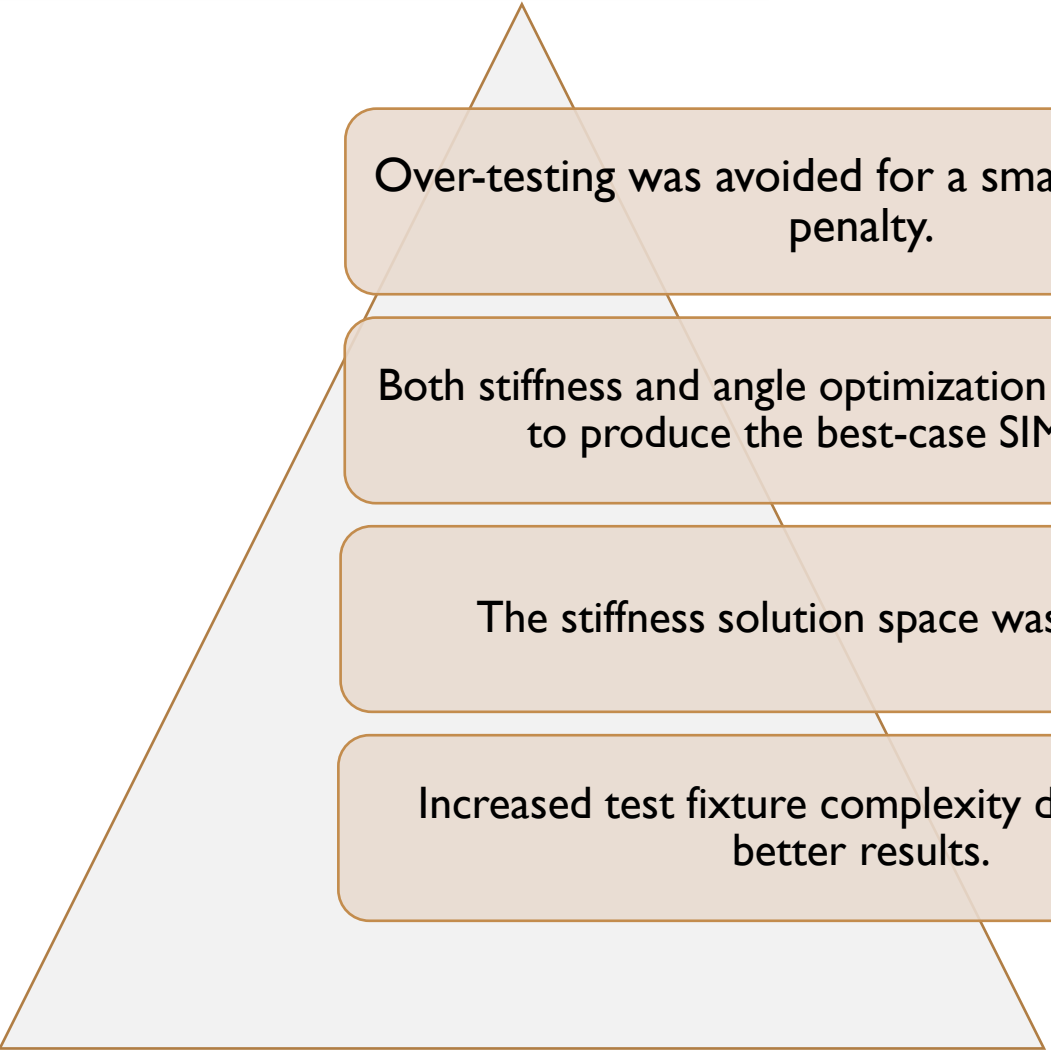
- i. To assess the quality of the proposed method.
 - The proposed method allowed a single test to approximate a multi-axis test with on-axis errors only slightly worse than sequential testing.
- ii. To understand how much test quality improvement is possible with a well-designed test fixture.
 - A dynamically optimized test fixture improved both the sequential test and SIMO test.
- iii. To determine the effect of increasing the number of test fixture optimization parameters.
 - For this specific model, target, and optimization strategy, more stiffness parameters showed no improvement beyond the initial optimization.

CONCLUSIONS

Summary

Future work

SUMMARY



Over-testing was avoided for a small on-axis error penalty.

Both stiffness and angle optimization were necessary to produce the best-case SIMO test.

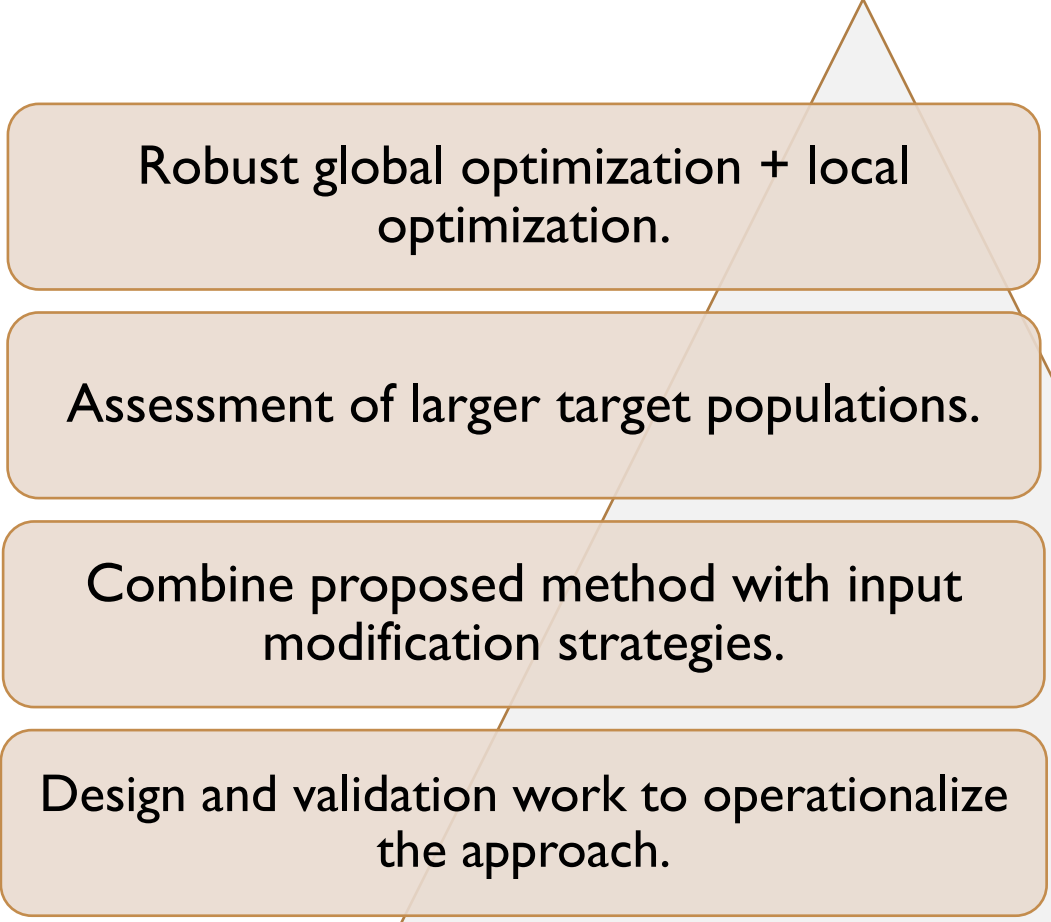
The stiffness solution space was tractable.

Increased test fixture complexity did not lead to better results.

- A fully validated method would enable wide access to rapid, approximate multi-axis vibration testing and eliminate the over-conservatism of sequential single-axis testing.

FUTURE WORK

- The proposed method produced promising results, but more work is needed to fully validate the method...



Robust global optimization + local optimization.

Assessment of larger target populations.

Combine proposed method with input modification strategies.

Design and validation work to operationalize the approach.

The background of the slide is a dark, textured surface composed of numerous thin, vertical lines of varying heights and colors, including shades of blue, teal, and gold, creating a dynamic, rain-like or digital data effect.

REFERENCES

K. Ahlin, “Comparison of Test Specifications and Measured Field Data,” *SOUND AND VIBRATION*, p. 3, 2006.

M. P. Bendsøe and O. Sigmund, “Material interpolation schemes in topology optimization,” *Archive of Applied Mechanics (Ingenieur Archive)*, vol. 69, no. 9–10, pp. 635–654, Nov. 1999, doi: 10.1007/s004190050248.

Bouma, A. Campbell, T. Roberts, S. Taylor, C. Haynes, and D. Harvey, “Accumulated Lifetimes in Single-Axis Vibration Testing,” in *Sensors and Instrumentation, Aircraft/Aerospace, Energy Harvesting & Dynamic Environments Testing, Volume 7*, vol. 7, 2019.

Chandrasekhar and K. Suresh, “TOuNN: Topology Optimization using Neural Networks,” *Struct Multidisc Optim*, vol. 63, no. 3, pp. 1135–1149, Mar. 2021, doi: 10.1007/s00158-020-02748-4.

P. M. Daborn, “Replicating Aerodynamic Excitation in the Laboratory,” in *Topics in Modal Analysis II, Volume 7: Proceedings of the 31st IMAC, A Conference and Exposition on Structural Dynamics*, 2014.

P. M. Daborn, C. Roberts, D. J. Ewins, and P. R. Ind, “Next-Generation Random Vibration Tests,” in *Topics in Modal Analysis II, Volume 8: Proceedings of the 32nd IMAC, A Conference and Exposition on Structural Dynamics*, 2014, pp. 397–410.

W. de Silva, “Vibration: Fundamentals and Practice Chapter 10,” in *Vibration: Fundamentals and Practice*, 2nd ed., 2006.

R. M. French, R. Handy, and H. L. Cooper, “A COMPARISON OF SIMULTANEOUS AND SEQUENTIAL SINGLE-AXIS DURABILITY TESTING,” *Exp Techniques*, vol. 30, no. 5, pp. 32–37, Sep. 2006, doi: 10.1111/j.1747-1567.2006.00083.x.

J. Gatscher, “Comparison of Mechanical Impedance Methods for Vibration Simulation,” 1996.

F. Gomez and B. F. Spencer, “Topology optimization framework for structures subjected to stationary stochastic dynamic loads,” *Struct Multidisc Optim*, vol. 59, no. 3, pp. 813–833, Mar. 2019, doi: 10.1007/s00158-018-2103-3.

Gregory, A. Nm, and D. O. Smallwood, "Comparison of the Response of a Simple Structure to Single Axis and Multiple Axis Random Vibration Inputs," p. 9.

T. M. Hall, "Analytically Investigating Impedance-Matching Test Fixtures," in *Sensors and Instrumentation, Aircraft/Aerospace, Energy Harvesting & Dynamic Environments Testing, Volume 7*, vol. 7, Society of Experimental Mechanics, 2020, pp. 21–31.

J. M. Harvie, "Using Modal Substructuring to Improve Shock & Vibration Qualification," in *Topics in Modal Analysis & Testing, Volume 9*, M. Mains and B. J. Dilworth, Eds. Cham: Springer International Publishing, 2017, pp. 227–239. doi: 10.1007/978-3-319-74700-2_24.

J. M. Harvie and M. van der Seijs, "Application of Transfer Path Analysis Techniques to the Boundary Condition Challenge Problem," in *Sensors and Instrumentation, Aircraft/Aerospace, Energy Harvesting & Dynamic Environments Testing, Volume 7*, C. Walber, P. Walter, and S. Seidlitz, Eds. Cham: Springer International Publishing, 2020, pp. 157–166. doi: 10.1007/978-3-030-12676-6_15.

S. Hoyer, J. Sohl-Dickstein, and S. Greydanus, "Neural reparameterization improves structural optimization," arXiv:1909.04240 [cs, stat], Sep. 2019, Accessed: Nov. 10, 2021. [Online]. Available: <http://arxiv.org/abs/1909.04240>

R. Jones, D. Soine, J. Harvie, T. Schoenherr, T. Skousen, and M. Starr, "Boundary Conditions in Environmental Testing Round Robin," p. 53, 2018.

A. J. Knight, "More Representative Spacecraft Random Vibration Testing," p. 158, 2019.

C. Knight, M. Remedios, G. S. Aglietti, and G. Richardson, "Satellite Vibration Testing: Angle optimisation method to Reduce Overtesting," *Acta Astronautica*, vol. 147, pp. 205–218, Jun. 2018, doi: 10.1016/j.actaastro.2018.04.004.

R. Kolaini, W. Tsuha, and J. P. Fernandez, "Spacecraft vibration testing: Benefits and potential issues," *Advances in aircraft and spacecraft science*, vol. 5, no. 2, pp. 165–175, Mar. 2018, doi: 10.12989/AAS.2018.5.2.165.

Z. Nie, T. Lin, H. Jiang, and L. B. Kara, "TopologyGAN: Topology Optimization Using Generative Adversarial Networks Based on Physical Fields Over the Initial Domain," p. 13.

J. On, "Mechanical impedance analysis for lumped parameter multi-degree of freedom/multi-dimensional systems," p. 41.

M. G. Ostergaard, A. R. Ibbotson, O. L. Roux, and A. M. Prior, "Virtual testing of aircraft structures," *CEAS Aeronaut J*, vol. 1, no. 1–4, pp. 83–103, Sep. 2011, doi: 10.1007/s13272-011-0004-x.

Z. Qiao, Z. Weihong, Z. Jihong, and G. Tong, "Layout optimization of multi-component structures under static loads and random excitations," *Engineering Structures*, vol. 43, pp. 120–128, Oct. 2012, doi: 10.1016/j.engstruct.2012.05.013.

S. Rawat, "A Novel Topology Optimization Approach using Conditional Deep Learning," p. 8.

M. Remedia, G. S. Aglietti, M. Appolloni, A. Cozzani, and A. Kiley, "An Enhanced Methodology for Spacecraft Correlation Activity Using Virtual Testing Tools," p. 41, 2017.

C. Roberts and D. Ewins, "Multi-axis vibration testing of an aerodynamically excited structure," *Journal of Vibration and Control*, vol. 24, no. 2, pp. 427–437, Jan. 2018, doi: 10.1177/1077546316642064.

D. P. Rohe. Modal data for the BARC challenge problem Test Report. United States: N. p., 2018. Web. doi:10.2172/1418738.

D. P. Rohe, R. A. Schultz, T. F. Schoenherr, T. J. Skousen, and R. J. Jones, "Comparison of Multi-Axis Testing of the BARC Structure with Varying Boundary Conditions," in *Sensors and Instrumentation, Aircraft/Aerospace, Energy Harvesting & Dynamic Environments Testing, Volume 7*, C. Walber, P. Walter, and S. Seidlitz, Eds. Cham: Springer International Publishing, 2020, pp. 179–193. doi: 10.1007/978-3-030-12676-6_17.

T. D. Scharton, "Vibration and Acoustic Testing of Spacecraft," p. 5, 2002.

T. D. Scharton, “DEVELOPMENT OF IMPEDANCE SIMULATION FIXTURES FOR SPACECRAFT VIBRATION TESTS,” p. 64.

T. F. Schoenherr, “Designing an Impedance Matched Test Fixture Using Parameterized Optimization and the Modal Projection Error,” p. 25.

T. F. Schoenherr, “Derivation of Six Degree of Freedom Shaker Inputs Using Sub-structuring Techniques,” in *Topics in Modal Analysis & Testing*, Volume 9, M. Mains and B. J. Dilworth, Eds. Cham: Springer International Publishing, 2019, pp. 5–14. doi: 10.1007/978-3-319-74700-2_2.

T. F. Schoenherr, P. Coffin, and B. Clark, “Use of Topology Optimization to Design Shock and Vibration Test Fixtures,” in *Sensors and Instrumentation, Aircraft/Aerospace, Energy Harvesting & Dynamic Environments Testing*, Volume 7, C. Walber, P. Walter, and S. Seidlitz, Eds. Cham: Springer International Publishing, 2020, pp. 77–92. doi: 10.1007/978-3-030-12676-6_8.

D. O. Smallwood, “The challenges of multiple input vibration testing and analysis,” p. 46, 2013.

D. E. Soine, R. J. Jones, J. M. Harvie, T. J. Skousen, and T. F. Schoenherr, “Designing Hardware for the Boundary Condition Round Robin Challenge,” p. 4.

B. H. V. Topping, Ed., *Optimization and Artificial Intelligence in Civil and Structural Engineering*. Dordrecht: Springer Netherlands, 1992. doi: 10.1007/978-94-017-2490-6.

P. S. Varoto and L. P. R. de Oliveira, “Interaction Between a Vibration Exciter and the Structure Under Test,” *SOUND AND VIBRATION*, p. 6, 2002.

S. Waimer, S. Manzato, B. Peeters, M. Wagner, and P. Guillaume, “Modelling and simulation of a closed-loop electrodynamic shaker and test structure model for spacecraft vibration testing,” *Advances in aircraft and spacecraft science*, vol. 5, no. 2, pp. 205–223, Mar. 2018, doi: 10.12989/AAS.2018.5.2.205.

Wein, P. D. Dunning, and J.A. Norato, “A review on feature-mapping methods for structural optimization,” *Struct Multidisc Optim*, vol. 62, no. 4, pp. 1597–1638, Oct. 2020, doi: 10.1007/s00158-020-02649-6.

Y. Yang, M. Zhu, M. D. Shields, and J. K. Guest, “Topology optimization of continuum structures subjected to filtered white noise stochastic excitations,” *Computer Methods in Applied Mechanics and Engineering*, vol. 324, pp. 438–456, Sep. 2017, doi: 10.1016/j.cma.2017.06.015.

“MIL-STD-810H, Department of Defense Test Method Standard: Environmental Engineering Considerations and Laboratory Tests.” Defense Logistics Agency, United States. 31 Jan 2019.

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