

# Updates to R-matrix Evaluations of Fissile Actinides: $^{233}\text{U}$ , $^{235}\text{U}$ , $^{239}\text{Pu}$

Marco T. Pigni

Nuclear Data and Criticality Safety

Oak Ridge National Laboratory, Oak Ridge, TN

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# Introduction

## Importance and Significance of Cross Section Fluctuations

- Criticality benchmark calculations are highly sensitive to cross section fluctuations (usually in the keV neutron energy region for fissile actinides)
  - Average resonance parameters and related probability tables are commonly used to generate cross section fluctuations
- Measured cross sections possess a rapidly varying resonant behavior depending on the incident neutron energy
  - At low energies, the resonance-like structure from the quasi-bound state of the compound nucleus can be measured fairly easily since the experimental resolution is higher than the spacing of the level states
  - However, as the energy increases, the number of levels is so large that only fluctuations related to very closely-spaced levels can be measured
- These data are generally evaluated with a relatively simple method (R-matrix theory)
- For resonance that cannot be resolved experimentally, the cross section fluctuations are usually described in terms of S-matrix elements calculated from average quantities obtained from R-matrix analyses

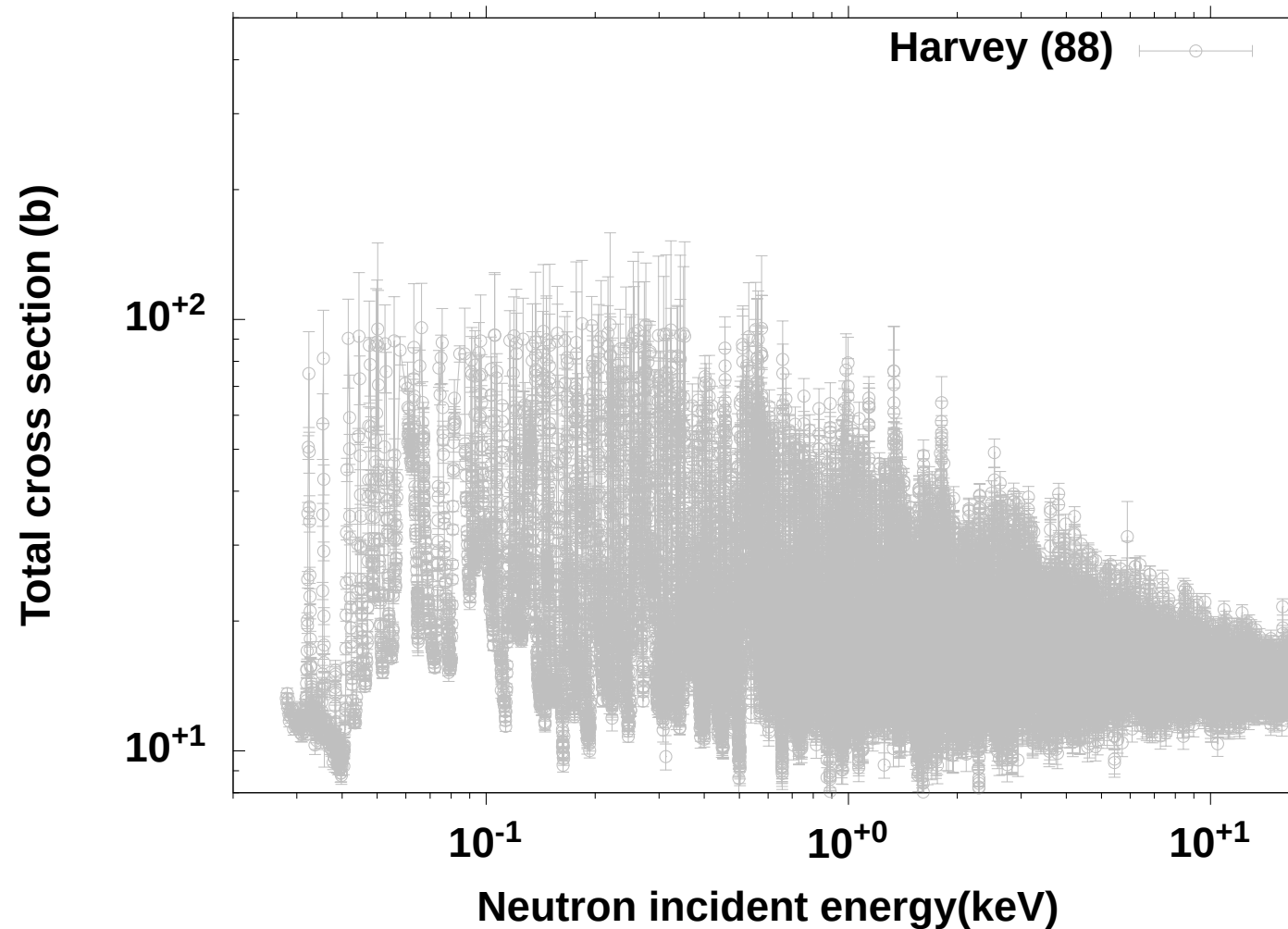
# Motivation and Overarching Goals

## To Establish the Highest Fidelity Nuclear Evaluated Data

- Evaluated data files do not entirely describe the cross section fluctuations available in measured data
  - This information could improve the performance in benchmark calculations
- High-fidelity description of measured data in neutron energy regions where fluctuations are important and relevant for applications
- Fluctuating cross sections evaluated within a consistent theoretical formalism
  - The energy averaged cross sections are defined in terms of the average resonance parameters simply because the Single Level Breit Wigner approximation is used
- Inclusion of fluctuating cross section aiming to limit the size of the evaluated data files as well as maintaining performance in benchmark calculations

# Experimental Data

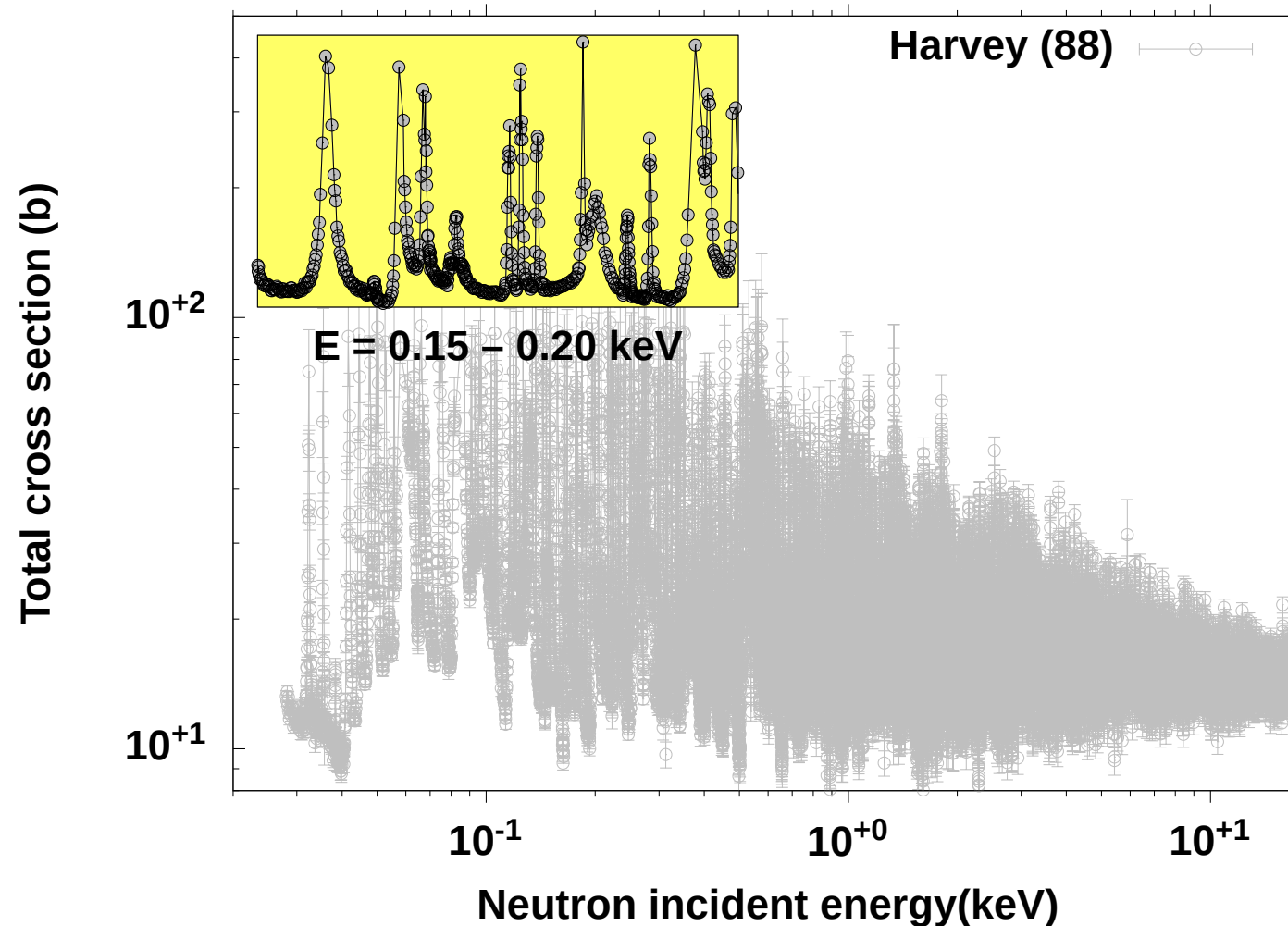
## Fluctuations in Measured Cross Section ( $n+^{239}\text{Pu}$ )





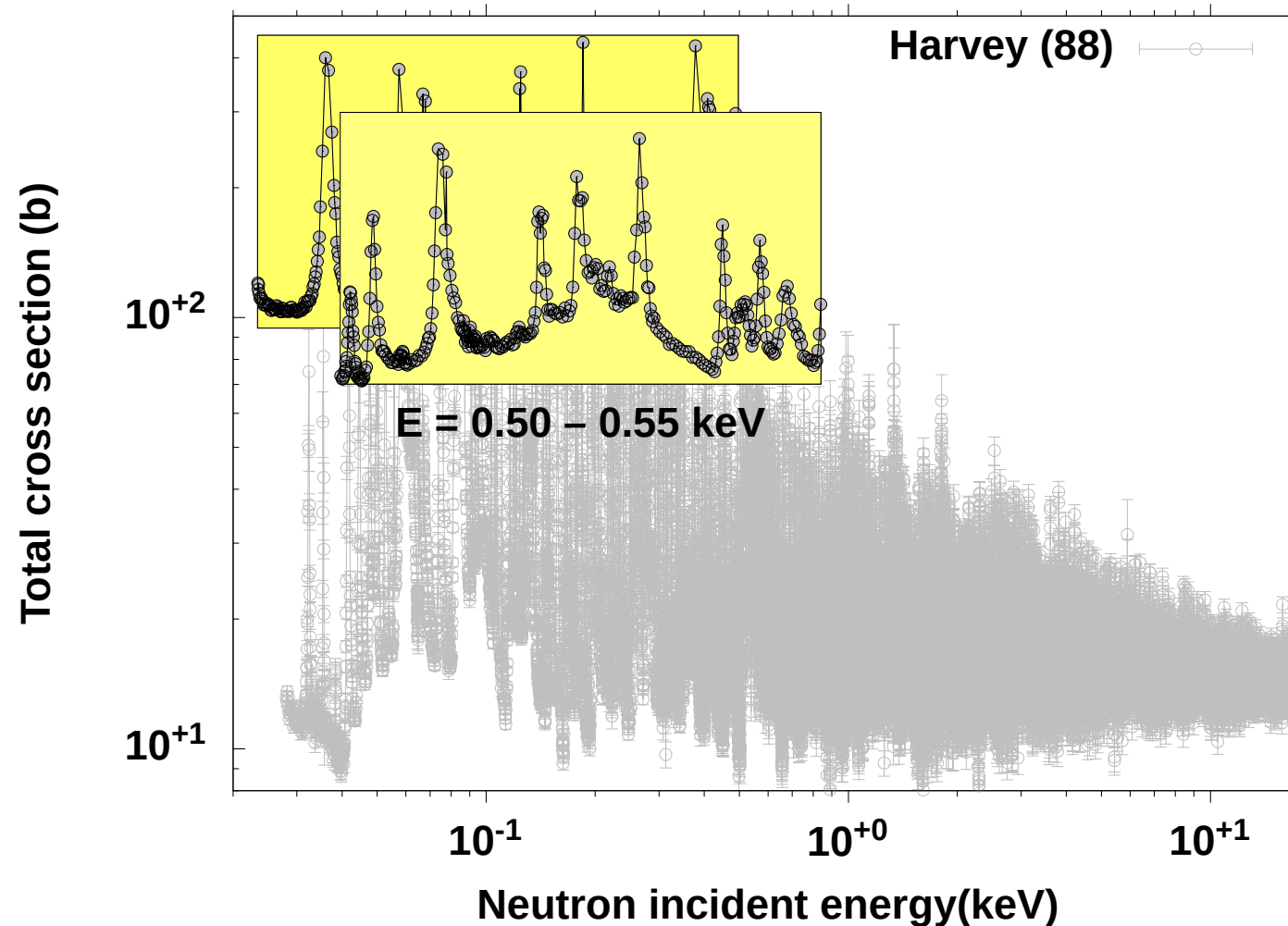
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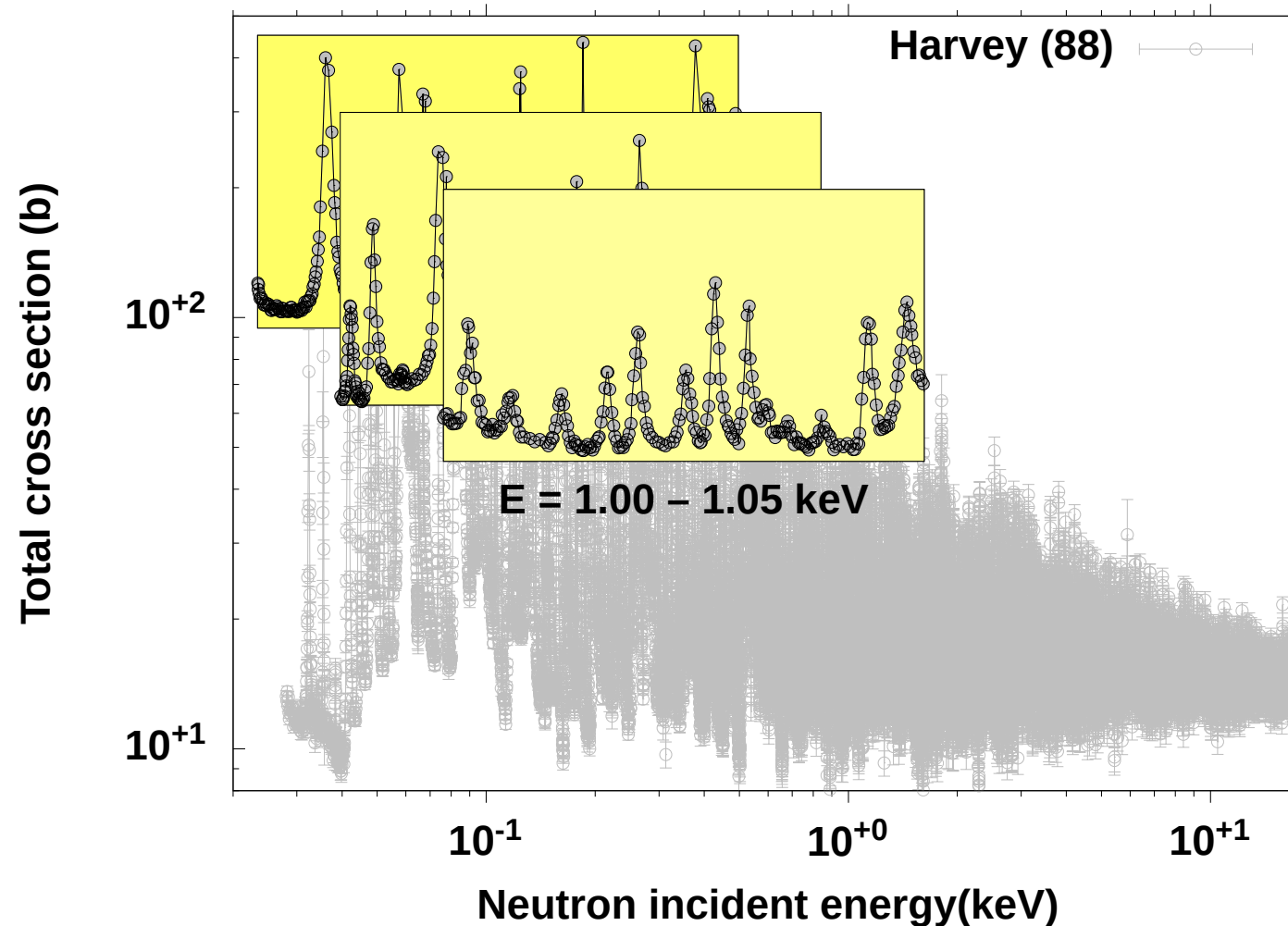
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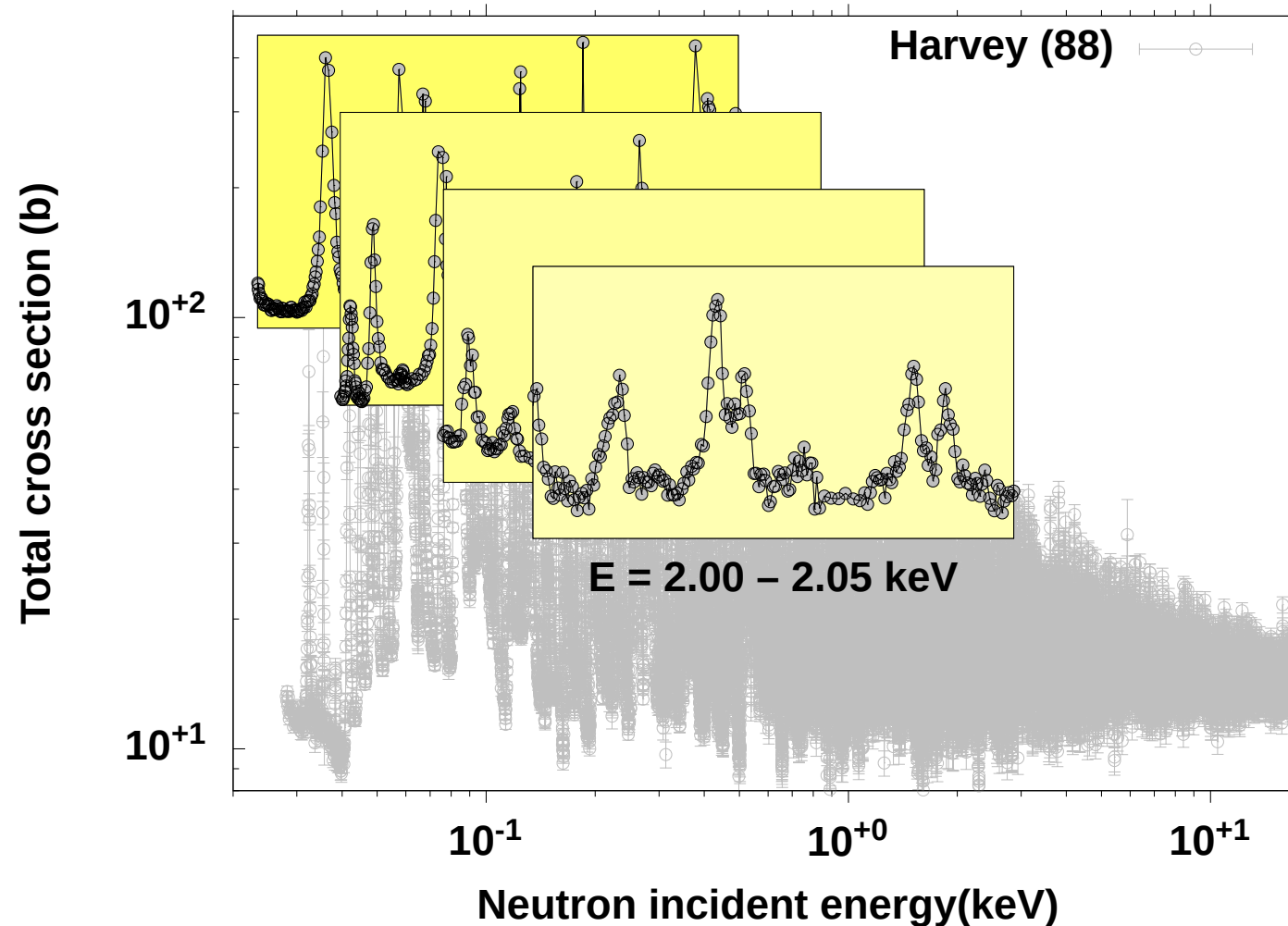
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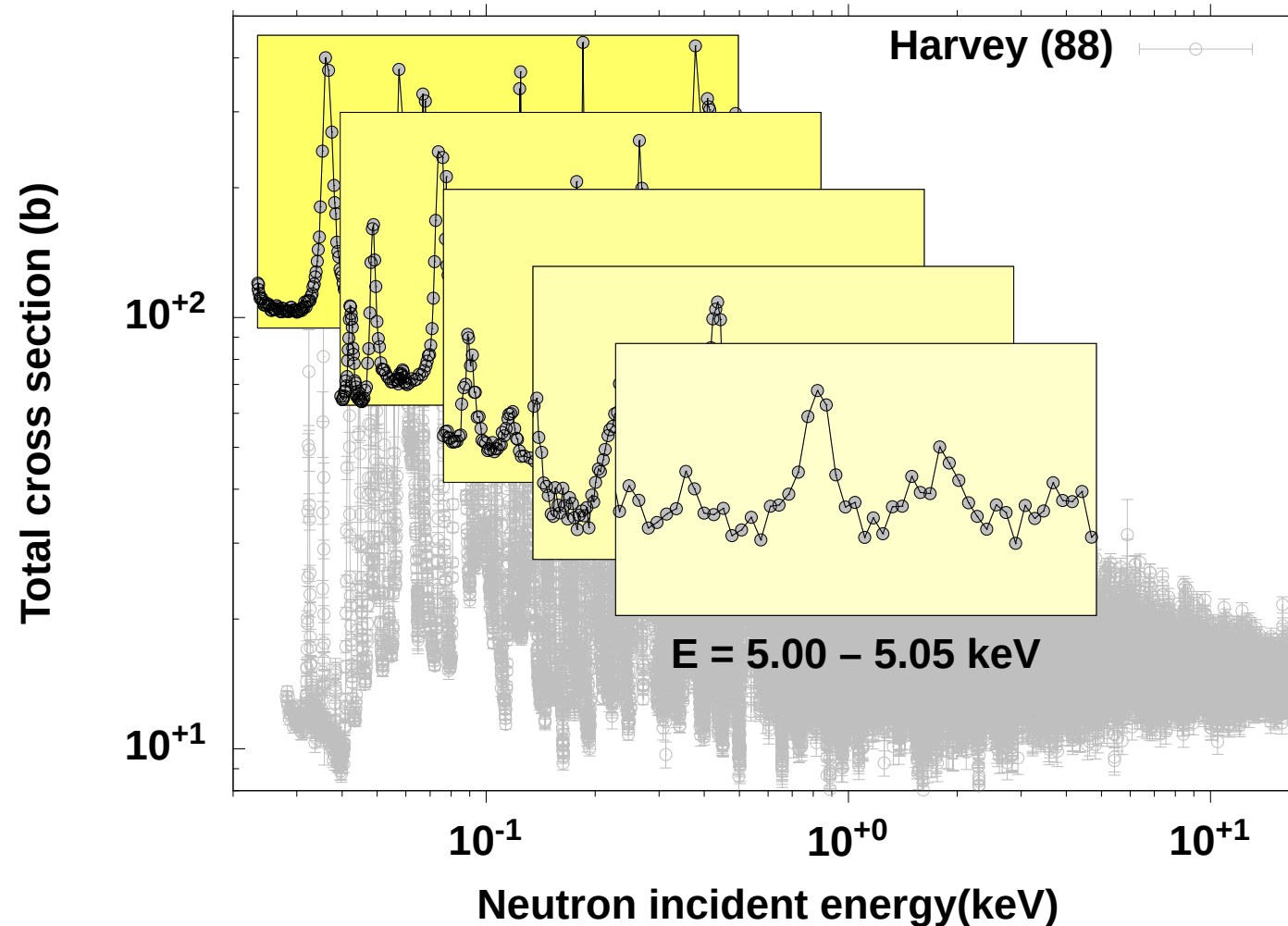
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## Fluctuations in Measured Cross Section ( $n+^{239}\text{Pu}$ )



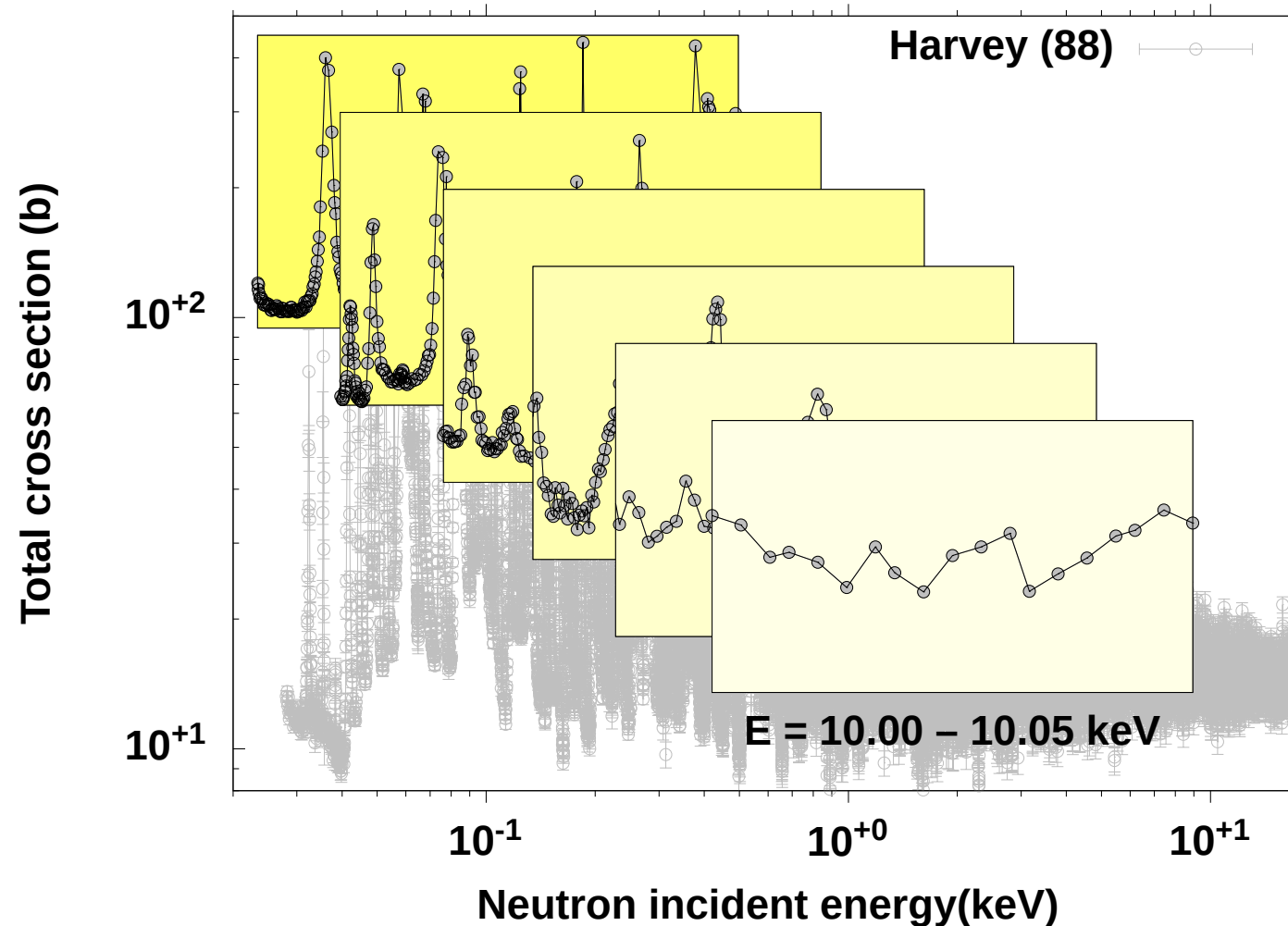
# Experimental Data

## Fluctuations in Measured Cross Section ( $n+^{239}\text{Pu}$ )



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## Fluctuations in Measured Cross Section ( $n+^{239}\text{Pu}$ )





# Theory: S-matrix Elements

## Reich-Moore Representation of the Reduced R-matrix Function

All physical quantities defined by  $\gamma_\lambda$  (reduced-width amplitudes) and  $E_\lambda$  (resonance energies)

$$R_{cc'}(E) = \sum_{\lambda} \frac{\gamma_{\lambda c} \gamma_{\lambda c'}}{E_{\lambda} - E - i\gamma_{\lambda}} \quad (1)$$

and the S-matrix elements are given by

$$S_{cc'}(E) = e^{-i(\phi_c + \phi_{c'})} \sqrt{P_c} \{ [\mathbf{1} - \mathbf{R}(\mathbf{L} - \mathbf{B})]^{-1} [\mathbf{1} - \mathbf{R}(\mathbf{L}^* - \mathbf{B})] \}_{cc'} \sqrt{P_{c'}}, \quad (2)$$

where  $\phi_c$  is the hard-sphere phase-shift and  $L_{cc'} = (S_c + iP_c)\delta_{cc'}$  is related to  $S_c$  (hard-sphere shift factor) and  $P_c$  (hard-sphere penetrability) for chosen  $B_c$  (boundary parameters)

The cross section for the entrance channel  $c$  (neutron) and exit channel  $c'$  (neutron,  $\gamma$ , ...) is

$$\sigma_{cc'}(E) = \frac{\pi}{k_c^2} g_c |\delta_{cc'} - S_{cc'}|^2 \quad (3)$$

where  $g_c$  is the statistical spin factor

# Link S-matrix Function to Average Total Cross Sections

## Lorentzian Energy Averaging

The total cross section averaged over the energy interval  $I$  can be calculated from the average S-matrix elements  $S_{cc}$  with a Lorentzian weight function

$$\langle S_{cc}(E) \rangle = \int_{-\infty}^{+\infty} dE' \frac{I/\pi}{(E' - E)^2 + I^2} S_{cc}(E') \quad (4)$$

Since there are no poles above the real axis due to the causality of the S matrix

$$\langle S_{cc}(E) \rangle = S_{cc}(E + iI) \quad (5)$$

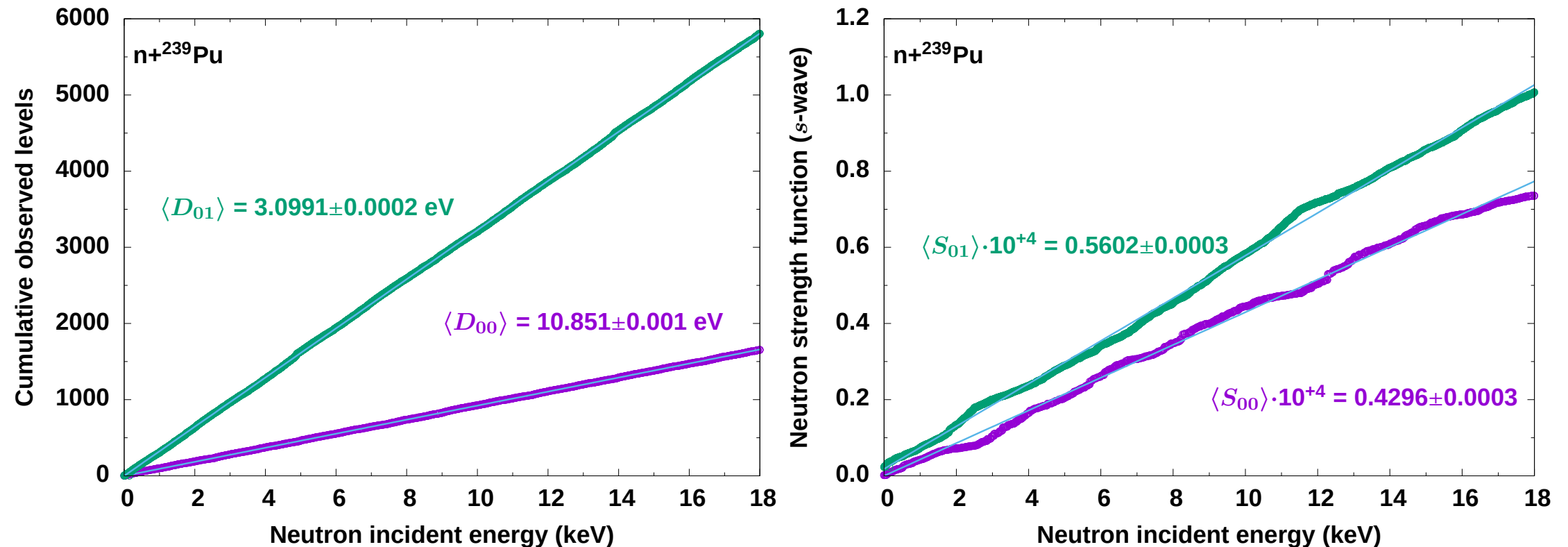
A common approximation is to average only the R-matrix function

$$\langle S_{cc}(E) \rangle \approx e^{-2i\phi_c} \sqrt{P_c} [1 - \langle R_{cc} \rangle (L_c - B_c)]^{-1} [1 - \langle R_{cc} \rangle (L_c^* - B_c)] \sqrt{P_c}, \quad (6)$$

where  $\langle R_{cc}(E) \rangle \equiv R_{cc}(E + iI)$  and the energy dependence of  $\phi_c, P_c$  and  $L_c$  is neglected

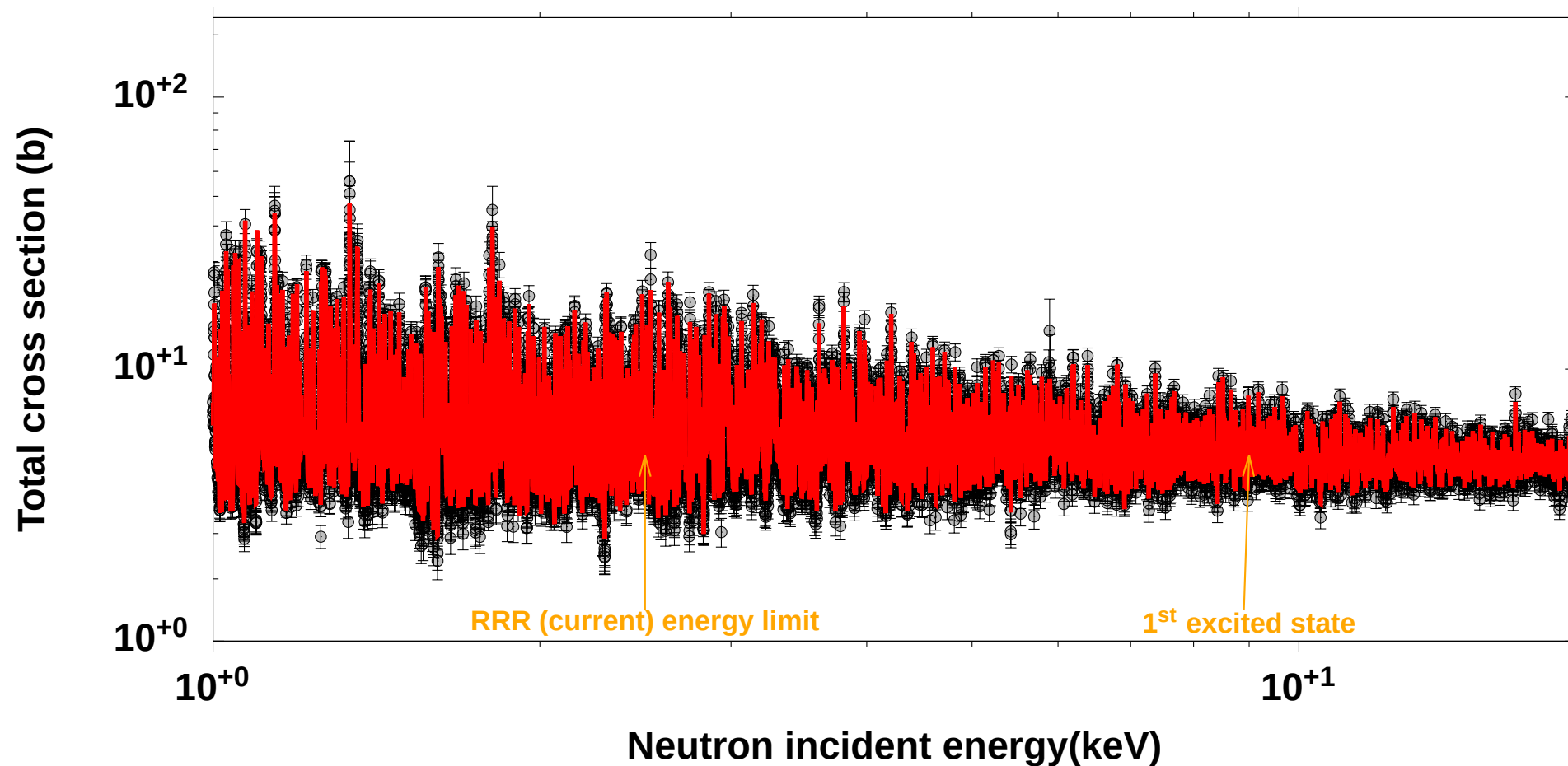
# Extrapolating Energy Levels and Resonance Amplitudes

## Statistics from Resonance Parameters below 2.5 keV



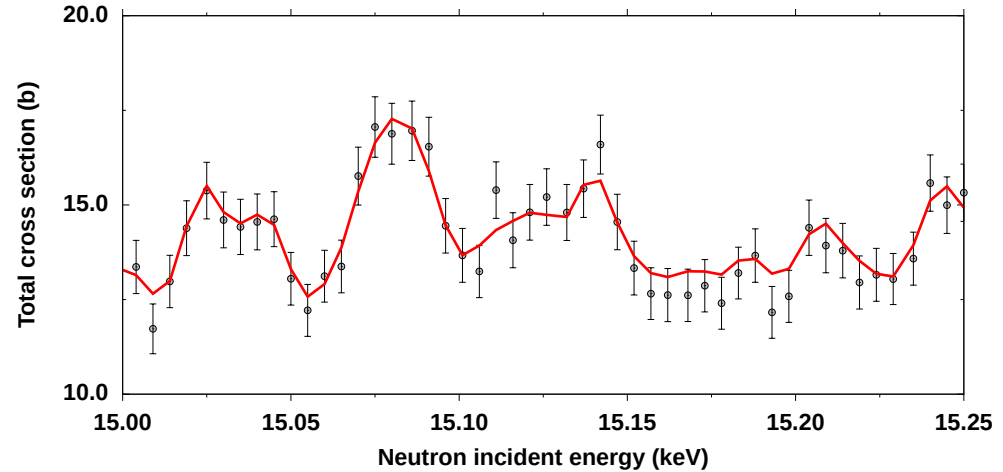
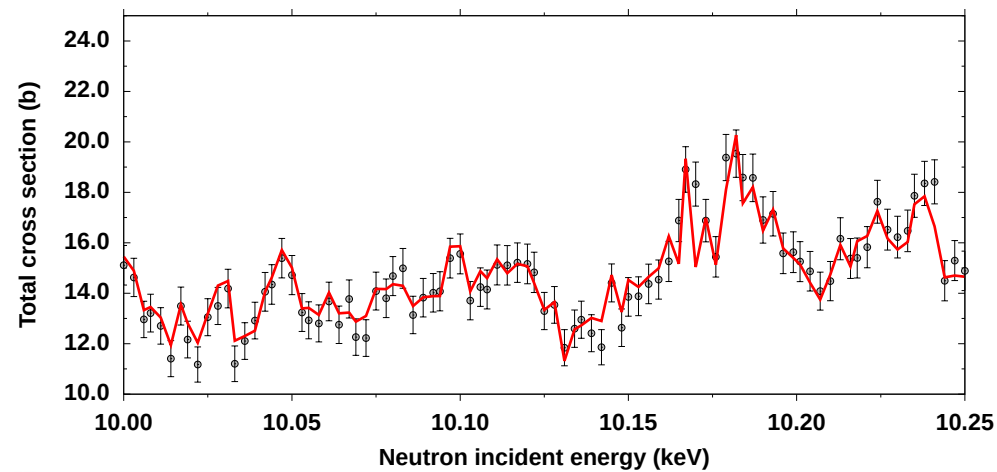
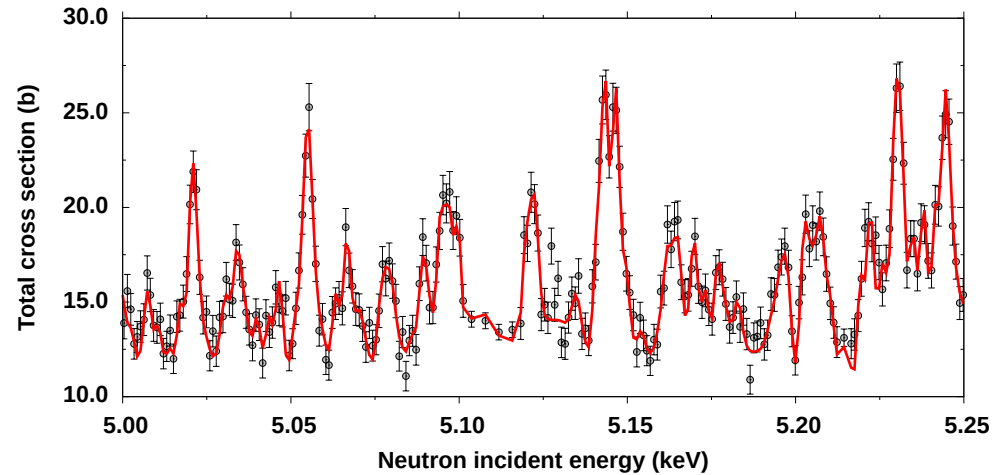
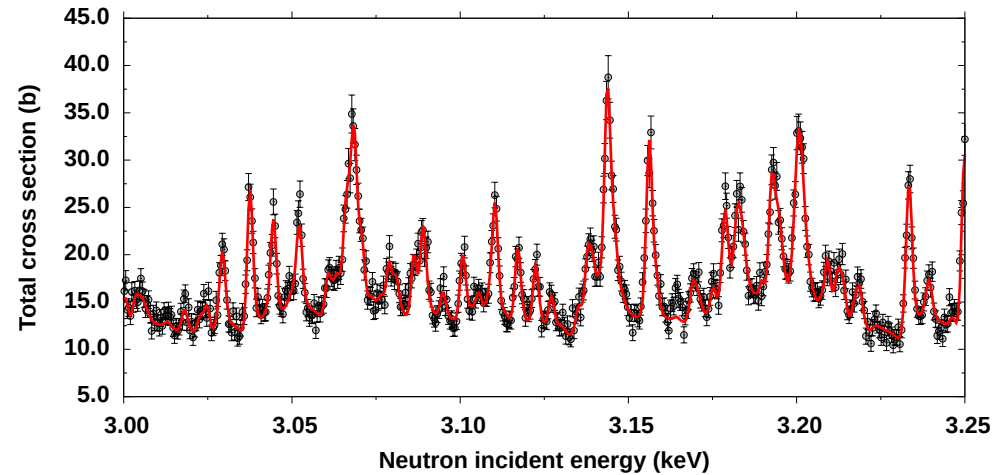
# $^{239}\text{Pu}$ Total Cross Section in the Resonance Region

## Fit to Measured Data

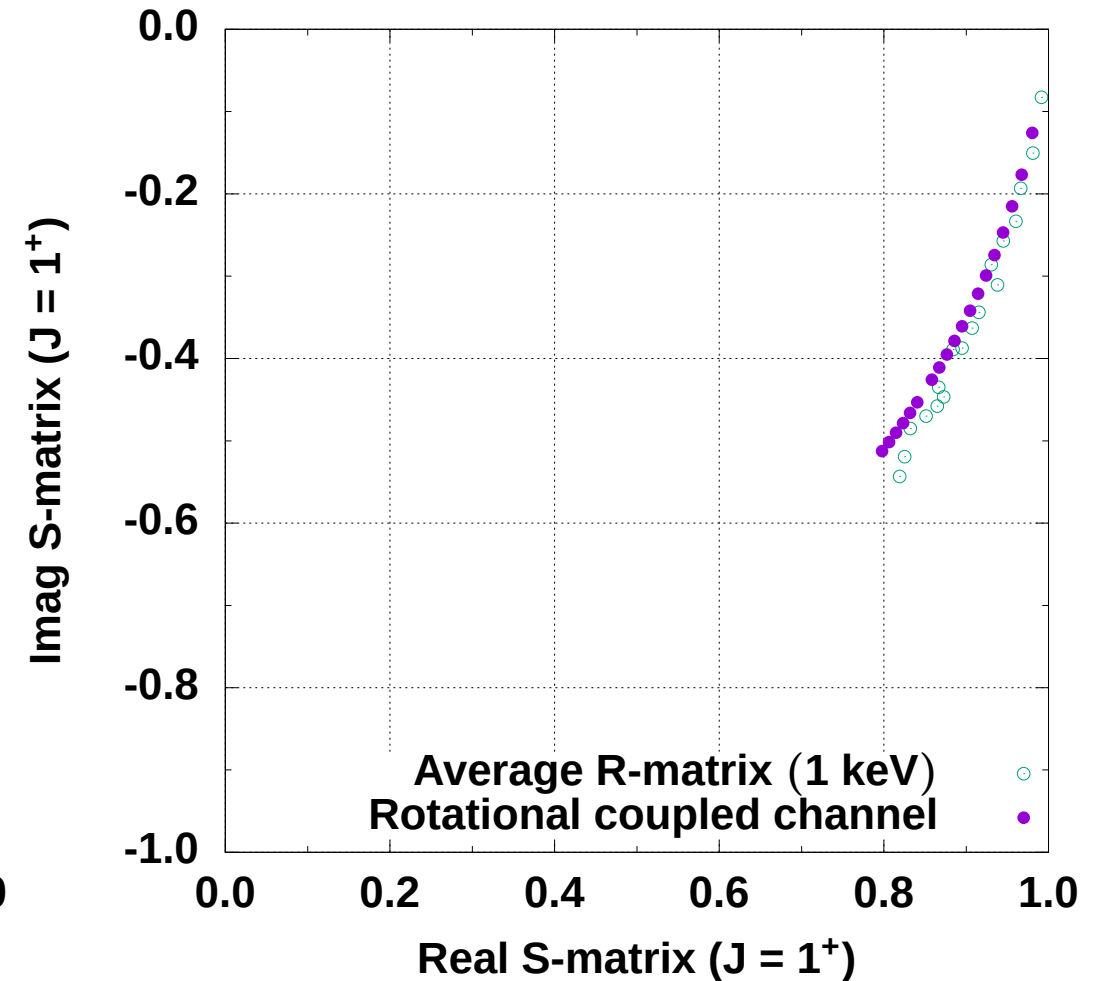
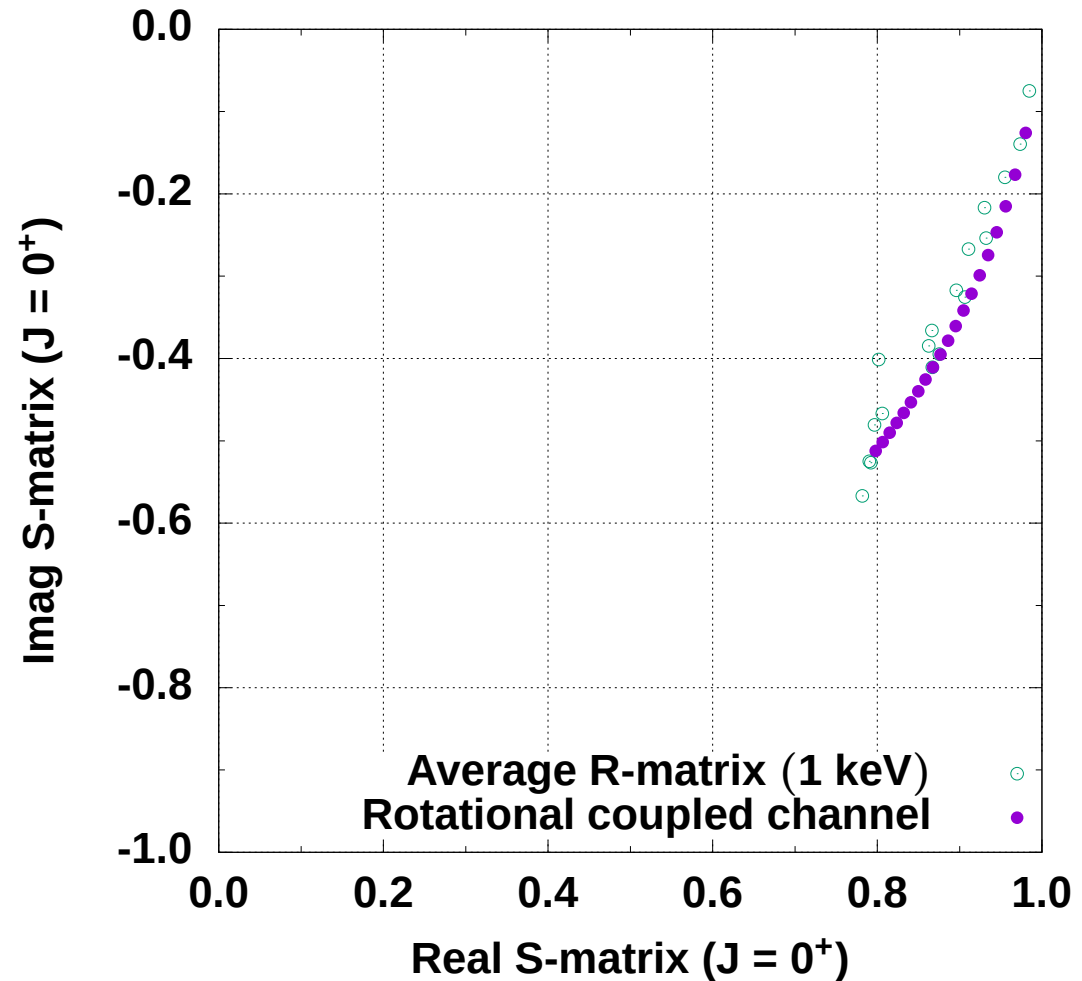


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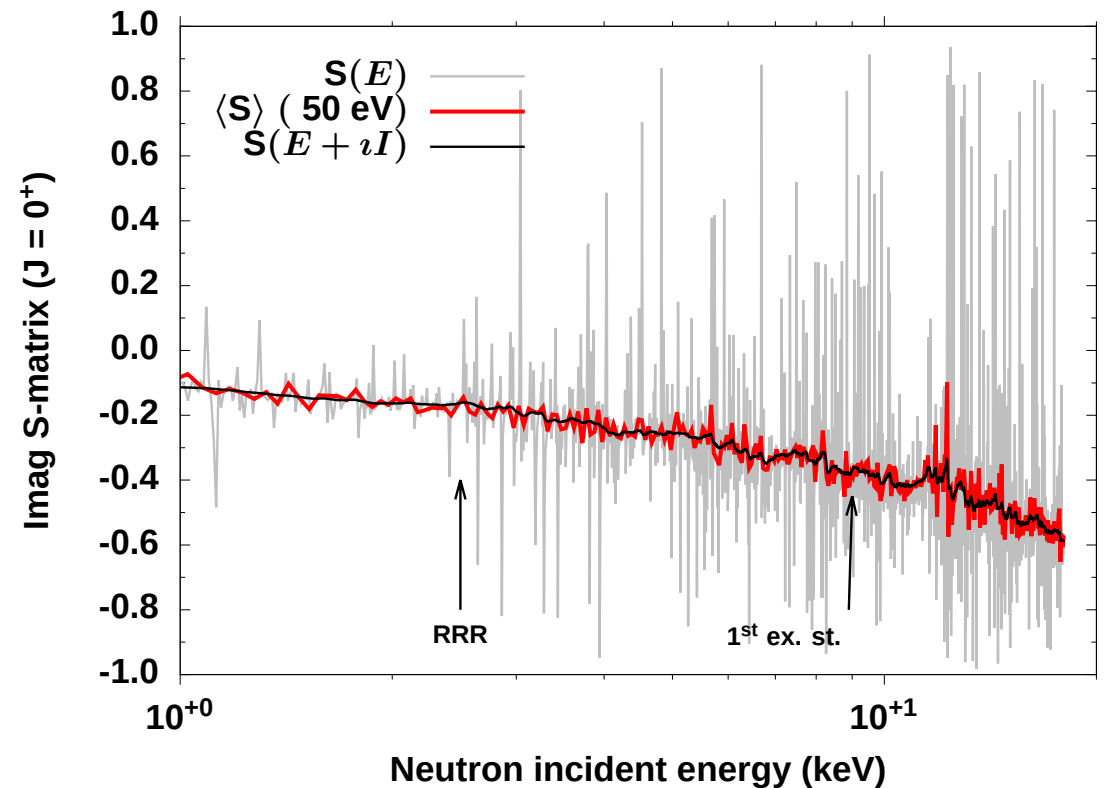
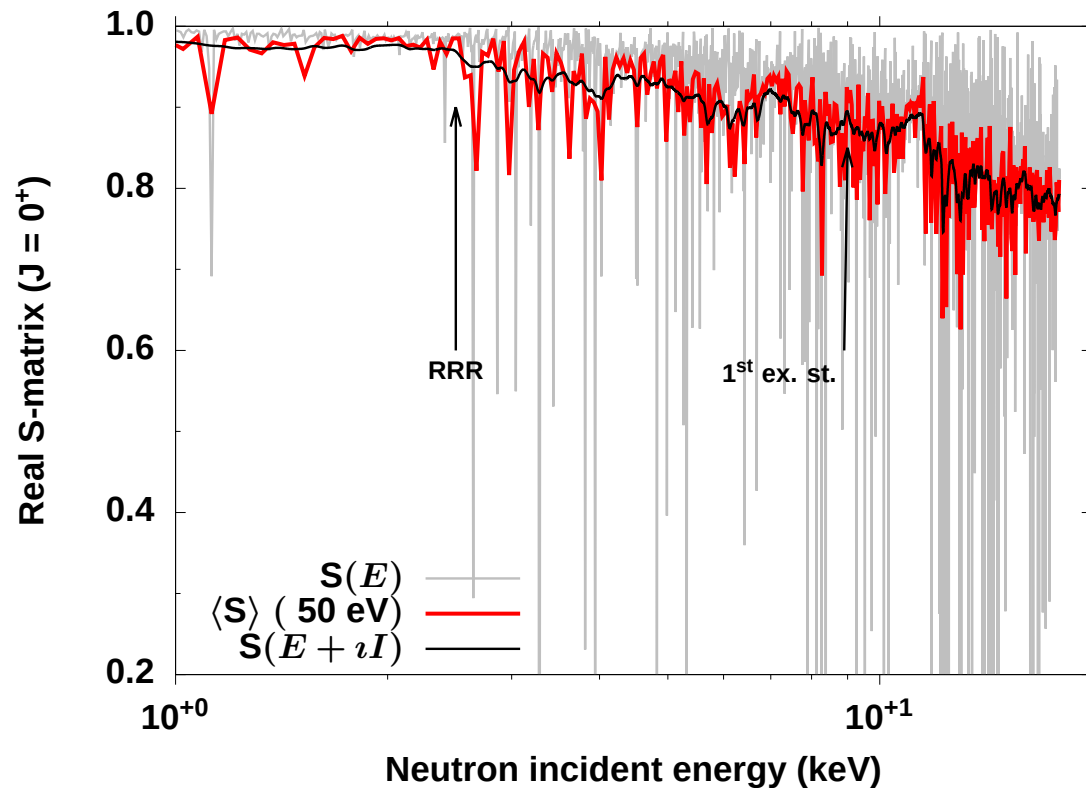
# Link R-matrix to Optical Model Calculations





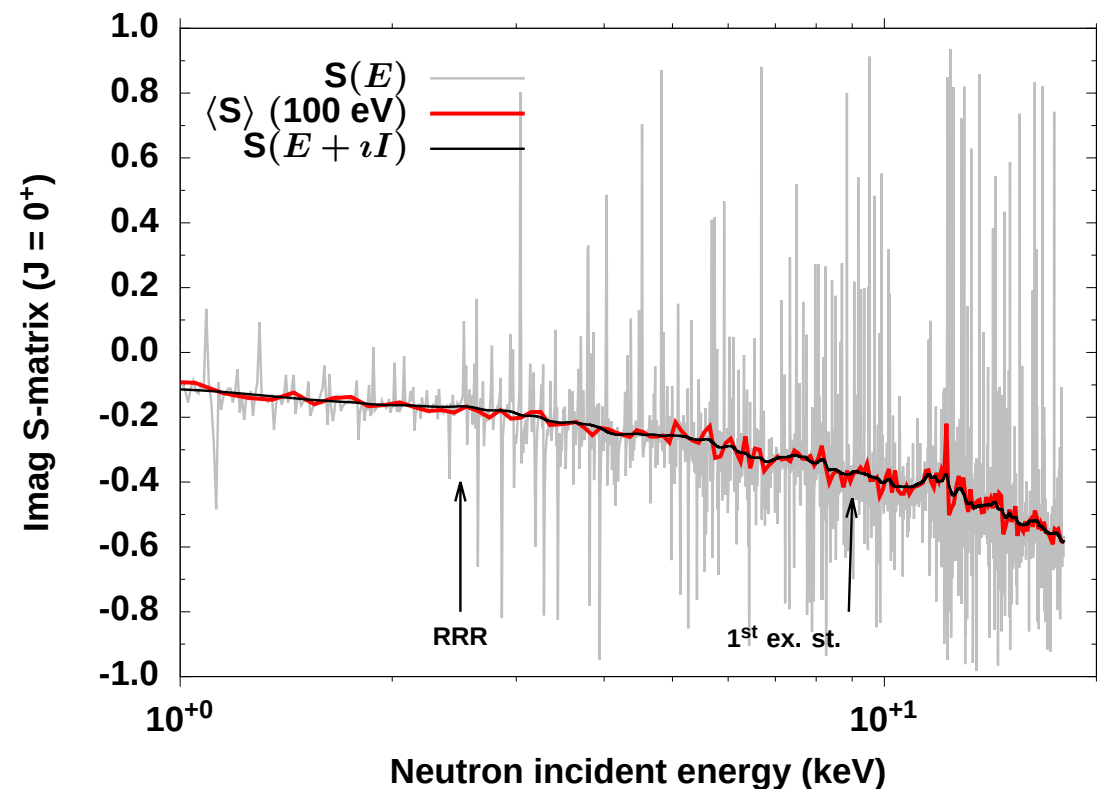
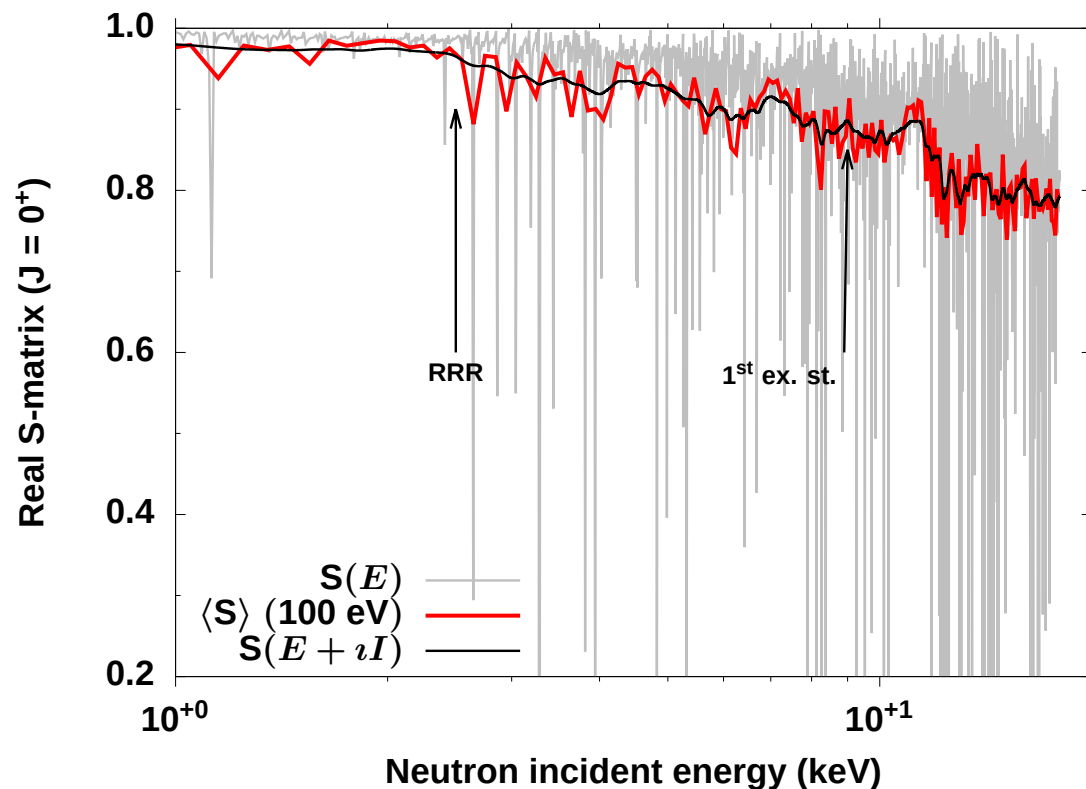
# S-matrix Elements from Extrapolated Populations

## Real and Imaginary Components ( $J=0^+$ Population)



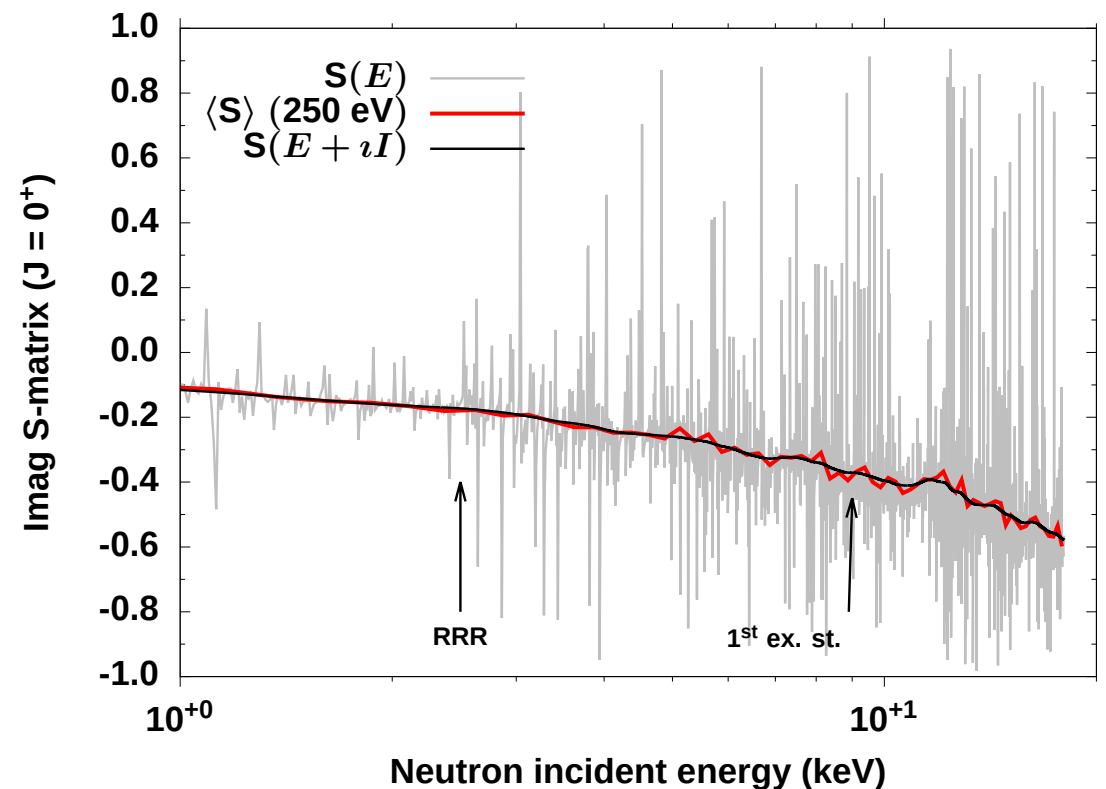
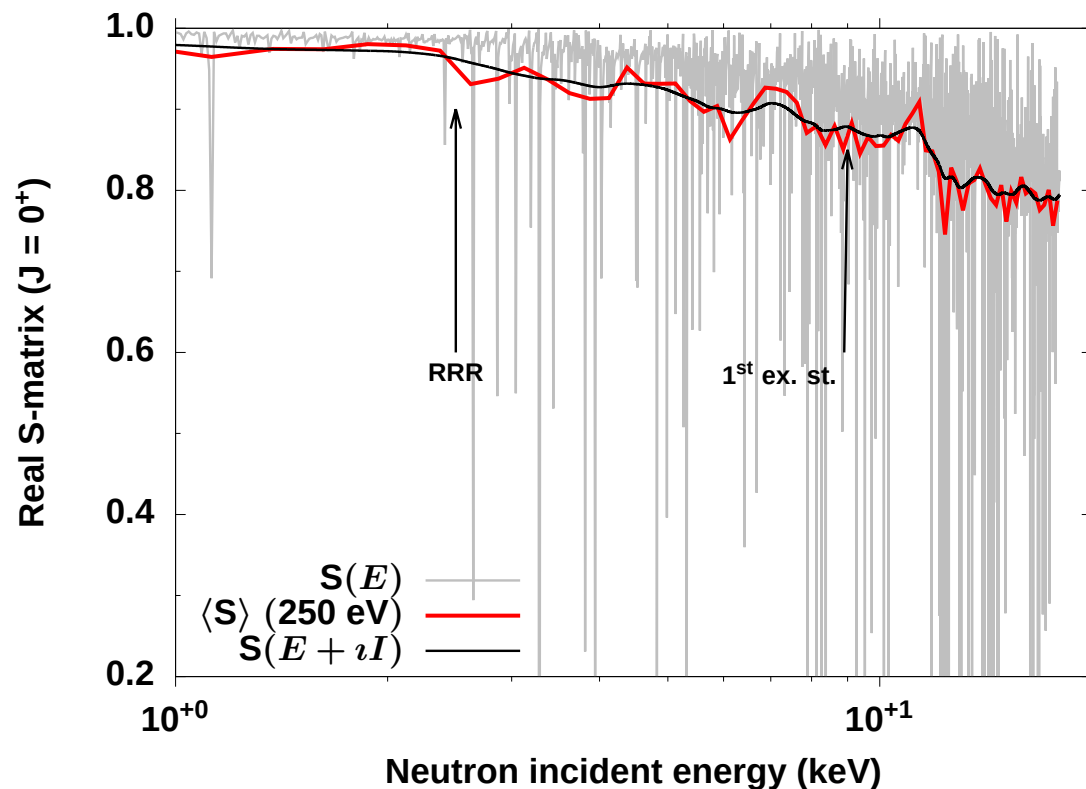
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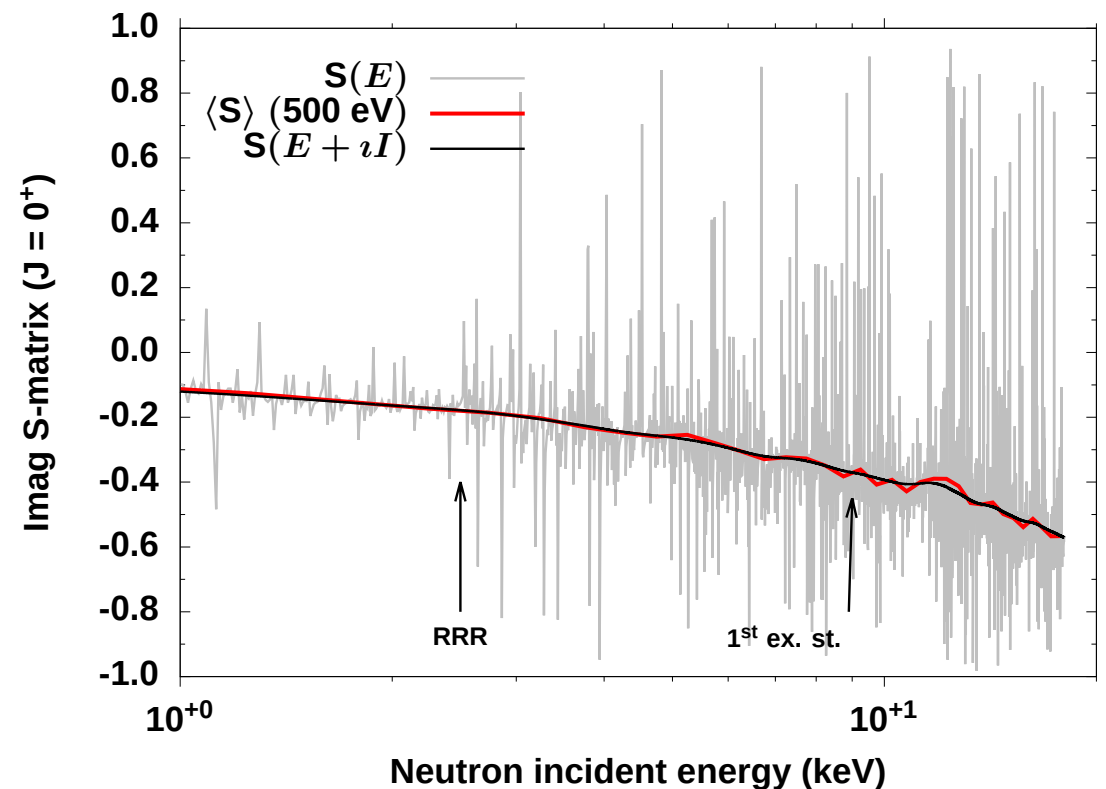
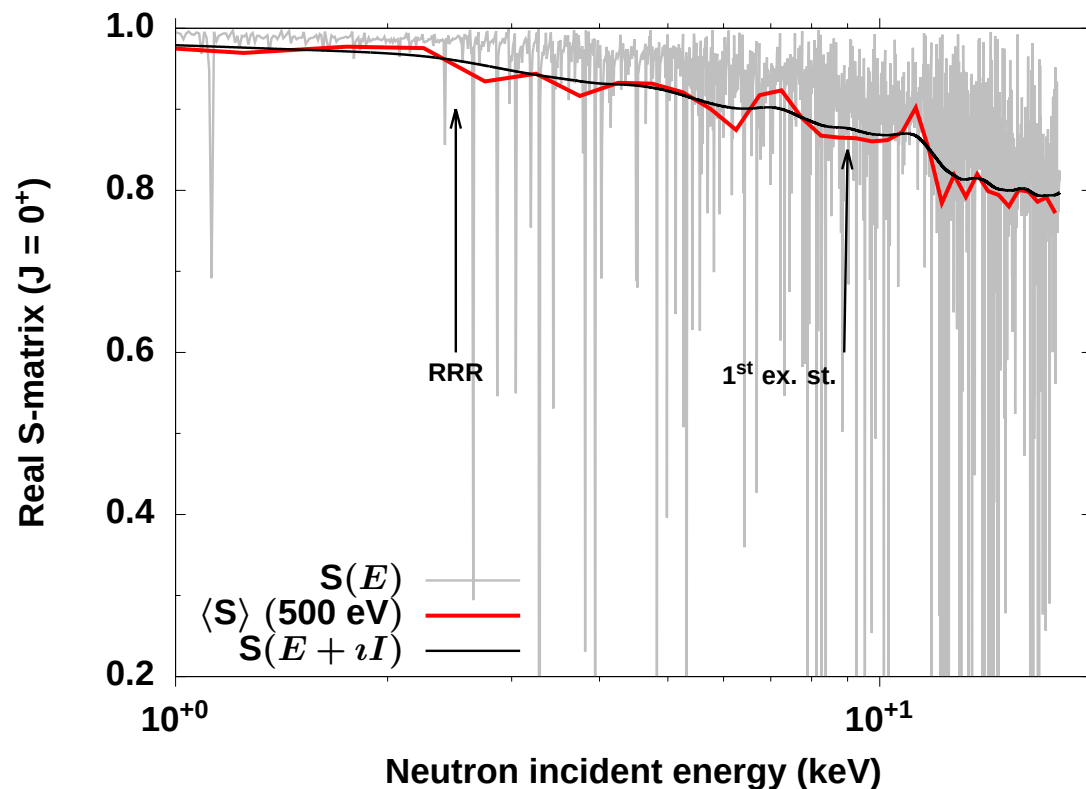
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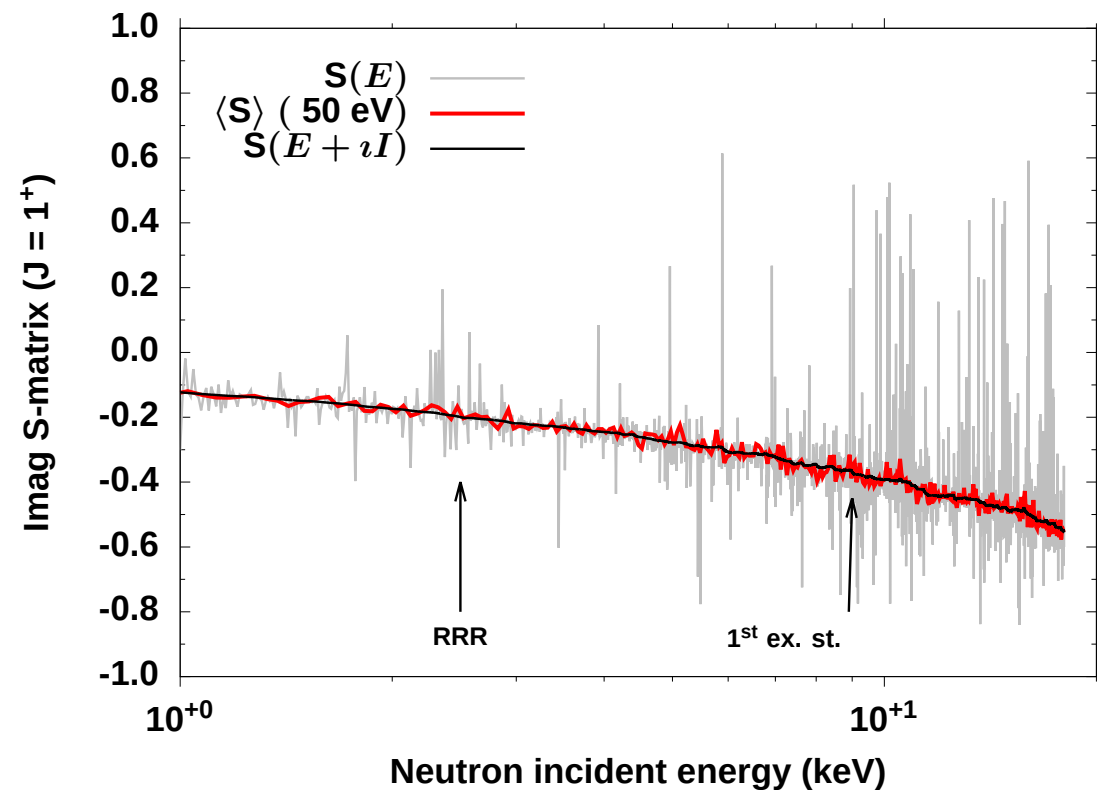
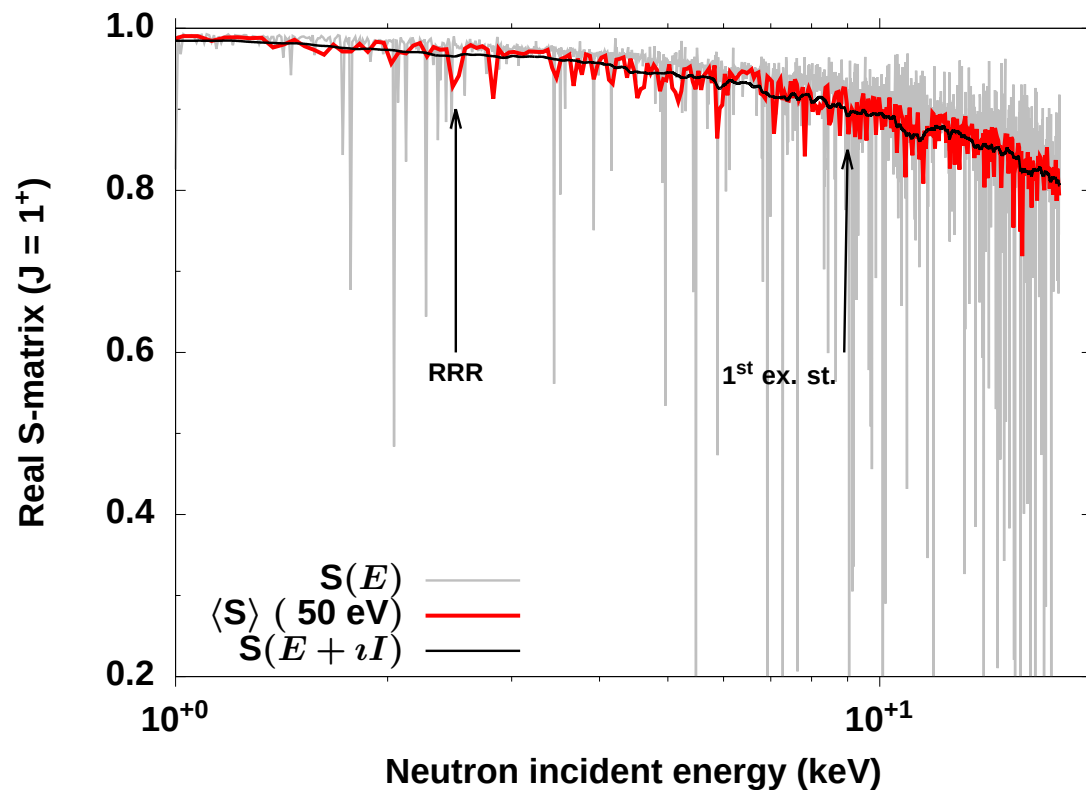
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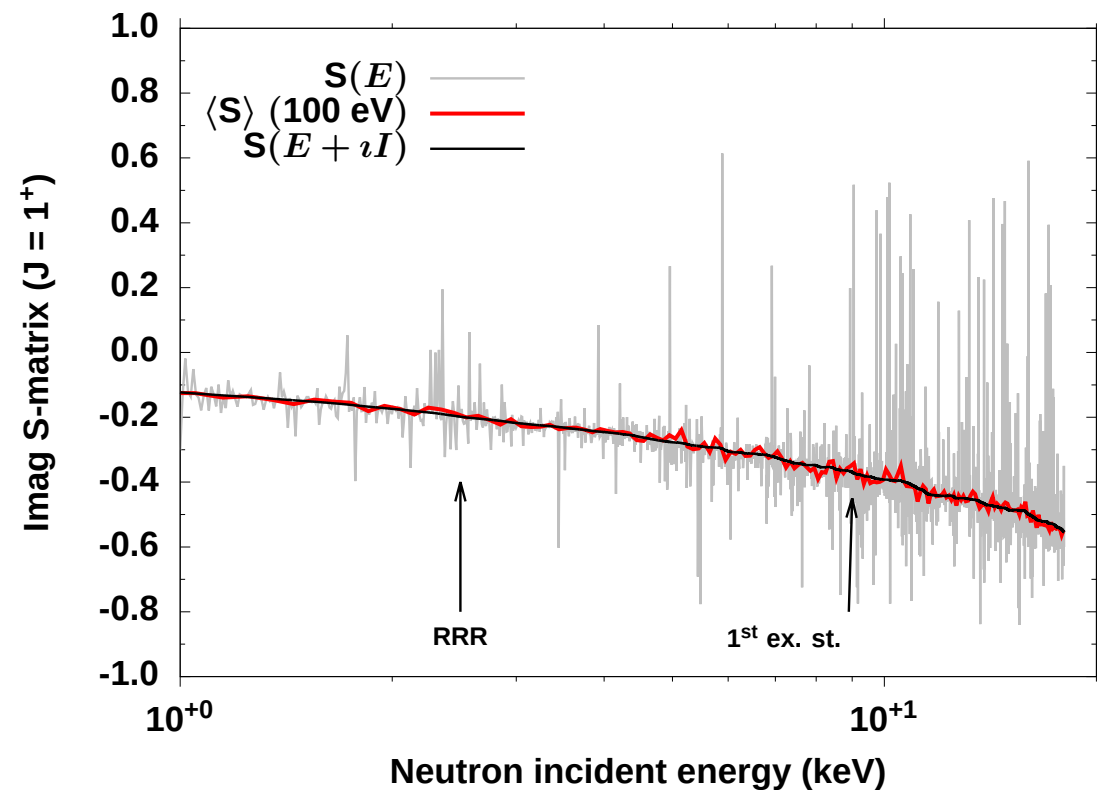
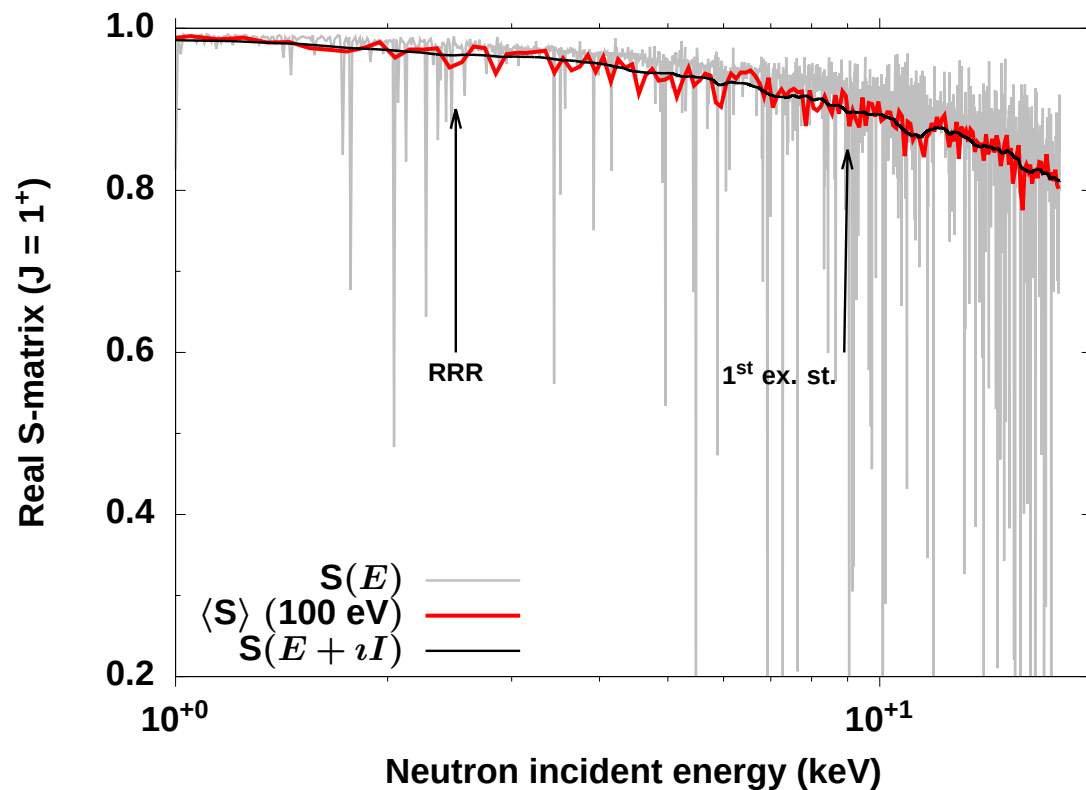
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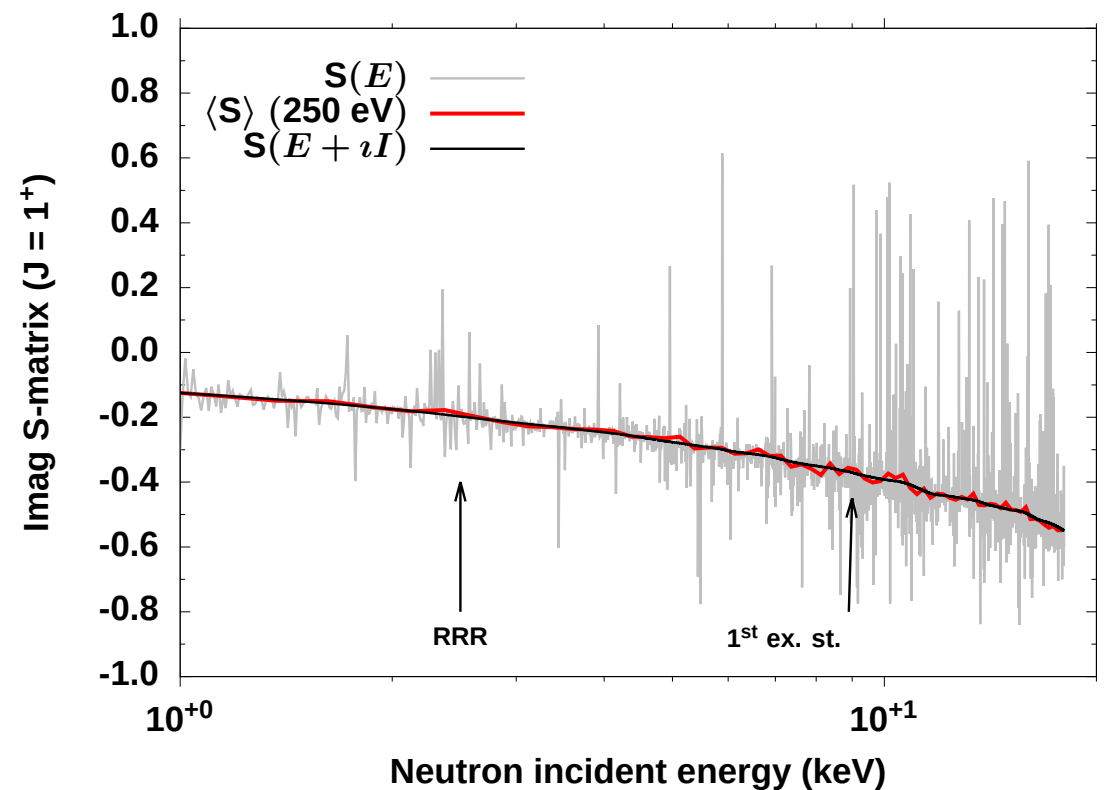
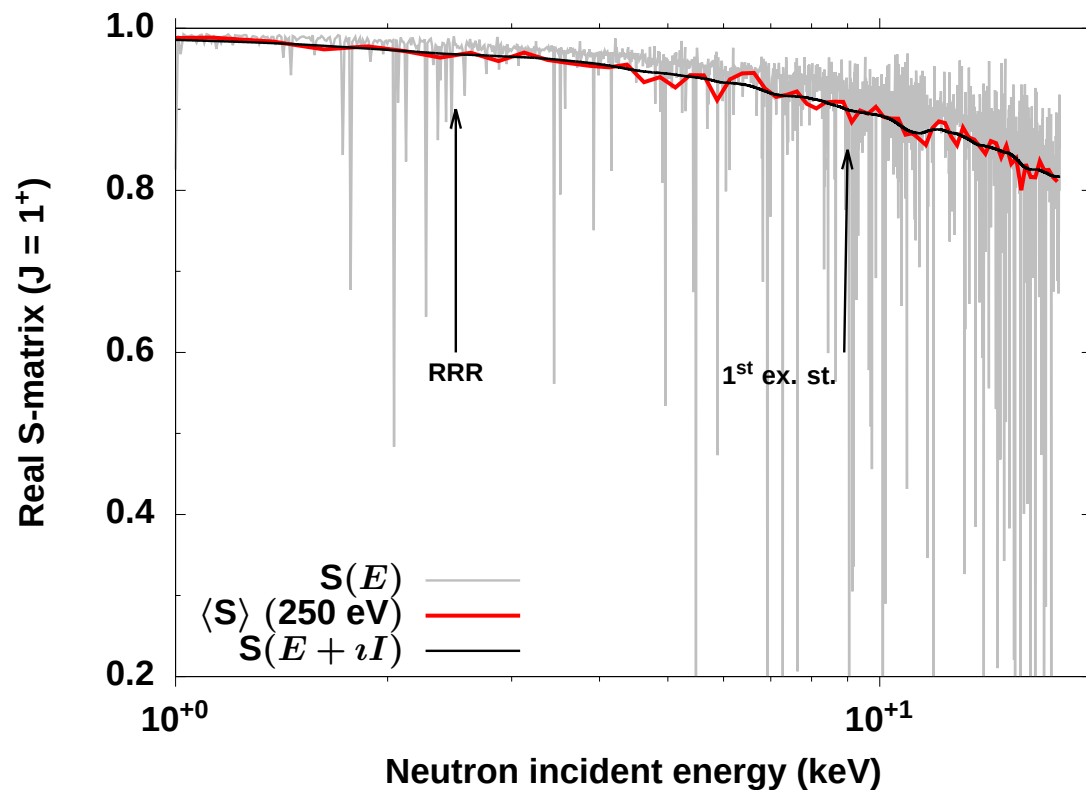
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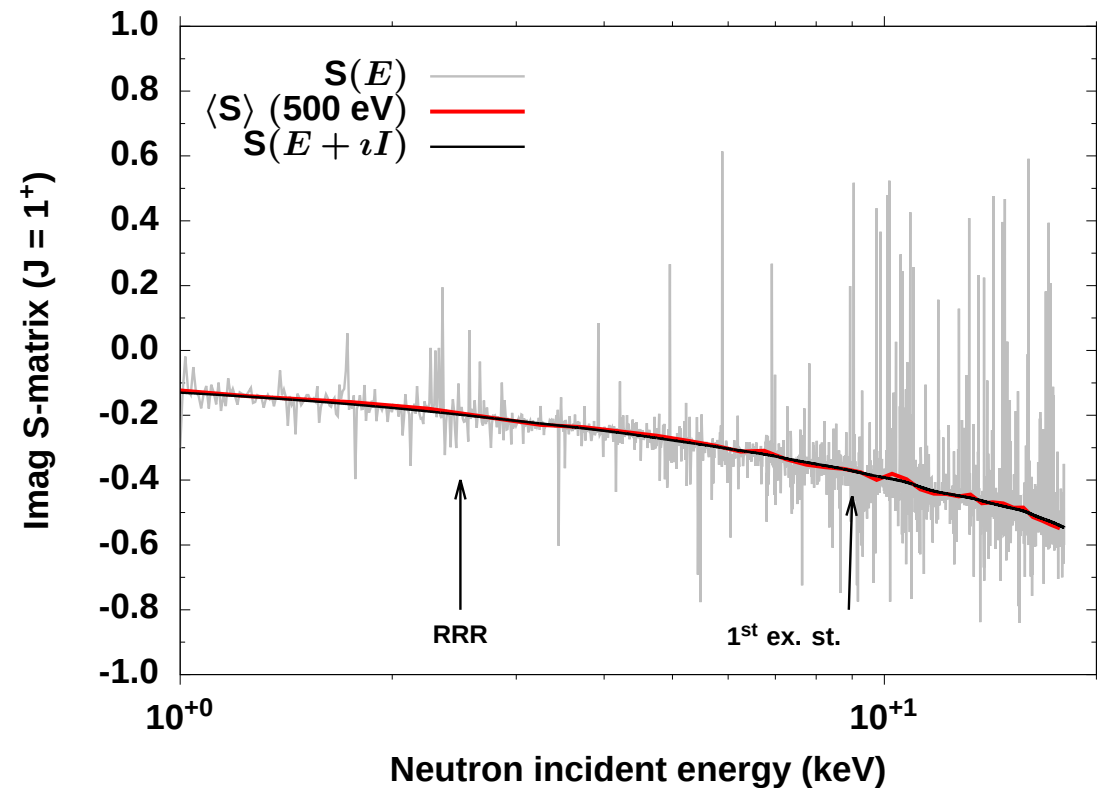
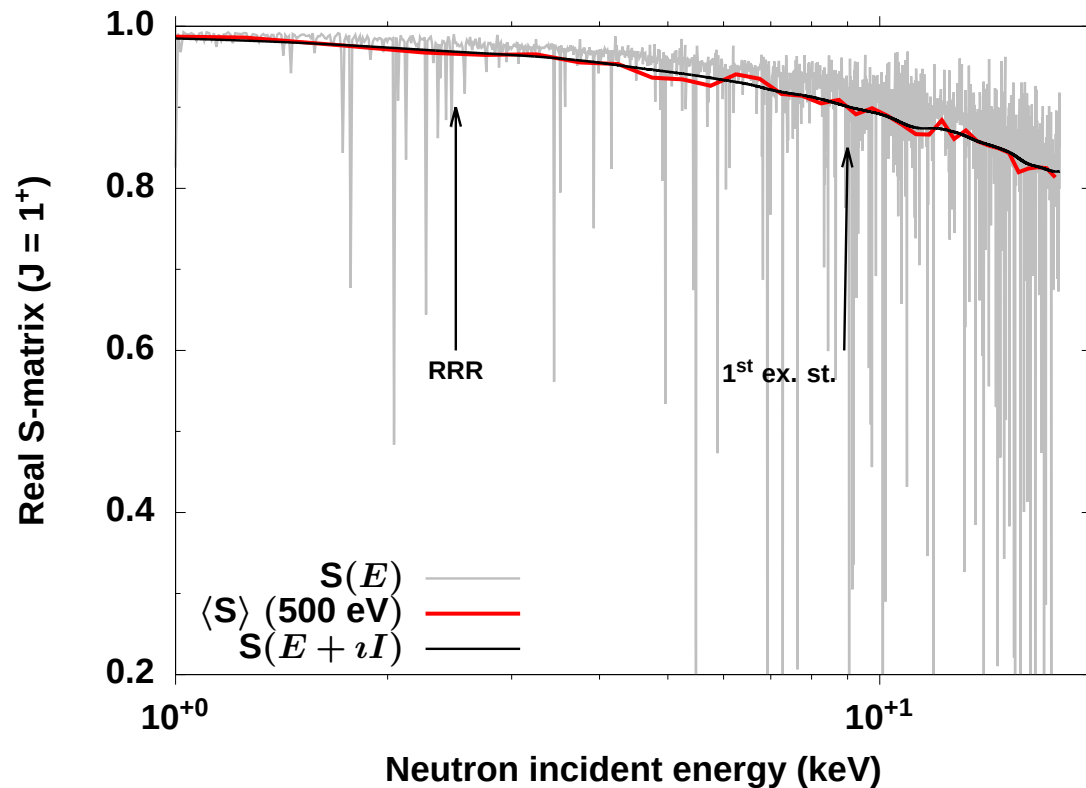
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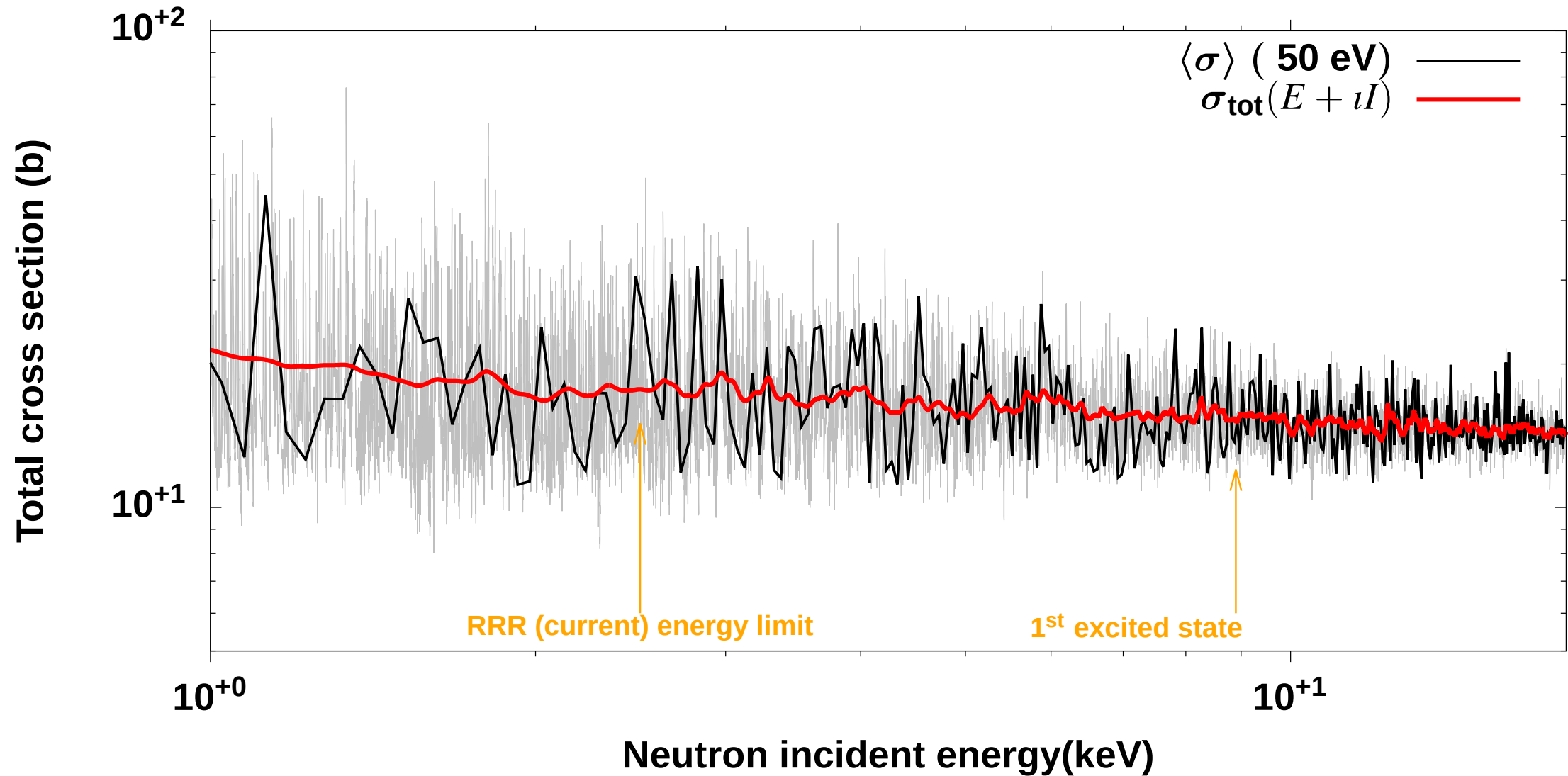


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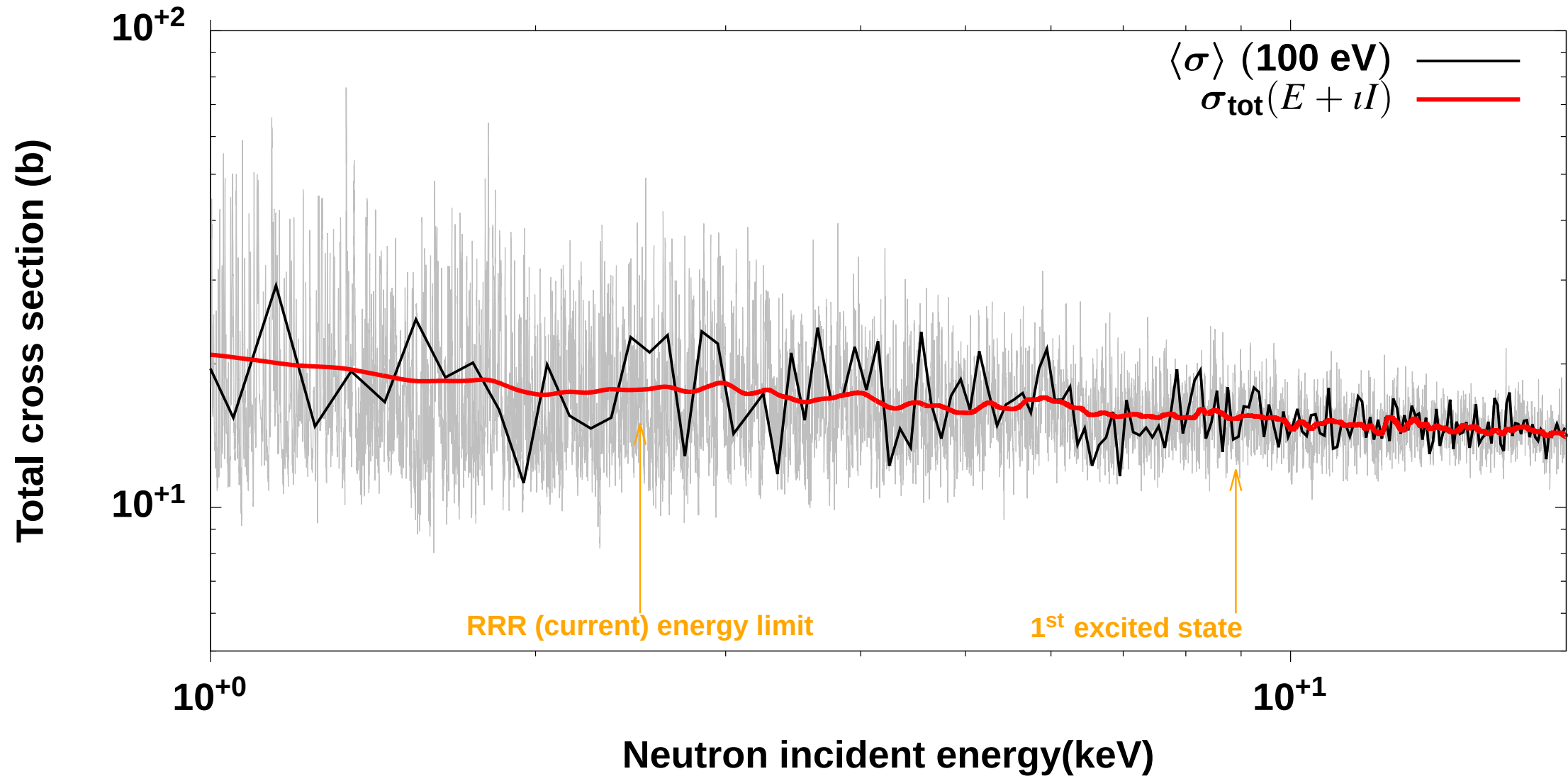
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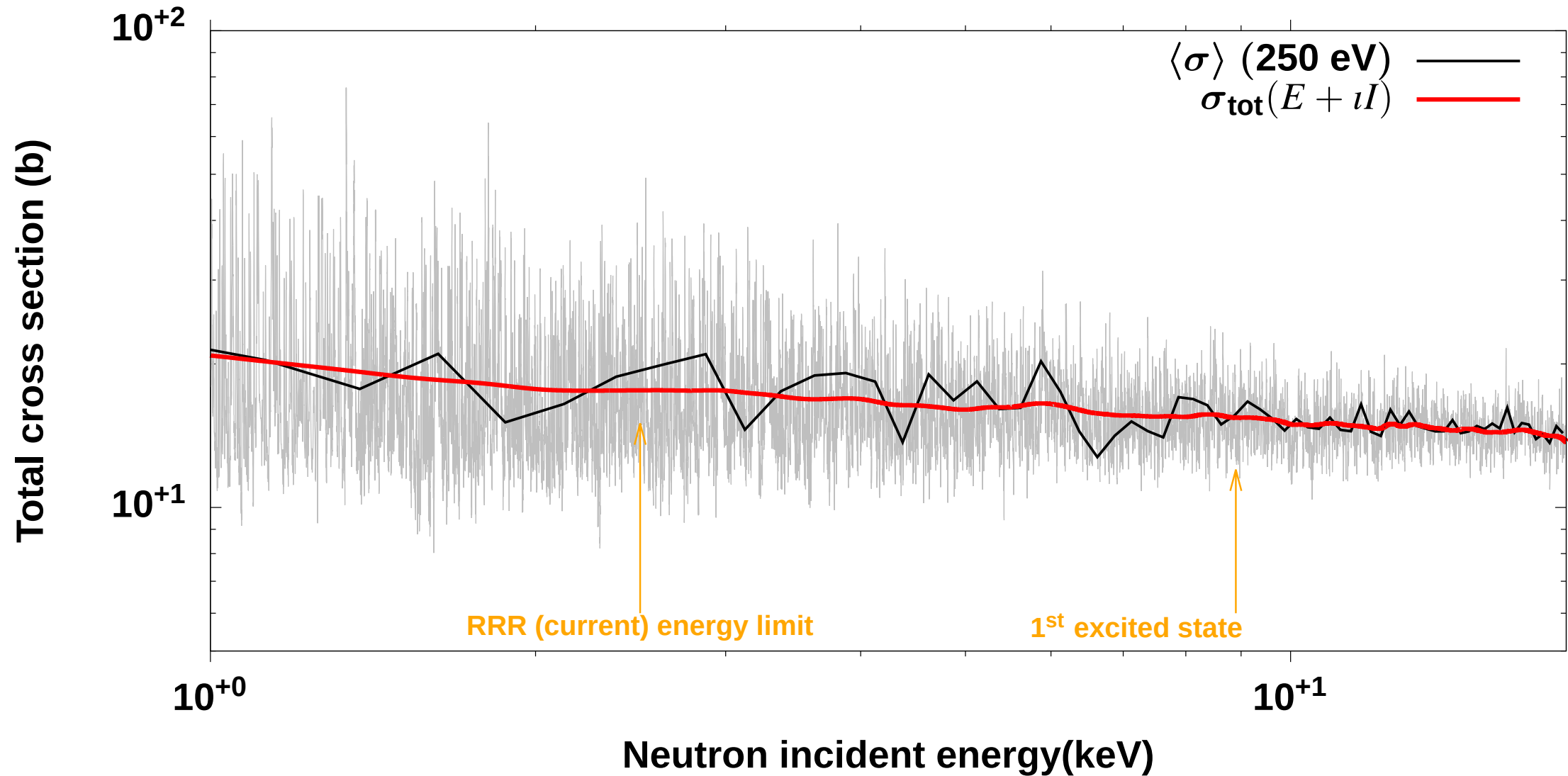
# $^{239}\text{Pu}$ Energy-average Total Cross Section



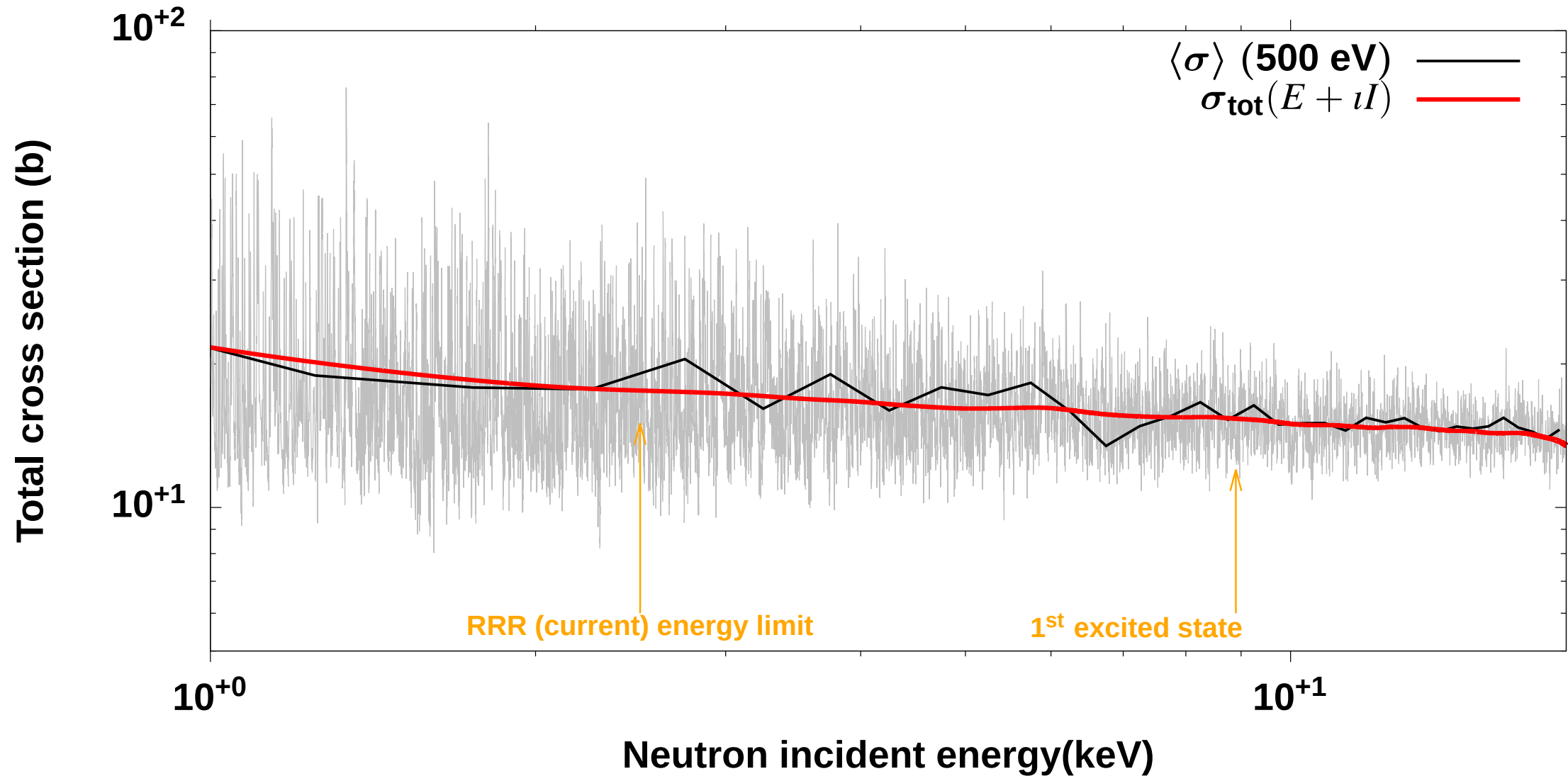
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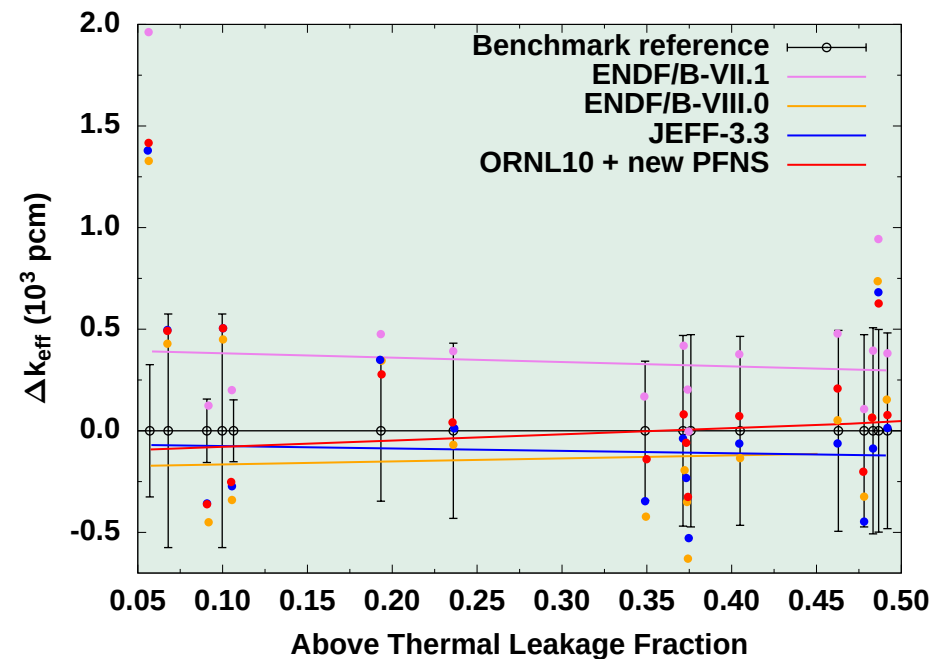
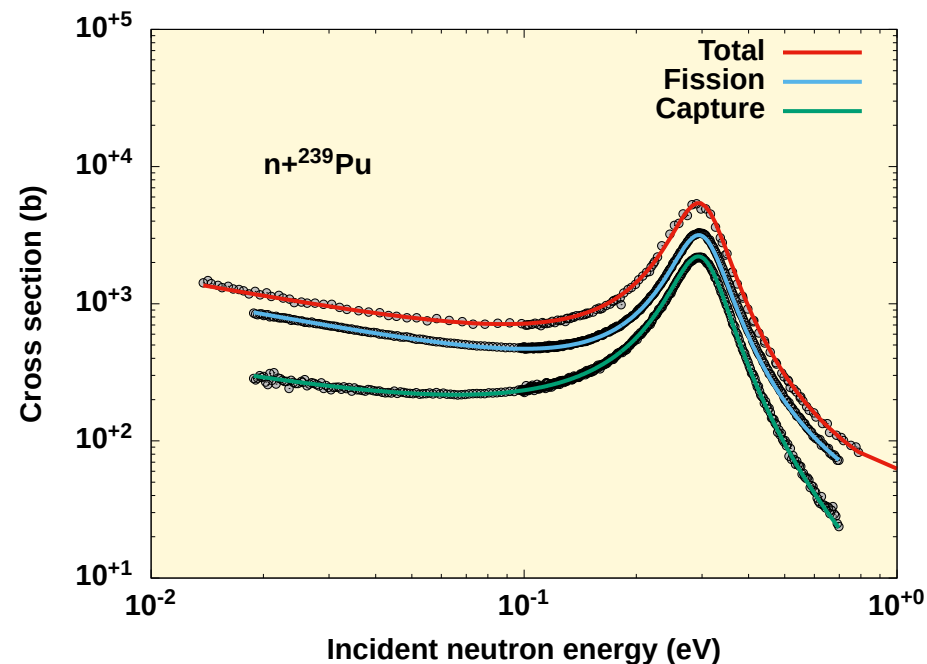


# Resolving Key Issues in the Intermediate Structure Region

- The intermediate structure observed in the measured data can be described by including in the evaluated data files average cross sections calculated from the average S-matrix function
- With a proper energy grid the average cross sections can be broadened and calculated at the desired temperature
- In the case of selected applications for which the temperature dependence is particularly important and sensitive to the fluctuations, the resonance parameters could be used instead of the average cross sections
- Theoretical development to rigorously quantify the shape and compound component of the cross sections might be still needed

# Coupling the Thermal and RRR Region to New PFNS

- **Background:** The recently released ENDF/B-VIII.0 was based on evaluations performed within the international collaboration CIELO aiming to improve nuclei of fundamental importance such as  $^{235}\text{U}$  and  $^{239}\text{Pu}$ . The  $^{235}\text{U}$  R-matrix evaluation (ORNL) was updated with the latest thermal constants and prompt fission neutron spectra (PNFS) improving the benchmark performance of the thermal solutions. However, for  $^{239}\text{Pu}$  evaluation the focus was in the high energy range and the prediction on the thermal solution benchmarks was underpredicted
- **Results:** Within IAEA coordinated research activities, newly evaluated PFNS showed a reduction of 1.8% on the average energy : PFNS( $\langle E_{\text{av}} \rangle = 2.08 \text{ MeV}$ ). This changes were combined to recent work on  $^{239}\text{Pu}$  R-matrix evaluation (ORNL) aimed to update the thermal constants. This led to improved benchmark performance in the thermal solutions



# Conclusion and Remarks

- A procedure to accurately model the measured cross section fluctuations for inclusion into evaluated nuclear data files was developed and tested on  $^{239}\text{Pu}$  total cross sections
- The procedure consisted on defining population of levels as well as related amplitudes extrapolated from the systematics of the resonance parameters in the resolved resonance region
- By using realistic population of extrapolated levels, the fit of measured total cross section produced a continuous S-matrix function  $S(E)$  in Reich-Moore approximation
- The S-matrix function averaged over a suitable energy interval was used to obtain averaged cross sections consistent with the measured data
- In the thermal energy region preliminary results coupling the RRR and the new PFNS evaluation were generated

# Acknowledgments

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Thank you!