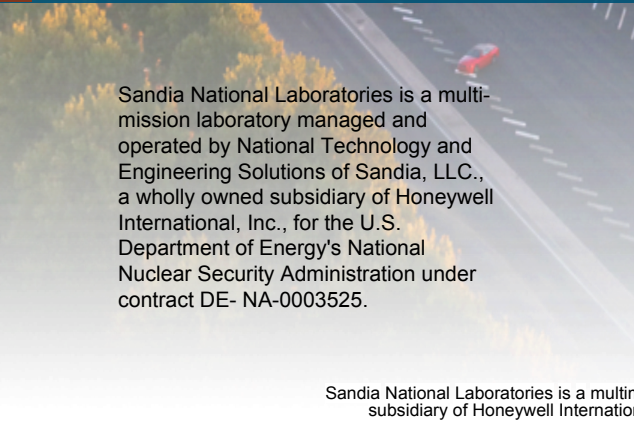


Experimental Modal Analysis of a Resonant Plate During a Mid-Field Pyroshock Replication Test



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Sandia National Laboratories

Date: 02/08/2022

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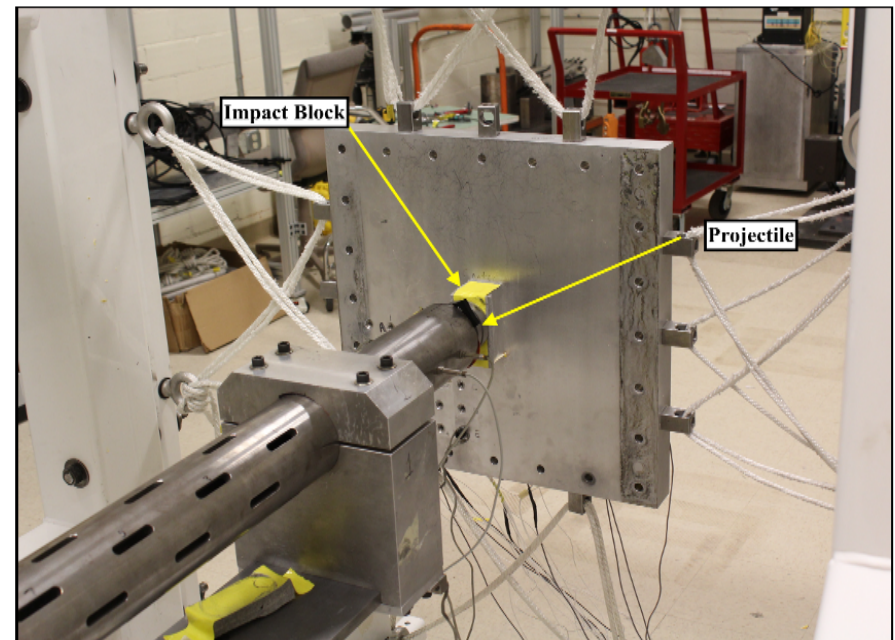
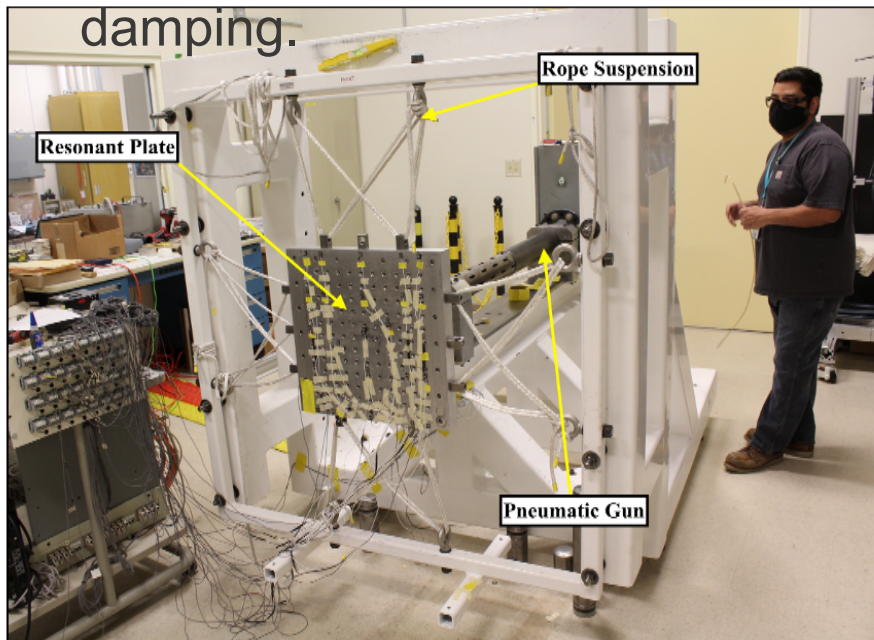
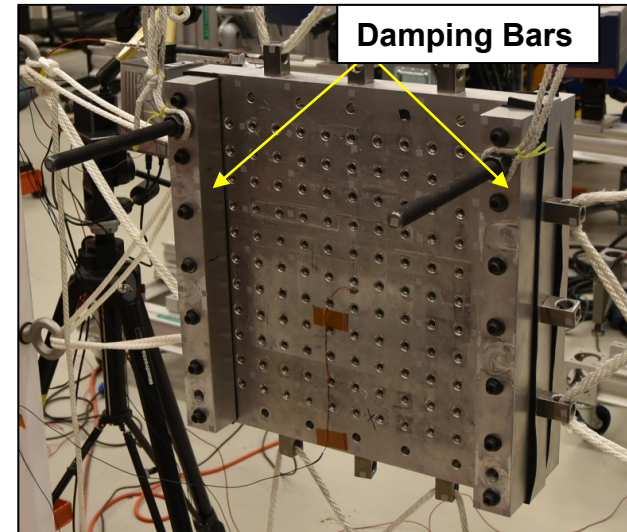
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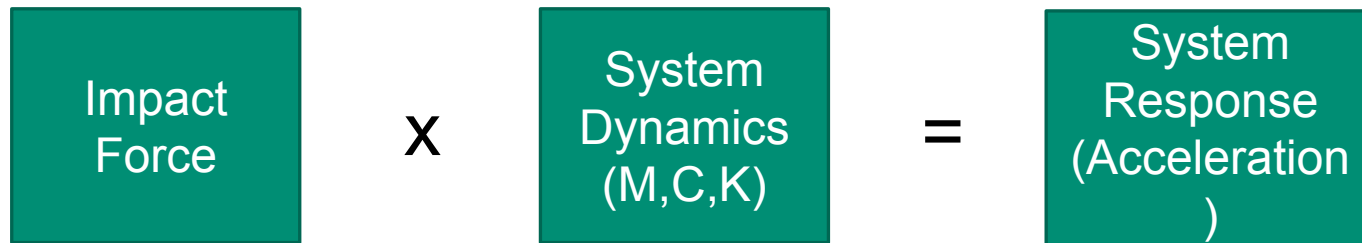
The Resonant Plate Test



- A high velocity projectile strikes the resonant plate and excites a response.
- Programmer (card stock; felt) is placed on the impact block to shape the pulse.
- Rubber is sandwiched between the plate and damping bars to increase damping.



The Resonant Plate Environment



Force

- The projectile imparts a pulse-like force on the resonant plate upon impact. This force is calculated via an inverse method, SWAT-TEEM (Sum of Weighted Accelerations Technique-Time Eliminated Elastic Motion). Load cell transducers are impractical for this application.

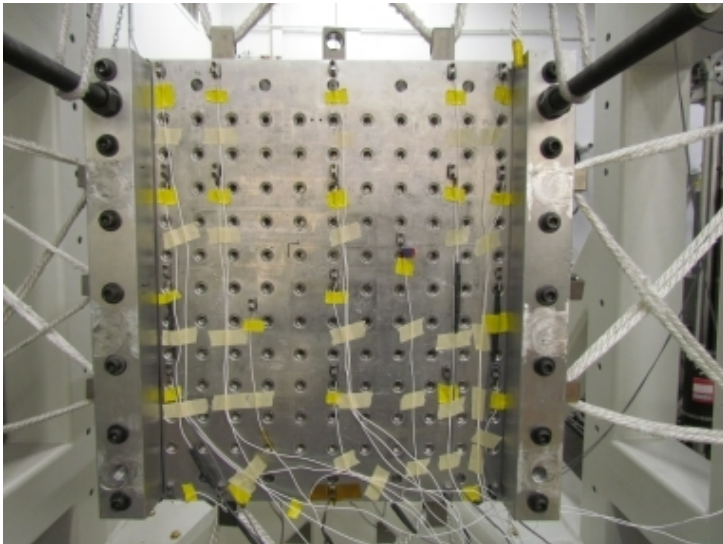
System Dynamics

- The dynamics are properties of the structure and can be measured with an experimental modal analysis test. Experimental modal analysis requires measuring the force and the response to that force and calculates the dynamic properties from that data.

Using SWAT-TEEM to Calculate the Force



- The SWAT-TEEM algorithm is used to calculate the force acting on the resonant plate from measured accelerations.
- SWAT-TEEM is particularly useful as the location of the input force is known.



The measured accelerations are projected into the rigid body motion and elastic motion.

$$\begin{bmatrix} M_r & 0 \end{bmatrix} \begin{bmatrix} \phi_r & \ddot{x}_{fd} \end{bmatrix}^+ \ddot{x} = \phi_r^T \bar{F}$$

M_r = Modal mass of rigid body mode r

$\phi_r = r^{\text{th}}$ rigid body mode shape

\ddot{x}_{fd} = Acceleration of the structure during free decay

\ddot{x} = Measured accelerations in the time domain

F = Vector of all external forces

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The elastic modal accelerations are made zero and the rigid body accelerations are scaled by the mass.

$$\begin{bmatrix} M_r & 0 \end{bmatrix} \begin{bmatrix} \phi_r & \ddot{x}_{fd} \end{bmatrix}^+ \ddot{x} = \phi_r^T \bar{F}$$

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The mass times the rigid body acceleration gives us the sum of the external forces acting at the CG.

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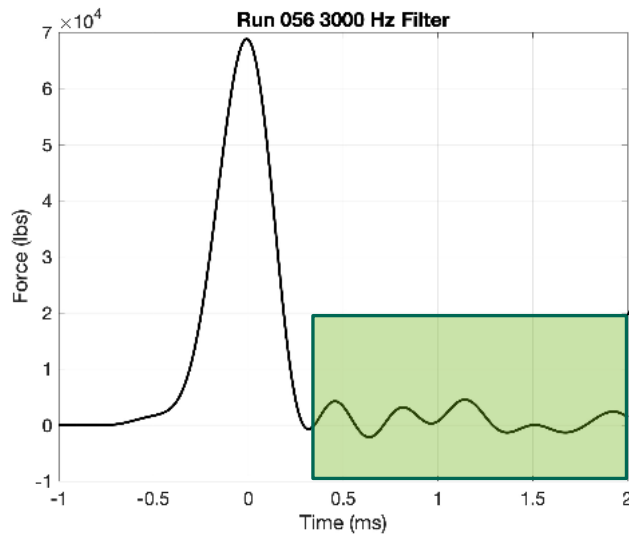
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Using SWAT-TEEM to Calculate the Force



- The SWAT-TEEM algorithm is used to calculate the force acting on the resonant plate from measured accelerations.
- SWAT-TEEM is particularly useful as the location of the input force is known.

The measured accelerations that don't project into the elastic modes results in error in the calculated force.



$$\begin{bmatrix} M_r & 0 \end{bmatrix} \begin{bmatrix} \phi_r & \ddot{x}_{fd} \end{bmatrix}^+ \ddot{x} = \phi_r^T \bar{F}$$

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Calibrating the Model System Dynamics



- An experimental modal test was on the resonant plate was executed at 'modal' levels. The test was executed with a modal hammer and laser doppler vibrometer. The model modal parameters were calibrated to match that data.

We are done, right?

Calibrating the Model System Dynamics



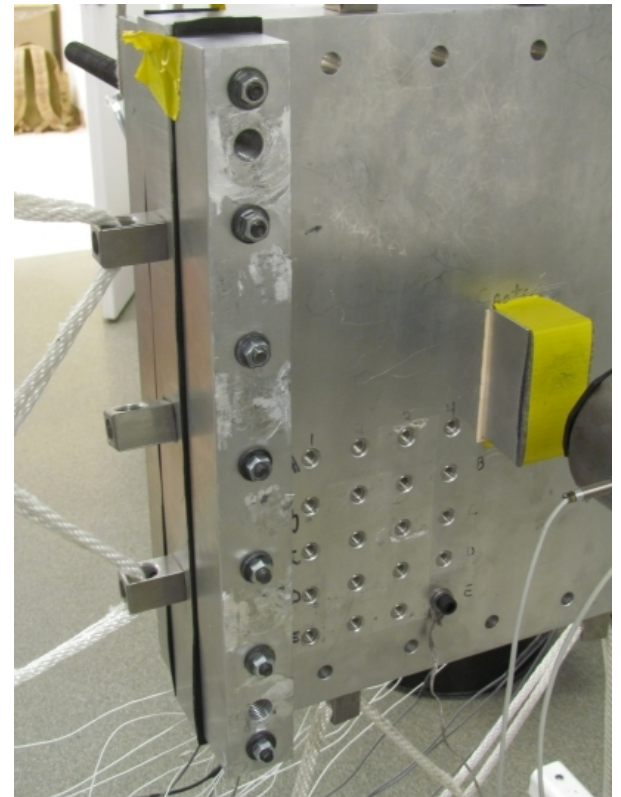
- An experimental modal test was on the resonant plate was executed at 'modal' levels. The test was executed with a modal hammer and laser doppler vibrometer. The model modal parameters were calibrated to match that data.

The system is non-linear!!!

- We know that the damping bars that are loosely torqued with 6" bolts and rubber as a member material will behave differently at different input levels.
- The modal force is around 200lbs whereas the shock pulse is in the 10s of thousands of lbs.

We are done, right?

WRONG



Have no fear!



- During the shock tests, we measured the response of 24 degrees of freedom and we have a calculated input force. Tests were repeated 4 times for test to test variation and FRF averaging.
- With measured responses and a good estimation of the force that caused the response, the FRFs are calculated and modal parameters are fit to the FRFs.

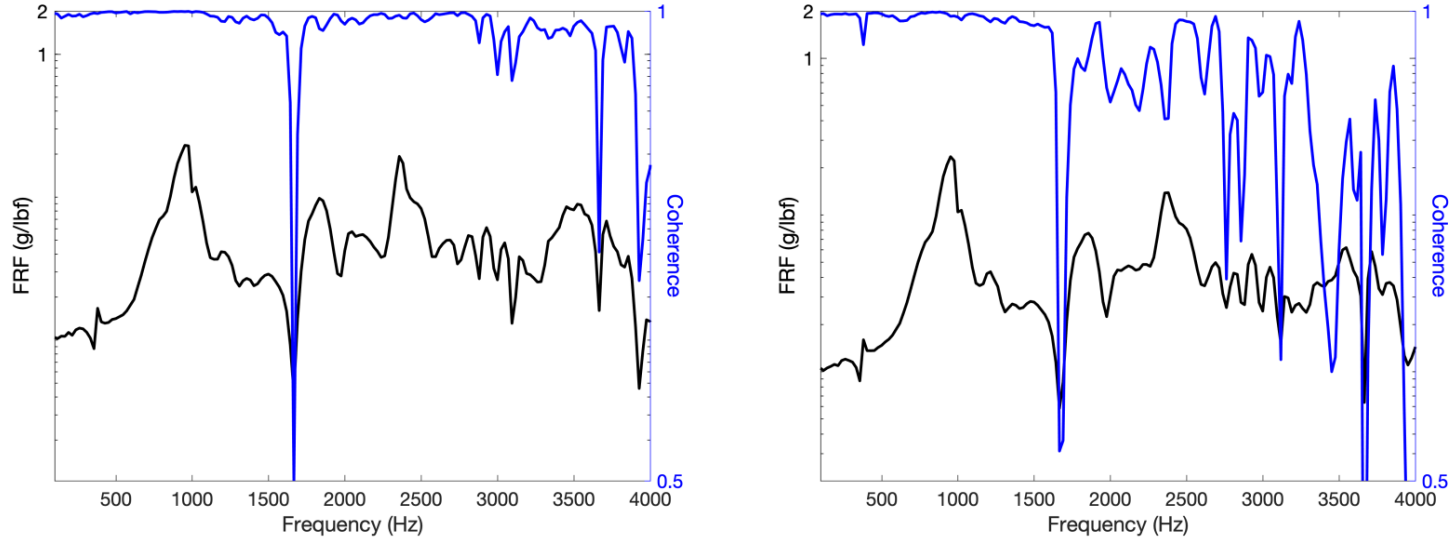


Figure: FRF and coherence of nominally identical shocks (left) and low level and high level shock (right)

Modal Comparisons



- The mode shapes between the experimental modal test and the shock tests remained mostly unchanged, so their frequencies and damping are compared.
- The data showed that the structure is linear during the different shock tests within the errors of the data.
- Having measured damping values, albeit just estimates, is critical in modeling the environment.

Table: Modal parameter comparison between different tests

Mode Number	Exp Modal Test Nat Freq (Damping)	Low Level Shock Nat Freq (Damping)	High Level Shock Nat Freq (Damping)
Mode 1 Frequency	391 Hz (0.35%)	379 Hz (0.41%)	383 Hz (0.40%)
Mode 2 Frequency	582 Hz (1.4%)	697 Hz (9.6%)	656 Hz (6.9%)
Mode 3 Frequency	1001 Hz (2.6%)	953 Hz (5.1%)	952 Hz (3.9%)
Mode 4 Frequency	1288 Hz (2.5%)	1167 Hz (4.5%)	NF
Mode 5 Frequency	2087 Hz (1.0%)	1822 Hz (4.0%)	NF
Mode 6 Frequency	2397 Hz (0.93%)	2359 Hz (1.2%)	NF



Having a good estimation of the input force, either from measurement or calculation is critical in modeling a response to an environment.

This body of work uses SWAT-TEEM to estimate the input force on the resonant plate and uses that force to calculate FRFs and consequentially modal parameters.

Having modal parameters extracted during high input forces allows us to not use extrapolation methods to model the hardware's frequencies and damping.

Bonus Information:

Many resonant plate tests were executed with different programmers, projectile sizes, and projectile speeds. The forces calculated for the different shock tests are related to each other in form. This means that I can take a low level, 'slow' force and modify its amplitude and pulse width to get a high level, 'fast' forcing function.