

Updates to R-matrix Evaluations of Fissile Actinides: $^{233,235}\text{U}$, ^{239}Pu

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Introduction

Importance and Significance of Cross Section Fluctuations

- Criticality benchmark calculations are highly sensitive to cross section fluctuations (usually in the keV neutron energy region for fissile actinides)
 - Average resonance parameters and related probability tables are commonly used to generate cross section fluctuations
- Measured cross sections possess a rapidly varying resonant behavior depending on the incident neutron energy
 - At low energies, the resonance-like structure from the quasi-bound state of the compound nucleus can be measured fairly easily since the experimental resolution is higher than the spacing of the level states
 - However, as the energy increases, the number of levels is so large that only fluctuations related to very closely-spaced levels can be measured
- These data are generally evaluated with a relatively simple method (R-matrix theory)
- For resonance that cannot be resolved experimentally, the cross section fluctuations are usually described in terms of S-matrix elements calculated from average quantities obtained from R-matrix analyses

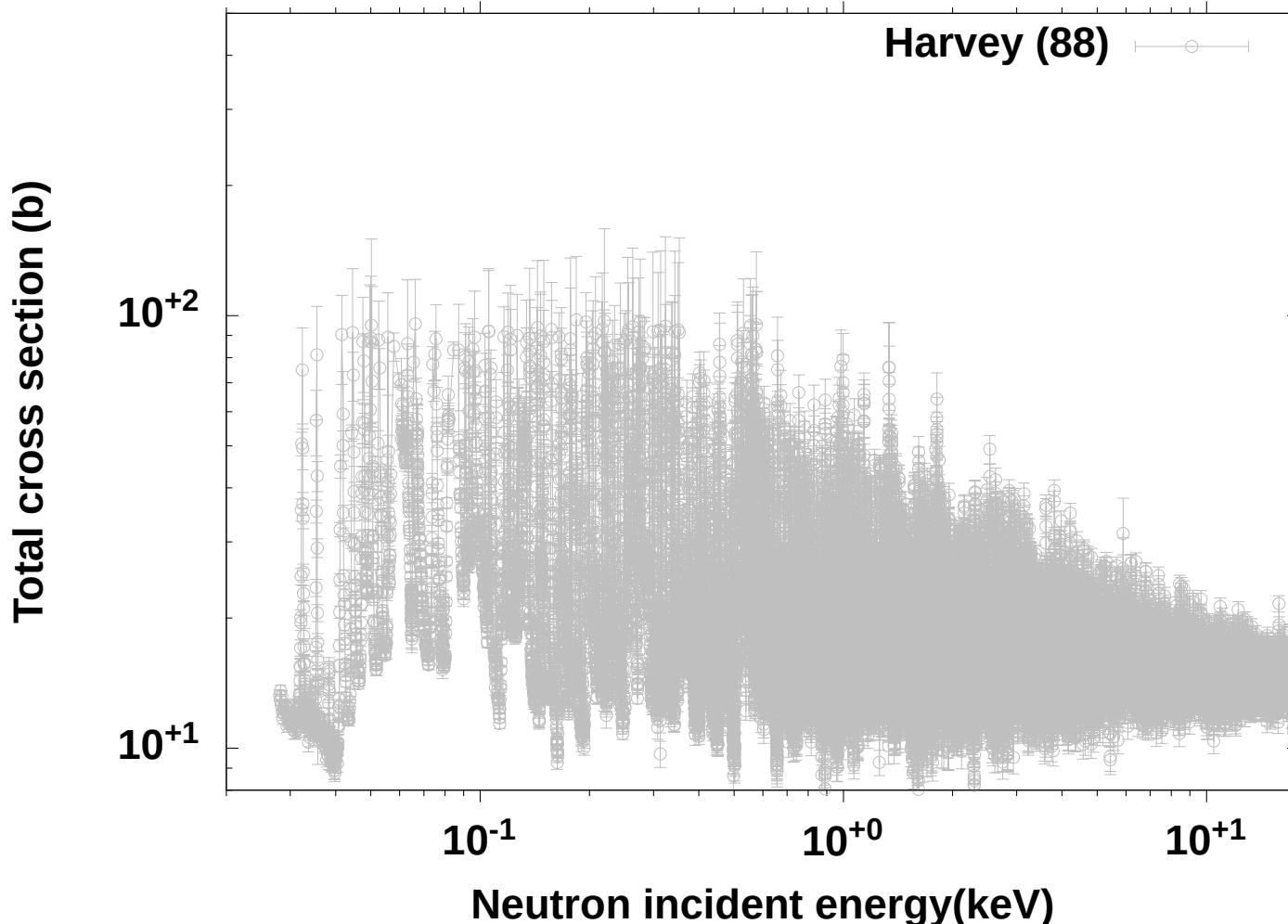
Motivation and Overarching Goals

To Establish the Highest Fidelity Nuclear Evaluated Data

- Evaluated data files do not entirely describe the cross section fluctuations available in measured data
 - This information could improve the performance in benchmark calculations
- High-fidelity description of measured data in neutron energy regions where fluctuations are important and relevant for applications
- Fluctuating cross sections evaluated within a consistent theoretical formalism
 - The energy averaged cross sections are defined in terms of the average resonance parameters simply because the Single Level Breit Wigner approximation is used
- Inclusion of fluctuating cross section aiming to limit the size of the evaluated data files as well as maintaining performance in benchmark calculations

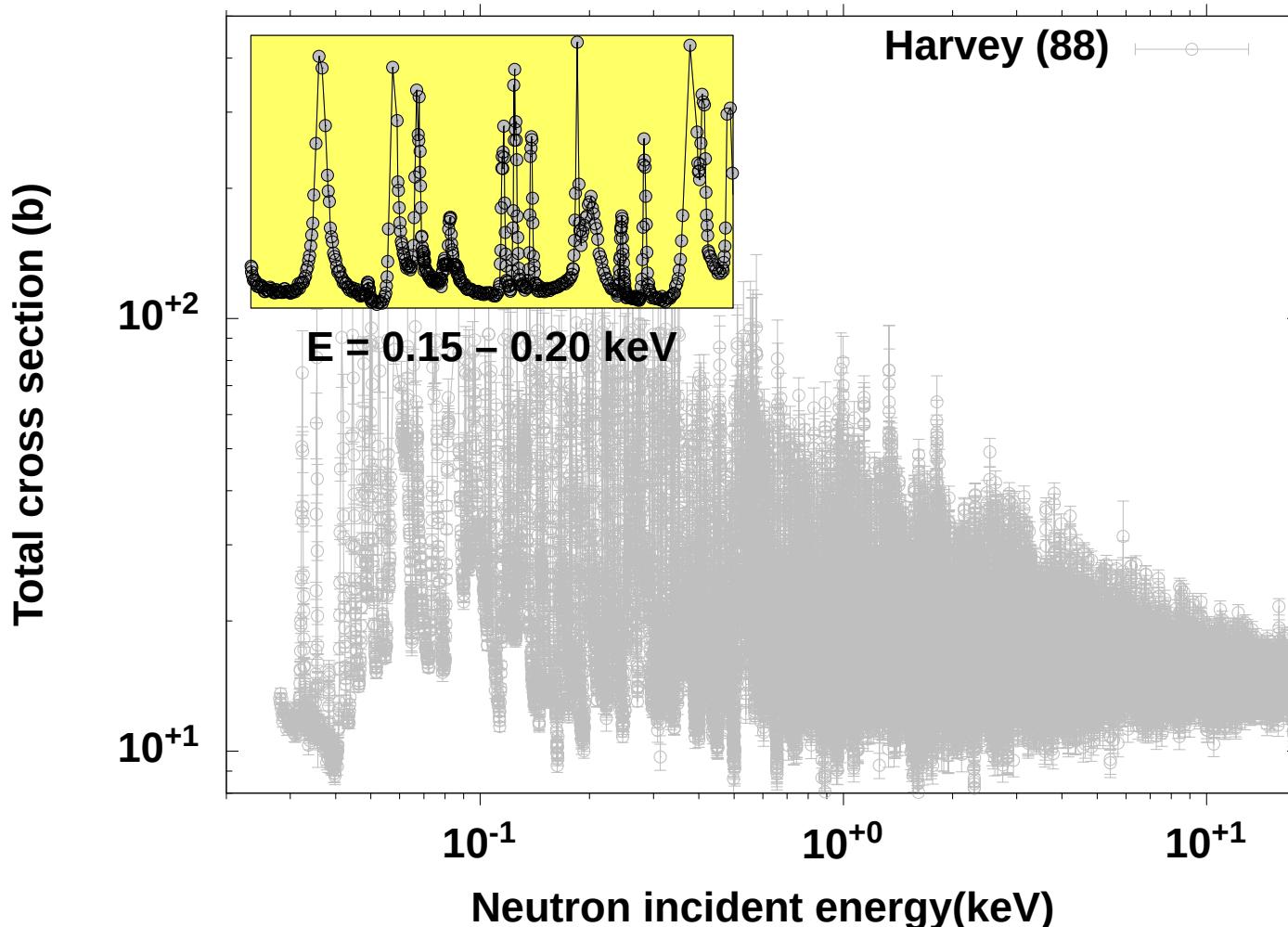
Experimental Data

Fluctuations in Measured Cross Section ($n+^{239}\text{Pu}$)



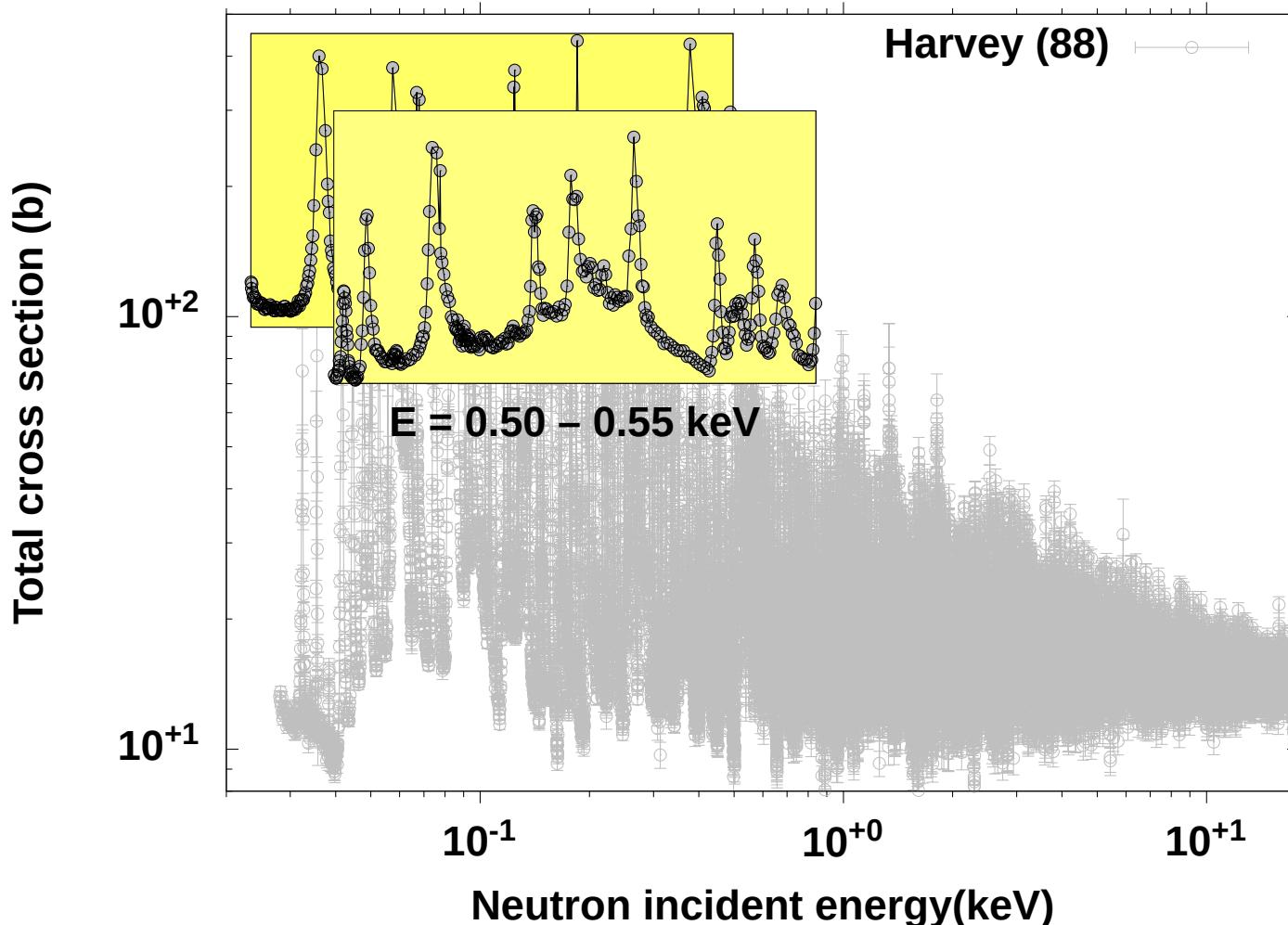
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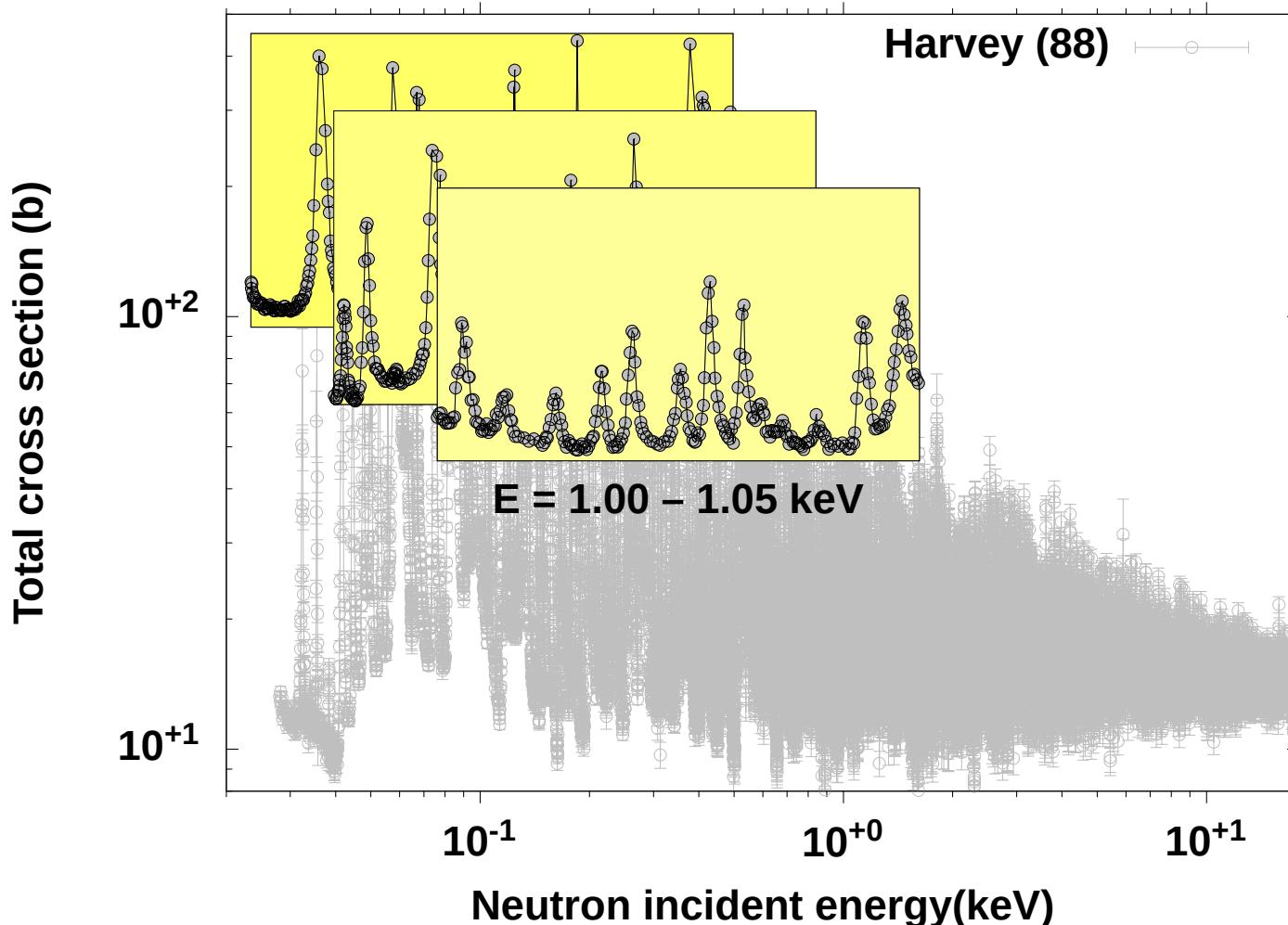
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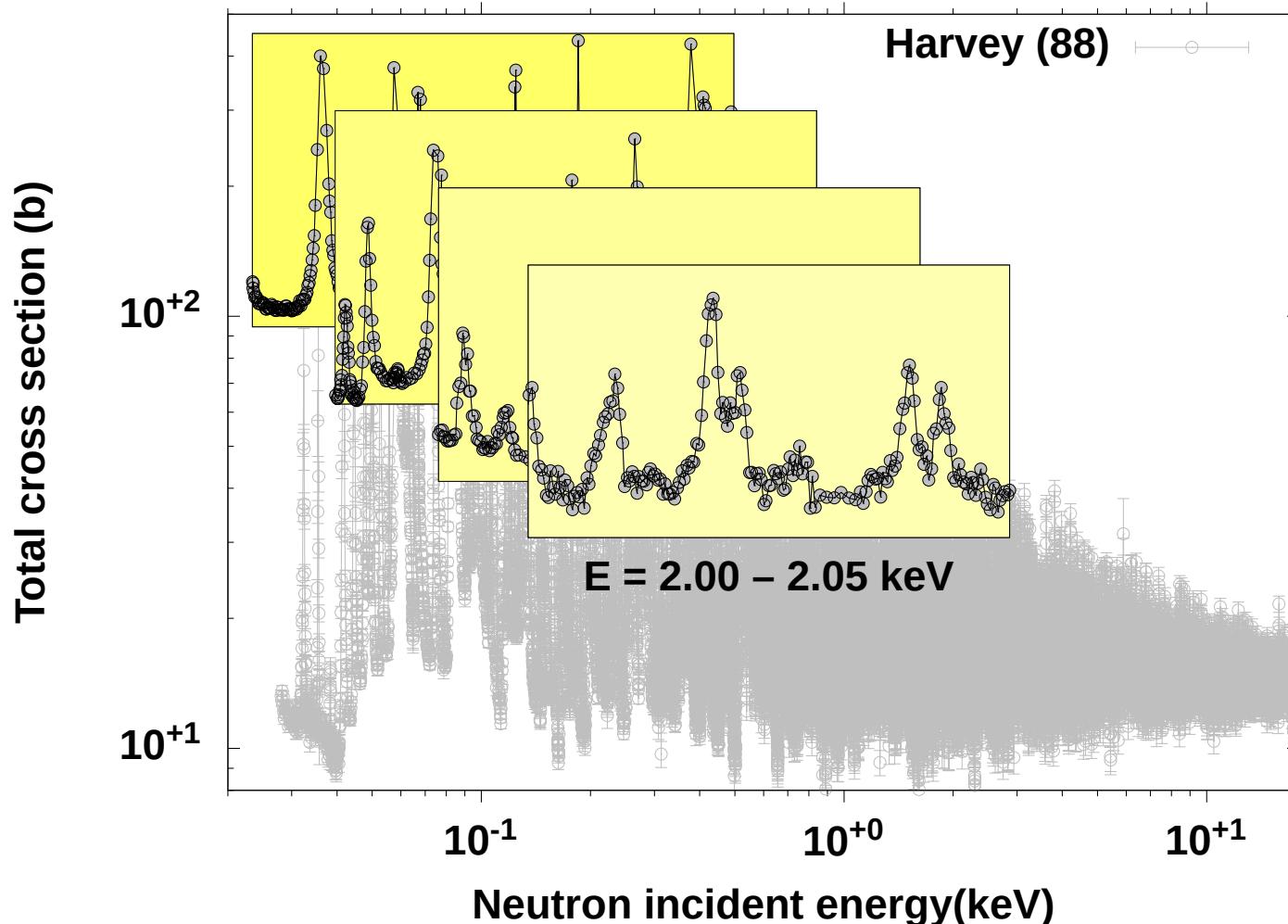
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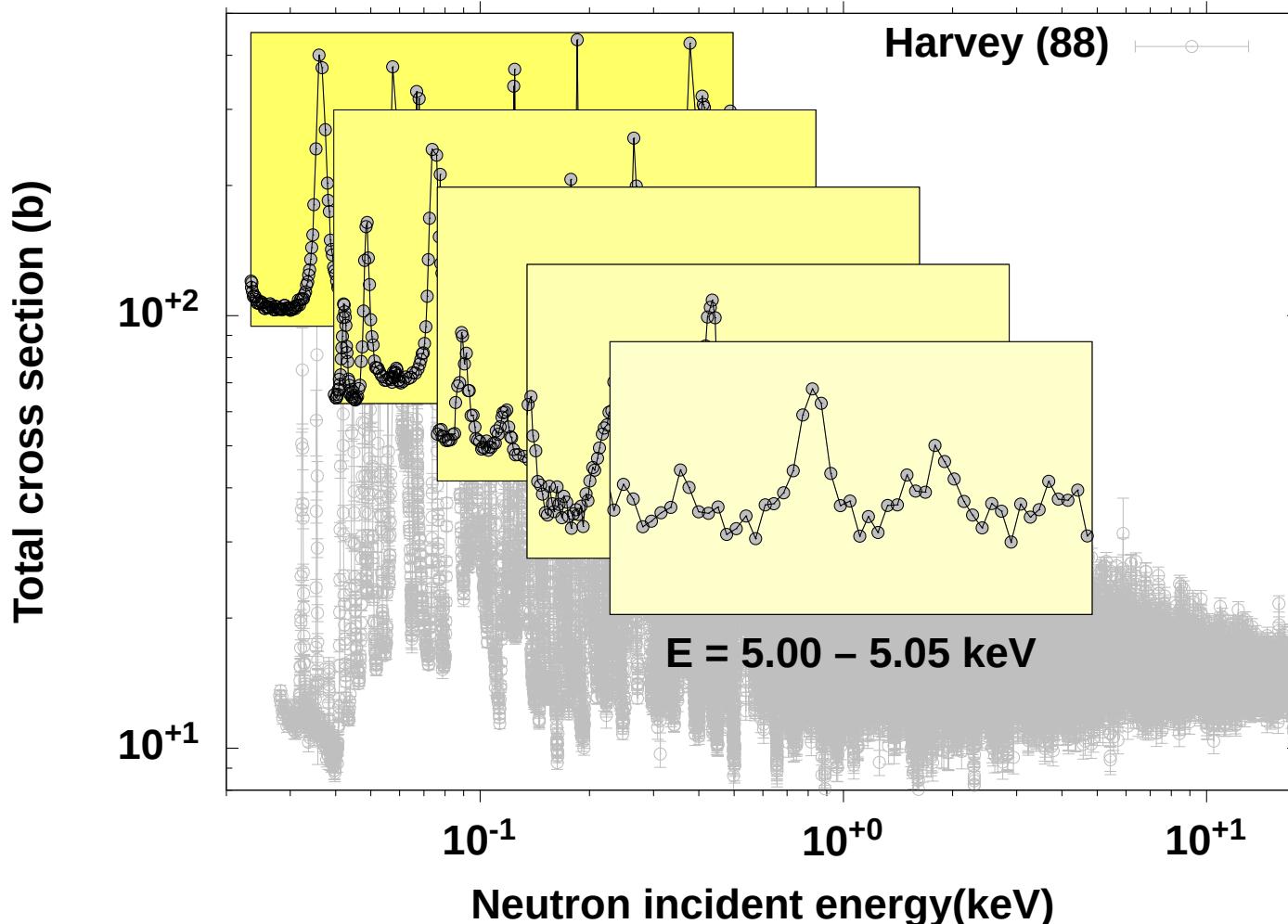
Experimental Data

Fluctuations in Measured Cross Section ($n+^{239}\text{Pu}$)



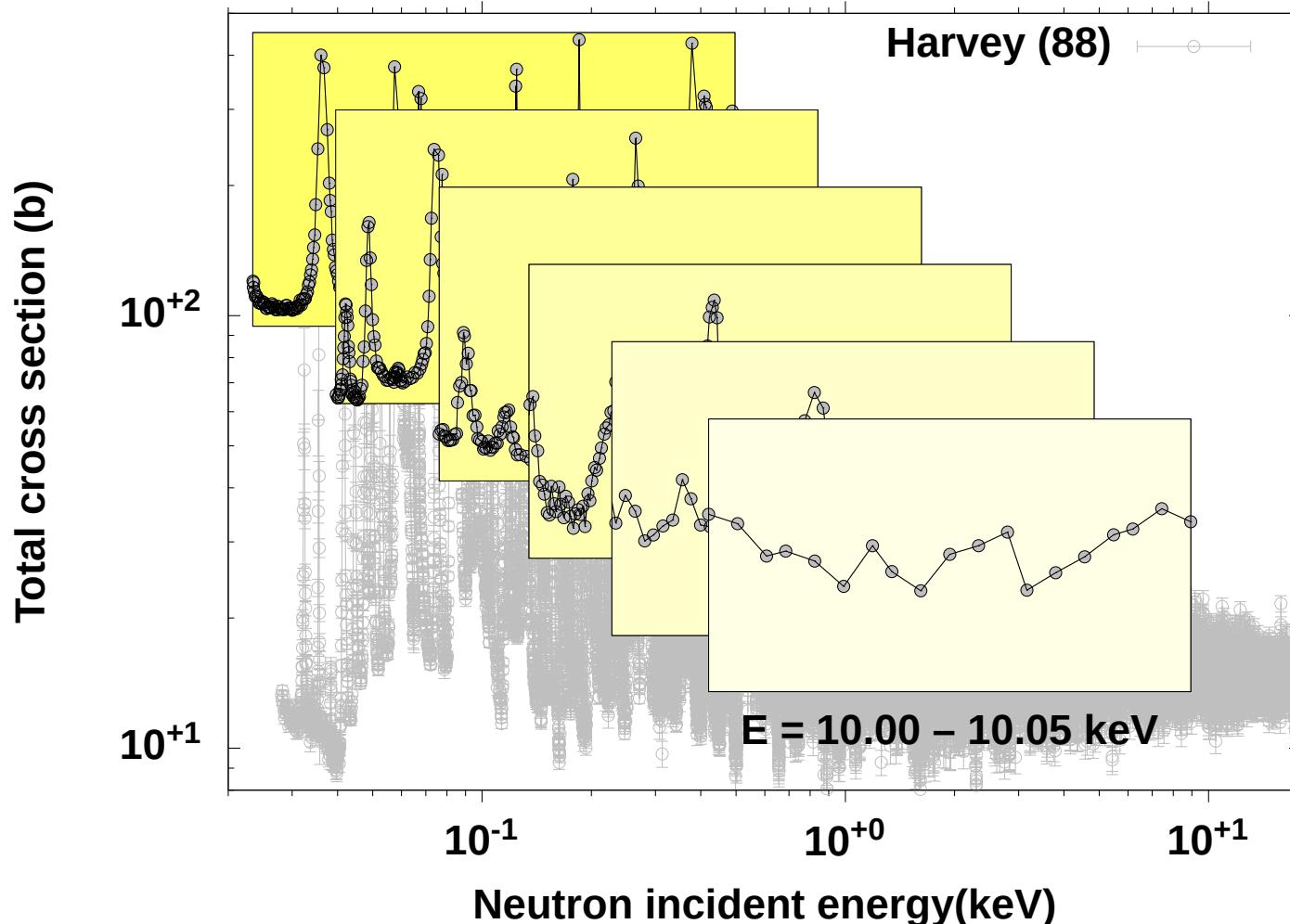
Experimental Data

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Experimental Data

Fluctuations in Measured Cross Section ($n+^{239}\text{Pu}$)



Theory: S-matrix Elements

Reich-Moore Representation of the Reduced R-matrix Function

All physical quantities defined by γ_λ (reduced-width amplitudes) and E_λ (resonance energies)

$$R_{cc'}(E) = \sum_\lambda \frac{\gamma_{\lambda c} \gamma_{\lambda c'}}{E_\lambda - E - i\gamma_{\lambda \gamma}} \quad (1)$$

and the S-matrix elements are given by

$$S_{cc'}(E) = e^{-i(\phi_c + \phi_{c'})} \sqrt{P_c} \{ [1 - \mathbf{R}(\mathbf{L} - \mathbf{B})]^{-1} [1 - \mathbf{R}(\mathbf{L}^* - \mathbf{B})] \}_{cc'} \sqrt{P_{c'}}, \quad (2)$$

where ϕ_c is the hard-sphere phase-shift and $L_{cc'} = (S_c + iP_c)\delta_{cc'}$ is related to S_c (hard-sphere shift factor) and P_c (hard-sphere penetrability) for chosen B_c (boundary parameters)

The cross section for the entrance channel c (neutron) and exit channel c' (neutron, γ , ...) is

$$\sigma_{cc'}(E) = \frac{\pi}{k_c^2} g_c | \delta_{cc'} - S_{cc'} |^2 \quad (3)$$

where g_c is the statistical spin factor

Link S-matrix Function to Average Total Cross Sections

Lorentzian Energy Averaging

The total cross section averaged over the energy interval I can be calculated from the average S-matrix elements S_{cc} with a Lorentzian weight function

$$\langle S_{cc}(E) \rangle = \int_{-\infty}^{+\infty} dE' \frac{I/\pi}{(E' - E)^2 + I^2} S_{cc}(E') \quad (4)$$

Since there are no poles above the real axis due to the causality of the S matrix

$$\langle S_{cc}(E) \rangle = S_{cc}(E + I) \quad (5)$$

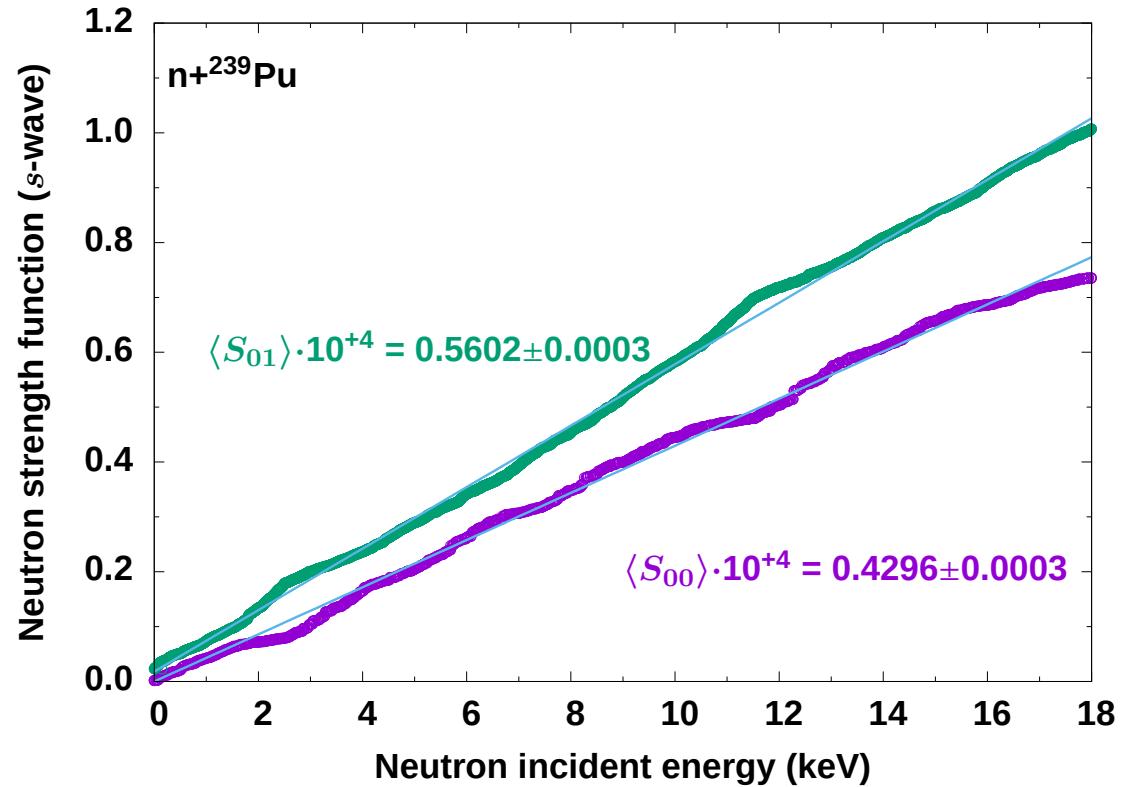
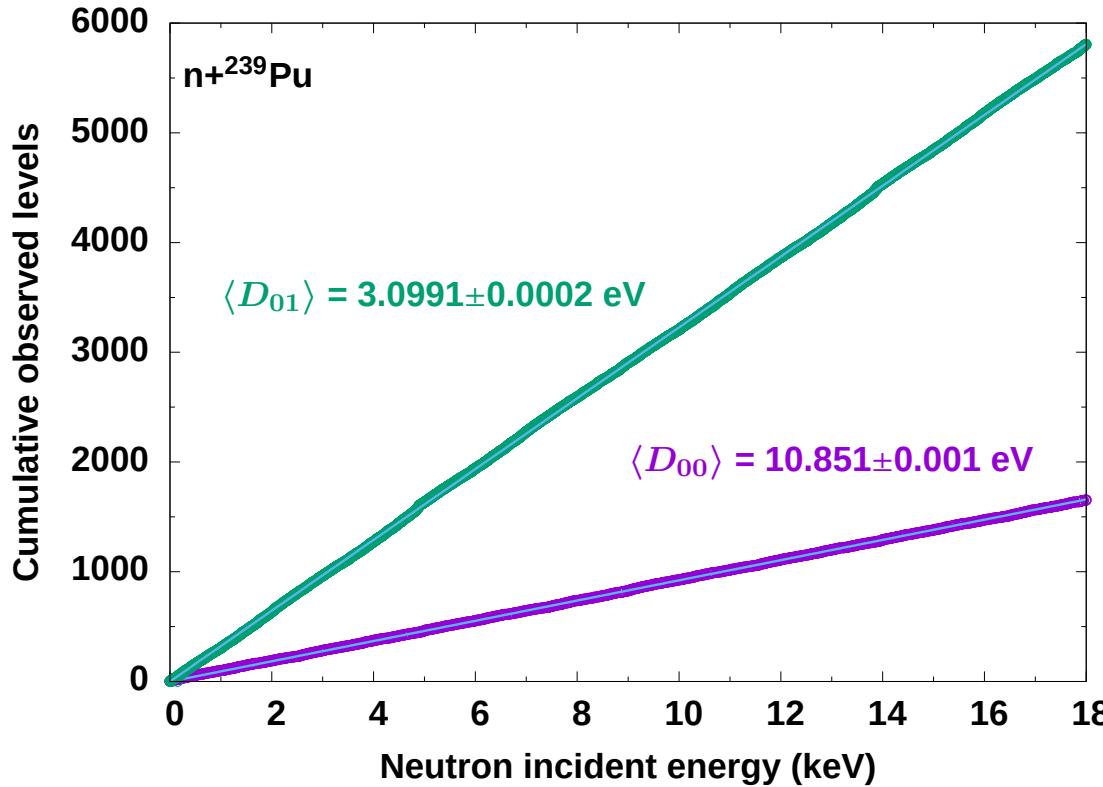
A common approximation is to average only the R-matrix function

$$\langle S_{cc}(E) \rangle \approx e^{-2I\phi_c} \sqrt{P_c} [1 - \langle R_{cc} \rangle (L_c - B_c)]^{-1} [1 - \langle R_{cc} \rangle (L_c^* - B_c)] \sqrt{P_c}, \quad (6)$$

where $\langle R_{cc}(E) \rangle \equiv R_{cc}(E + I)$ and the energy dependence of ϕ_c, P_c and L_c is neglected

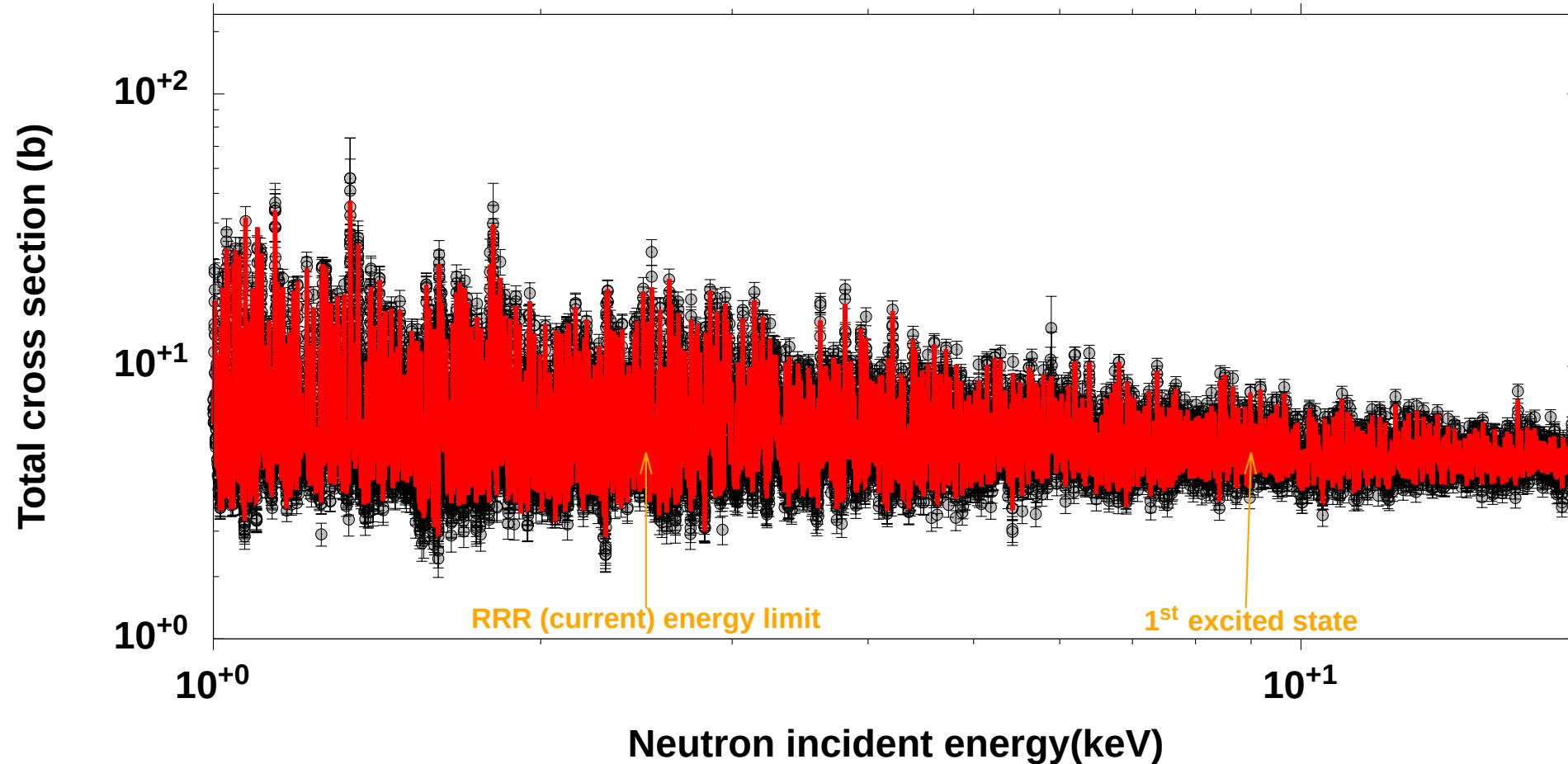
Extrapolating Energy Levels and Resonance Amplitudes

Statistics from Resonance Parameters below 2.5 keV



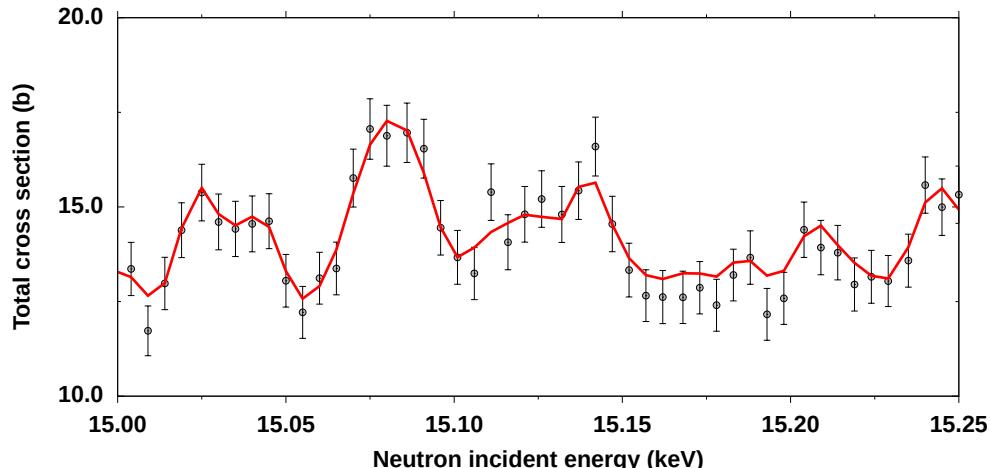
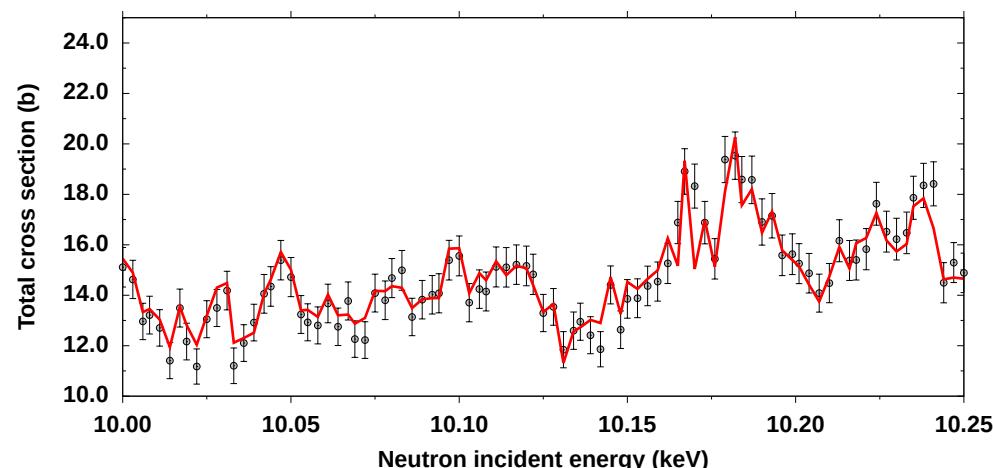
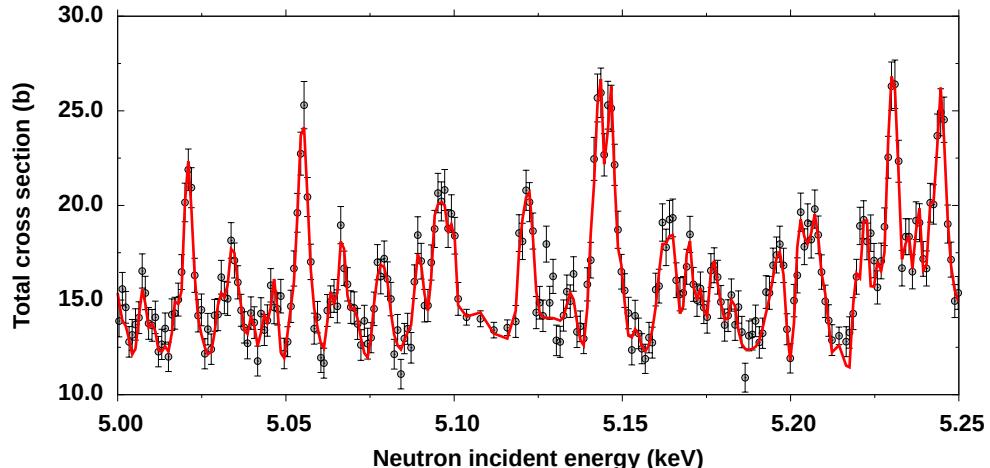
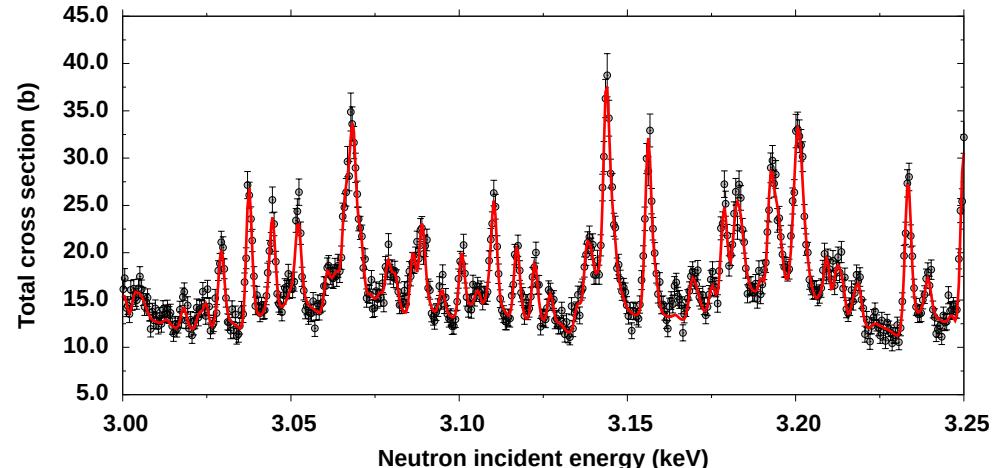
^{239}Pu Total Cross Section in the Resonance Region

Fit to Measured Data

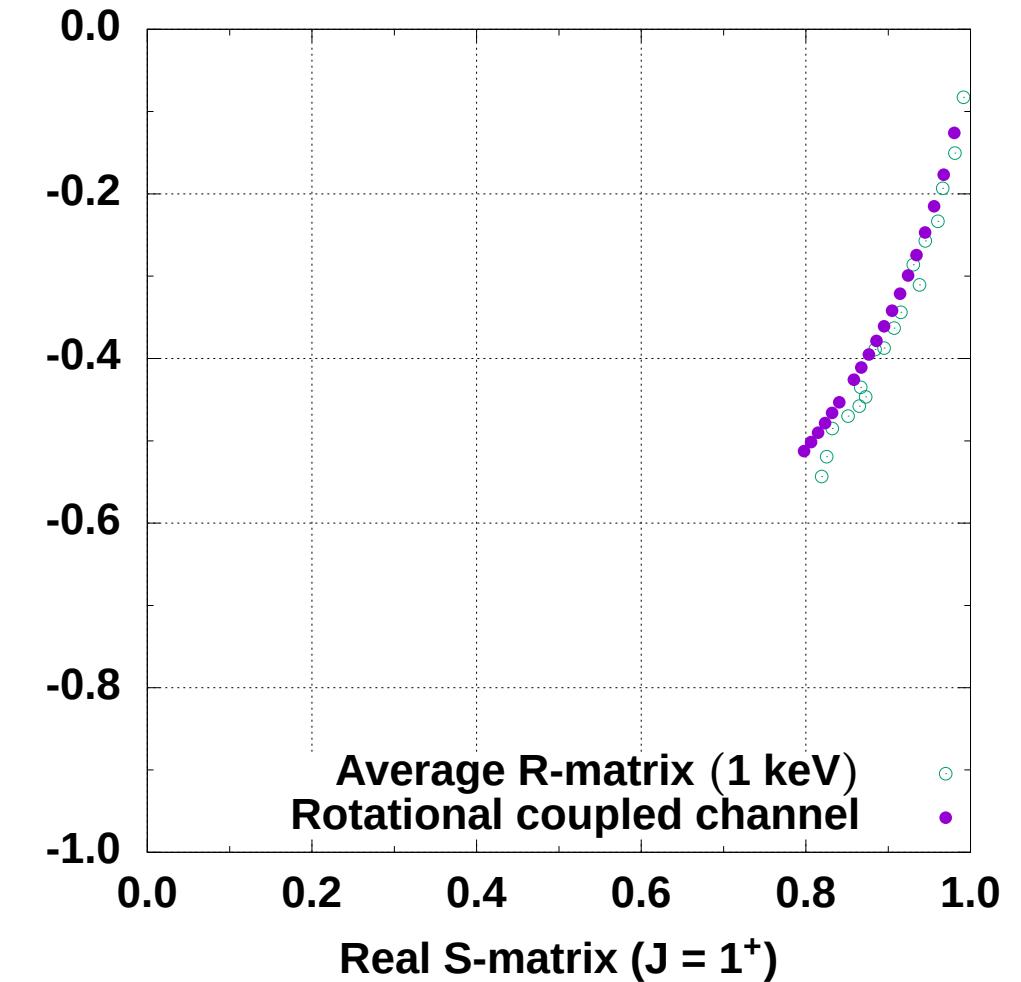
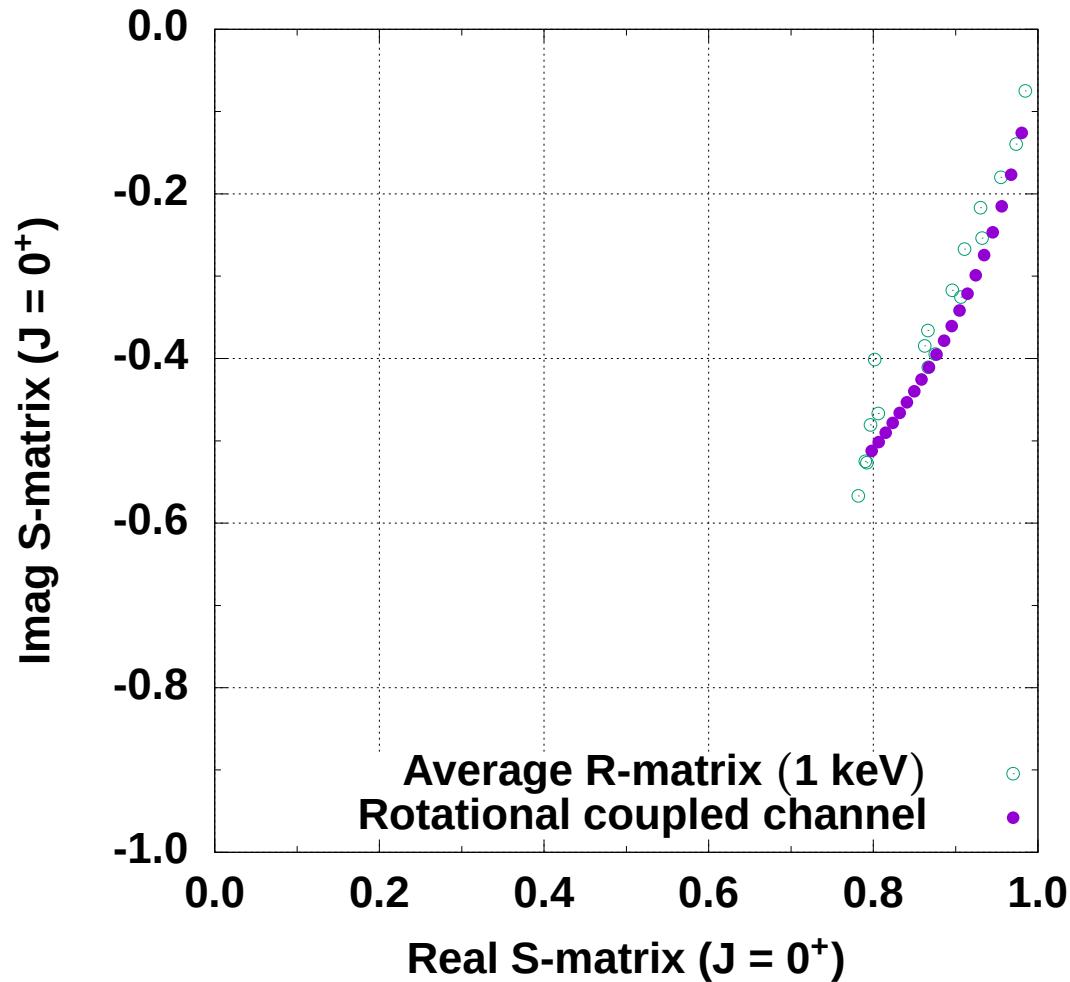


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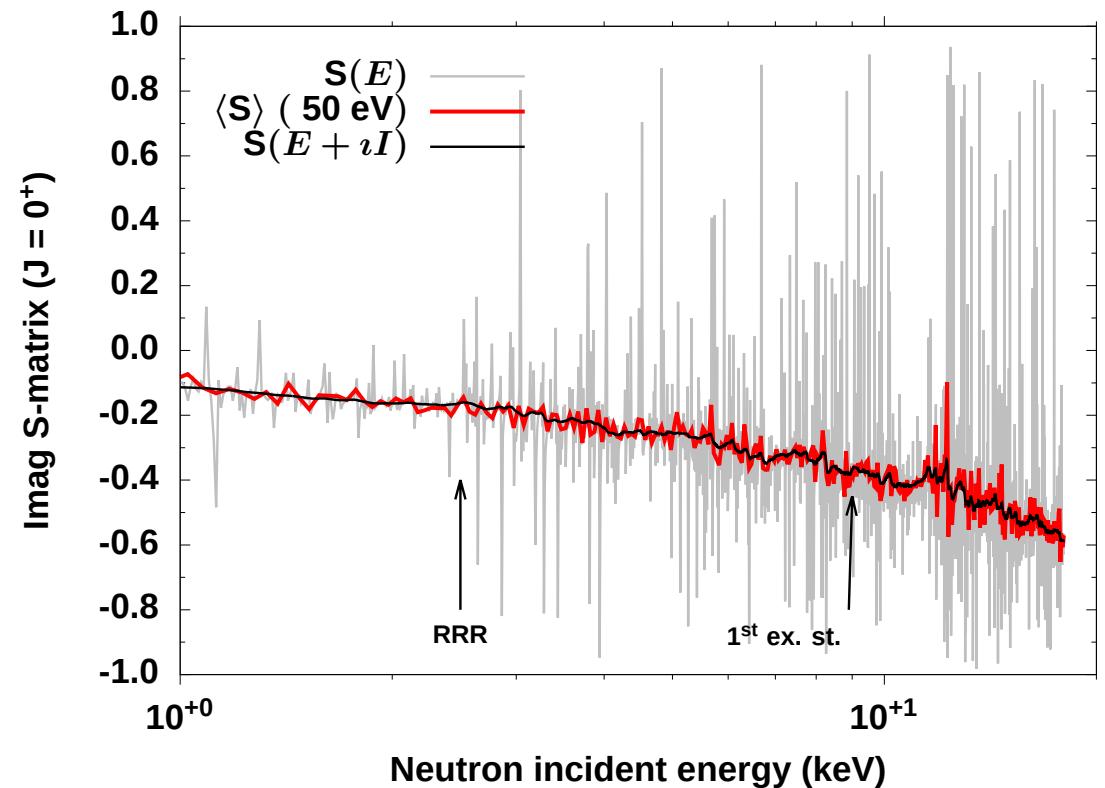
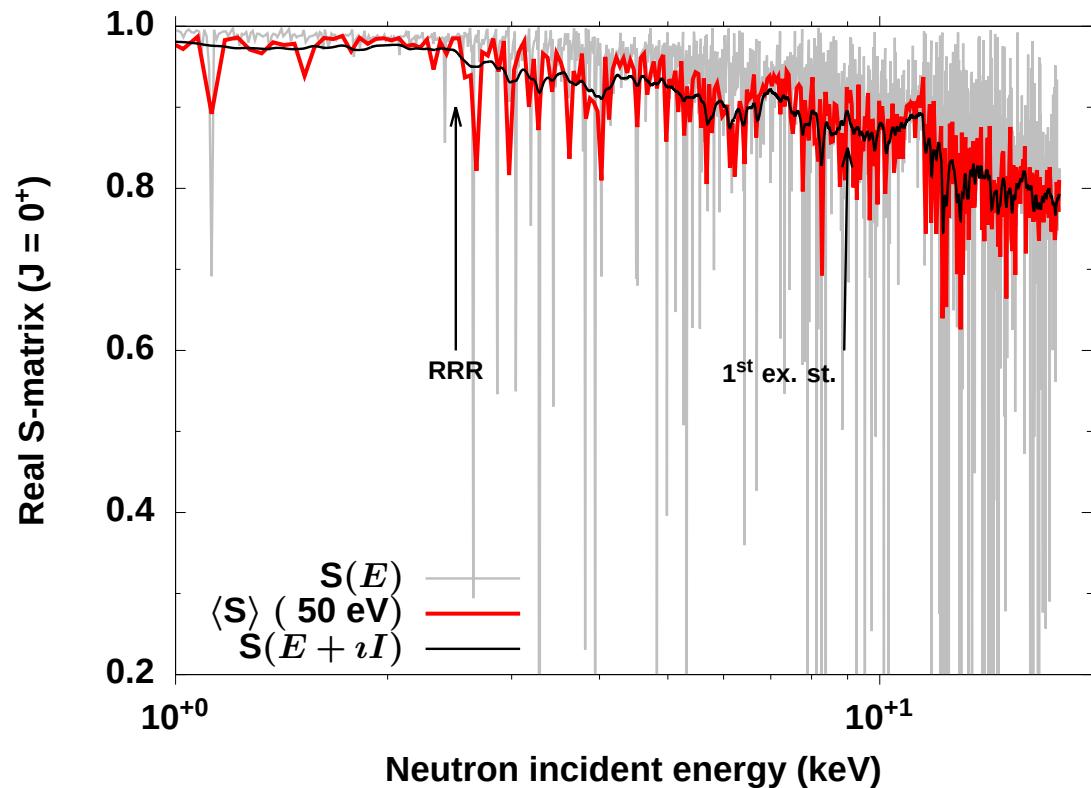


Link R-matrix to Optical Model Calculations



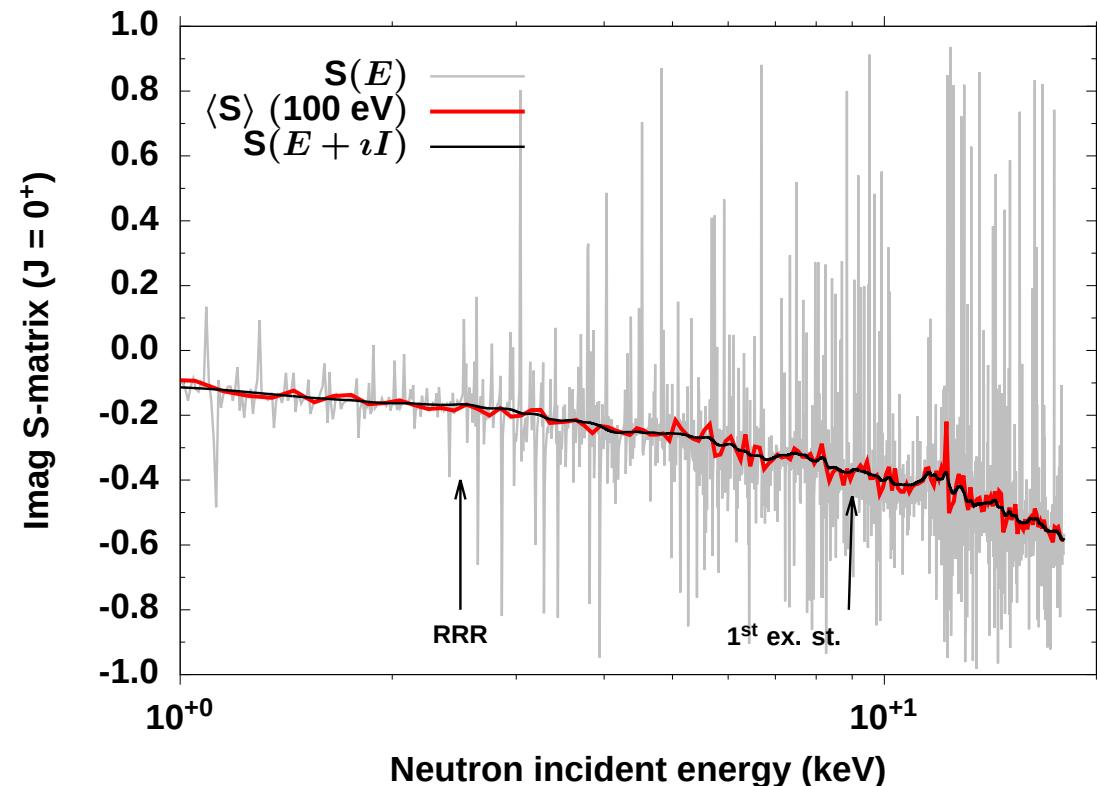
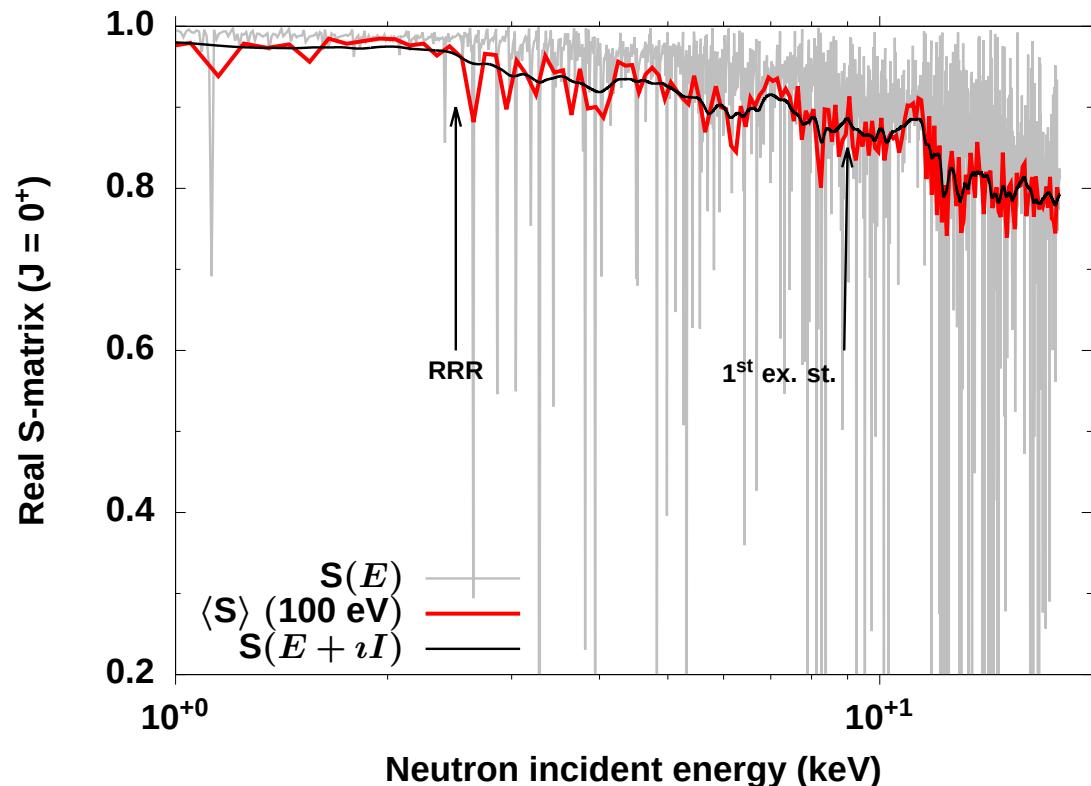
S-matrix Elements from Extrapolated Populations

Real and Imaginary Components (J=0⁺ Population)



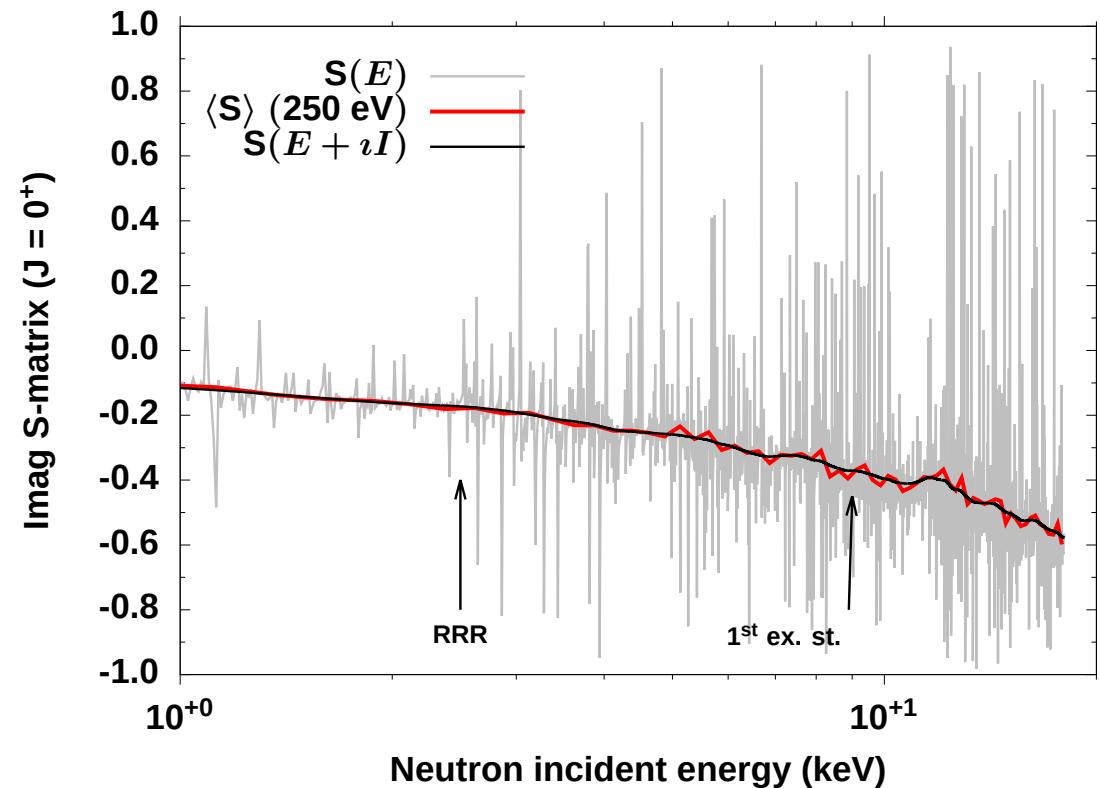
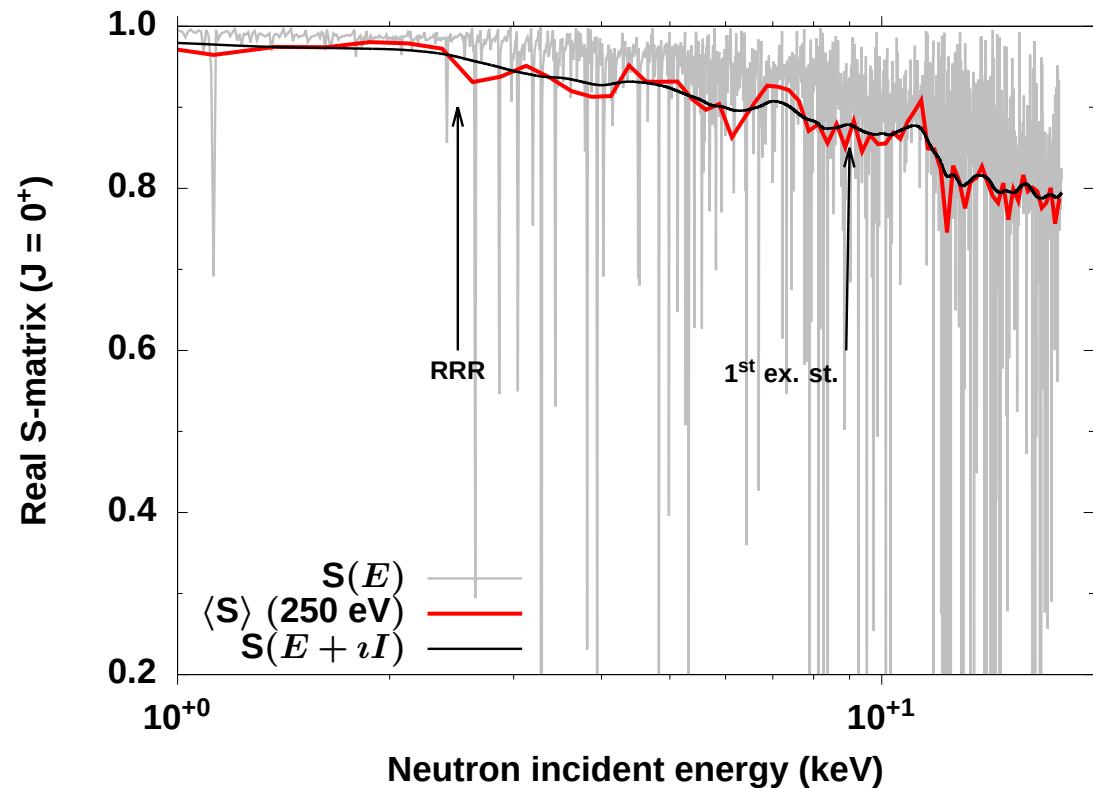
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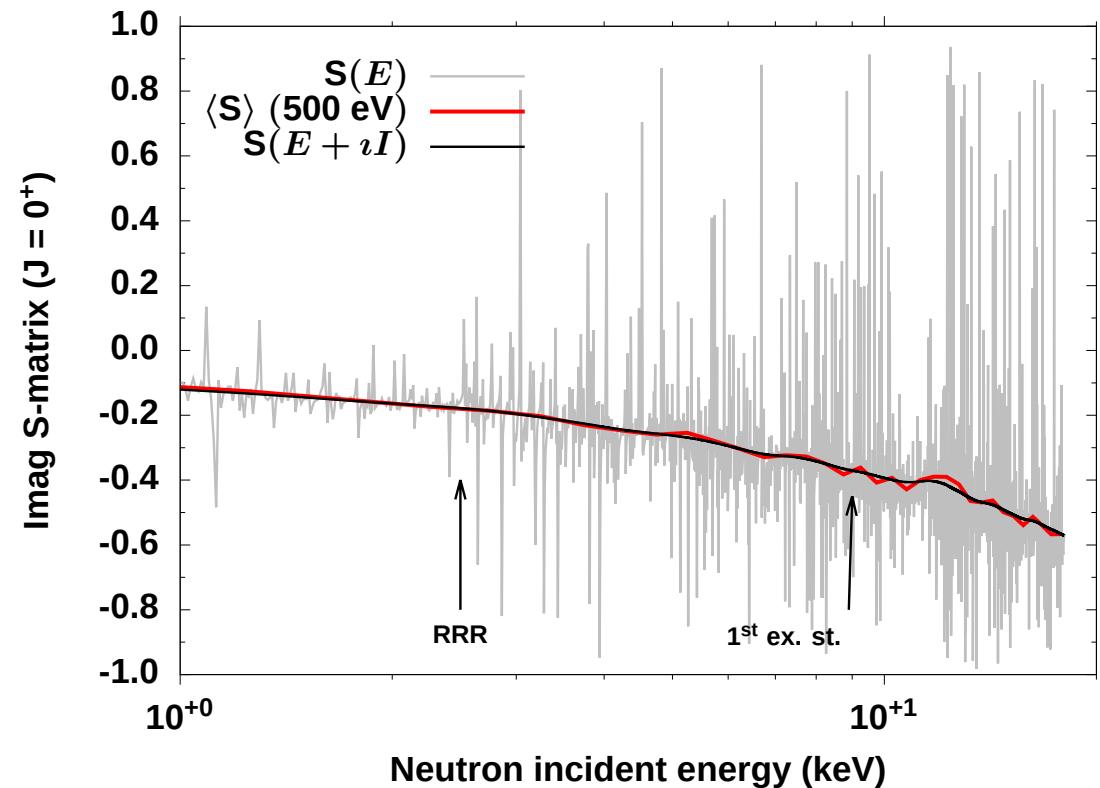
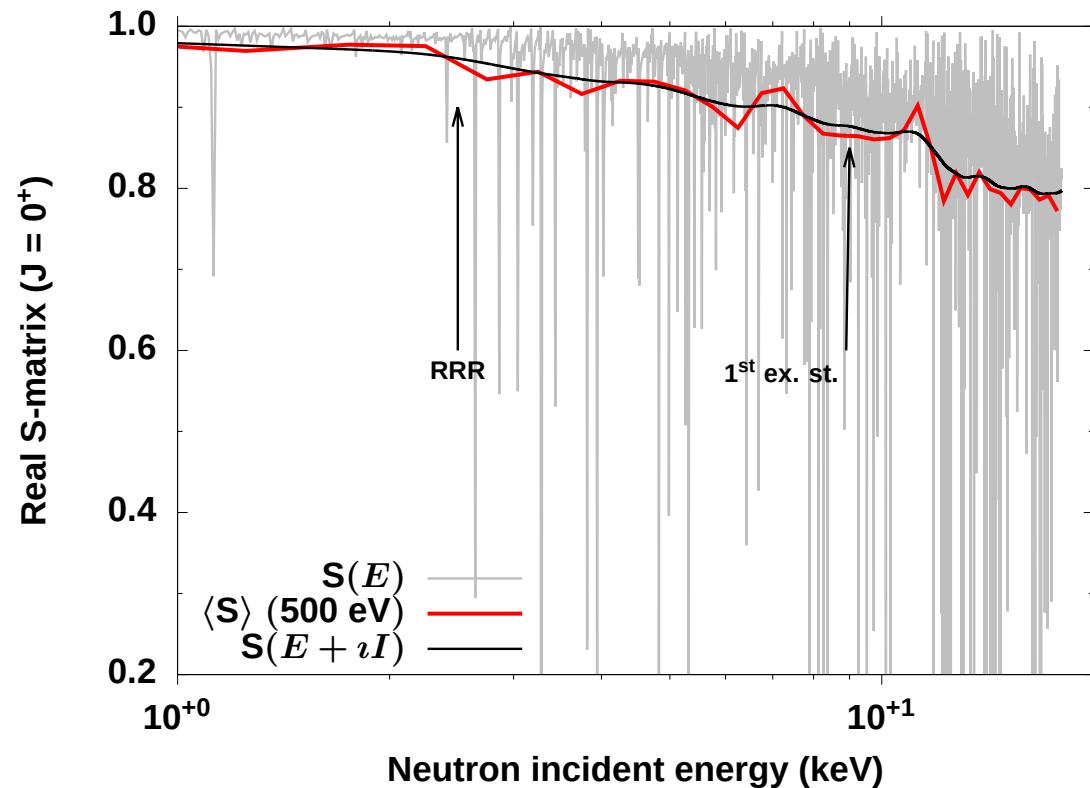
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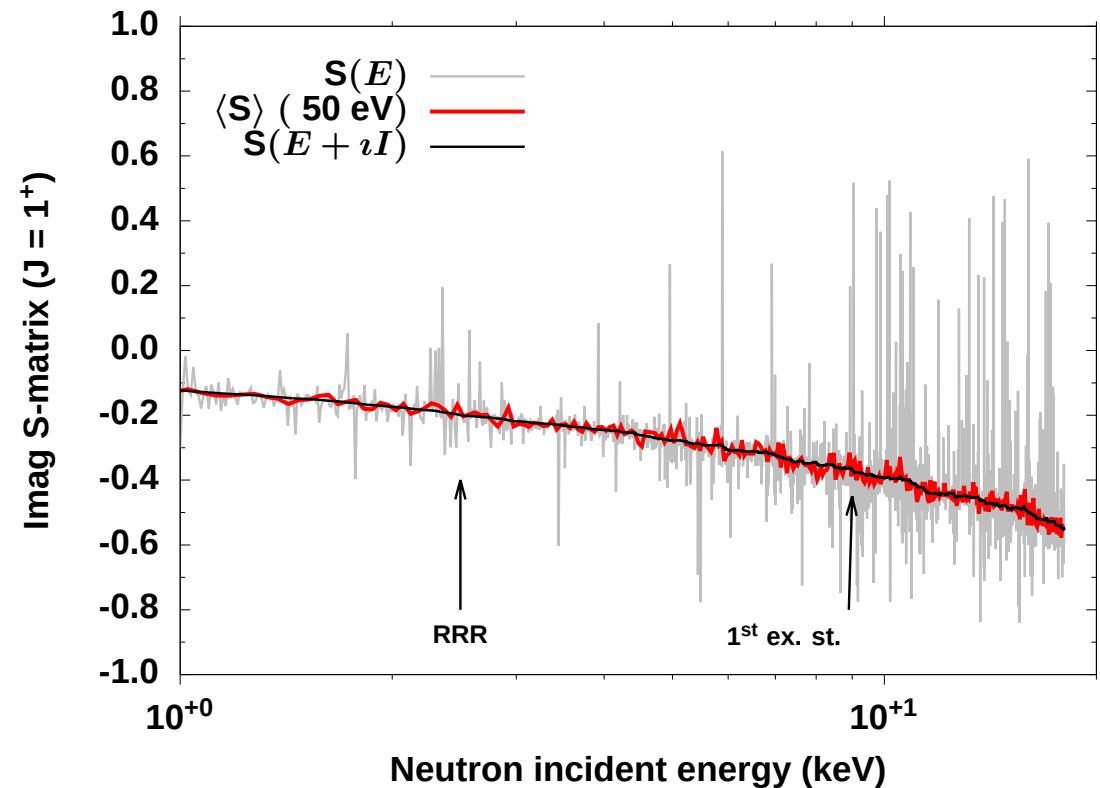
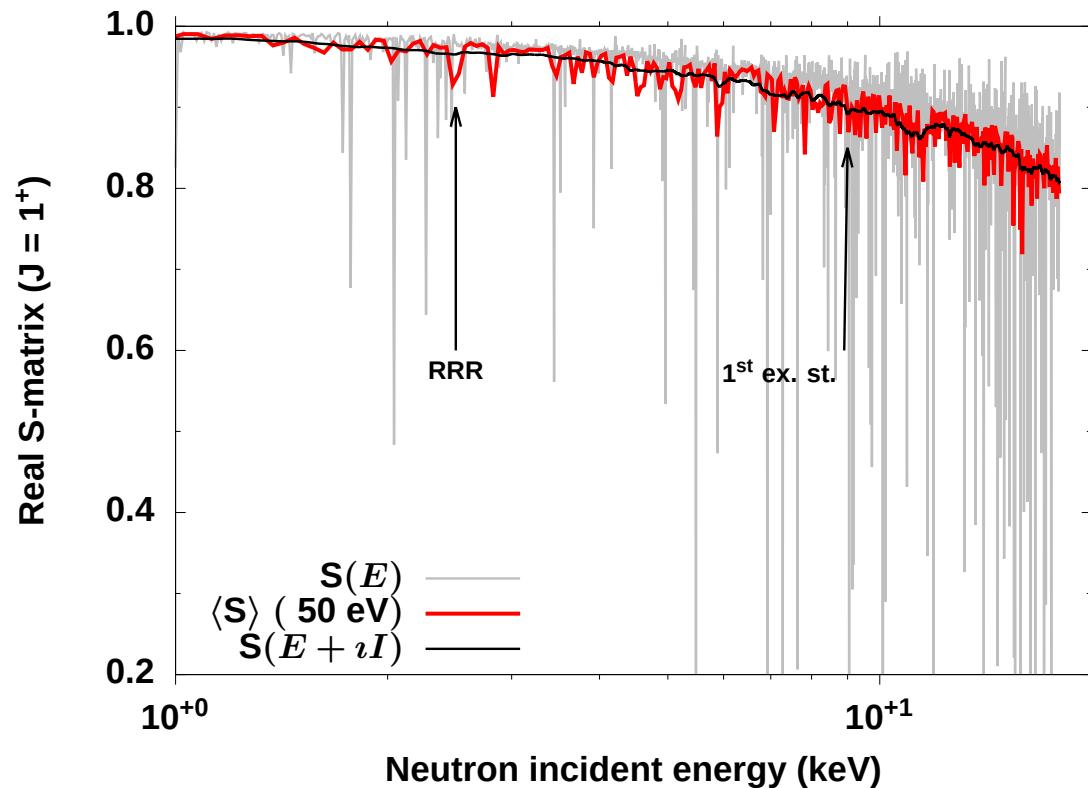
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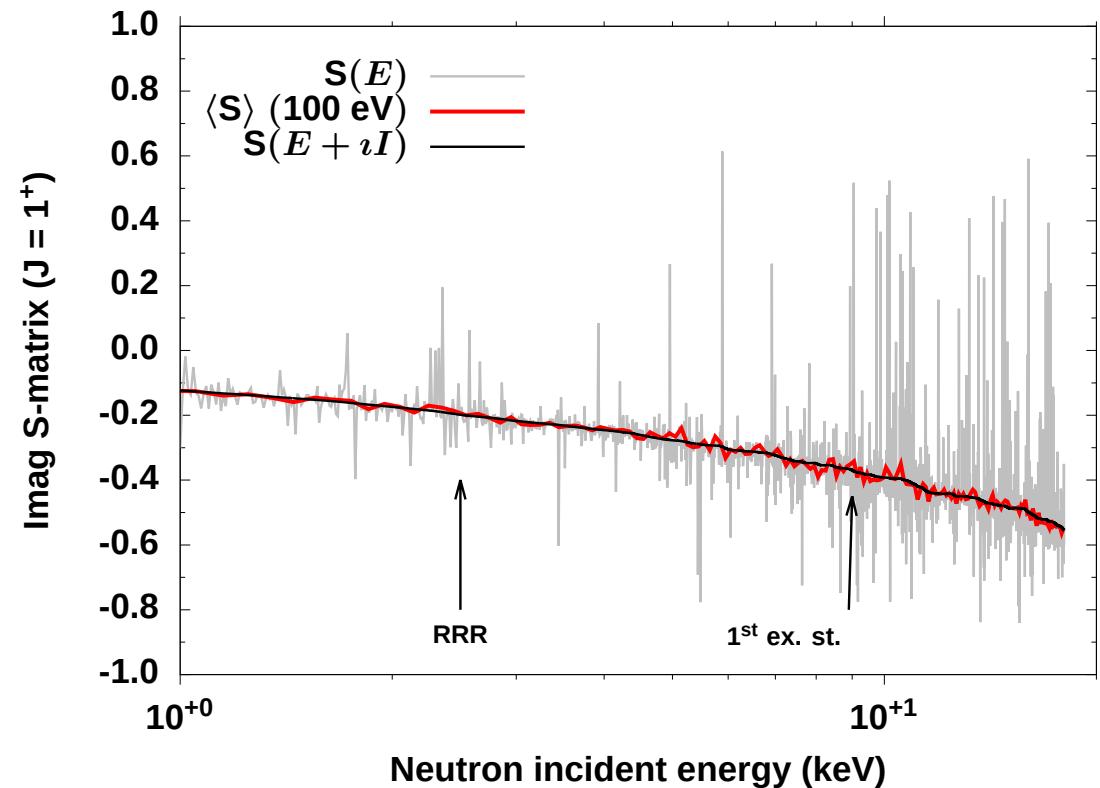
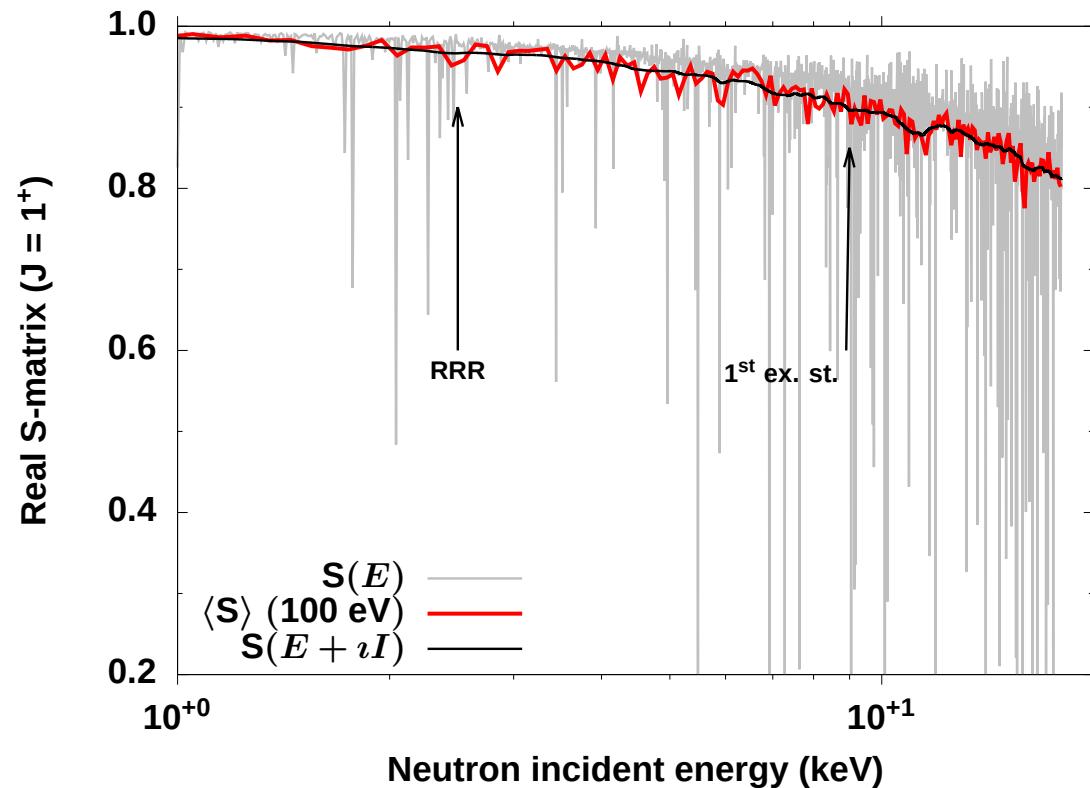
S-matrix Elements from Extrapolated Populations

Real and Imaginary Components (J=1⁺ Population)



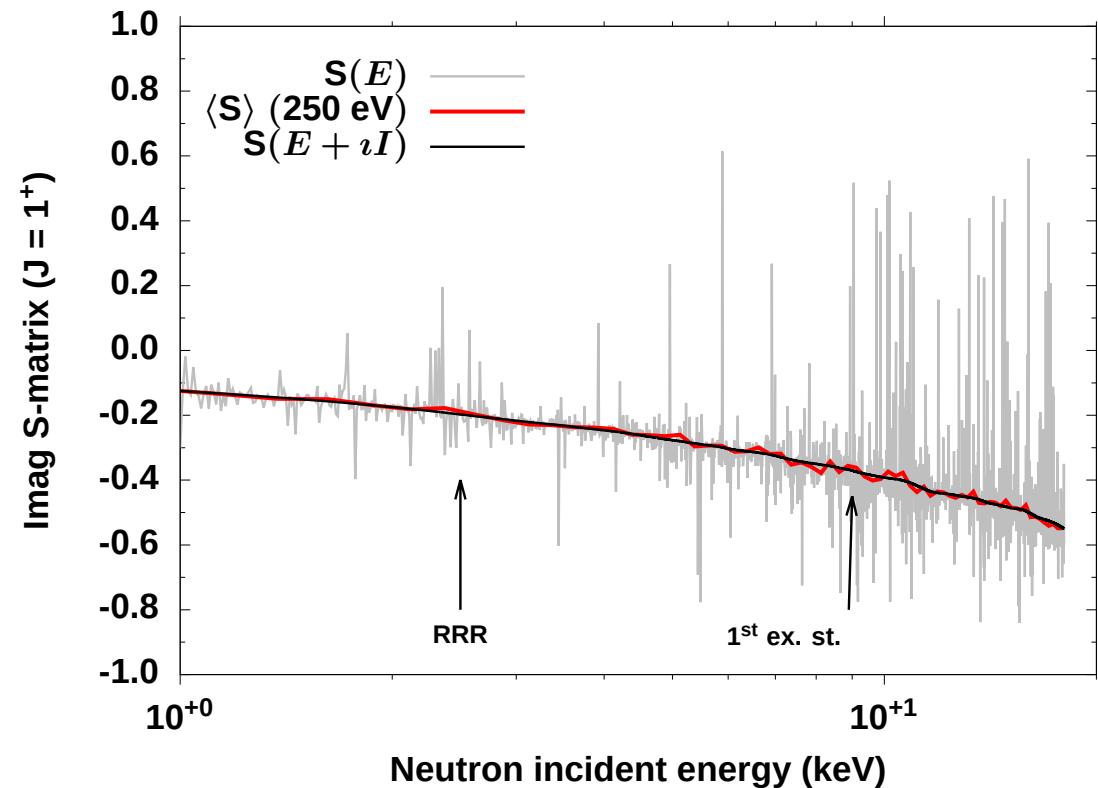
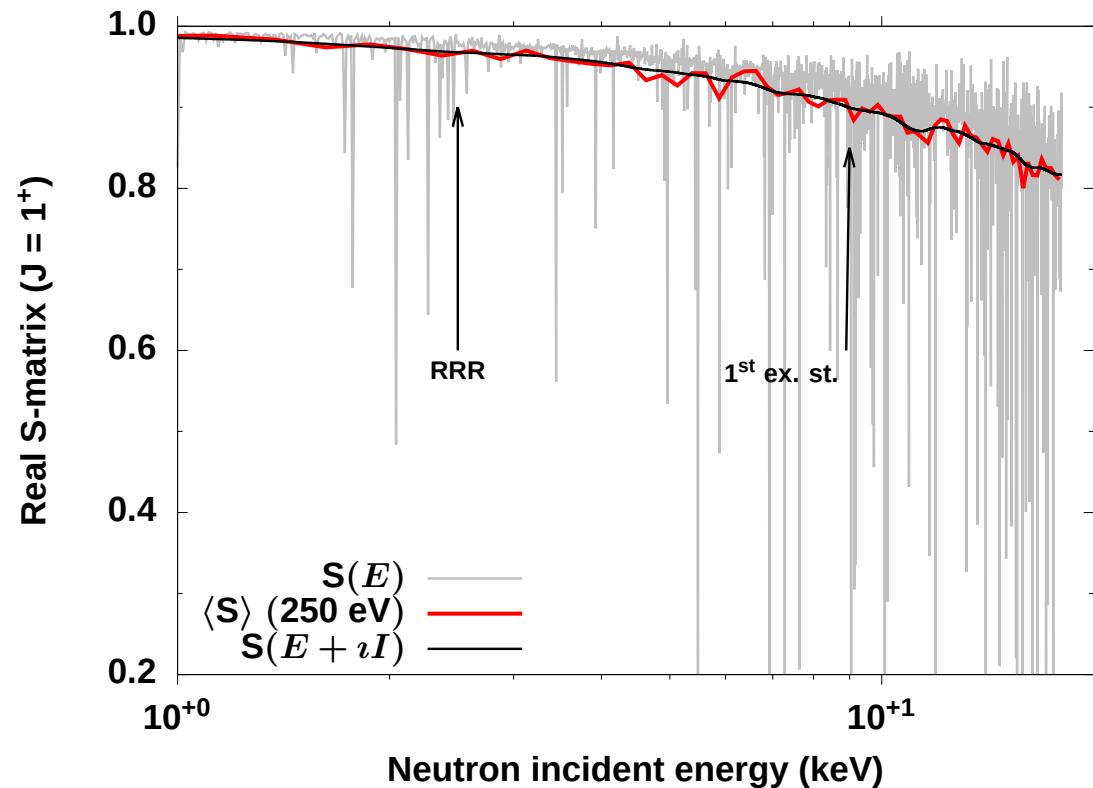
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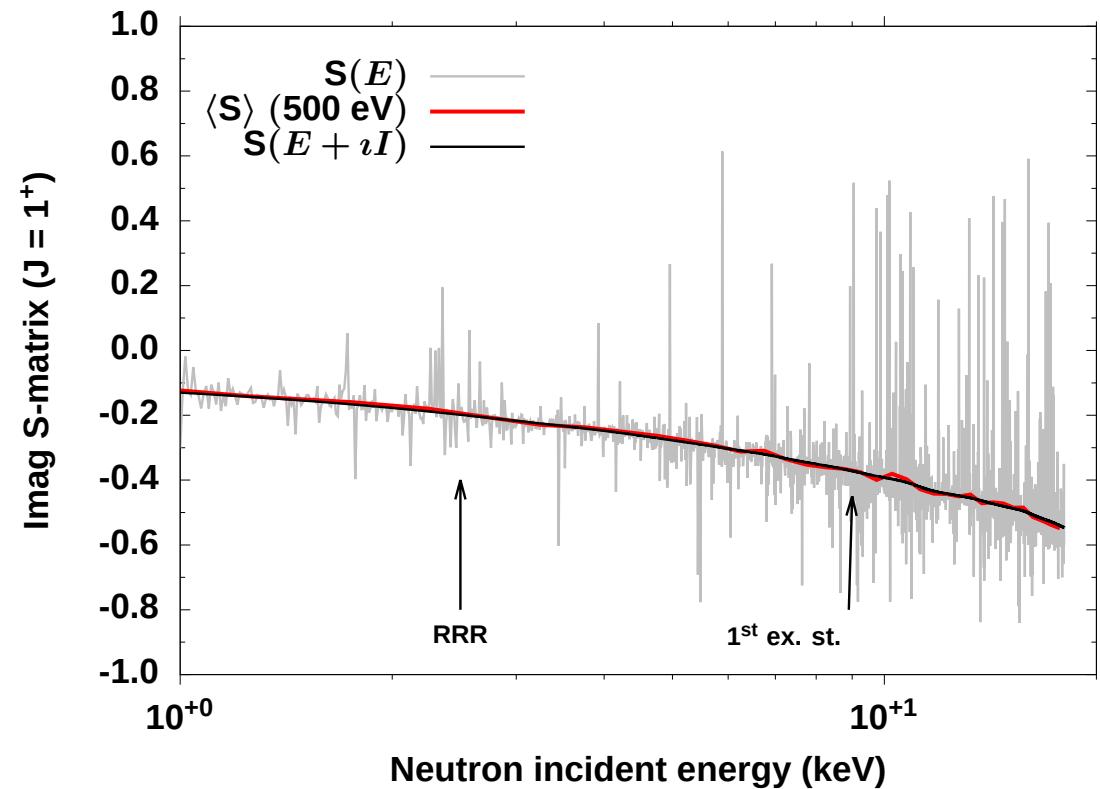
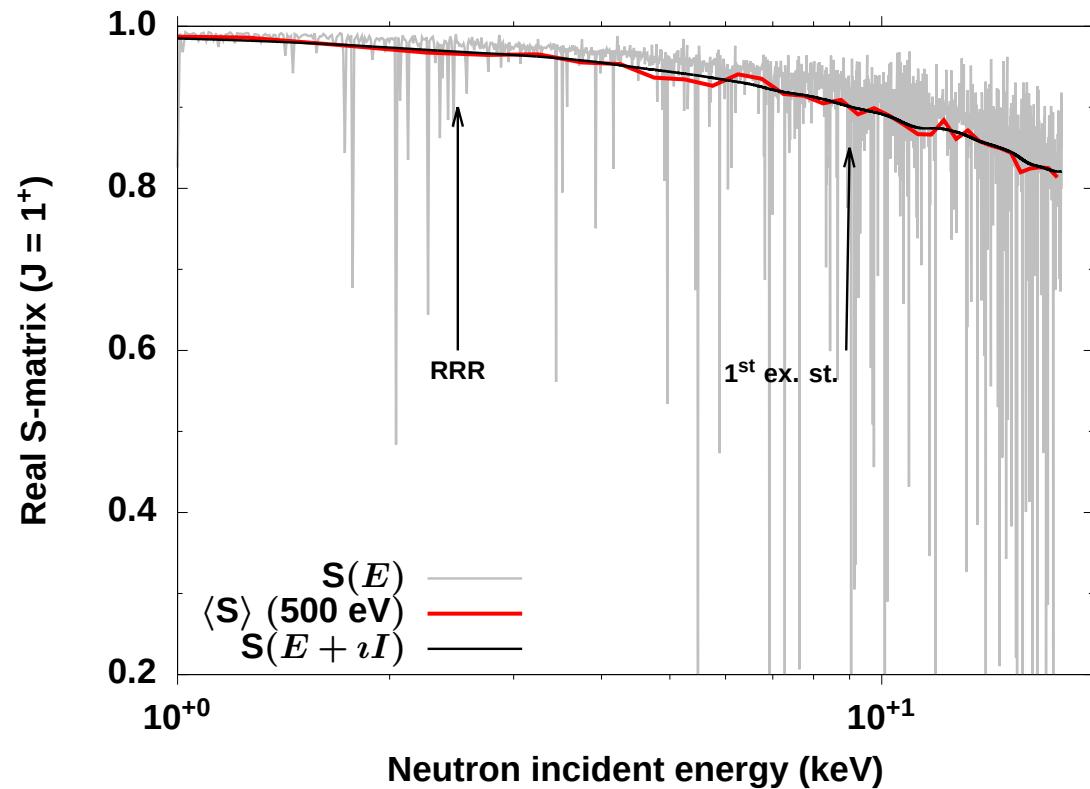
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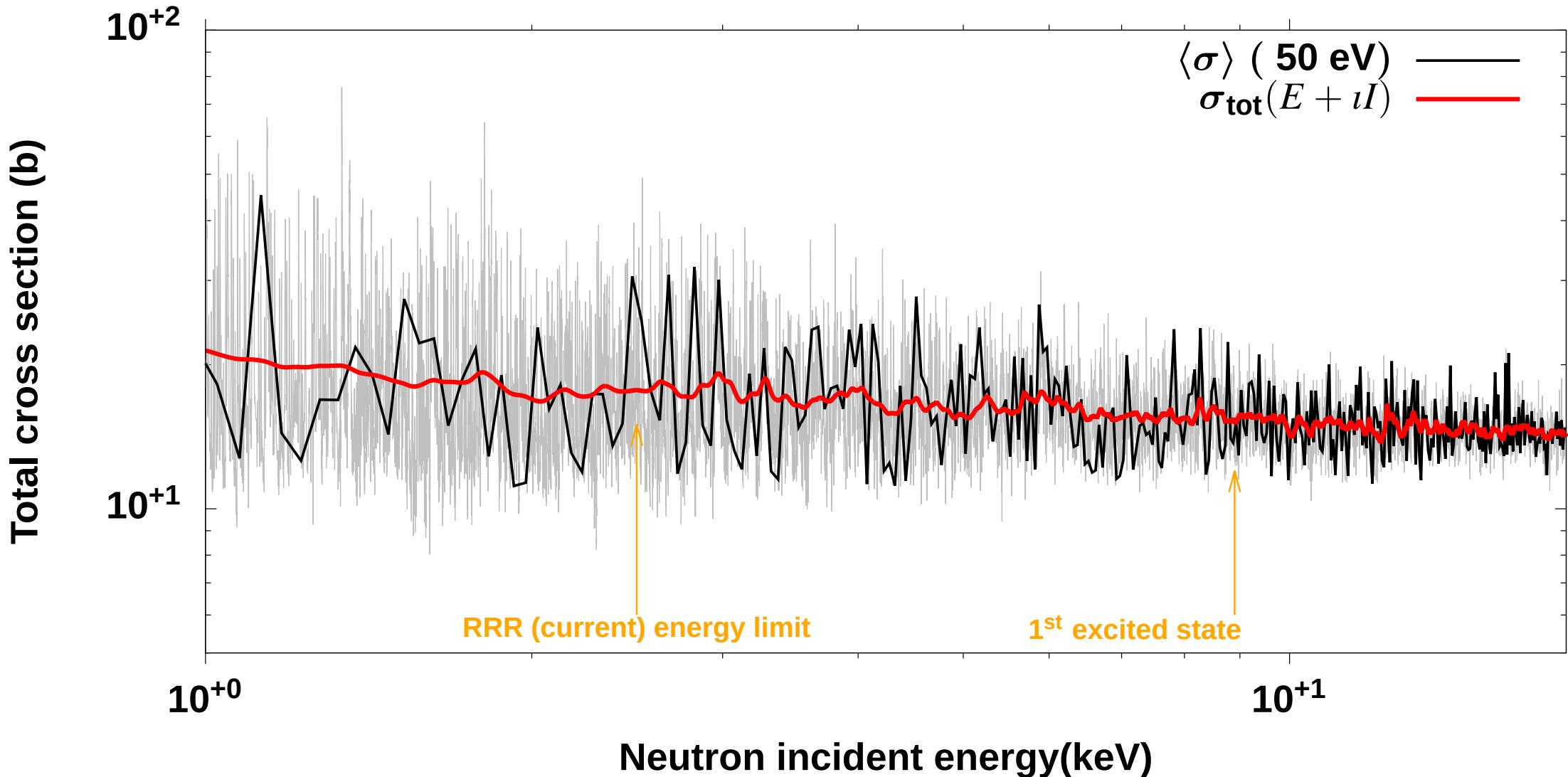


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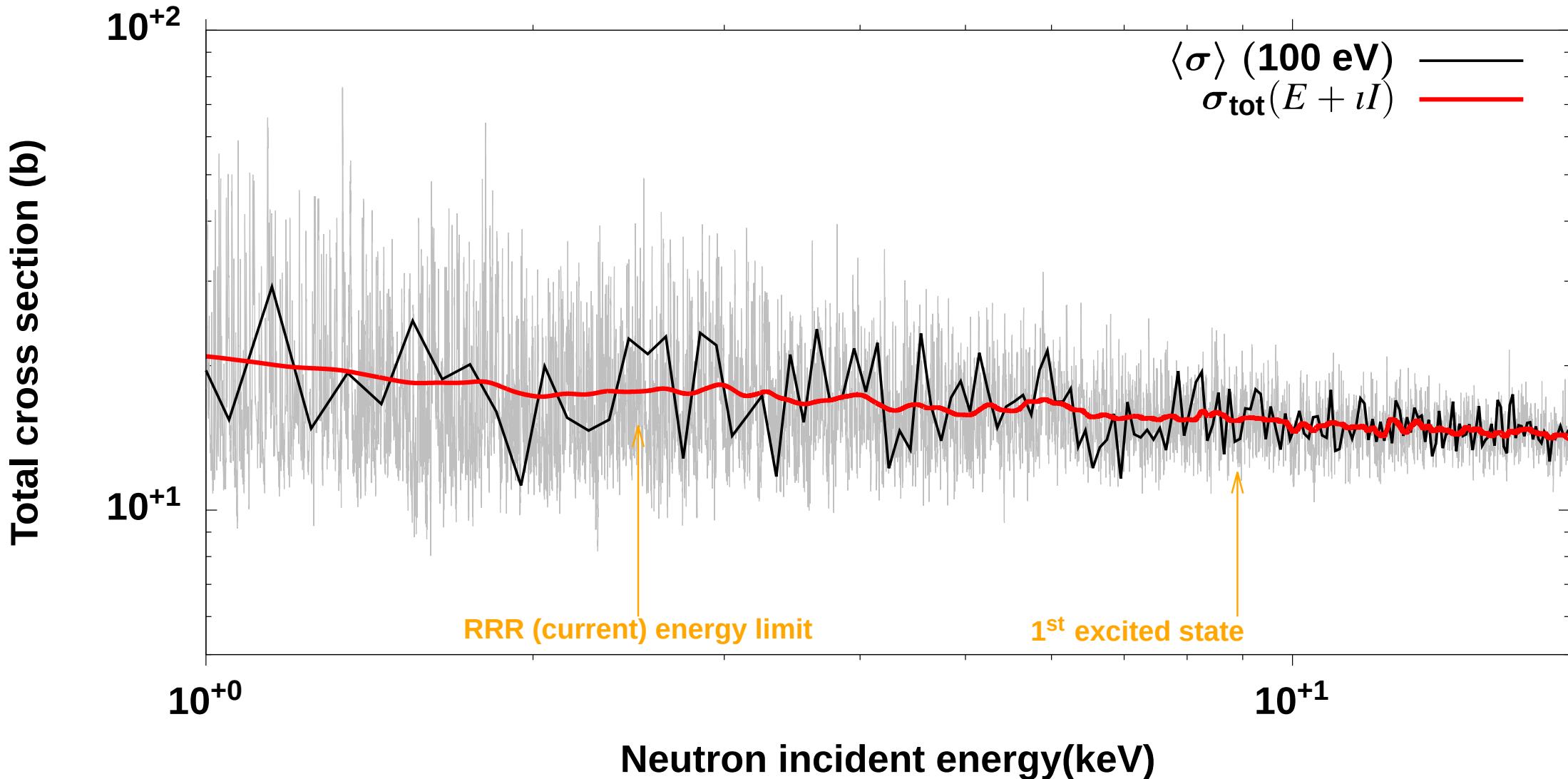
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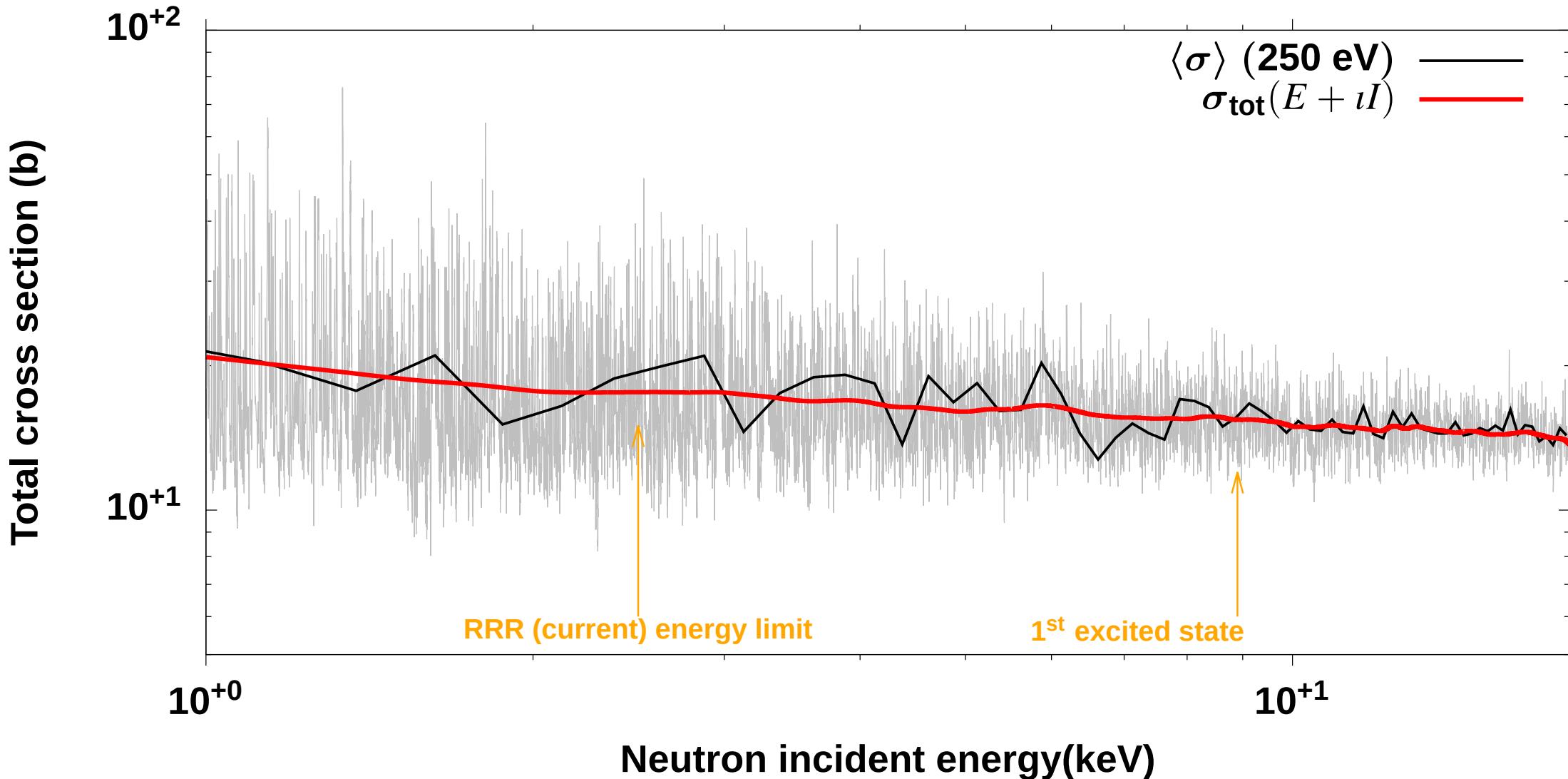
^{239}Pu Energy-average Total Cross Section



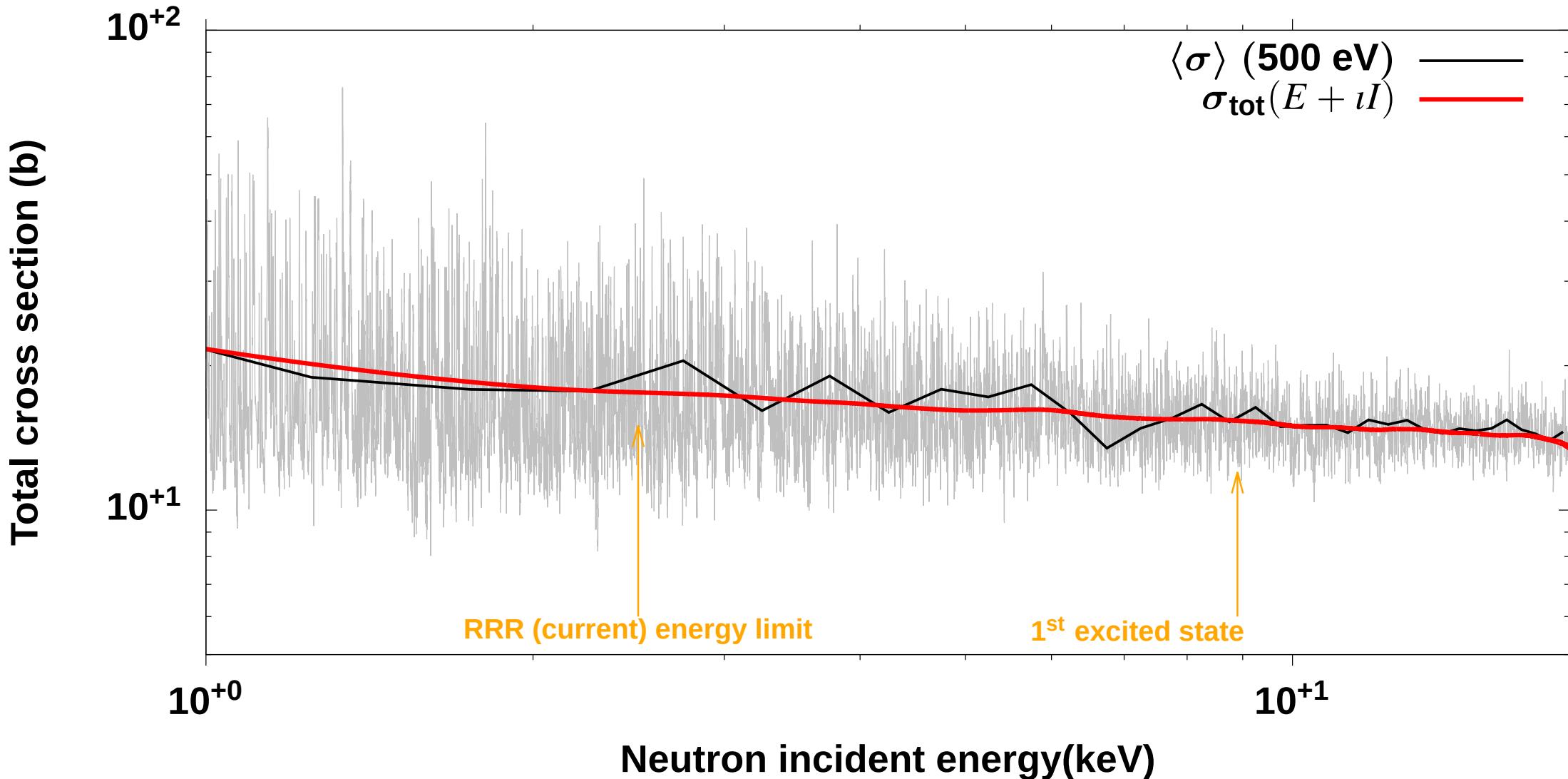
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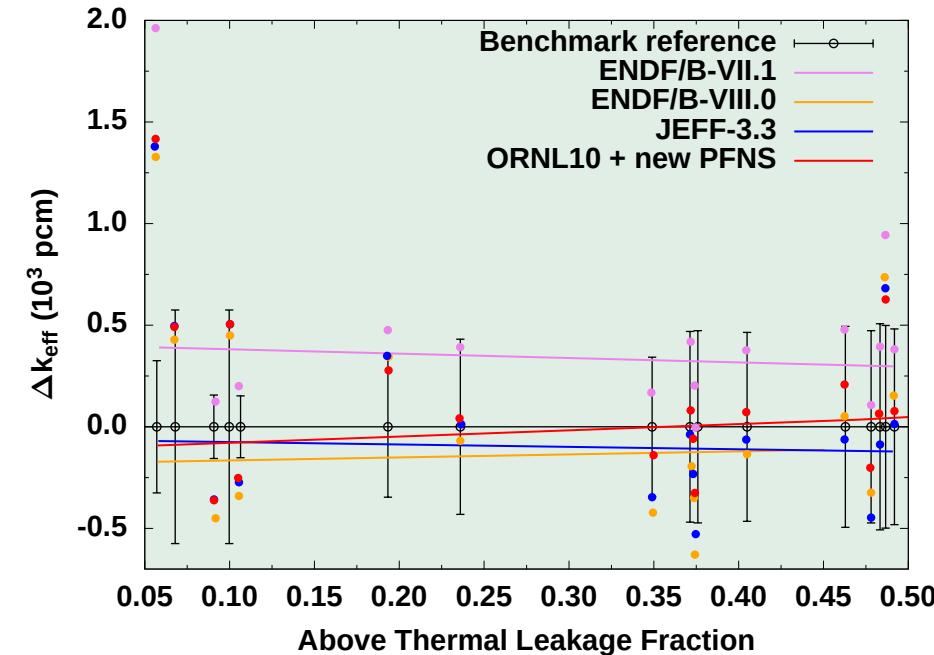
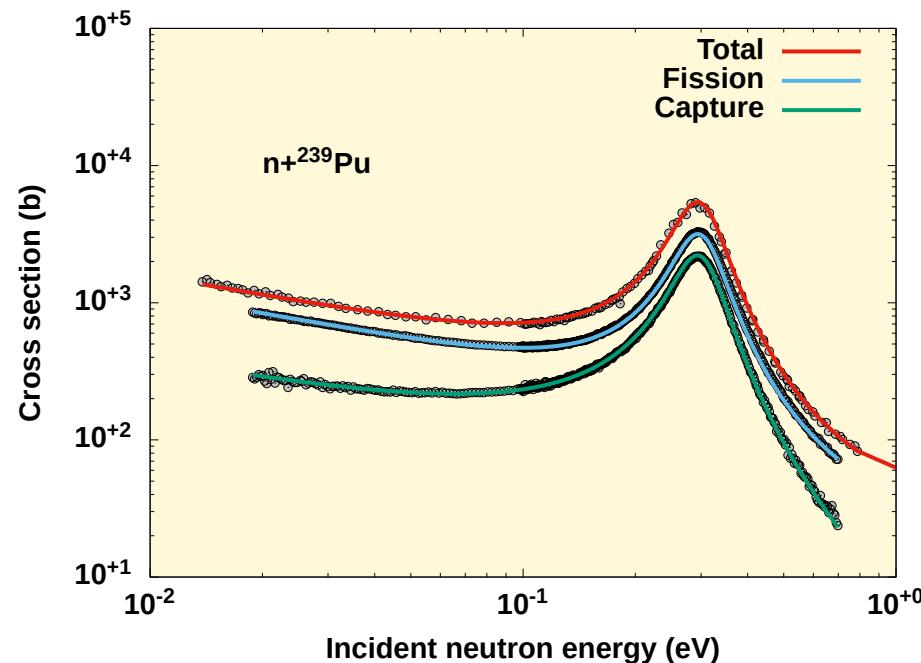


Resolving Key Issues in the Intermediate Structure Region

- The intermediate structure observed in the measured data can be described by including in the evaluated data files average cross sections calculated from the average S-matrix function
- With a proper energy grid the average cross sections can be broadened and calculated at the desired temperature
- In the case of selected applications for which the temperature dependence is particularly important and sensitive to the fluctuations, the resonance parameters could be used instead of the average cross sections
- Theoretical development to rigorously quantify the shape and compound component of the cross sections might be still needed

Coupling the Thermal and RRR Region to New PFNS

- Background:** The recently released ENDF/B-VIII.0 was based on evaluations performed within the international collaboration CIELO aiming to improve nuclei of fundamental importance such as ^{235}U and ^{239}Pu . The ^{235}U R-matrix evaluation (ORNL) was updated with the latest thermal constants and prompt fission neutron spectra (PFNS) improving the benchmark performance of the thermal solutions. However, for ^{239}Pu evaluation the focus was in the high energy range and the prediction on the thermal solution benchmarks was underpredicted
- Results:** Within IAEA coordinated research activities, newly evaluated PFNS showed a reduction of 1.8% on the average energy : PFNS($\langle E_{\text{av}} \rangle$)=2.08 MeV). This changes were combined to recent work on ^{239}Pu R-matrix evaluation (ORNL) aimed to update the thermal constants. This led to improved benchmark performance in the thermal solutions



Conclusion and Remarks

- A procedure to accurately model the measured cross section fluctuations for inclusion into evaluated nuclear data files was developed and tested on ^{239}Pu total cross sections
- The procedure consisted on defining population of levels as well as related amplitudes extrapolated from the systematics of the resonance parameters in the resolved resonance region
- By using realistic population of extrapolated levels, the fit of measured total cross section produced a continuous S-matrix function $S(E)$ in Reich-Moore approximation
- The S-matrix function averaged over a suitable energy interval was used to obtain averaged cross sections consistent with the measured data
- In the thermal energy region preliminary results coupling the RRR and the new PFNS evaluation were generated

Acknowledgments

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Thank you!