DOE/ER/14100-2

Hydrodynamic Instabilities and Coherent Strustures

Progress Report

January 1, 1995 - December 31, 1995

Alexander Frenkel, Principal Investigator

Department of Mathematics

University of Alabama

Tuscaloosa, Alabama 35487-0350

August 1995

Prepared for the U.S. department of Energy

Under Grant Number DE-FG05-90ER14100

During the project year under consideration, the following progress has been made:

1. We continued to study three-dimensional waves on films flowing down solid surfaces which are governed by an evolution equation we derived earlier (see the previous progress report). Our results were reported recently by the principal investigator in an invited lecture at the Joint AMS-IMS-SIAM Summer Research Conference on Multi-Fluid Flows and Interfacial Instabilities (Seattle, July 1995), and are described in an appended paper entitled "Spatiotemporal Patterns in 3-D film flows" (referred below as I).

The most unexpected result during the last year was our discovery that a new type of spatiotemporal patterns can exist on attractors of driven non-equilibrium dissipative dynamical systems. Although pattern formation in such systems has been a topic of active research for a considerable time now, the patterns under consideration were always, at least locally, almost periodic in space and stationary in an appropriate reference frame. Our numerical simulations have shown that under conditions corresponding to large values of the dispersion coefficient λ of our equation, the system starting from a random infinitesimal disturbance of the film surface approaches an attractor on which the surface pattern is highly ordered, but consists of two *subpatterns,* each of these moving as a whole with a different velocity. Thus, the overall pattern cannot be stationary in any reference frame. Also, one of the subpatterns—and hence the overall pattern as well—is definitely far from being periodic. Also, this highly ordered dynamical pattern nevertheless exhibits small-amplitude chaotic fluctuations, so the attractor should be a strange one. The appended paper entitled "Complex spatiotemporal patterns on dynamical attractors" has been submitted to *Physical Review Letters* (at the end of July 1995).

2. We have analytically found 2-D travelling-wave solutions of our equation. They are made of one fundamental Fourier harmonic which is unstable as a disturbance to a uniform basic state, and one or two stable overtones with smaller and smaller amplitudes. As was anticipated in our proposal of 1993 (see II-2 of Summary there), we were able to investigate the secondary instabilities of such equilibria to infinitesimal 3-D disturbances by the methods we used in the framework of the present project to study stability of driven space-time-periodic flows.

Similar work done by Chang and his collaborators at the University of Notre-Dame for the particular case of a vertical film (which corresponds to the value zero of the other parameter—κ— of our equation) showed that the equilibrium wave is always unstable to 3-D disturbances. We have however found that a certain window of stable wavelengths appears for *nonzero* κ. It grows with κ and reaches its maximum extent at $\kappa \approx 1.55$. Our numerical simulations confirmed the existence of traveling-wave attractors with such a set of wavelengths (see I for more detail). The linear-theory growth rates of disturbances for traveling waves which are outside of the stability island give satisfactory estimates of life times of quasi-stationary states. However, they fail to predict the character of patterns which emerge as a result of the secondary instabilities, because the nonlinearity quickly brings to prominence modes which are insignificant in the linear-theory stage of the disturbance growth.

3. In accordance with II-1 of the Summary, we have refined our multi-parametric perturbation approach in two directions. First, we obtain now, along with the evolution equation and explicit expressions for velocities and pressures in terms of film thickness, the conditions of both local (in time) and global validity of the theory (see I for more detail). Second, the procedure was made more algorithmic, and we can claim that now it yields the most general of the possible leading-order evolution equations.

4. We have completed the work planned in the proposal (see II-5 of the Summary), the derivation of the next-order correction to our Benney-type equation for a film falling down a vertical cylinder (for the leading-order equation, see A.

Frenkel, "On evolution equations for thin films flowing down solid surfaces", *Phys. Fluids A* 5 (1993) 2342). This manuscript is being prepared for publication.

5. The manuscript with G. Rudenko on instabilities of the oscillating triangular-eddy flow (see the preceding Report) is to be submitted for publication in the near future.

6. The work with A. Oron (Technion) on the Benney equation for falling planar films has led to some interesting results which need to be further sorted out. This work, as well as our joint investigation of the sheared and non-isothermal core-annular flow has been temporarily interrupted because of his sabbatical leave (to work with Dr. Bankoff at Northwestern).

7. We (with my Ph.D. student, Mr. X. Zhang) have obtained the critical conditions for onset of large-scale instability in generalized Kolmogorov flows, which are periodic, but not necessarily sinusoidal, in both space and time, as was planned in the Proposal (see I-2 of the Summary). Mr. Zhang is scheduled to have his defense in the middle of August 1995. Later, we will prepare this work for publication, as well as the other joint work, on the Landau-type equation of the nonlinear evolution of disturbances of a triangular-eddy flow, mentioned in the previous report; it has been reported last year in the annular APS-DFD meeting.

8. A detailed account of the work on large-amplitude waves in core-annular

flows previously reported by A. Frenkel and V. Kerchman in *Proceedings of the 14th IMACS World Congress*, Atlanta, 1994 (appended as paper IV) was published by V. Kerchman in the Journal of Fluid Mechanics (see appended paper V).

Plans for the next year

(a) We will continue to study the 3-D film waves governed by our evolution equation. In case of large dispersion, we will study the more realistic limit of a rectangular domain with a large streamwise dimension. Also, it would be interesting to implement simulations with a boundary excitation of disturbances rather than periodic boundary conditions, which would more directly correspond to the experiments.

(b) The work (with A. Oron) on the Benney evolution equation for falling planar films (II-1 Of the proposal Summary) and on sheared and nonisothermal core-annular flow in horizontal pipes (II-4 and III-5 of the proposal Summary) will be continued.

(c) We will prepare and submit for publication several papers with previously obtained results.

(d) As time permits, we will work on the cases of (i) a large-Reynolds-number core-annular flow (III-2 of the Summary), (ii) horizontal core-annular flow with the essential effects of gravity (III-3 of the Summary), and (iii) inclined core-annular flow (III-4 of the Summary of our 1993 proposal).

Alex Frenkel   Alexander Frenkel, Principal Investigator