SAND2022-0193C

IMAC XL

IT'S NOT JUST MODAL ANYMORE

Insights on the nonlinear modeling and characterization of bolted lap joint interface of beams

Gabrielle Graves[1], Abdessattar Abdelkefi[1], Michael Ross[2]

[1]Department of Mechanical and Aerospace Engineering, New Mexico State University, Las Cruces, NM 88003, USA

[2]Sandia National Laboratories, Albuquerque, NM, 87123, USA

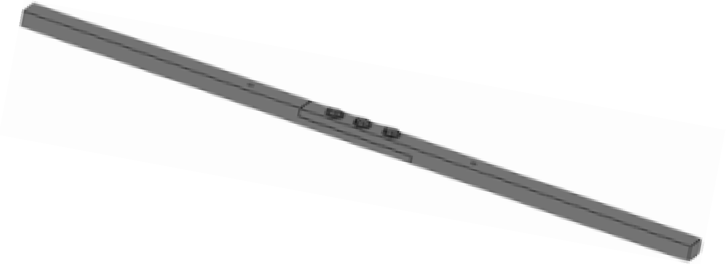
Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

Outline

- Background & Motivation
- Literature review & Significance of work
- Nonlinear reduced-order modeling
- Effects of lap joint on linear characteristics of system
- Nonlinear characterization of lap joint system
- Conclusion & Future work

Background & Motivation

- Bolted joints are used in simple and complex structures due to their short assembly time and simplicity
- Many complex structures have high consequences if failure occurs
  - Automobiles
  - Airplanes
  - Steel building frames
- Considering joint effects to accurately predict dynamics of structures is necessary due to their inherent nonlinearities



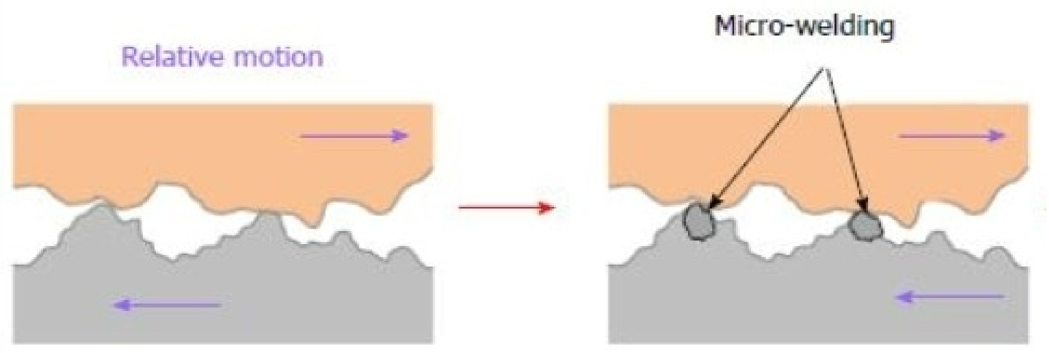
Background & Motivation

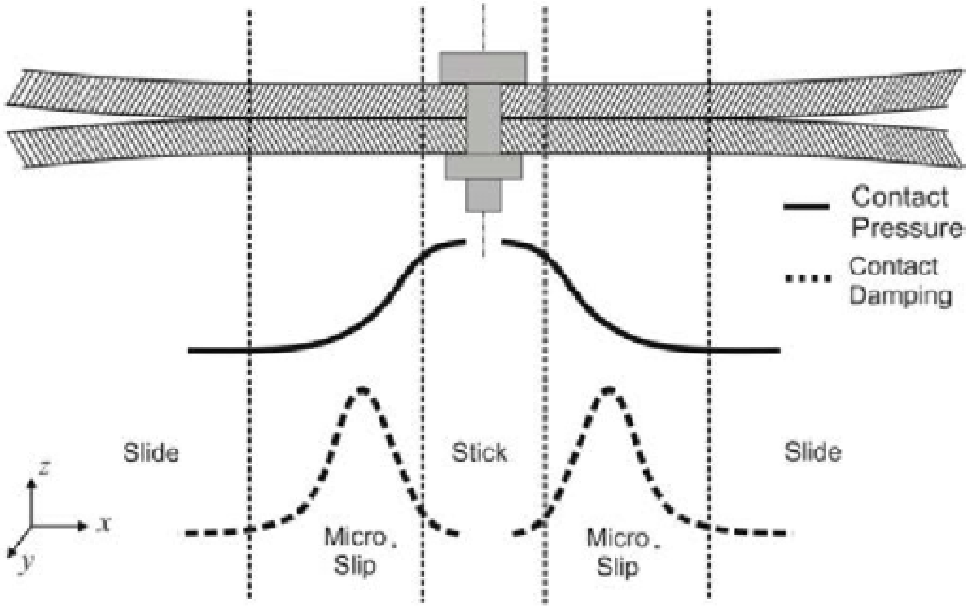
Energy Dissipative behavior and associated nonlinearities:

- Energy dissipation is not well understood
- No model for how energy is dissipated

❑ <u>Factors of Energy Dissipation from friction:</u>

- Surface Roughness
- Geometry
- Material properties
- Preload
- Propagating Stress waves
- Slippage
- Multiple Asperity Contact

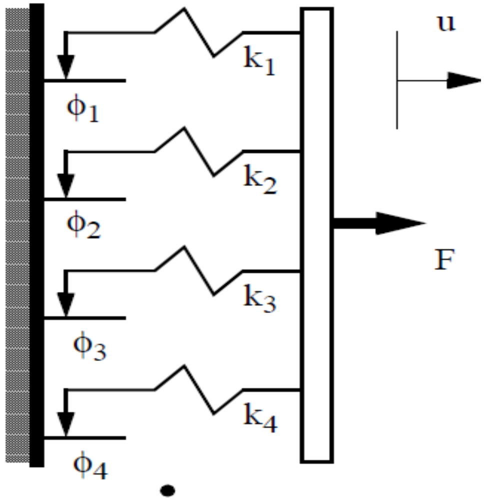




Segalman, D., Lacayo, R., Allen, M.S., Schwingshackl, C., Barber, J. (2018). Mechanics of Jointed Structures, 15-16, 26-27, 255-264

Background & Motivation

Discrete Iwan Model



Four Parameters:

1. Macro-slip: $\boldsymbol{F_s}$
2. Tangential Joint Stiffness: $\boldsymbol{K_T}$
3. Energy Dissipation Slope: χ
4. Related to shape of energy dissipation curve: β

Iwan Model are composed of Jenkins elements in parallel

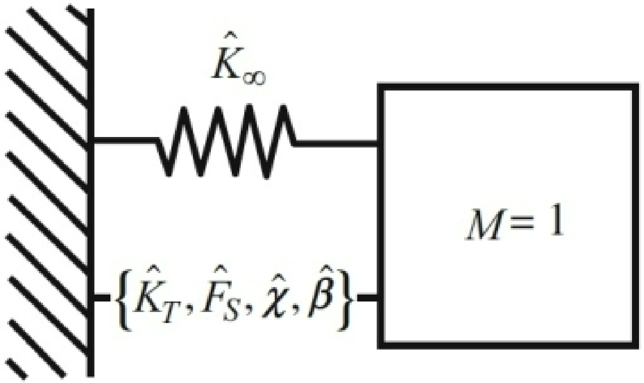
- Model Individual Joint

Jenkins Elements:

1. Frictional slider with strength ϕ
2. Linear spring

Segalman, D., Lacayo, R., Allen, M.S., Schwingshackl, C., Barber, J. (2018). Mechanics of Jointed Structures, 15-16, 26-27, 255-264

Modal Iwan Model



Models modes of the entire system with 1-DOF spring-mass-Iwan model system

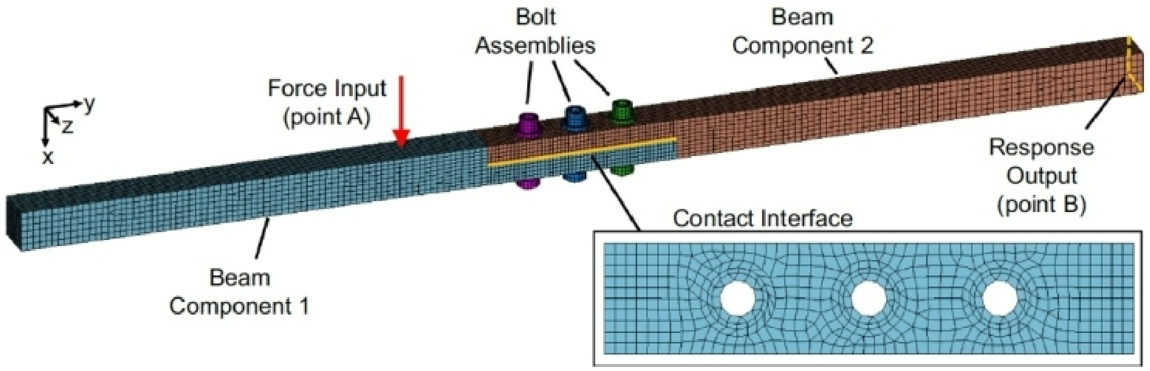
Assumptions:

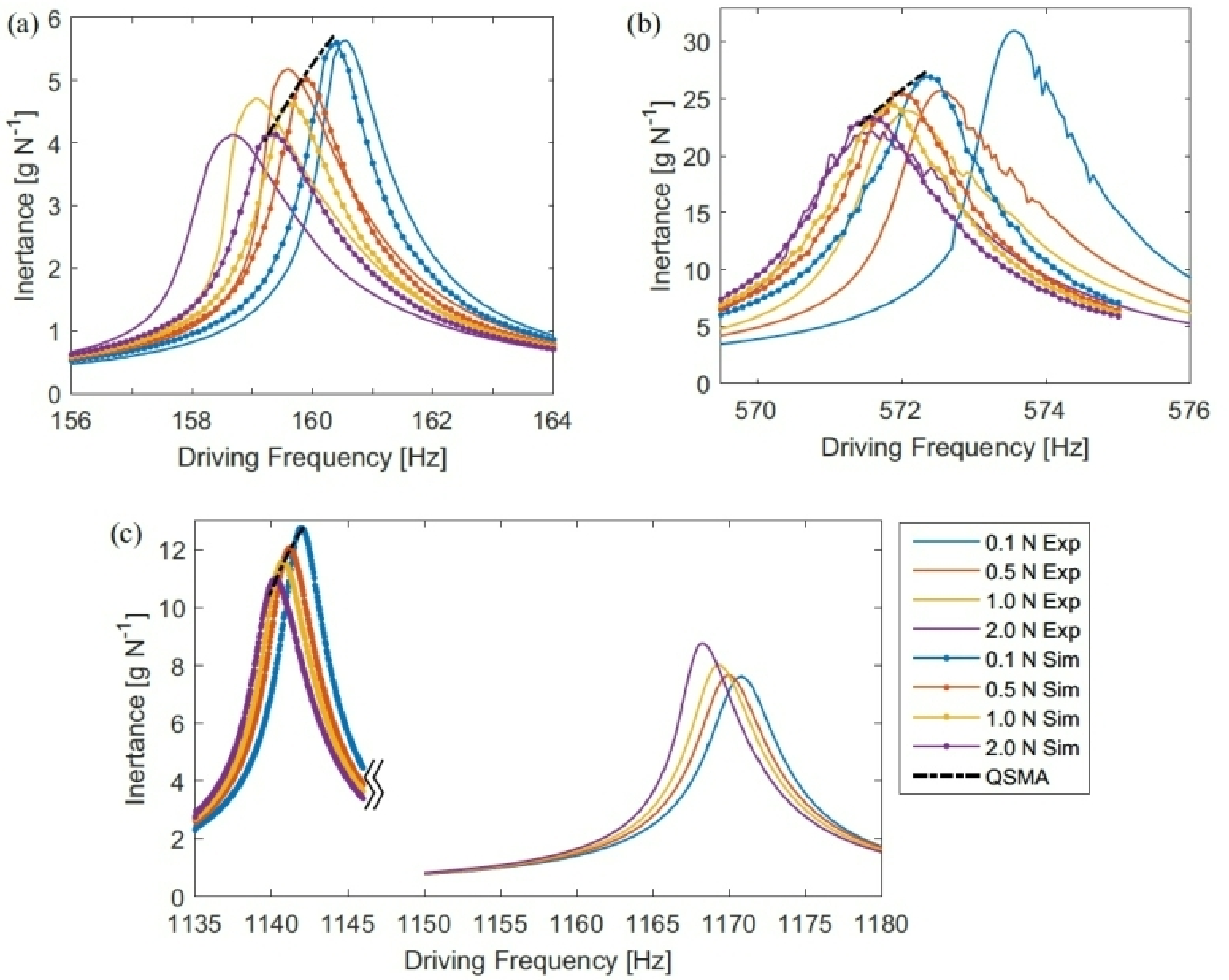
1. Negligible coupling between modes
    - Joints typically have linear responses
2. Modes shapes do not change significantly with the excitation amplitude

Segalman, D., Allen, M. S., Deaner, B. J., Starr, M. J., (2013) Investigation of Modal Iwan Models for Structure with Bolted Joints, 5-7

Background & Motivation

- ❑ Iwan model used to model nonlinear characteristics of Brake Rueβ beam
  - Iwan parameters tuned to first mode
  - Produces acceptable agreement for first two modes
- ❑ High variation between simulated and experimental results of mode 3

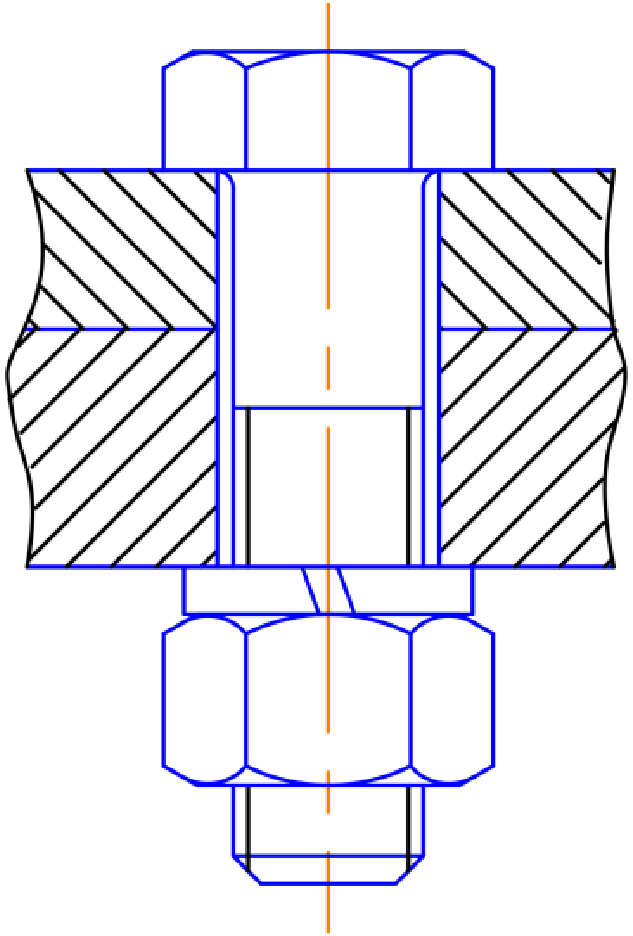




Lacayo, R., Pesaresi, L., Groß, J., Fochler, D., Armand, J., Salles, L., Schwingshackl, C., Allen, M. and Brake, M., 2019. Nonlinear modeling of structures with bolted joints: a comparison of two approaches based on a time-domain and frequency-domain solver. *Mechanical Systems and Signal Processing*, *114*, pp.413-438.

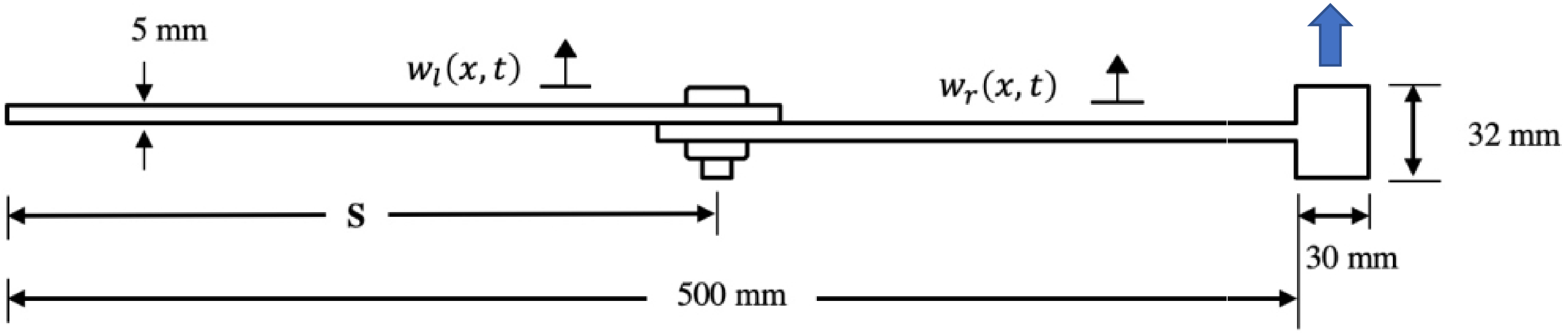
Background & Motivation

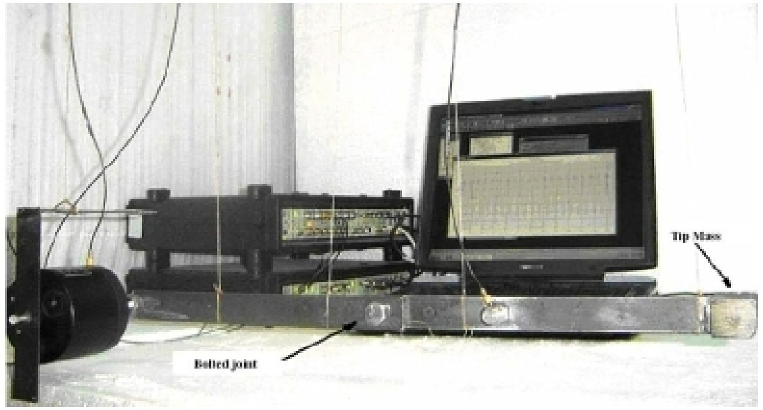
- ❑ Joints cause nonlinearities and are a major source of damping in a structure
- ❑ Joint interface can have major impacts on the dynamics of entire structure
- ❑ Linear damping joint models are insufficient for modeling friction/energy dissipation and nonlinearities
- ❑ Current models are difficult to use and define linear and nonlinear characteristics implicitly
  - Iwan parameters are difficult to determine
  - Desire to quantify value of nonlinear changes in frequency and damping

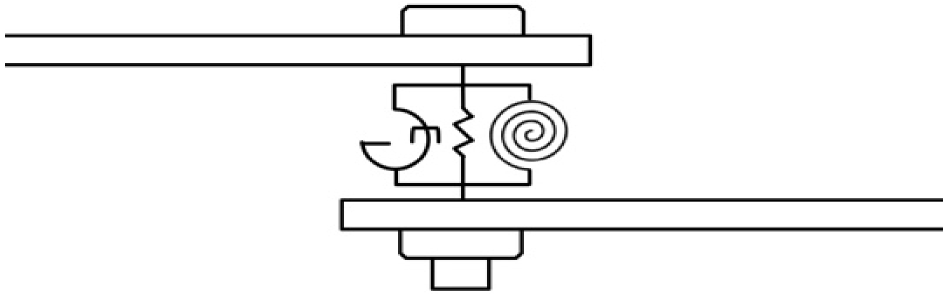


Reduced order model

GOAL: Simulate dynamic response of bolted beam with lap joint and free-free and cantilever BCs by explicitly defining the linear and nonlinear characteristics of the joint interface







$C_l, C_\theta, K_\theta, K_l, K_{3\theta}, K_3$

Interface Components

- $K_{3\theta}$, K_3: nonlinear stiffness effects at joint (torsional & translational)
- K_θ, K_l: Linear stiffness parameters at joint (torsional & translational)
- C_θ, C_l: Linear damping effects at joint (torsional & translational)

“Ahmadian, H., & Jalali, H. (2007). Identification of bolted lap joints parameters in assembled structures. *Mechanical Systems and Signal Processing*, *21*(2), 1041-1050.”

System’s modeling

$$= -C_\theta\left(\dot{w}'_r(s,t)-\dot{w}'_l(s,t)\right)\left(\delta w'_r(s,t)-\delta w'_l(s,t)\right)-C_l\left(\dot{w}_r(s,t)-\dot{w}_l(s,t)\right)\left(\delta w_r(s,t)-\delta w_l(s,t)\right)$$

→ Linear damping

$$-C_{3\theta}\left(\dot{w}'_r(s,t)-\dot{w}'_l(s,t)\right)^3\left(\delta w'_r(s,t)-\delta w'_l(s,t)\right)-C_3\left(\dot{w}_r(s,t)-\dot{w}_l(s,t)\right)^3\left(\delta w_r(s,t)-\delta w_l(s,t)\right)$$

→ Nonlinear damping

$$-c\left[\int_0^s\left(\dot{w}_l(x)\right)^2dx+\int_s^L\left(\dot{w}_r(x)\right)^2dx\right]$$

→ Viscous Damping $= -2\xi_i\omega_i\dot{q}(t)$

Hamilton’s principle

$$\int(\delta T-\delta V+\delta W_{nc})$$

$$T = \frac{1}{2}\rho A\left[\int_0^s(\dot{w}_l(x,t)+\dot{y}(t))^2dx+\int_s^L(\dot{w}_r(x,t)+\dot{y}(t))^2dx\right]+\frac{1}{2}M_t(\dot{w}_r(L,t)+\dot{y}(t))^2+\frac{1}{2}J\left(\dot{w}_r'(L,t)\right)^2$$

→ Tip Mass Kinetic Energies

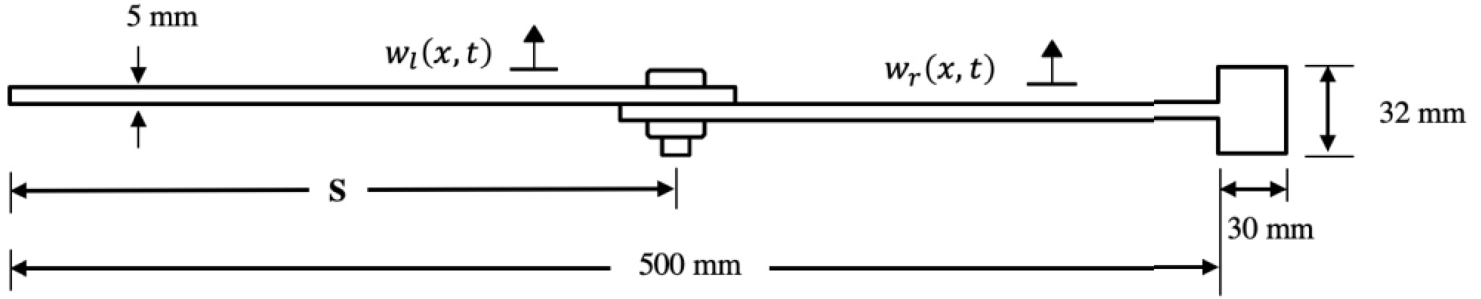
$$V = \frac{1}{2}EI\left[\int_0^s\left(w_l''(x,t)\right)^2dx+\int_s^L\left(w_r''(x,t)\right)^2dx\right]+\frac{1}{2}K_l\left(w_r(s,t)-w_l(s,t)\right)^2+\frac{1}{2}K_\theta\left(w_r'(s,t)-w_l'(s,t)\right)^2$$

$$+\frac{1}{4}K_{3\theta}\left(w_r'(s,t)-w_l'(s,t)\right)^4+\frac{1}{4}K_3\left(w_r(s,t)-w_l(s,t)\right)^4$$

→ Linear Joint stiffness

→ Nonlinear Stiffness

System's modeling



$\emptyset_l(x) = A\sin(kx) + B\cos(kx) + C\sinh(kx) + D\cosh(kx)$

$\emptyset_r(x) = E\sin(kx) + F\cos(kx) + G\sinh(kx) + H\cosh(kx)$

Continuity Relationships

$$\emptyset_l''(s) = \emptyset_r''(s)$$

$$\emptyset_l'''(s) = \emptyset_r'''(s)$$

$$EI\,\emptyset_l''(s) + K_\theta\left(\emptyset_l'(s) - \emptyset_r'(s)\right) = 0$$

$$-EI\,\emptyset_l'''(s) + K_l\left(\emptyset_l(s) - \emptyset_r(s)\right) = 0$$

Boundary Conditions

Free-Free Beam

$\emptyset_l''(0) = 0$ $\quad EI\,\emptyset_r''(L) - \omega^2 J\,\emptyset_r'(L)$

$\emptyset_l'''(0) = 0$ $\quad EI\emptyset_r''(L) - \omega^2 M_t \emptyset_r(L)$

Cantilever Beam

$\emptyset_l(0) = 0$ $\quad EI\,\emptyset_r''(L) - \omega^2 J\,\emptyset_r'(L)$

$\emptyset_l'(0) = 0$ $\quad EI\emptyset_r''(L) - \omega^2 M_t \emptyset_r(L)$

Reduced-Order Model Equation

Left Beam

$$w_l(x,t) = \sum_{i=1}^{n} \left(\emptyset_{l,i}(x) q_i(t) \right)$$

$$\ddot{q}_i + 2\omega_i \zeta_i \dot{q}_i + \omega_i^2 q_i + C_i(\dot{q}_j) + K_{NL_i}(q_j) = f_i(t)$$

Right Beam

$$w_r(x,t) = \sum_{i=1}^{n} \left(\emptyset_{r,i}(x) q_i(t) \right)$$

$$C_i = \left(\frac{\delta W_{nc}}{\delta q} \right)_i = - C_\theta \left[\sum_{j=1}^{n} \left(\emptyset'_{r,j}(s) - \emptyset'_{l,j}(s) \right) \dot{q}_j(t) \right] \left(\emptyset'_{r,i}(s) - \emptyset'_{l,i}(s) \right) - C_l \left[\sum_{j=1}^{n} \left(\emptyset_{r,j}(s) - \emptyset_{l,j}(s) \right) \dot{q}_j(t) \right] \left(\emptyset_{r,i}(s) - \emptyset_{l,i}(s) \right)$$

$$- C_{3\theta} \left[\sum_{j=1}^{n} \left(\emptyset'_{r,j}(s) - \emptyset'_{l,j}(s) \right) \dot{q}_j(t) \right]^3 \left(\emptyset'_{r,i}(s) - \emptyset'_{l,i}(s) \right) - C_3 \left[\sum_{j=1}^{n} \left(\emptyset_{r,j}(s) - \emptyset_{l,j}(s) \right) \dot{q}_j(t) \right]^3 \left(\emptyset_{r,i}(s) - \emptyset_{l,i}(s) \right)$$

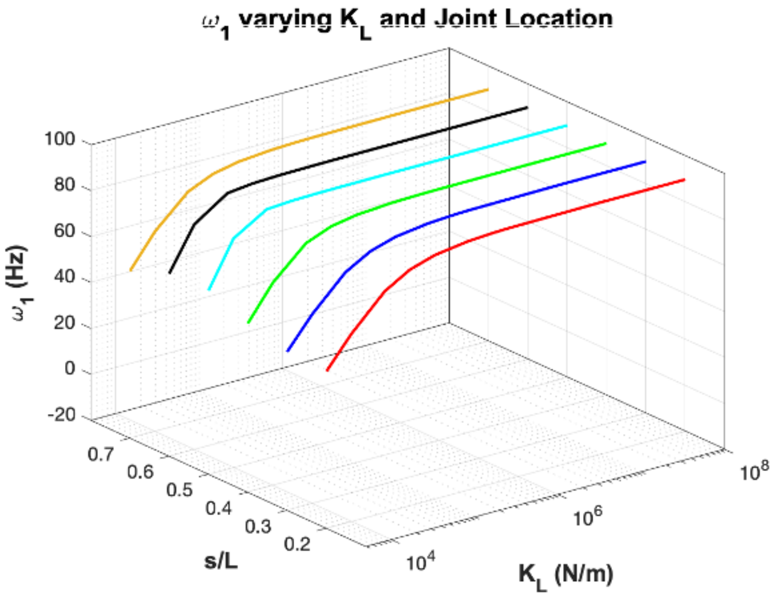
$$K_{NL_i} = K_{3\theta} \left[\sum_{j=1}^{n} \left(\emptyset'_{r,j}(s) - \emptyset'_{l,j}(s) \right) q_j(t) \right]^3 \left(\emptyset'_{r,i}(s) - \emptyset'_{l,i}(s) \right) + K_3 \left[\sum_{j=1}^{n} \left(\emptyset_{r,j}(s) - \emptyset_{l,j}(s) \right) q_j(t) \right]^3 \left(\emptyset_{r,i}(s) - \emptyset_{l,i}(s) \right)$$

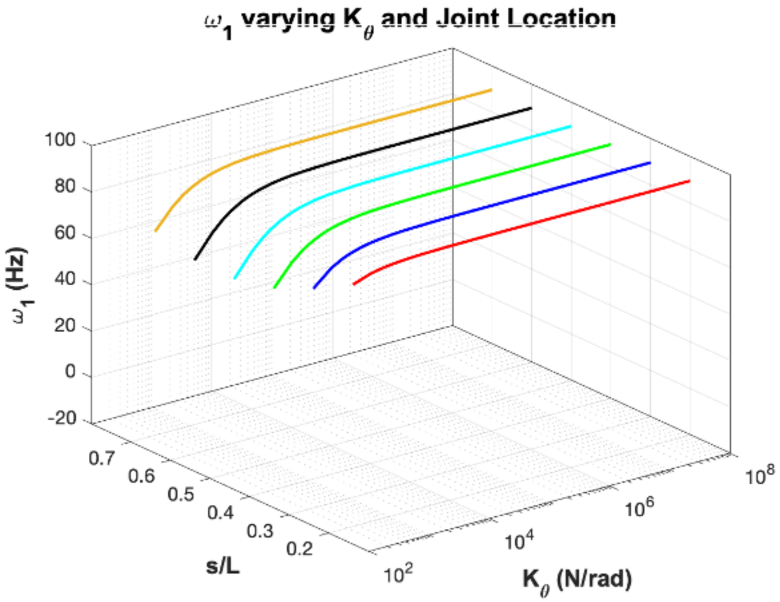
$$= EI \int_0^s \left(\emptyset''_{l,i}(x) \right)^2 dx + EI \int_s^L \left(\emptyset''_{r,i}(x) \right)^2 dx + K_l \left(\emptyset_{r,i}(s) - \emptyset_{l,i}(s) \right)^2 + K_\theta \left(\emptyset'_{r,i}(s) - \emptyset'_{l,i}(s) \right)^2$$

(orthogonality condition)

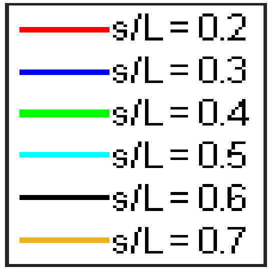
$$f_i(t) = -\ddot{y}(t) \left[\rho A \left(\int_0^s \emptyset_{l,i}(x) dx + \int_s^L \emptyset_{r,i}(x) dx \right) + M_t \emptyset_{r,i}(L) \right] = a\cos(wt) \left[\rho A \left(\int_0^s \emptyset_{l,i}(x) dx + \int_s^L \emptyset_{r,i}(x) dx \right) + M_t \emptyset_{r,i}(L) \right]$$

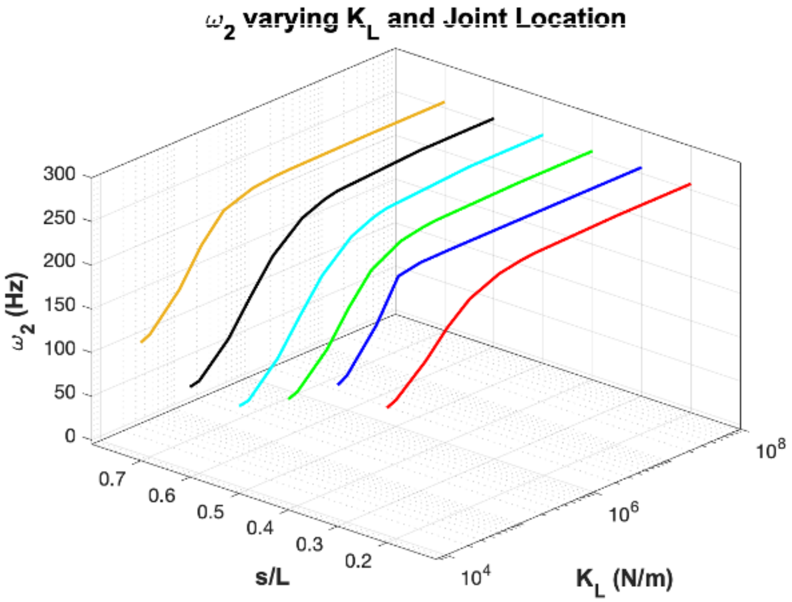
Frequency Effects Modes 1&2 (free-free)

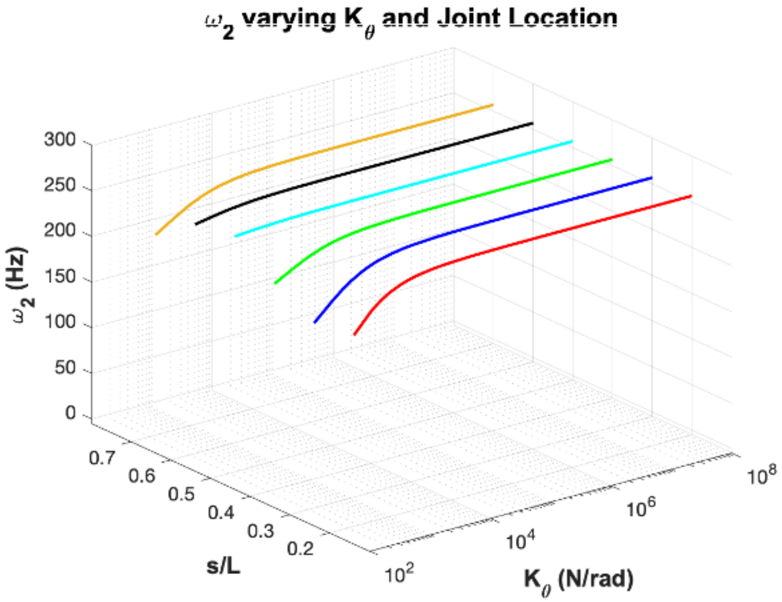




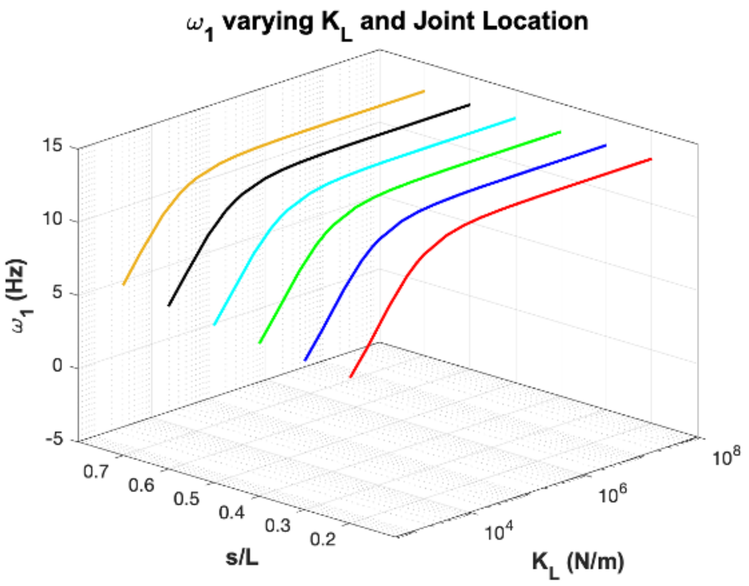
- K_θ has a lesser impact on the natural frequency than K_L
- Certain configurations are more sensitive to stiffness changes if joint is located at node

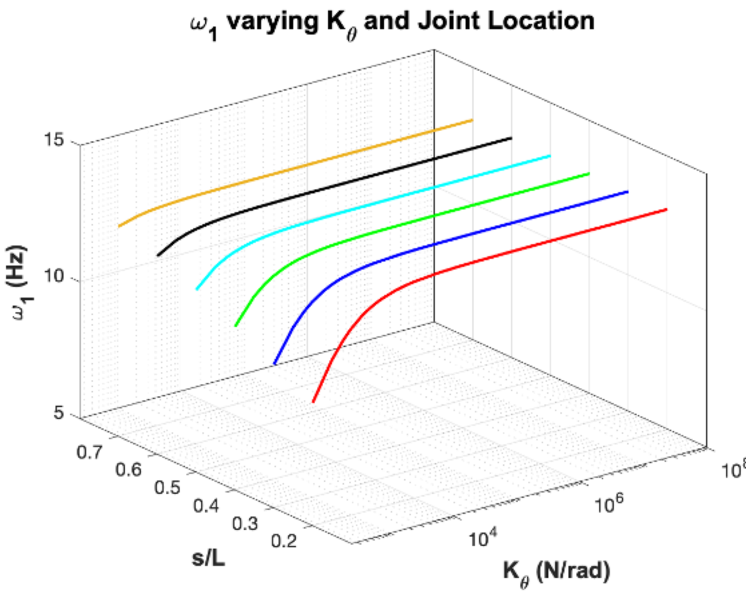




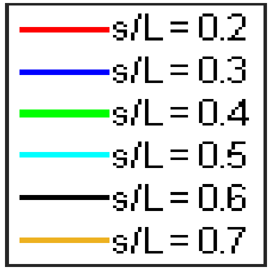


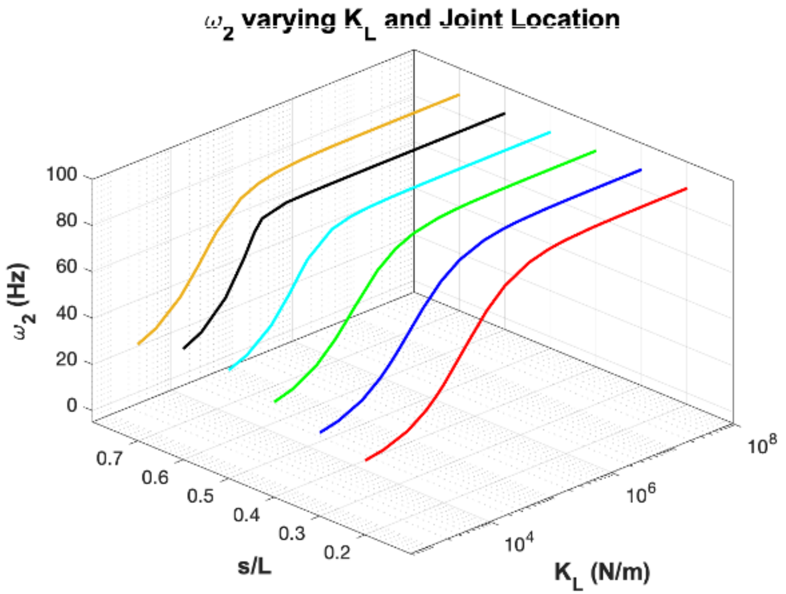
Frequency Effects Modes 1&2 (Cantilever)

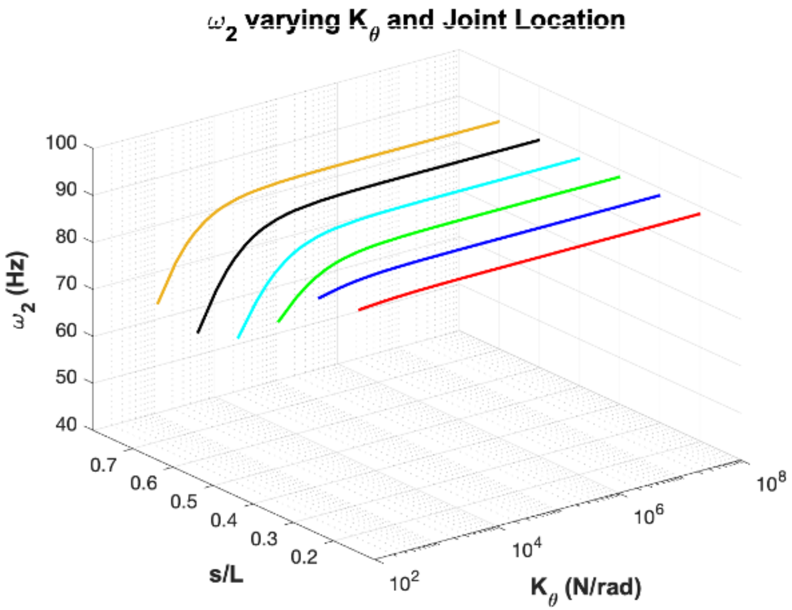




- ❑ K_θ has a lesser impact on the natural frequency than K_L
- ❑ ω_1 decreases as joint moves closer to fixed end
  - ❑ Increase in bending moment and slippage

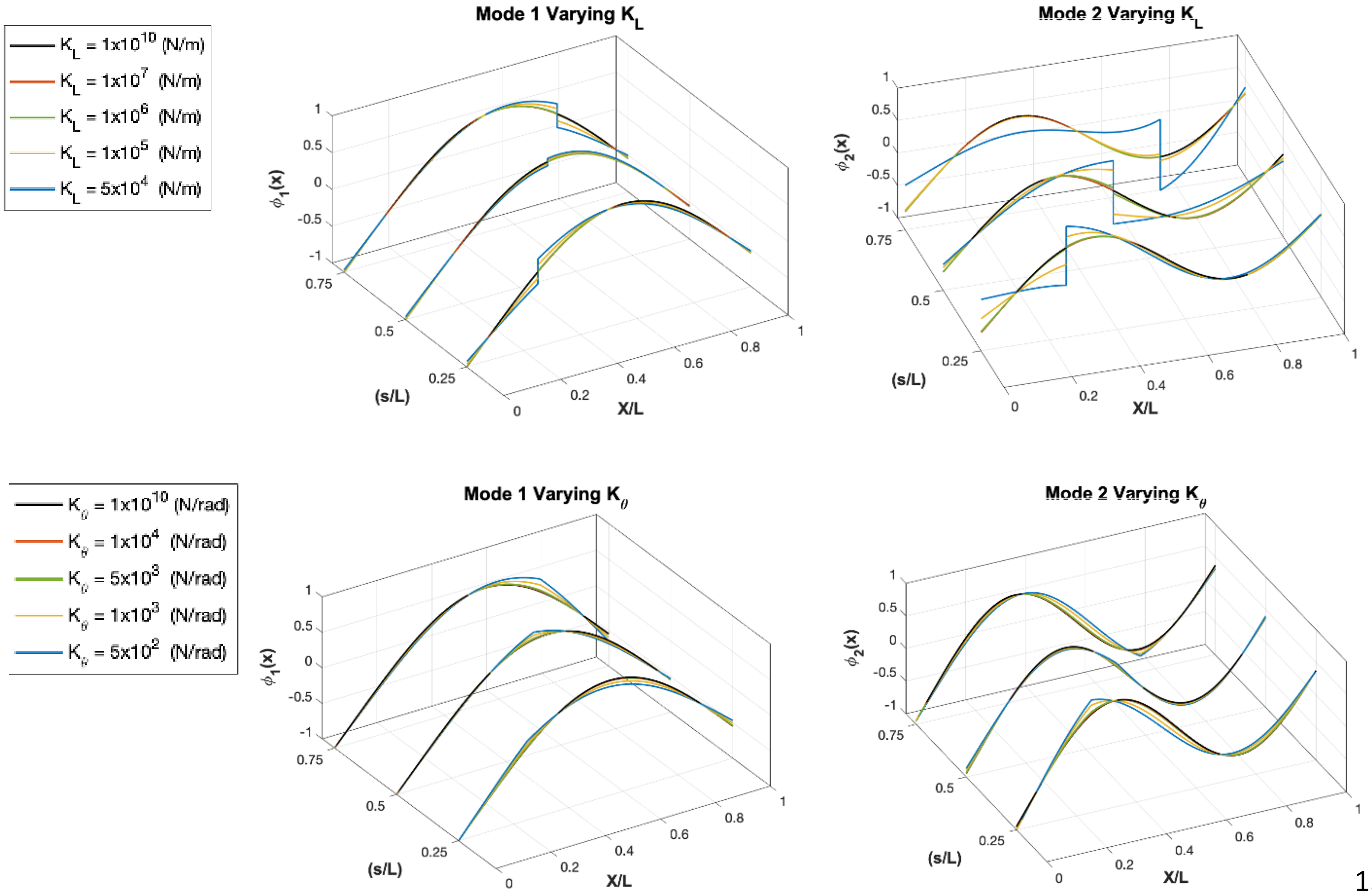






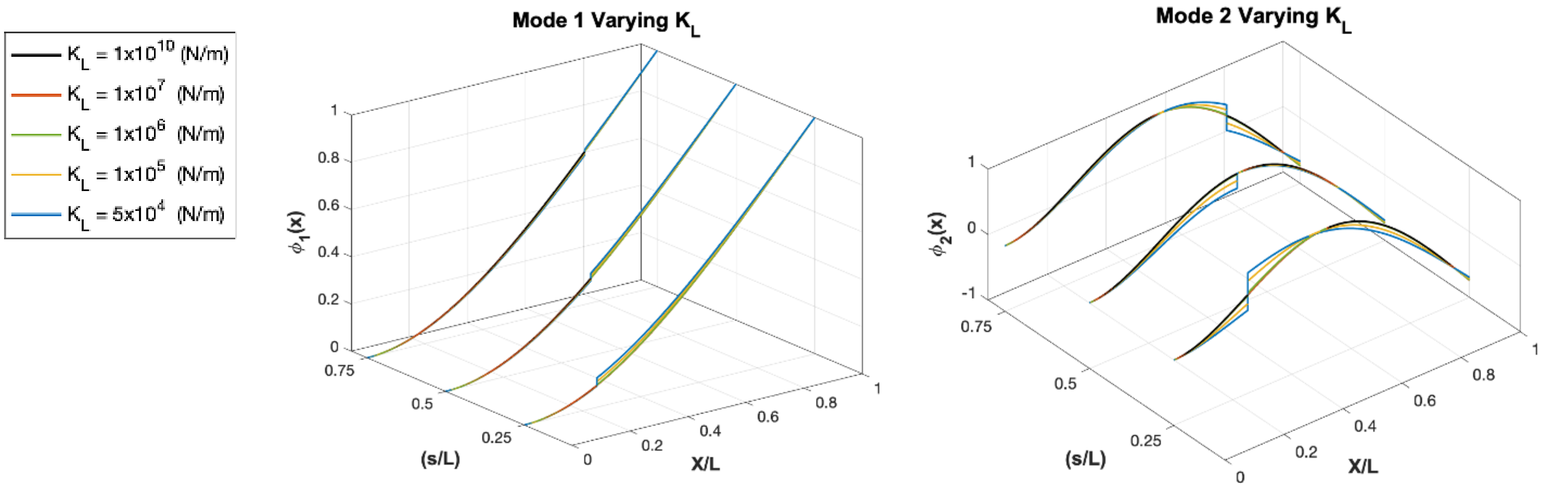
(Free-Free) Varying Linear Stiffness And Joint Location

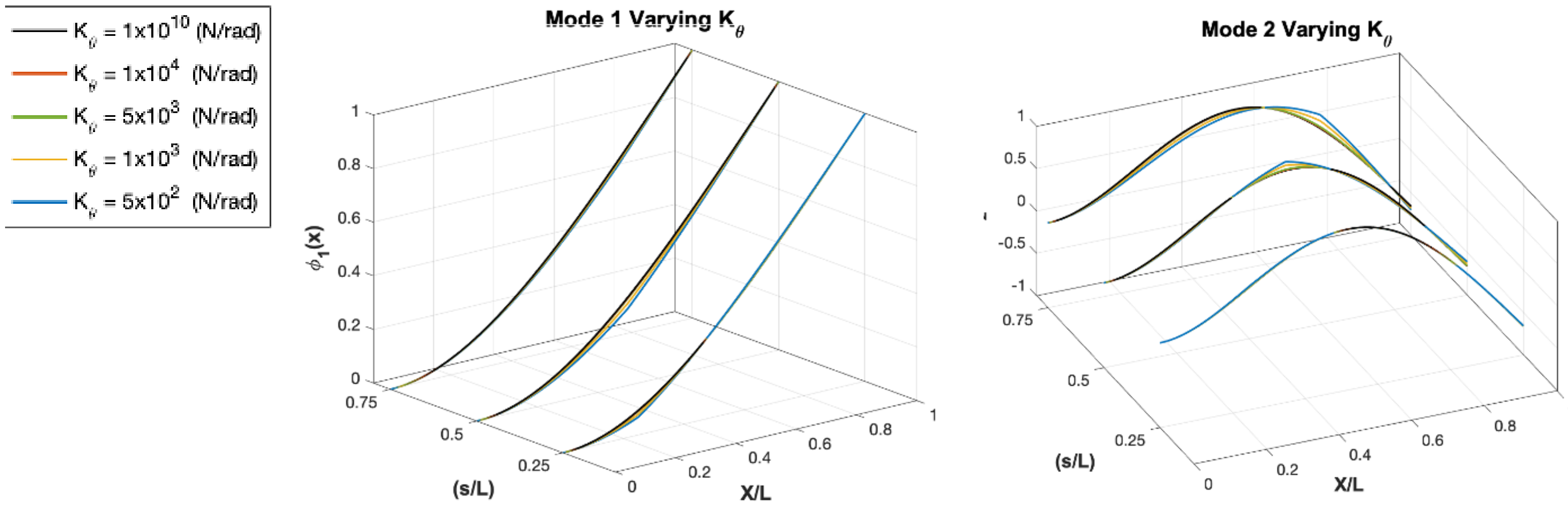
- ❑ High values of K_l & K_θ ≥ 10e10 simulate continuous beam
- ❑ Lower joint stiffness values effect the dynamics of the structure
  - ❑ Greater effects at node
  - ❑ Minimal effects at antinode
- ❑ K_l & K_θ ≤ 1e3 generate unreasonable mode shapes and resemble joint failure



(Cantilever) Varying Linear Stiffness And Joint Location

- High values of K_l & K_θ $\geq$ 10e10 simulate continuous beam
- Lower joint stiffness values effect the dynamics of the structure
  - Greater effects at node
  - Minimal effects at antinode
- K_l & K_θ $\leq$ 1e3 generate unreasonable mode shapes and resemble joint failure

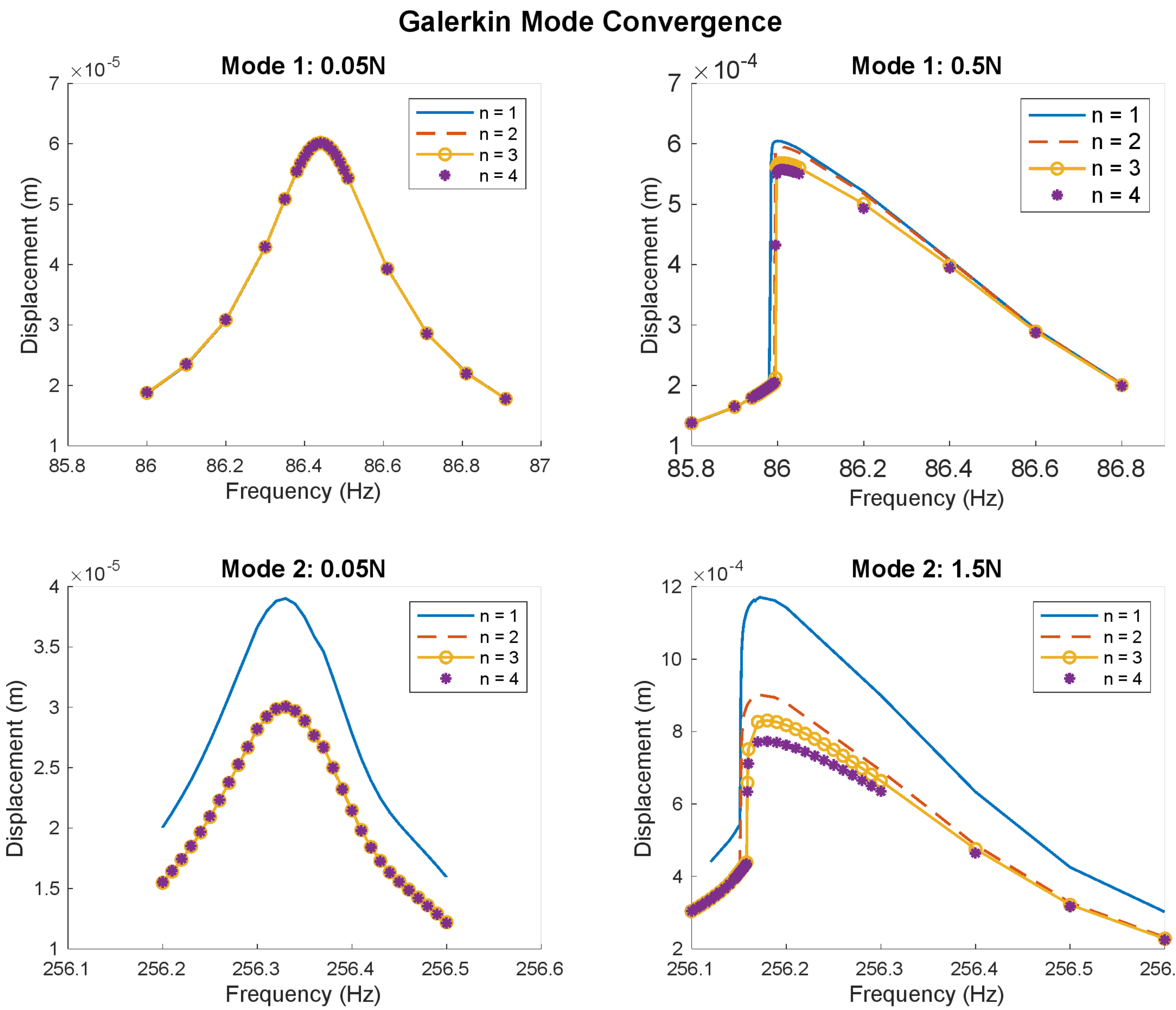




Convergence Analysis

- Increase number of modes of ROM until convergence of FRFs is reached

- ❑ NL regime requires more modes for accuracy
- ❑ Lesser modes required for linear regime
- ❑ Predicted convergence within 4-5 modes
    - Considering accuracy and computation time



Hilbert Transform-vibrations

-HT defines the relationship between real and imaginary parts of analytical signals

$$u_A(t) = u(t) + i\hat{u}(t)$$

$$u_A(t) = A(t)e^{j\psi(t)}$$

Phase Angle

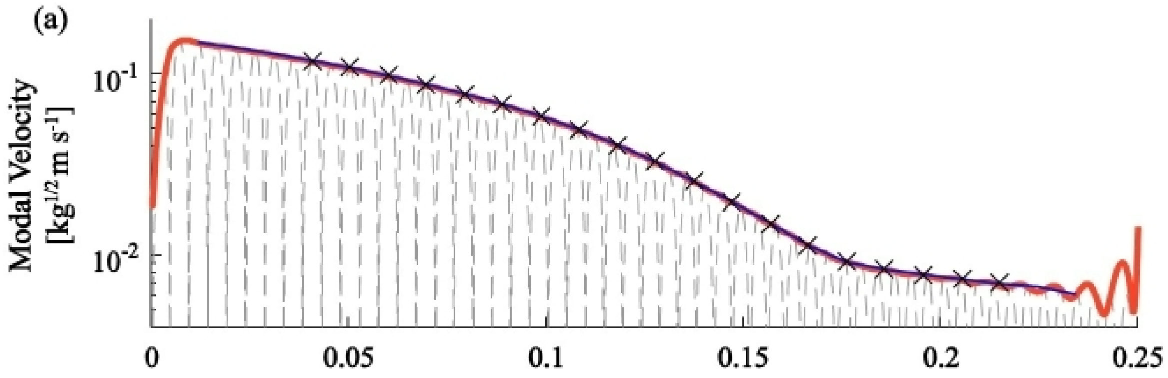
$$\psi(t) = tan^{-1}\left(\frac{\hat{u}(t)}{u(t)}\right)$$

Instantaneous Damping Frequency

$$\omega_d(t) = \frac{d\psi_j(t)}{dt} \longrightarrow \omega_d(t) \approx \left(\psi_{j,i}(t) - \psi_{j,i-1}(t)\right)\frac{F_s}{2\pi}$$

Amplitude Envelope

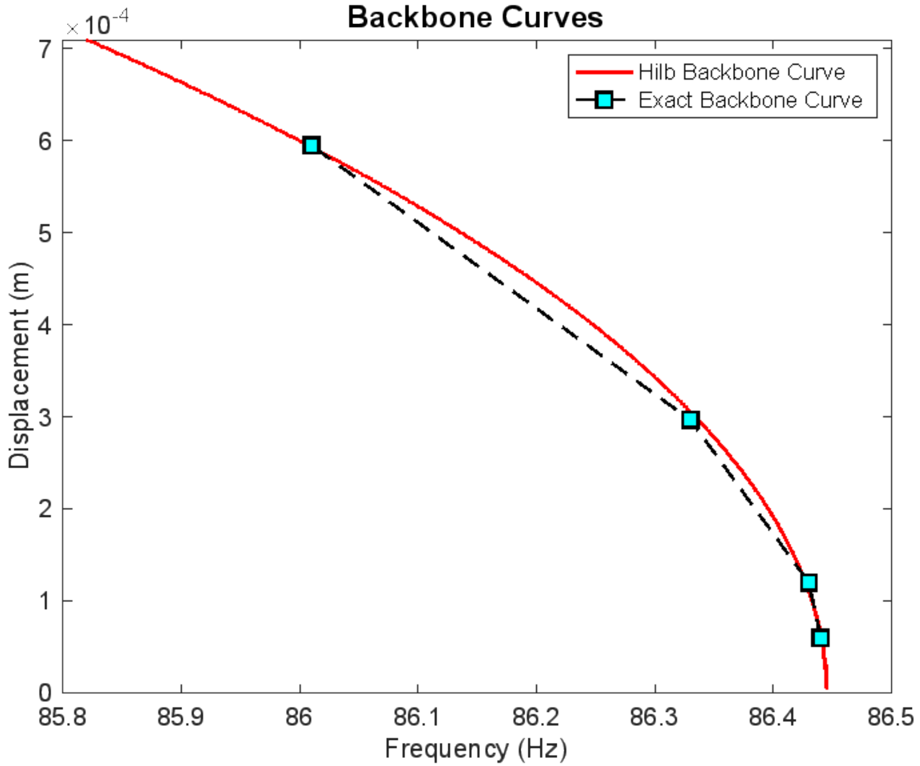
$$|u_A(t)| = \mathrm{A(t)} = |\sqrt{u(t)^2 + \hat{u}(t)^2}|$$

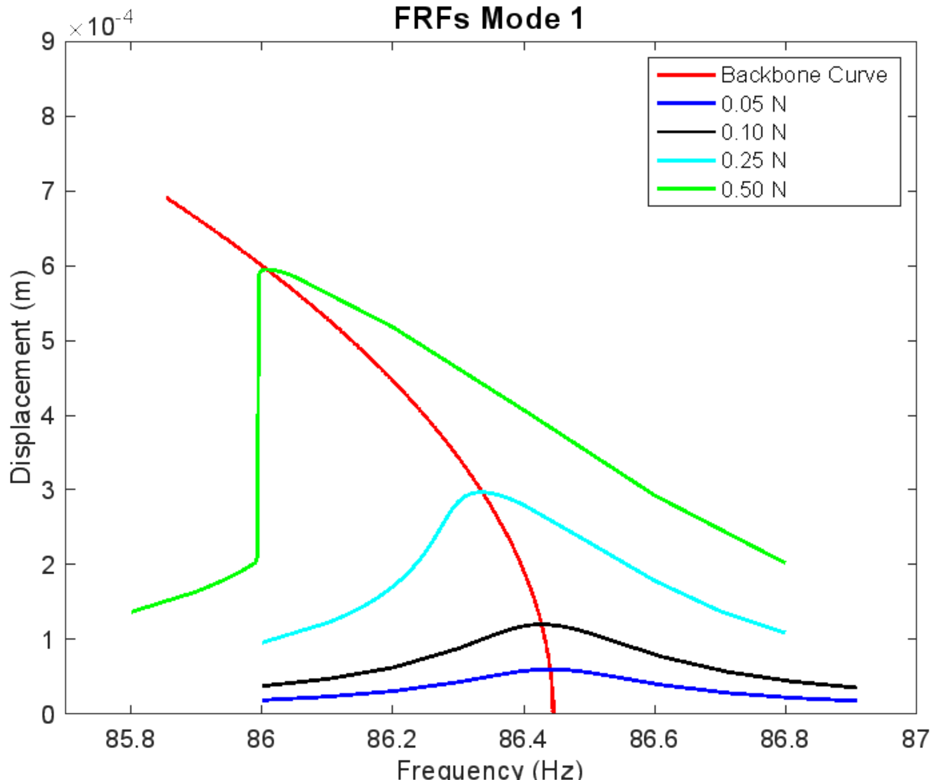


Backbone Curve: Hilbert Transform

- ❑ H-T used to determine instantaneous damped frequency
  - ❑ Approximates backbone curve
- ❑ Backbone curve relates nonlinear resonance frequency to various amplitudes

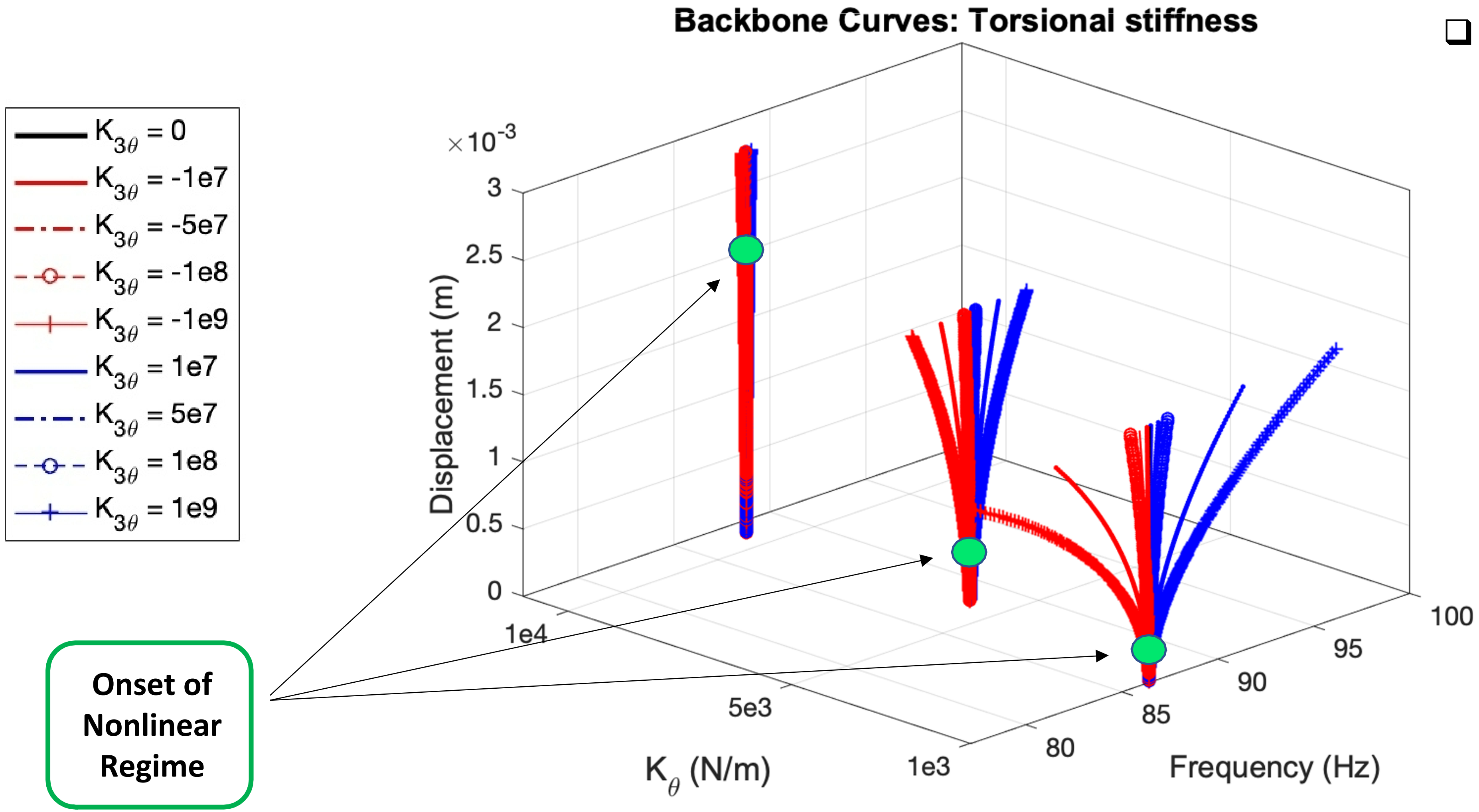
Parameters	Values	Units
K_L	8.089e8	N/m
K_θ	3264	N/rad
$K_{3\theta}$	3.277e8	N/rad³
K_3	0	N/m³
C_l	0	Ns/m
C_θ	0.281	Ns/rad





Backbone Curve: Hilbert Transform

Nonlinear Stiffness: $K_{3\theta}\left[\sum_{i=1}^{n}\left(\emptyset'_{r,i}(s)-\emptyset'_{l,i}(s)\right)q_i(t)\right]^3\left(\emptyset'_{r,i}(s)-\emptyset'_{l,i}(s)\right)$

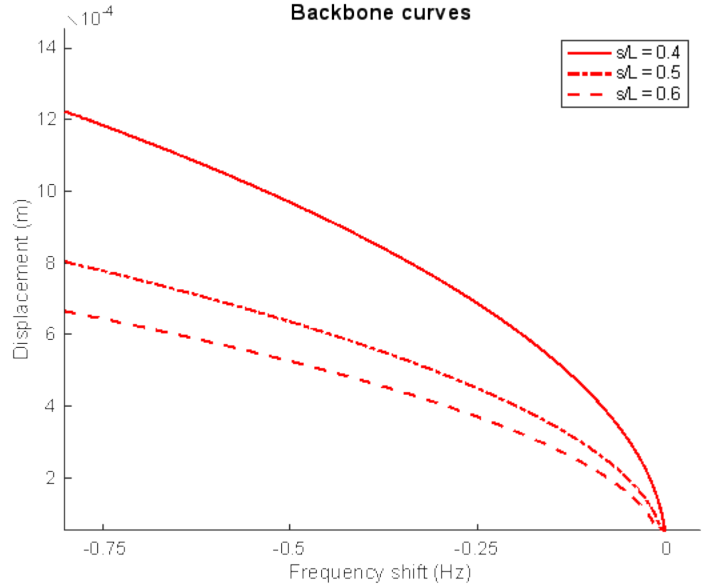


Increasing K_θ:

- ❑ Rate of nonlinear frequency decreases
- ❑ Higher excitation needed to activate nonlinear regime
  - Weaker nonlinearities
  - Stronger linear characteristics

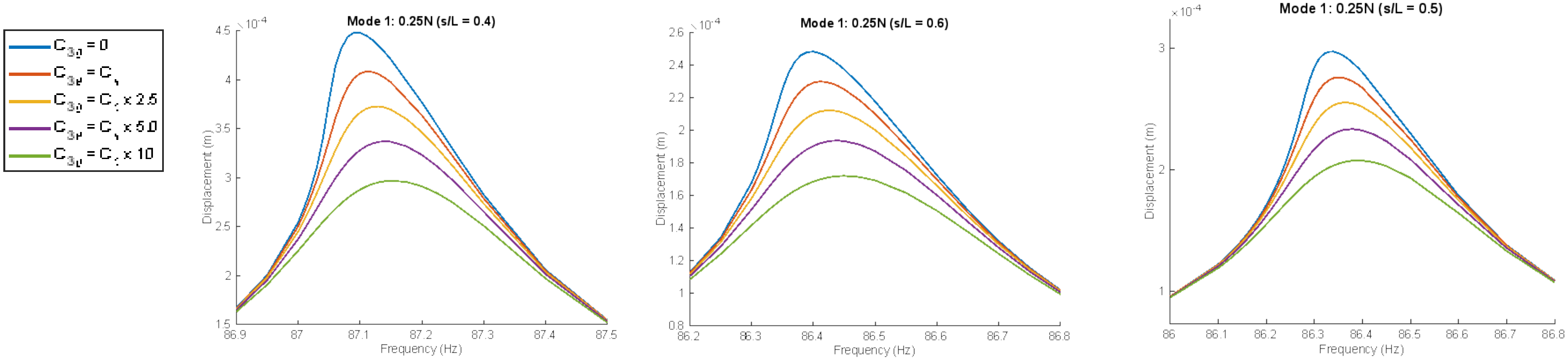
Uncertainty of Nonlinearities

- In the manufacturing process there are many uncertainties in the dimensions that can affect the linear and nonlinear characteristics



+/- 10% uncertainty in joint location:

- ❑ Drastic changes in linear natural frequency and damping are examined
- ❑ Nonlinear damping is not significantly affected by varying joint location
- ❑ Nonlinear softening effects weaken as s/L increases



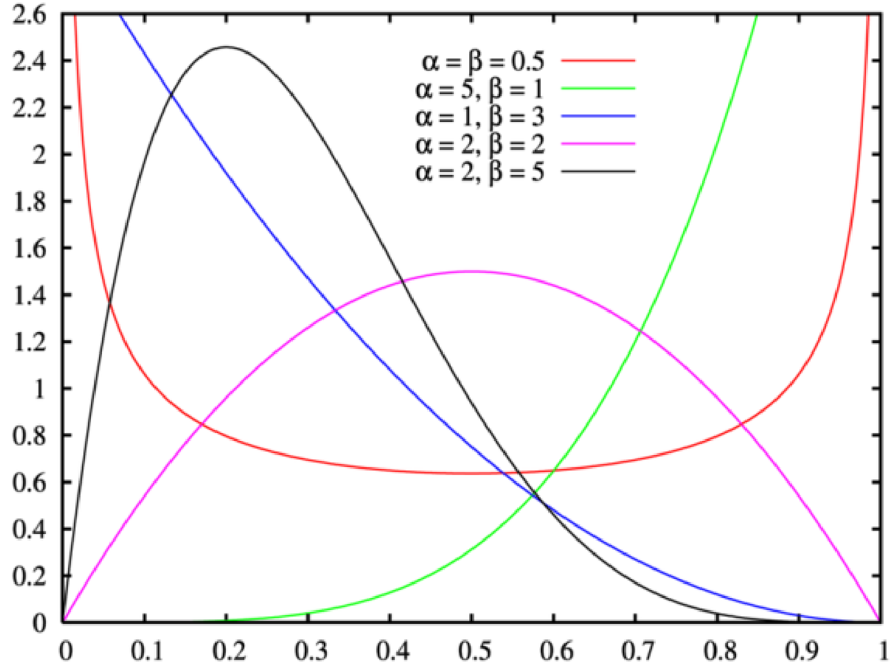
Conclusion & Future Work

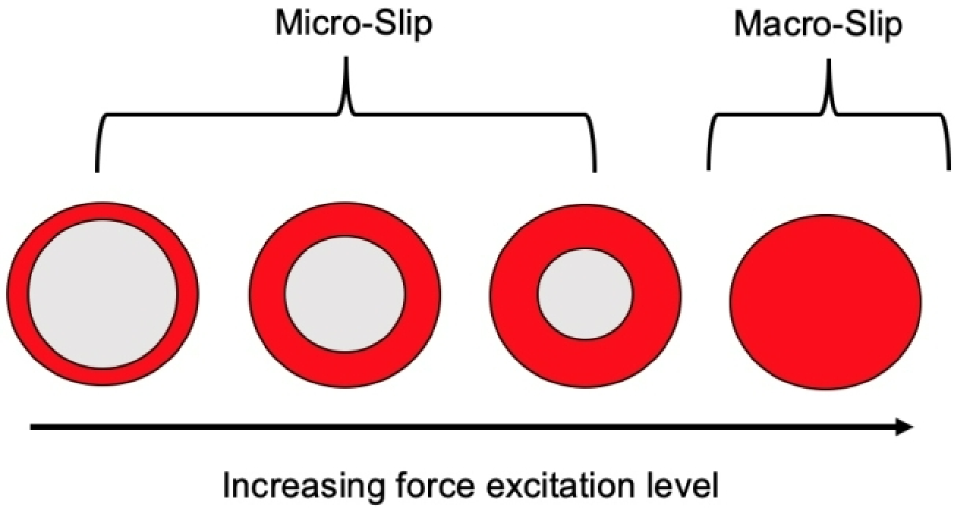
Conclusion:

- Torsional springs and dampers are effective in simulating nonlinear effects of joint interfaces
- Linear and nonlinear characteristics are affected by joint location
- Hilbert Transform can accurately approximate nonlinear trends (backbone curve)

Future Work:

- Implement torsional and out-of-plane bending modes
- Developed nonlocalized forces dependent on slip-stick areas of joint interface
  - Determine usefulness of probability density function shape
- Develop sub-routine for ROM into ABAQUS CAE
- Model beam structure with multiple bolts and joints





References

- Segalman, D., Lacayo, R., Allen, M.S., Schwingshackl, C., Barber, J. (2018). Mechanics of Jointed Structures, 15-16, 26-27, 255-264
- Lacayo, R., Deaner, B. J., Allen, M. S. (2017). An Investigation on Iwan Models for Capturing the Amplitude-Dependent Behvior of Structures with Bolted Joints, 20-24
- Segalman, D., Lacayo, R., Allen, M.S., Schwingshackl, C., Barber, J. (2018). Mechanics of Jointed Structures, 15-16, 26-27, 255-264
- Feldman, M., Hilbert Transform Applications in Mechanical Vibrations, 12-20
- Segalman, D., Lacayo, R., Allen, M.S., Schwingshackl, C., Barber, J. (2018). Mechanics of Jointed Structures, 15-16, 26-27, 255-264
- Segalman, D., Allen, M. S., Deaner, B. J., Starr, M. J., (2013) Investigation of Modal Iwan Models for Structure with Bolted Joints, 5-7
- Jamia, N., Jalali, H., Taghipour, J., Friswell, M.I. and Khodaparast, H.H., 2021. An equivalent model of a nonlinear bolted flange joint. *Mechanical Systems and Signal Processing*, *153*, p.107507.

Acknowledgements:

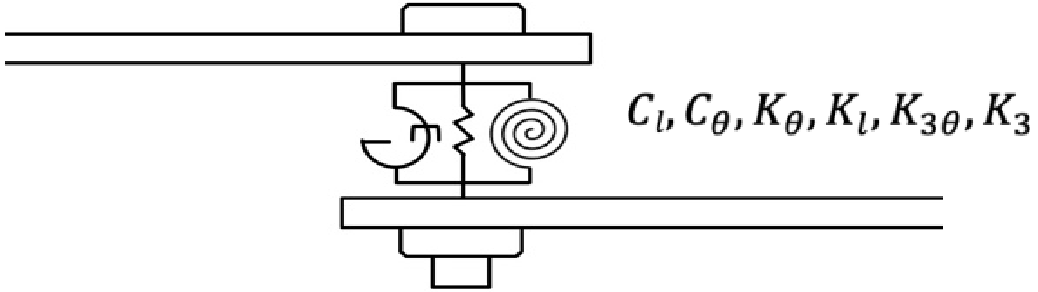
Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy National Nuclear Security Administration under contract DE-NA0003525.

Questions?

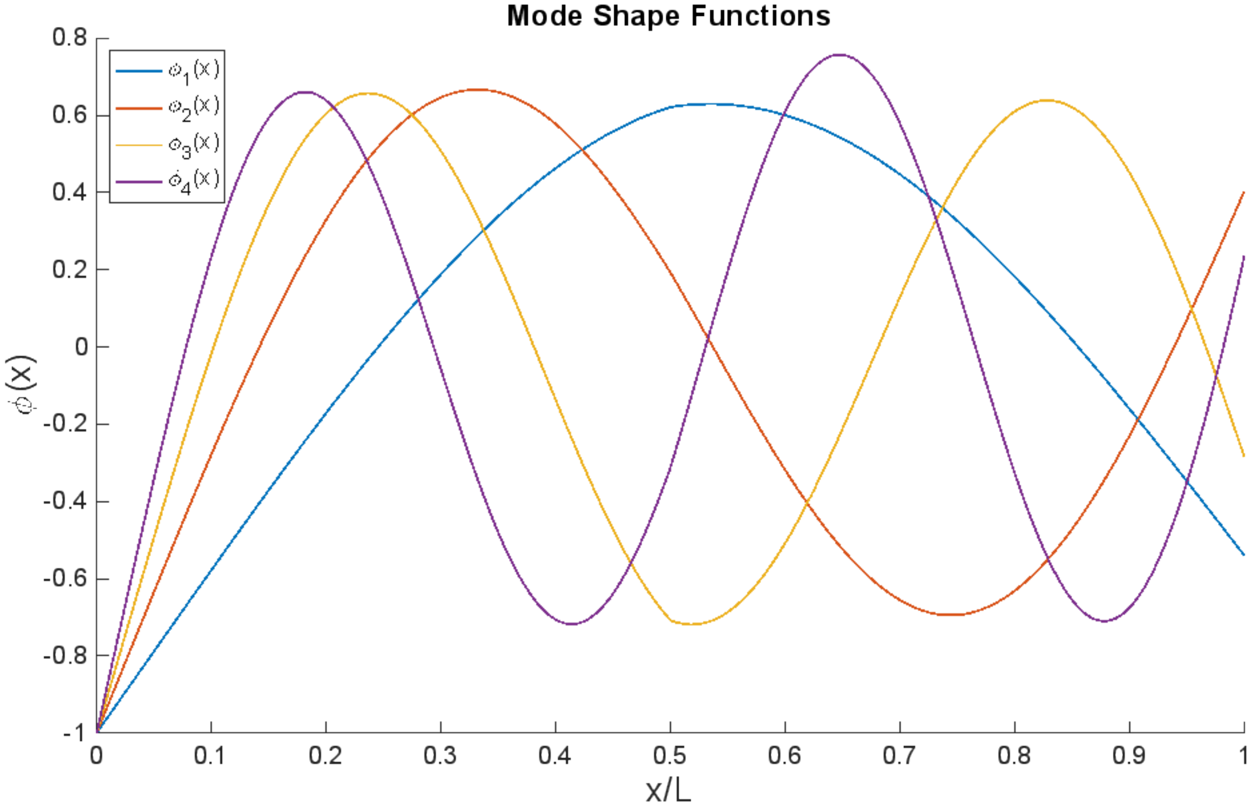
Extra Slides

LINEAR PROPERTY IDENTIFICATION

- **ROM validation with**: "Ahmadian, H., & Jalali, H. (2007). Identification of bolted lap joints parameters in assembled structures. *Mechanical Systems and Signal Processing*, *21*(2), 1041-1050."



Parameters	Values	Units
K_L	8.089e8	N/m
K_θ	3264	N/rad
$K_{3\theta}$	3.277e8	N/rad^3
K_3	0	N/m^3
C_l	0	Ns/m
C_θ	0.281	Ns/rad



Transverse Bending Modes	1	2	3
ω_n (Hz)	86.445	256.37	501.18

Uncertainty of Nonlinearities

- In the manufacturing process there are many uncertainties in the dimensions therefore, in the joint location.
  - +/- 10% uncertainty in joint location is evaluated

❑ Drastic changes in linear natural frequency and damping are examined

❑ Nonlinear damping has decreased effect as as s/L increases

❑ Nonlinear softening effects increase as s/L increases