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Inverse Calculation of Cross-Spectral Densities Using a Bayesian Inference Approach

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Motivation

- Inverse calculating random acoustic environments from small number of structural measurements has strong appeal

- A Bayesian approach has been shown to accurately inverse-calculate autospectra
  - Uses structural model and test measurements as input
  - Accuracy can be improved by including prior assumption about field's spatial correlation
    - But only in special cases are good spatial correlation assumptions available

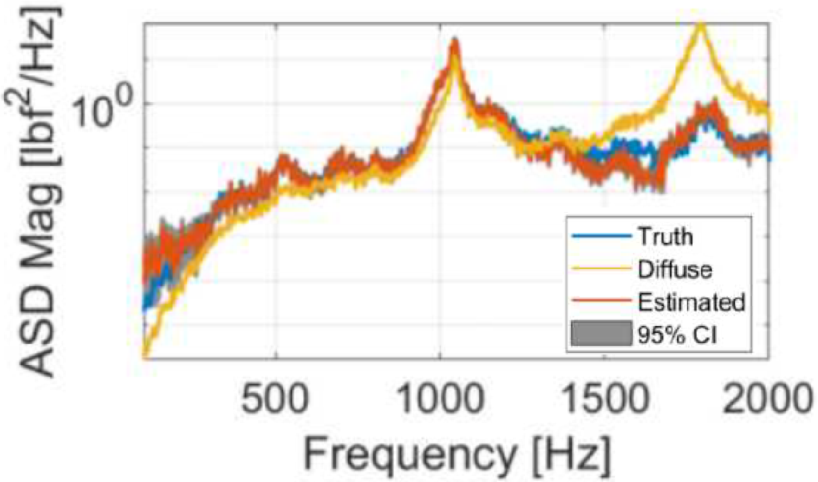
Questions:

1. How accurately can spatial correlation be calculated with Bayesian approach?

2. How important is it to use an accurate spatial correlation prior?

Technical Approach

- Approach follows Pereira et al. [1] and Lopp and Schultz [2]
- Use model-derived transfer functions and synthesized response measurements
- Assumed priors can also be supplied to indicate knowledge of:
  - Spatial correlation of environment
  - Measurement noise
- 1-D optimization identifies optimal regularization parameter
- Final output: spatially-resolved PSD matrix describing the acoustic environment
- Also includes Markov chain Monte Carlo (MCMC) to determine statistics and distribution of inverse-calculated quantities



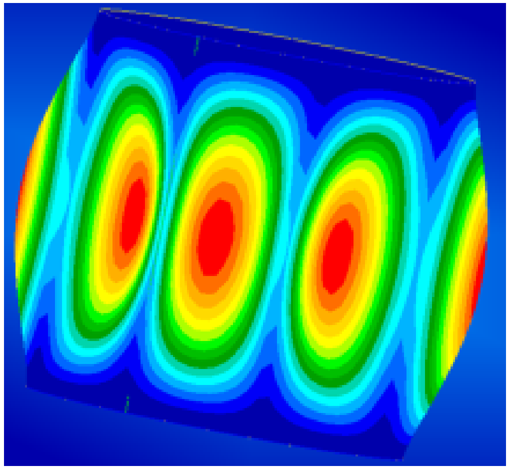
Representative acoustic ASD result from Bayesian approach. Shows Bayesian approach more accurate than assuming diffuse field (from Ref. [2])

[1] **Pereira, A., Antoni, J.** and **Leclere, Q.**, *Empirical Bayesian regularization of the inverse acoustic problem*, Applied Acoustics, Vol. 97, pp. 11–29, 2015.

[2] **Lopp, G. K.** and **Schultz, R.**, *A Bayesian Approach for Identifying the Spatial Correlation of Acoustic Loads during Vibroacoustic Testing.*, Tech. rep., Sandia National Lab.(SNL-NM), Albuquerque, NM (United States), 2019.

Structural Model

- The structural model a simply supported aluminum cylindrical shell with:
  - Radius = 15.24 cm
  - Length = 30.48 cm
  - $\rho = 2800\ kg/m^3$
  - E = 72 GPa
  - $\nu = 0.33$
- Natural frequencies and modes calculated analytically using Flügge theory [3]
- Acceleration-to-force transfer functions calculated at 180 non-boundary DOFs arranged in rings of 20 circumferential nodes at nine axial locations
- Flat 1% modal damping assumed



Four nodal diameter shell mode

[3] **Davis, R. B.**, Techniques to assess acoustic-structure interaction in liquid rocket engines, Duke University, 2008.

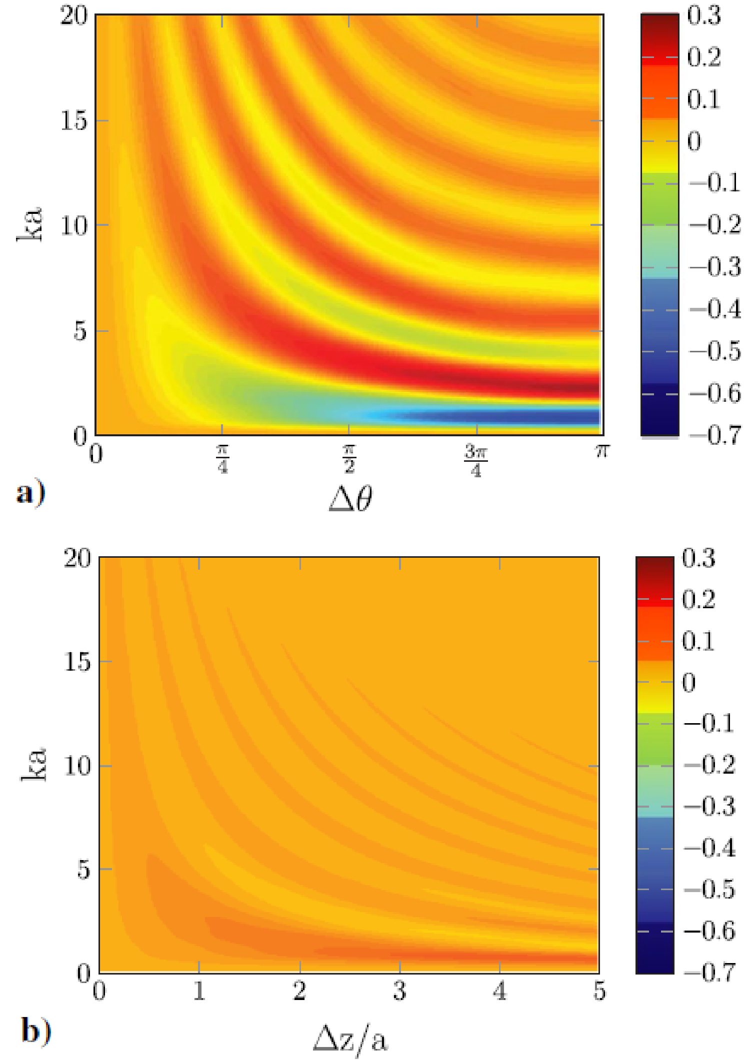
Truth Environment

- Diffuse acoustic field (DAF) including scattering effects of the cylinder
- DAF assumes plane waves of all frequencies arrive from all directions with random amplitude and phase
- Spatial correlation for DAF on infinite flat plate is

$$f(k,r) = \frac{\sin(kr)}{r}$$

k: acoustic wave number
r: distance between field point and reference point

- For cylindrical geometries in a DAF, scattering keeps the spatial correlation from being expressed in closed-form
- Differences between the cylinder spatial correlation and the *sinc* function are most significant for ka<4, where a is the radius of the cylinder

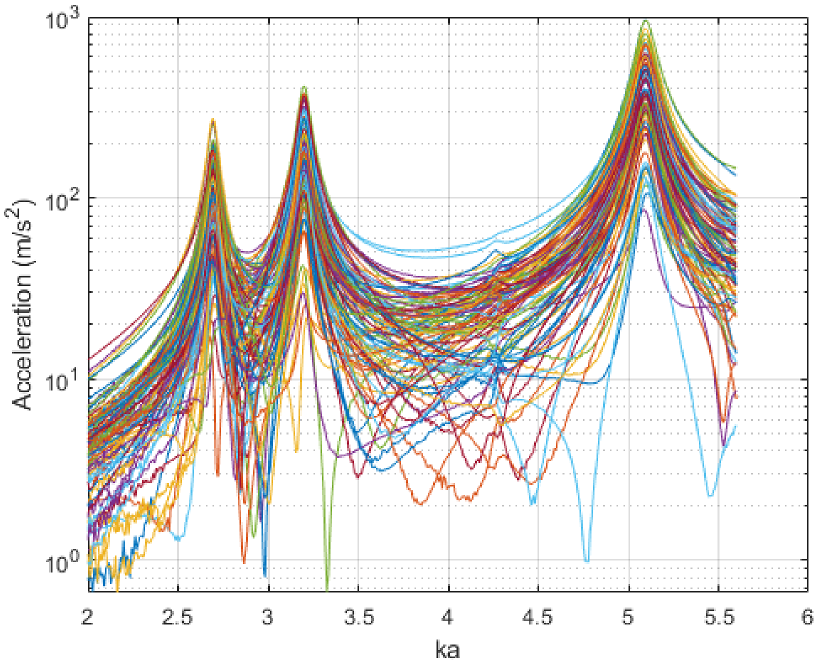


Difference between cylinder spatial correlation and sinc function (from Ref. [4])

[4] **Davis, R. B.**, *Spatial Correlation Function of Cylinders in Diffuse Acoustic Fields*, AIAA Journal, Vol. 55, No. 11, pp. 4005–4010, 2017.

Synthesized Measurements

- Force realizations corresponding the truth environment are synthesized using an approach described by Yuen et al. [5]
- Realizations are multiplied by the model-derived transfer functions to generate synthesized response measurements
- Uncorrelated random noise with specified standard deviation is added to synthesized measurements
- To check implementation, standard deviation of the noise is accurately recovered from the inverse calculation



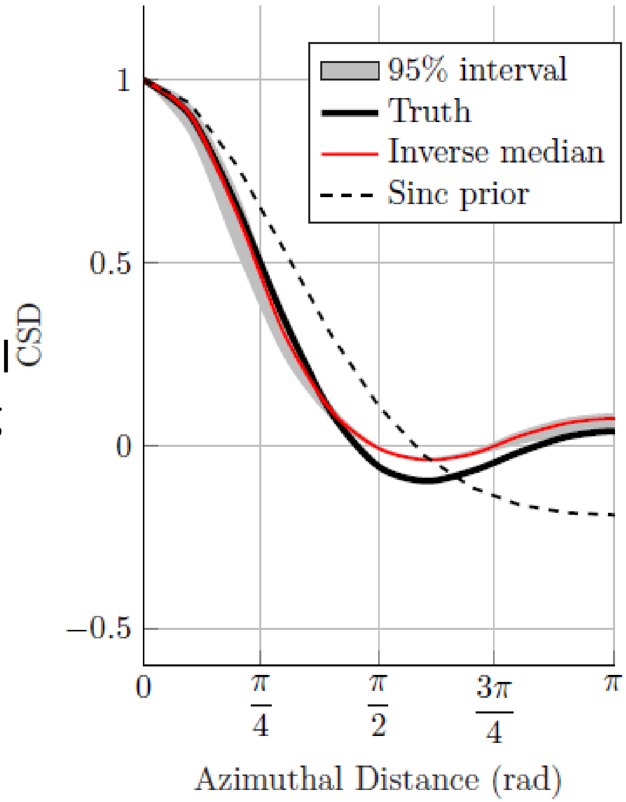
100 realizations of a synthesized sensor measurement with noise and truth environment with specified spatial correlation

[5] **Yuen, K.-V., Katafygiotis, L. S.** and **Beck, J. L.**, *Spectral density estimation of stochastic vector processes*, Probabilistic Engineering Mechanics, Vol. 17, No. 3, pp. 265–272, 2002.

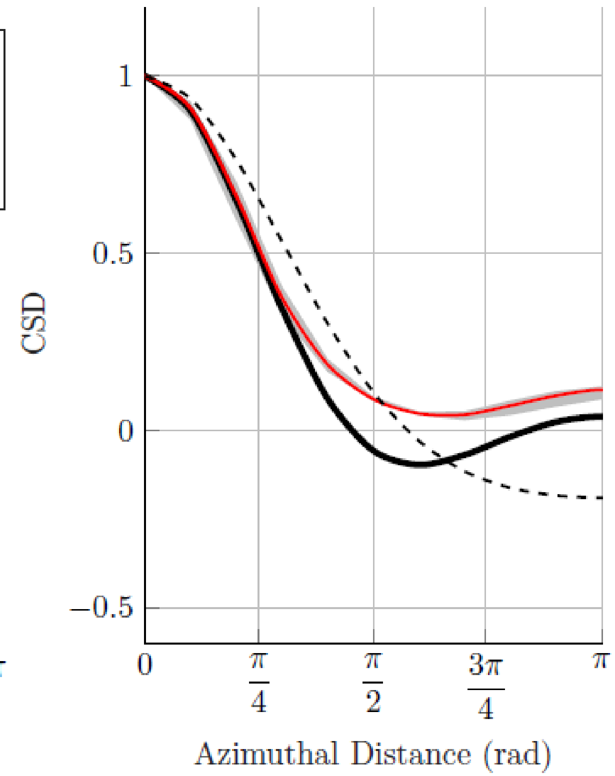
Results

- Inverse calculation performed with:
  - ka=2
  - 12 randomly placed sensors
  - *sinc* prior
- Nine nodes at zero azimuthal angle at the taken as the reference points and the spatial correlation of field points along a half-circumference are considered
- Out of the nine results, the spatial correlations providing the best and worst match to the truth are shown

Best Match

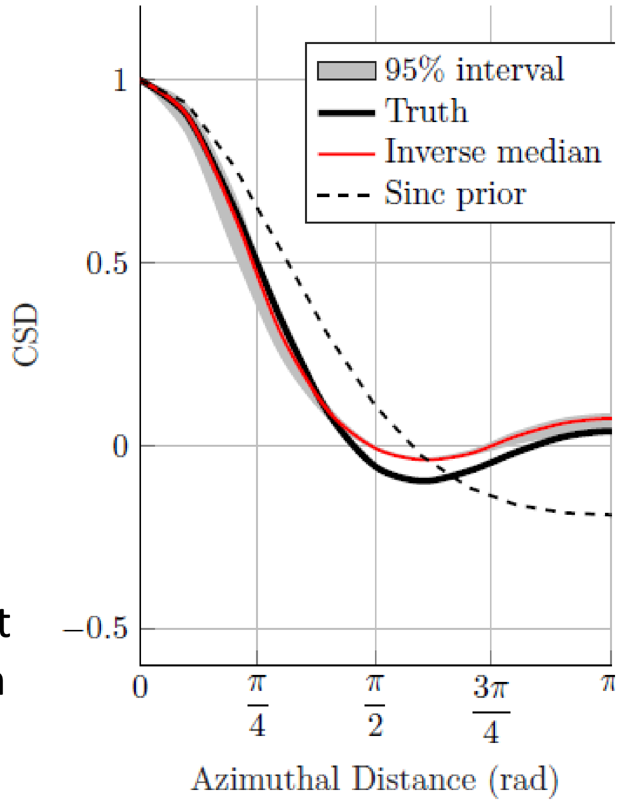


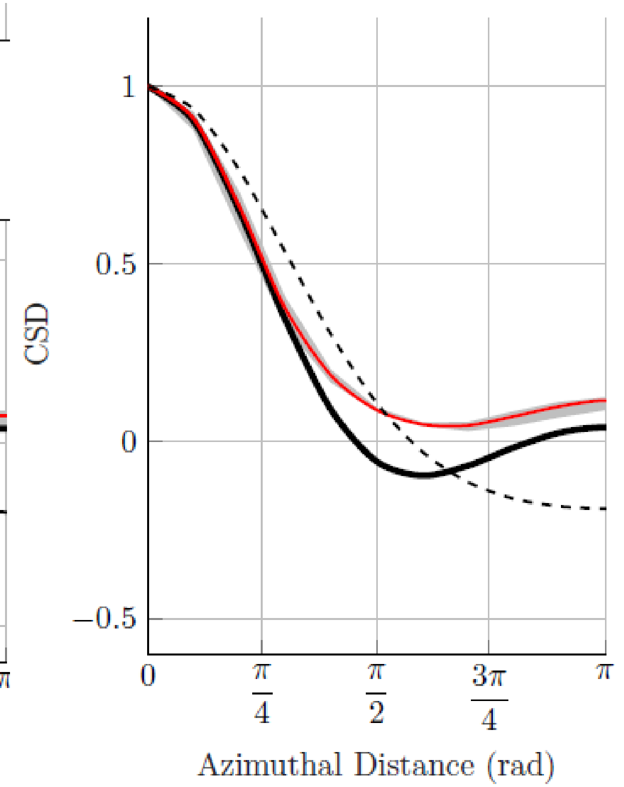
Worst Match



Results

- Providing a reasonable prior estimate of the spatial correlation (i.e., the *sinc*) can result in accurate inverse calculated CSD
- In the best match, the band representing 95% of the inverse calculations encompasses the truth CSD for the majority of the nodes along the half-circumference
- Same cannot be said for worst match, but inverse calculation is closer to the truth than it is to the prior.



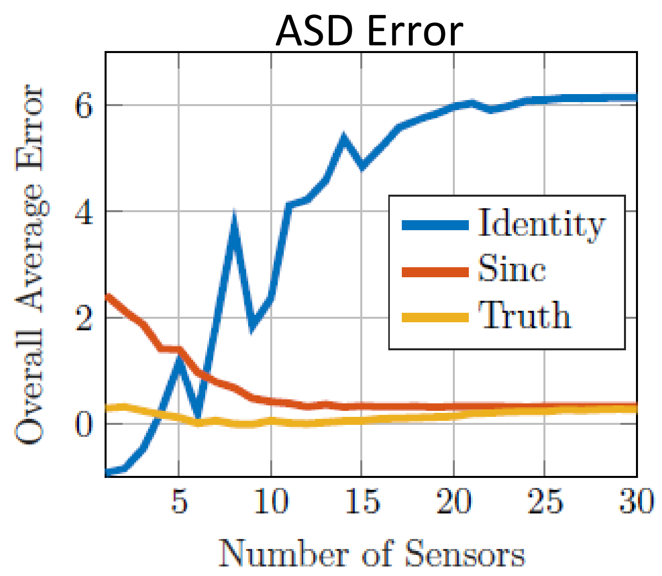


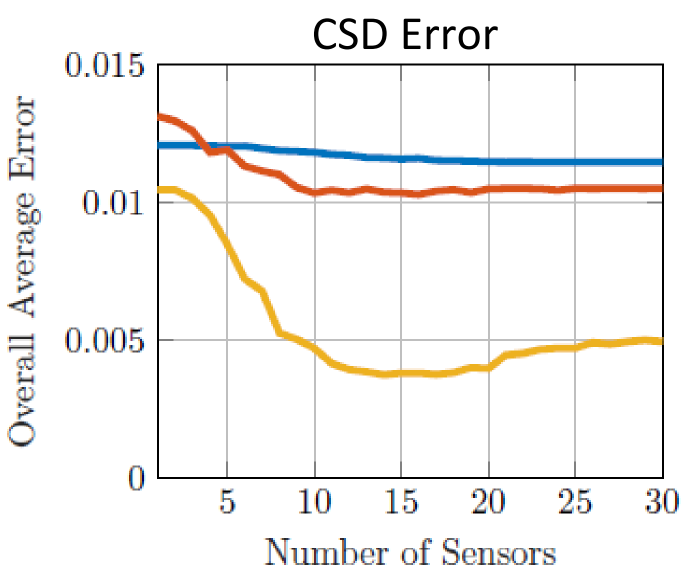
Results

- Averaged error in the inverse-calculated auto-spectra, and the CSDs for ka=2 and different quantities of sensors

- If a reasonable prior is not supplied, the inverse autospectra are very much over-estimated because the truth is more correlated than the prior indicates

- The accuracy of the inverse-calculated CSD improves with the quality of prior, with the largest improvement coming between the *sinc* and truth prior





Conclusions & Future Work

- Results indicate that the Bayesian inference approach can inverse-calculate reasonably accurate CSD matrices using a relatively small number of sensor measurements

- The quality of the inverse-calculated results depends on a number of factors
  - Quality of the prior
  - Number and placement of sensors
  - Frequency of interest

- All else being equal, the use of a refined prior appears to produce substantially more accurate CSDs

- Future work may focus on developing suitably refined priors for environments that cannot be assumed to be diffuse, such as turbulent boundary layers or direct acoustic fields