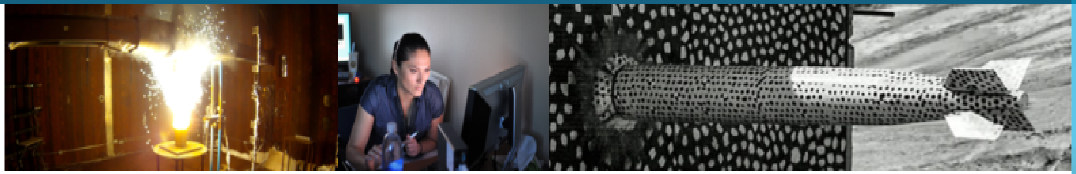
SAND2021-15981PE

Sandia National Laboratories

Explosive Analysis of Nuclear Packaging



Environmental Management

safety ✣ performance ✣ cleanup ✣ closure

DOE Packaging Certification Program

- ***Module 7 – Explosive Shock***

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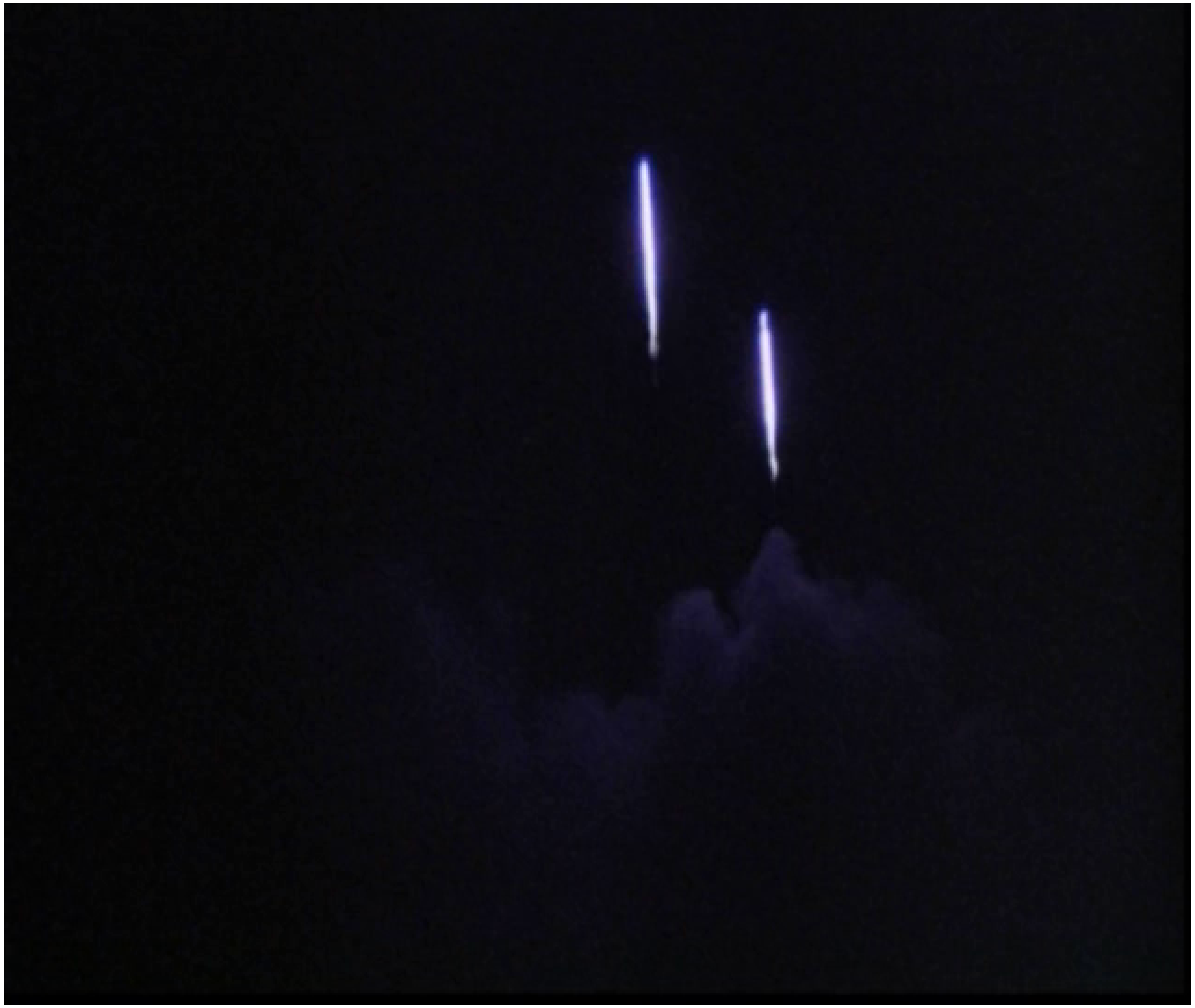
Module 7

- **Detonation Shock Theory**



Thirteen Days Introduction Sequence

Four Classic Shock Problems

1. Impact of one material into another material

   The shock interface pressure will depend on material properties and impact velocity

Shock crosses an interface between two different materials

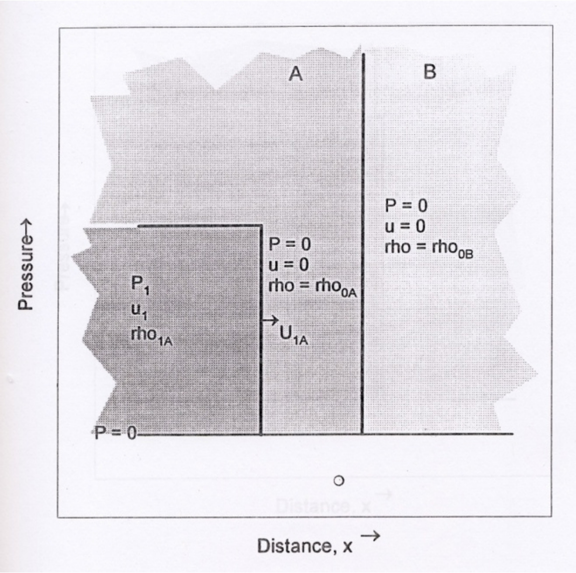
2. Case 1, where the shock travels from a low impedance material into a higher impedance material
   - The interface pressure will ***INCREASE*** from the incident pressure
3. Case 2, where the shock travels from a high impedance material into a lower impedance material
   - The interface pressure will ***DECREASE*** from the incident pressure

4. Collision of two shock waves within a material
   - The collision interface pressure will be ***GREATER*** than the sum of the two incident waves

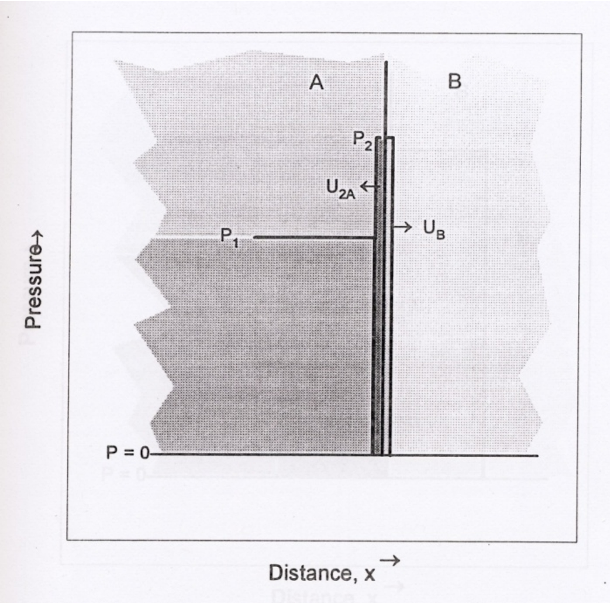
2. Shock Traveling from Low to High Impedance Material

- Here, a shock wave is traveling through A (low impedance material) towards B (high impedance material)
- At the interface, a shock will be transmitted to B, and reflected back into A
- The resulting pressure will be higher (for this configuration) than the input pulse.

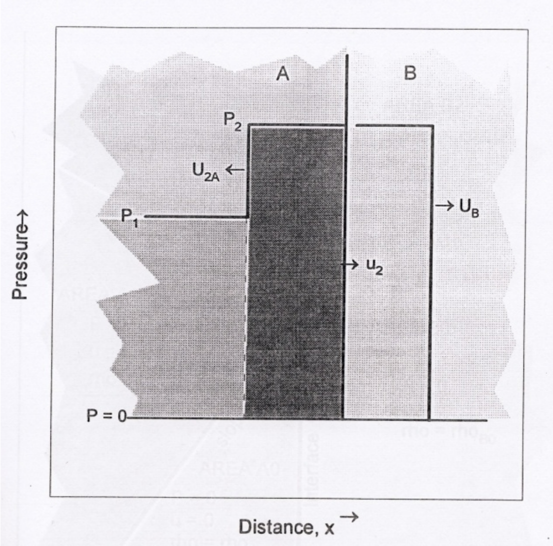
2. Shock Traveling from Low to High Impedance Material



Shock approaches interface through low impedance material

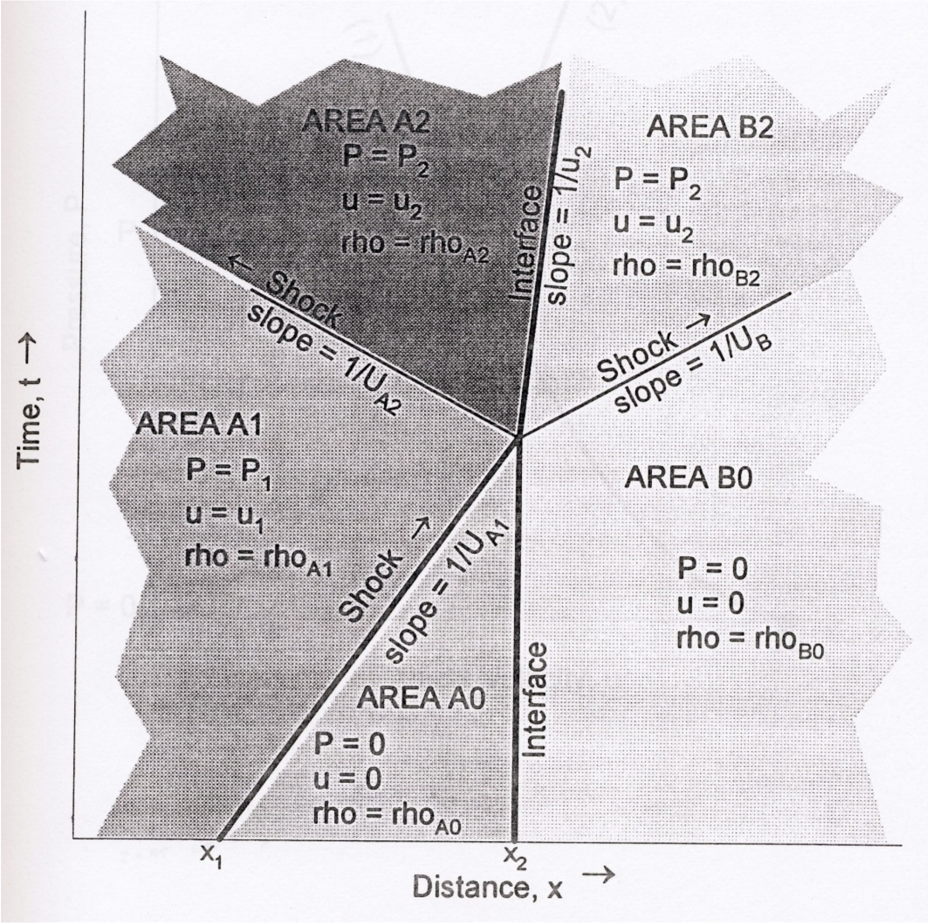


Shock reaches interface sending a Shock into B and a reflection back into A, with an increase in pressure



The reflected (left-going) and transmitted pressure (right-going) waves progress into the material

2. Shock Traveling from Low to High Impedance Material



The XT Diagram

2. Shock Traveling from Low to High Impedance Material

- What do we know?
  - Often the amplitude of the pressure
- We can develop or construct the left-facing Hugoniot (in material A before the interface) from the right-facing *P-u* Hugoniot (in material A)
- Why do we need the Left-facing Hugoniot if we know what the right-going wave is doing?
  - In order to equate the pressures and particle velocities at the interface, we must equate P_L and P_R

Constructing the Left-Going Hugoniot

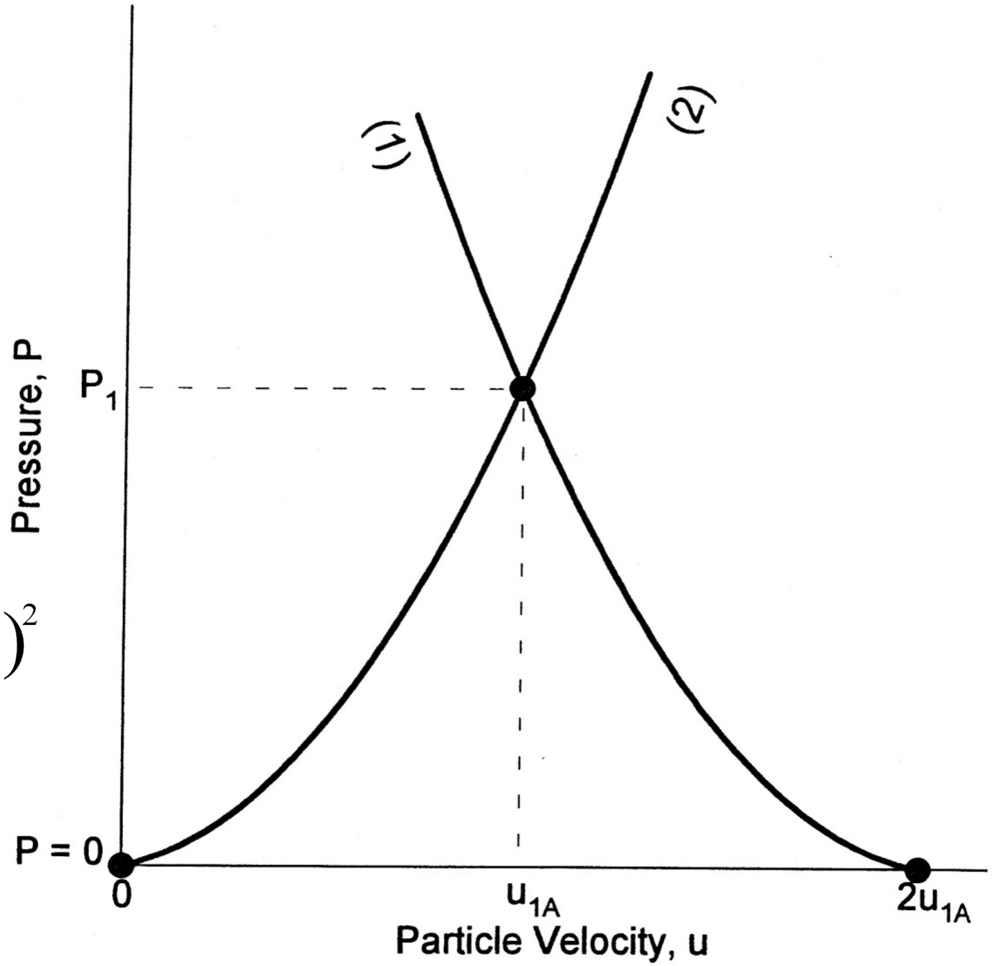
- Assuming a zero particle velocity in the slab

$$P_R = \rho_{0A} C_{0A} u + \rho_{0A} s_A u^2$$

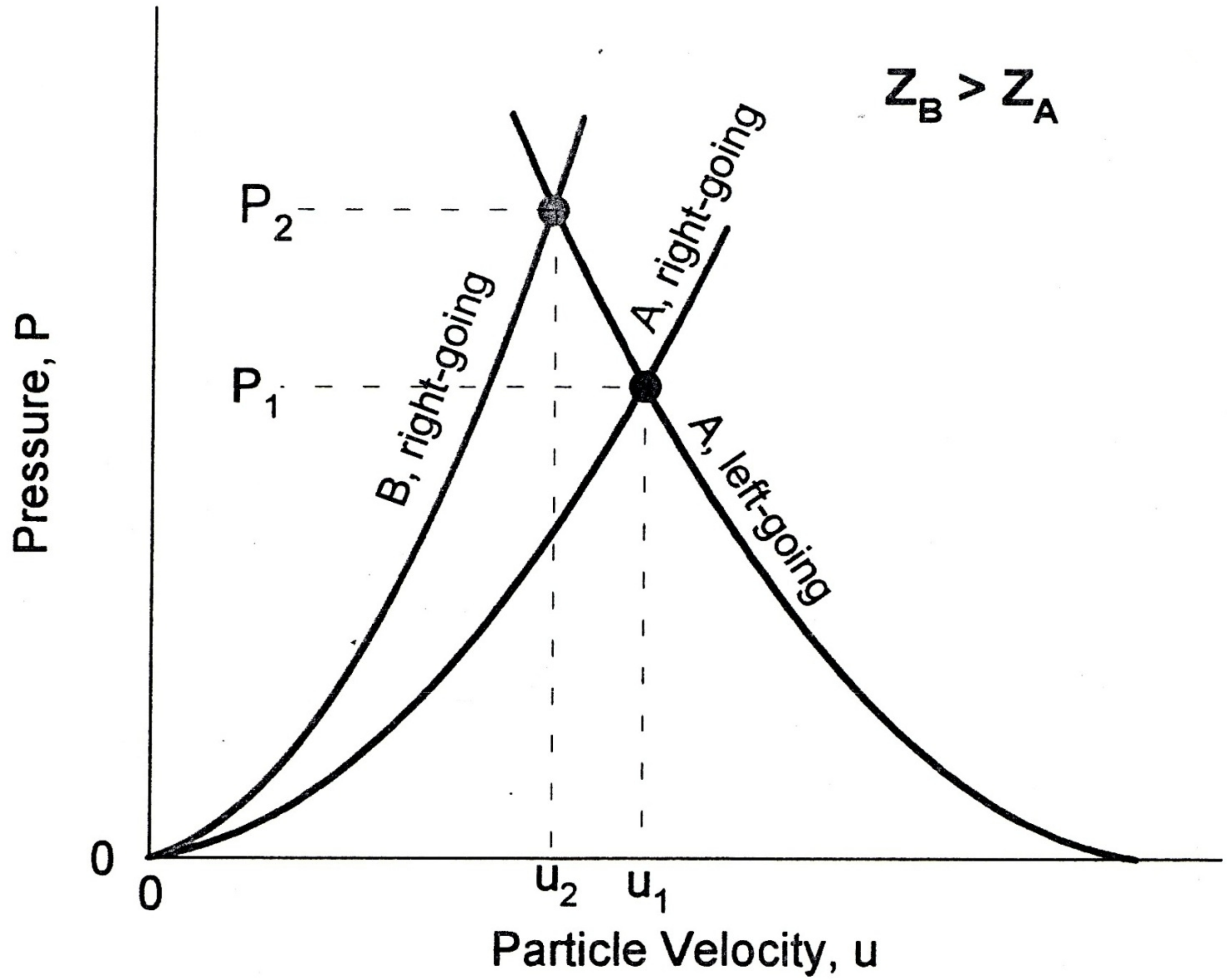
- The left-facing Hugoniot in the material will be a mirror image of the right-facing equation about the particle velocity at P_1

$$P_L = \rho_{0A} C_{0A} \left(u_{0A} - u\right) + \rho_{0A} s_A \left(u_{0A} - u\right)^2$$

- u_0 (left-going) will be twice the particle velocity at *P*



Using the Constructed Hugoniot



Example Problem

A = 921-T Aluminum
PA = 25 GPa
B = Copper

- A slab of 921-T aluminum (material A) is in contact with a slab of copper (material B).
- A long pulse shock wave traveling through the aluminum encounters the interface.
- The initial shock pressure in the aluminum was 25 GPa,
  - What pressure does this change to when the shock interacts at the interface?

Example Cont'd

First, we need to find material properties of the Aluminum and Copper

921-T Aluminum	Copper
$\rho_0 = 2.833 \ {}^{\text{g}}/_{\text{cm}^3}$	$\rho_0 = 8.930 \ {}^{\text{g}}/_{\text{cm}^3}$
$C_0 = 5.041 \ {}^{\text{km}}/_{\text{s}}$	$C_0 = 3.940 \ {}^{\text{km}}/_{\text{s}}$
$s = 1.420$	$s = 1.489$

Next, we need to find the particle velocity behind the oncoming shock in the Aluminum before it encounters the interface. This establishes the condition that will start to form the left facing Hugoniot after the interaction.

$$P_1 = \rho_0 C_0 u + \rho_0 s u^2$$

$$25 = (2.833)(5.041)u + (2.833)(1.420)u^2$$

$$4.023 \cdot u^2 + 14.28 \cdot u - 25 = 0$$

Solving the quadratic equation, we get

$$u_1 = 1.285 \ {}^{\text{km}}/_{\text{s}}$$

Example Cont'd

Now we can write the equation for the left-facing Hugoniot in the aluminum at the interface. Remember, u_0 will be double the particle velocity calculated for the incoming shock

$$P = \rho_0 C_0 (u_0 - u) + \rho_0 s (u_0 - u)^2$$

$$P = (2.833)(5.041)((2)1.285 - u) + (2.833)(1.420)((2)1.285 - u)^2$$

The Hugoniot equation for the shock in the copper after the interaction will be

$$P = \rho_0 C_0 u + \rho_0 s u^2$$

$$P = (8.93)(3.94)u + (8.93)(1.489)u^2$$

Example Cont'd

Since the shock pressure and particle velocity in both materials must be the same at the material interface at the time of the interaction, we can equate the pressure equations to solve for particle velocity.

$$(14.281)(2.57-u)+(4.023)(2.57-u)^2=(35.184)u+(13.30)u^2$$

Solving for particle velocity

$$u=0.814\ {}^{\mathrm{km}}\!/_{\mathrm{s}}$$

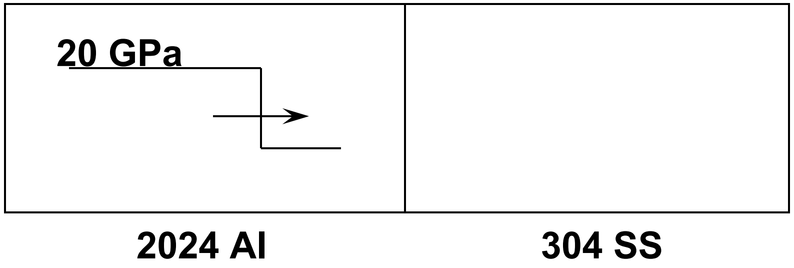
Then plugging back into one of the pressure equations:

$$P=(35.184)(0.814)+(13.30)(0.814)^2$$

$$P=37.5\ \mathrm{GPa}$$

Shock transmission example:

Starting with a right facing Hugoniot in Al

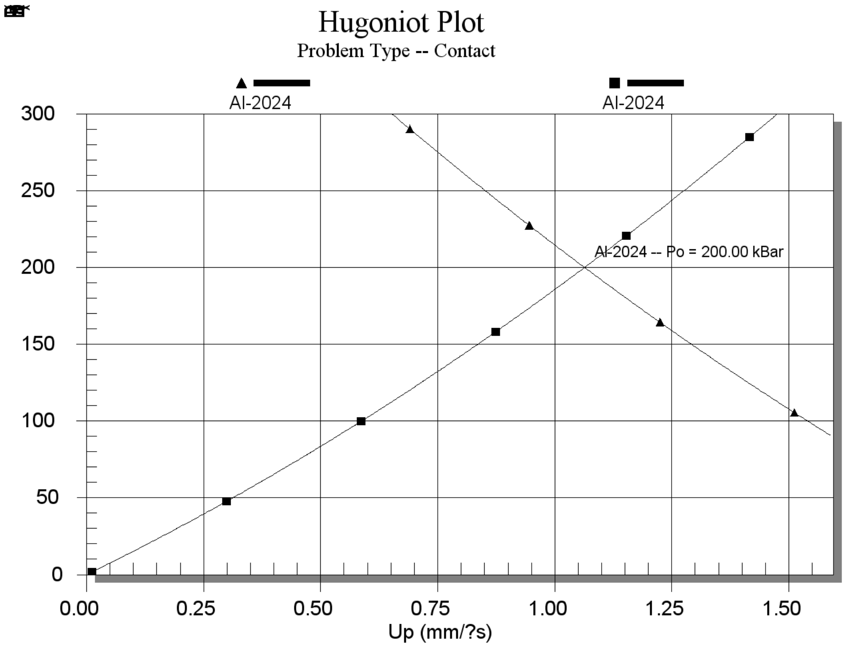


$$P = \rho_0 C_0 u + \rho_0 s u^2$$

$$20 = (14.388)u + (3.726)u^2$$

Solving for particle velocity (*u*)

$$u = 1.062 \,{}^{km}/_{s} \quad \text{or} \quad u = -5.006 \,{}^{km}/_{s}$$



We can now construct our left facing Hugoniot

$$P_{Al} = \rho_0 C_0 (u_0 - u) + \rho_0 s (u_0 - u)^2$$

Where u_0 is 2*u* = 2(1.062) = 2.124 km/s

$$P_{Al} = (3.726)u^2 - (30.666)u + 48.324$$

Shock transmission example (cont'd):

At the material interface, we know that there will be a right facing Hugoniot in the SS.

$$P = \rho_0 C_0 u + \rho_0 s u^2$$

$$P_{SS} = (36.077)u + (11.765)u^2$$

$$P_{Al} = P_{ss}$$

$$(3.726)u^2 - (30.666)u + 48.324 = (36.077)u + (11.765)u^2$$

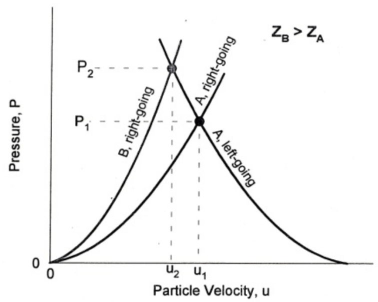
$$u = 0.670 \, \mathrm{km/s} \quad \text{or} \quad u = -9.010 \, \mathrm{km/s}$$

$$P = 29.452 \text{ GPa}$$

From here, we can find shock velocity (U) from the momentum equation

$$P = \rho_0 (u - u_0)(U - u_0)$$

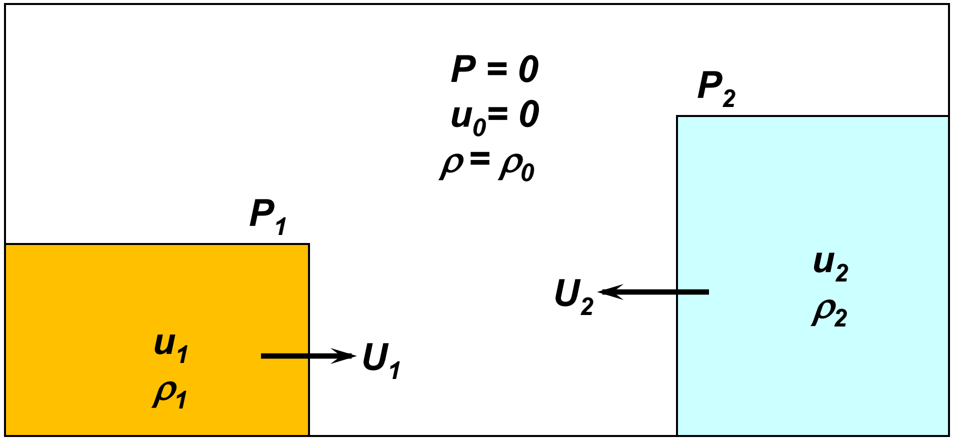
Shock Across a Material Interface



1. Diagram the problem
2. Find material properties
3. Construct the right facing Hugoniot for material 2 ---------- $P_R = \rho_0 c_0 (u_2 - u_0) + \rho_0 s (u_2 - u_0)^2$
4. Find the particle velocity behind shock in material 1 ---------- $P_1 = \rho_0 C_0 u_1 + \rho_0 s u_1^2$, $u_1 = x.xxx \, \mathrm{km/s}$
5. Construct the left facing Hugoniot for material 1 ---------- $P_L = \rho_0 c_0 (2 \cdot u_1 - u_2) + \rho_0 s (2 \cdot u_1 - u_2)^2$
6. Set Pressures equal to each other ---------- $P_R = P_L$
7. Solve for particle velocity (u) at interface ---------- $u_2 = x.xxx \, \mathrm{km/s}$
8. Solve for pressure at interface ---------- $P_R = \rho_0 c_0 (u_2 - u_0) + \rho_0 s (u_2 - u_0)^2$
9. Solve for shock velocity (U) in material 1 ---------- $U = c_0 + su$ or $P = \rho_0 (u - u_0)(U - u_0)$
10. Solve for shock velocity (U) in material 2 ---------- $U = c_0 + su$

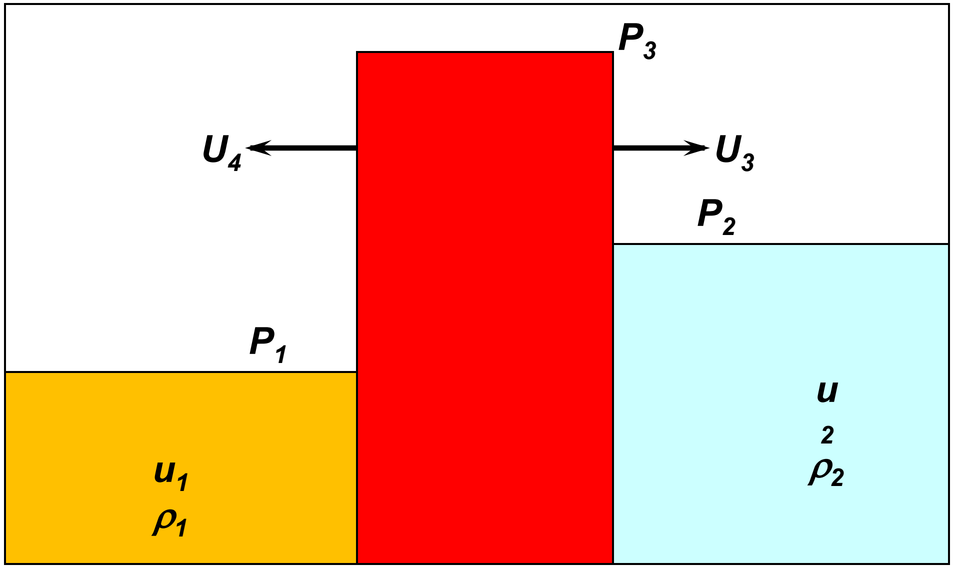
Shock Wave Interaction

- Imagine a scenario where two shock waves are traveling towards each other on a collision course
  - One-dimensional discussion
  - Single material (no interfaces
  - Potentially different pressure amplitudes, shock speeds
  - Left and right going waves each have their own shock properties



Shock Wave Interaction

- At the intersection, there will be an increase in pressure
- There will be a "transmitted and reflected wave traveling form the intersection
- To solve for resultant pressure,
  - Construct left facing Hugoniot from the original right going wave
  - Construct right facing Hugoniot from the original left going wave
    - Here we will be in the negative part of the plane since the particle velocity will be negative per our directional convention
  - Equate the two Hugoniots to solve for particle velocity at the interface



Shock Wave Interaction

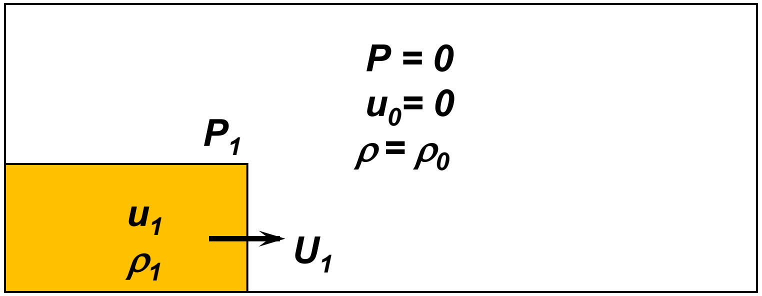
- Construct left facing Hugoniot from the original right going wave
  - Just like what we did in problem where the pressure wave encountered a material interface
  - Given material properties and an input pressure
    - Solve for particle velocity

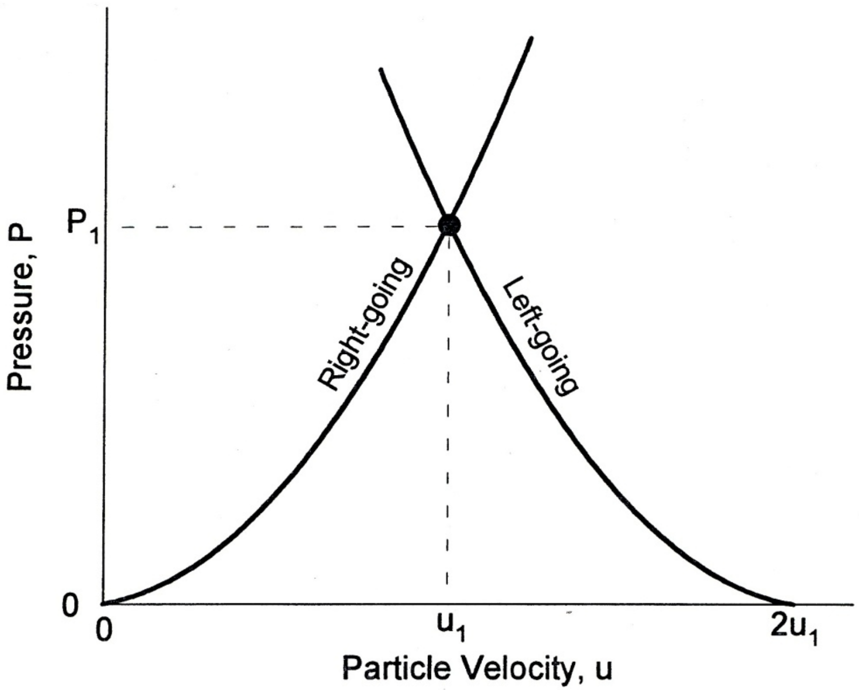
$$P = \rho_0 C_0 \left(u_1 - u_0\right) + \rho_0 s \left(u_1 - u_0\right)^2$$

$$P = \rho_0 C_0 u_1 + \rho_0 s u_1^2$$

    - The effective particle velocity for left-going Hugoniot will be $2u_1$

$$P_L = \rho_0 C_0 \left(2u_1 - u\right) + \rho_0 s \left(2u_1 - u\right)^2$$





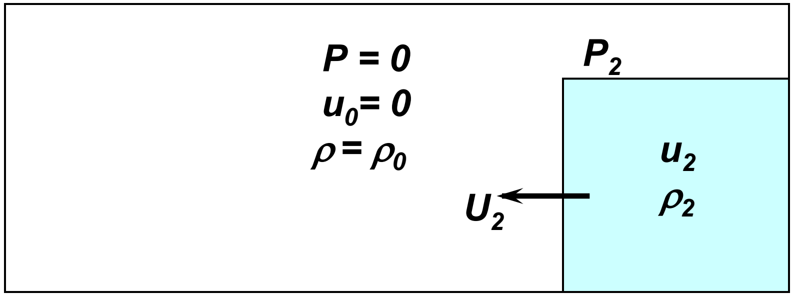
Shock Wave Interaction

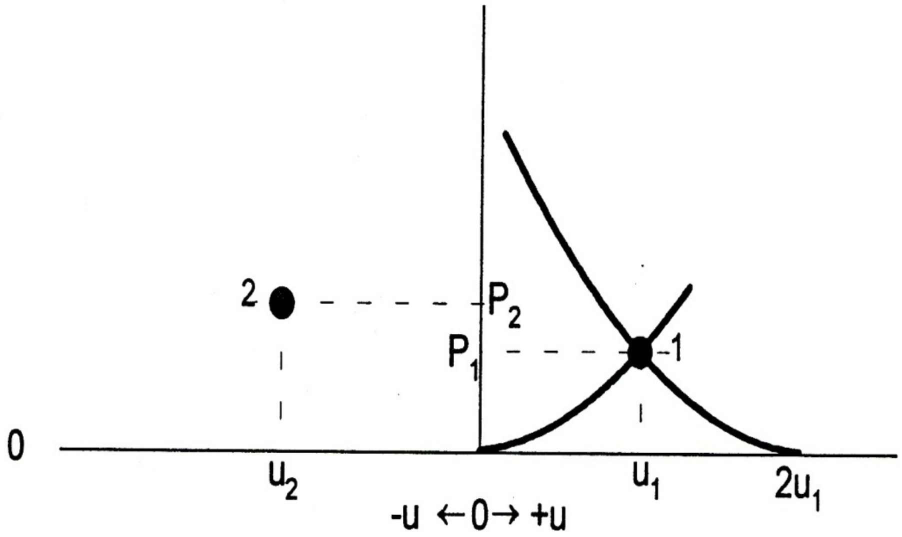
- Construct right facing Hugoniot from the original left going wave
  - Here, we must take the sign convention into account
  - The left going wave is traveling in the NEGATIVE direction
  - Therefore, constructing the right facing Hugoniot will be done in the negative velocity region of the plane

$$P = \rho_0 C_0 \left(u_0 - u_2\right) + \rho_0 s \left(u_0 - u_2\right)^2$$

$$P = \rho_0 C_0 \left(-u_2\right) + \rho_0 s \left(-u_2\right)^2$$

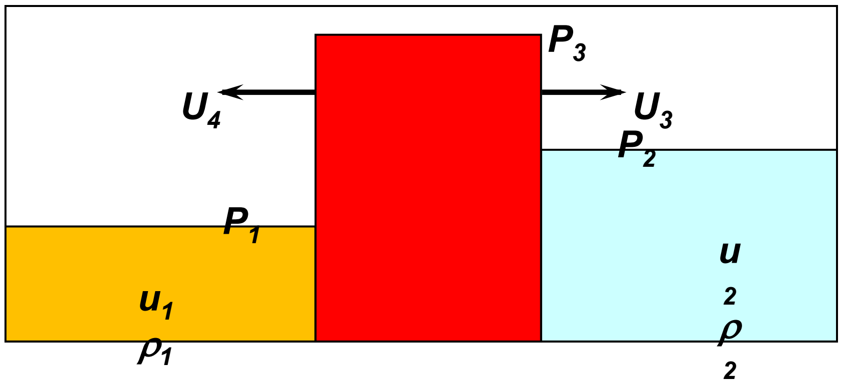
$$P_R = \rho_0 C_0 \left(u - 2u_2\right) + \rho_0 s \left(u - 2u_2\right)^2$$

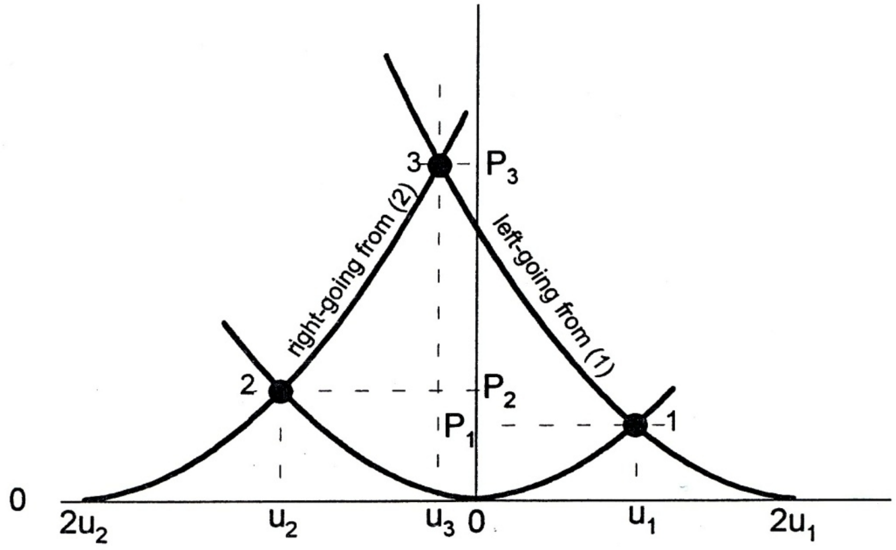




Shock Wave Interaction

- Equate the two Hugoniots (right going pressure = left going pressure) to solve for particle velocity at the interface
- What do we know?
  - In this case, the left going pressure is higher, resulting in a higher particle velocity (in this case to the left)
  - The intersection of the two waves results in a particle velocity slightly to the left
  - The pressure at the interaction is more than the sum of the two individual pressures
    - the Hugoniot is a quadratic





Shock Wave Interaction Example

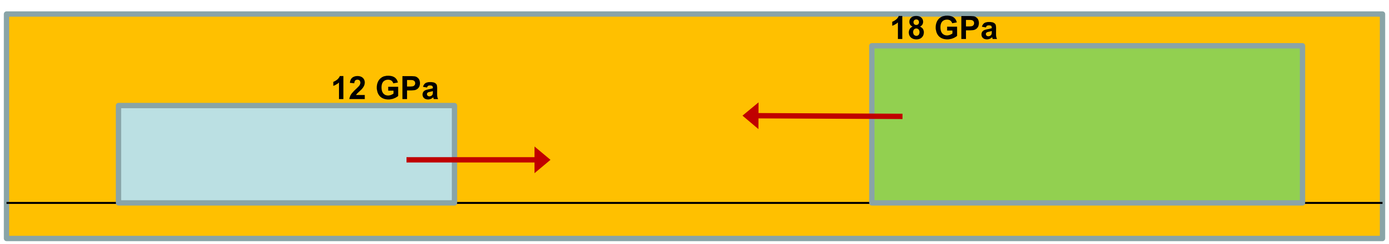
- In a slab of brass there is a shock traveling toward the right with a pressure of 12 GPa. In the same slab there is also a shock traveling towards the left with a pressure of 18 GPa on a head-on collision course with the other shock. When the two shocks collide, what will the resultant particle velocity and pressure be?

Solution

First, lets get the U-u Hugoniot values for brass

$$\rho_0 = 8.450 \text{ g/cm}^3$$
$$C_0 = 3.726 \text{ km/s}$$
$$s = 1.434$$

Next, diagram the problem



Shock Wave Interaction Example

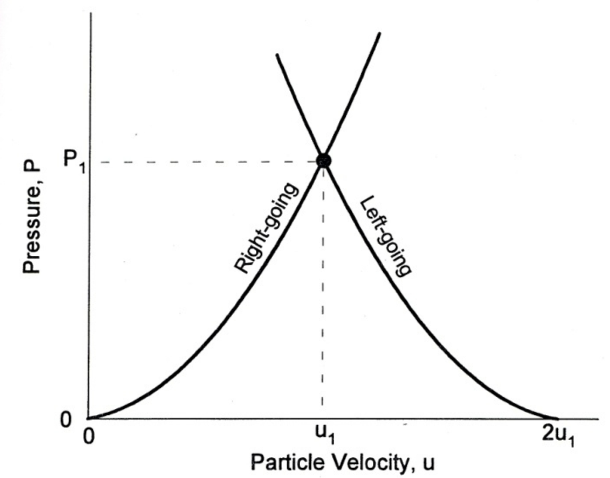
- After the interaction of the two shocks, there will be a left going wave whose P-u Hugoniot is the reflection around the P-u state of the original right-going shock. To find this, we must first find the particle velocity of the original shock at 12 GPa

$$P = \rho_0 C_0 (u_1 - u_0) + \rho_0 s (u_1 - u_0)^2$$

$$P = \rho_0 C_0 u_1 + \rho_0 s u_1^2$$

$$12 = (8.450)(3.762) u_1 + (8.450)(1.434) u_1^2$$

$$u_1 = 0.335 \,{}^{\text{km}}/_{\text{s}} \quad \text{or} \quad u_1 = -2.958 \,{}^{\text{km}}/_{\text{s}}$$



- But we know that since the original shock wave is moving to the right, so must the particle velocity. Here, the negative root cannot be correct. $u_1 = 0.335 \,{}^{\text{km}}/_{\text{s}}$ The resultant Left facing Hugoniot is:

$$P_L = \rho_0 C_0 (2u_1 - u) + \rho_0 s (2u_1 - u)^2$$

$$P_L = (8.450)(3.762)(2(0.335) - u) + (8.450)(1.434)(2(0.335) - u)^2$$

Shock Wave Interaction Example

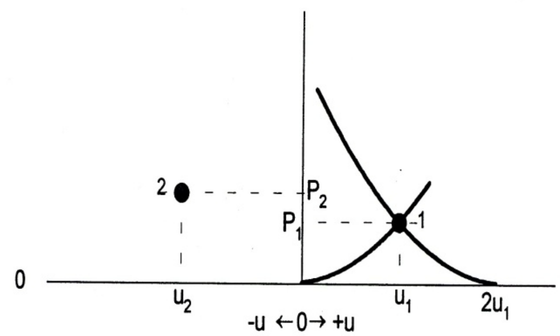
- After the interaction there will be a right facing Hugoniot whose P-u curve is a reflection of the original left-going shock whose pressure was 18 GPa. Since this original wave was left going into still material, its Hugoniot is

$$P = \rho_0 C_0 \left(u_0 - u_2\right) + \rho_0 s \left(u_0 - u_2\right)^2$$

$$P = \rho_0 C_0 \left(-u_2\right) + \rho_0 s \left(-u_2\right)^2$$

$$18 = (8.450)(3.762)(-u_2) + (8.450)(1.434)(-u_2)^2$$

$$u_2 = -0.4788 \, \text{km}/\text{s} \quad \text{or} \quad u_2 = 3.1023 \, \text{km}/\text{s}$$



- But we know that since the original shock wave is moving to the left, so must the particle velocity (negative). Here, the positive root must not be correct. $u_2 = -0.4788 \, \text{km}/\text{s}$ The resultant Right facing Hugoniot is:

$$P_R = \rho_0 C_0 \left(u - 2u_2\right) + \rho_0 s \left(u - 2u_2\right)^2$$

$$P_R = (8.450)(3.762)\left(u - 2(-0.4788)\right) + (8.450)(1.434)\left(u - 2(-0.4788)\right)^2$$

Shock Wave Interaction Example

- Now we can set our constructed left going Hugoniot to our constructed right going Hugoniot to solve for particle velocity of the resulting interaction

$$P_L = (8.450)(3.762)(2(0.335) - u) + (8.450)(1.434)(2(0.335) - u)^2$$

$$P_R = (8.450)(3.762)(u - 2(-0.4788)) + (8.450)(1.434)(u - 2(-0.4788))^2$$

$$P_L = P_R$$

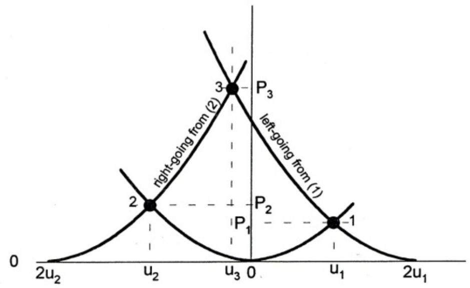
$$u_3 = -0.145\ {}^{\text{km}}\!/_{\text{s}}$$

- Finally, we can solve for pressure using u_3 in either P_L or P_R

$$P = 33.9\ \text{GPa}$$

- Note that our colliding shock waves were originally 12 GPa and 18 GPa, and the resulting shock was 33.9 GPa…more than the sum of the original two waves!

Collision of Two Shock Waves



1. Diagram the problem
2. Find material properties
3. Find particle velocity behind Shock 1 ---------- $P_1 = \rho_0 C_0 (u_1 - u_0) + \rho_0 s (u_1 - u_0)^2$

$P_1 = \rho_0 C_0 u_1 + \rho_0 s u_1^2$

$u_1 = x.xxx \, ^{\mathrm{km}}/_{\mathrm{s}}$

4. Construct left facing Hugoniot (Shock 1) ---------- $P_L = \rho_0 C_0 (2u_1 - u_3) + \rho_0 s (2u_1 - u_3)^2$
5. Find particle velocity behind Shock 2 ---------- $P_2 = \rho_0 C_0 (u_0 - u_2) + \rho_0 s (u_0 - u_2)^2$

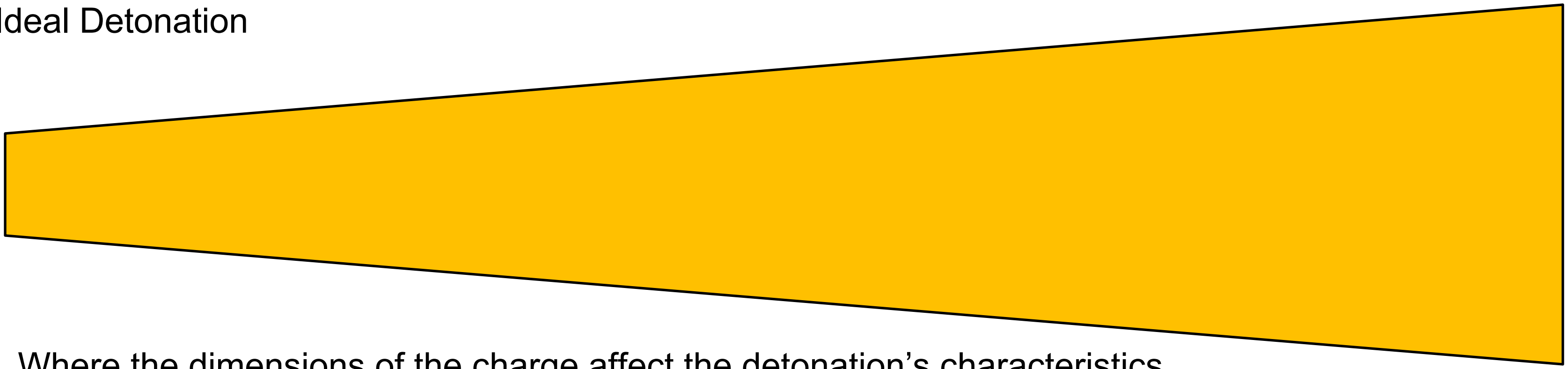
$P_2 = \rho_0 C_0 (-u_2) + \rho_0 s (-u_2)^2$

$u_2 = x.xxx \, ^{\mathrm{km}}/_{\mathrm{s}}$

6. Construct right facing Hugoniot (Shock 2) ---------- $P_R = \rho_0 C_0 (u_3 - 2u_2) + \rho_0 s (u_3 - 2u_2)^2$
7. Set Pressures equal to each other ---------- $P_R = P_L$
8. Solve for particle velocity (u) at collision ---------- $u_3 = x.xxx \, ^{\mathrm{km}}/_{\mathrm{s}}$
9. Solve for pressure at collision ---------- $P_L = \rho_0 C_0 (2u_1 - u_3) + \rho_0 s (2u_1 - u_3)^2$

Detonation

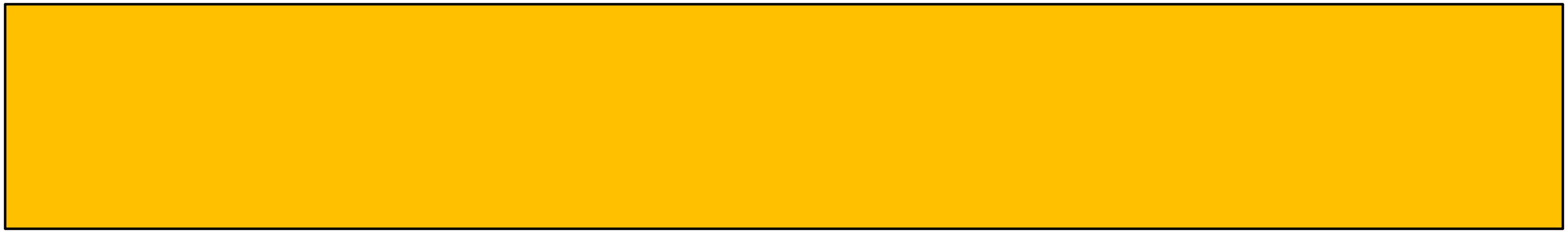
Non-Ideal Detonation



Where the dimensions of the charge affect the detonation's characteristics

- CJ Pressure can vary
- Detonation Velocity can vary

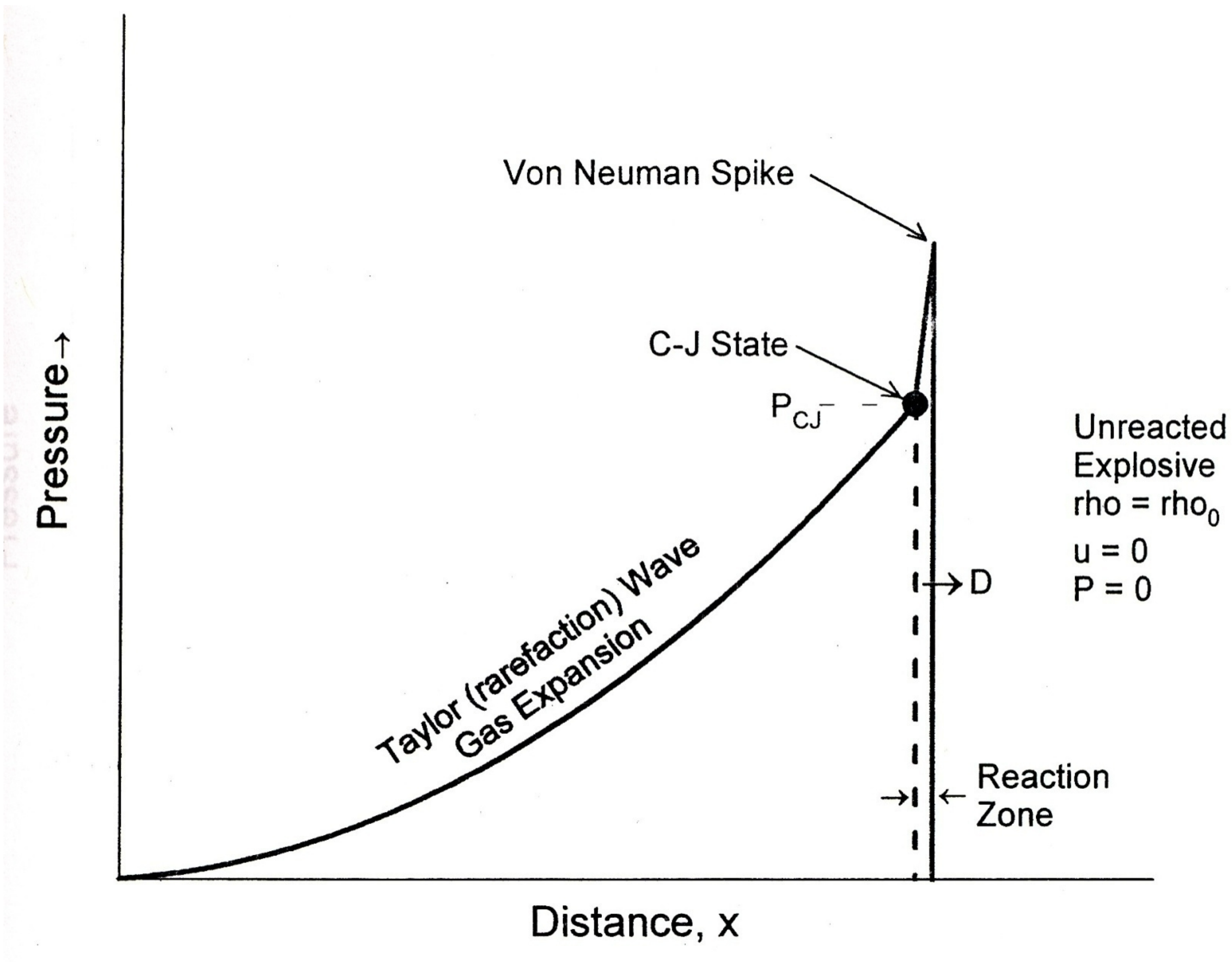
Ideal Detonation



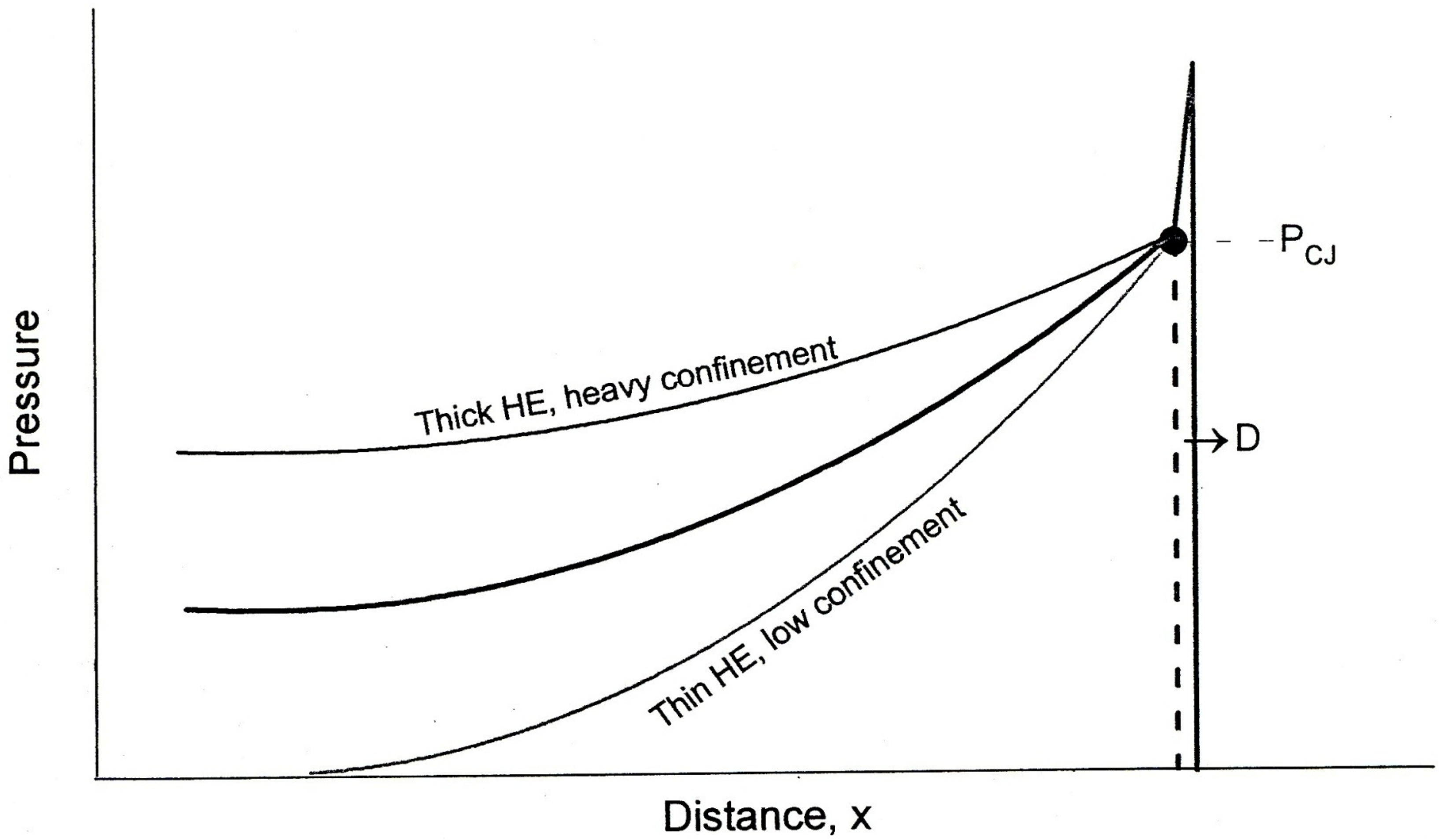
Where the cross section of the explosive is large enough to have no diameter effect

- CJ Pressure is constant
- Detonation Velocity is constant

Detonation – The Generalized Explosive Pressure Pulse



Detonation – The Taylor Wave



Simple Theory (ZND Model)

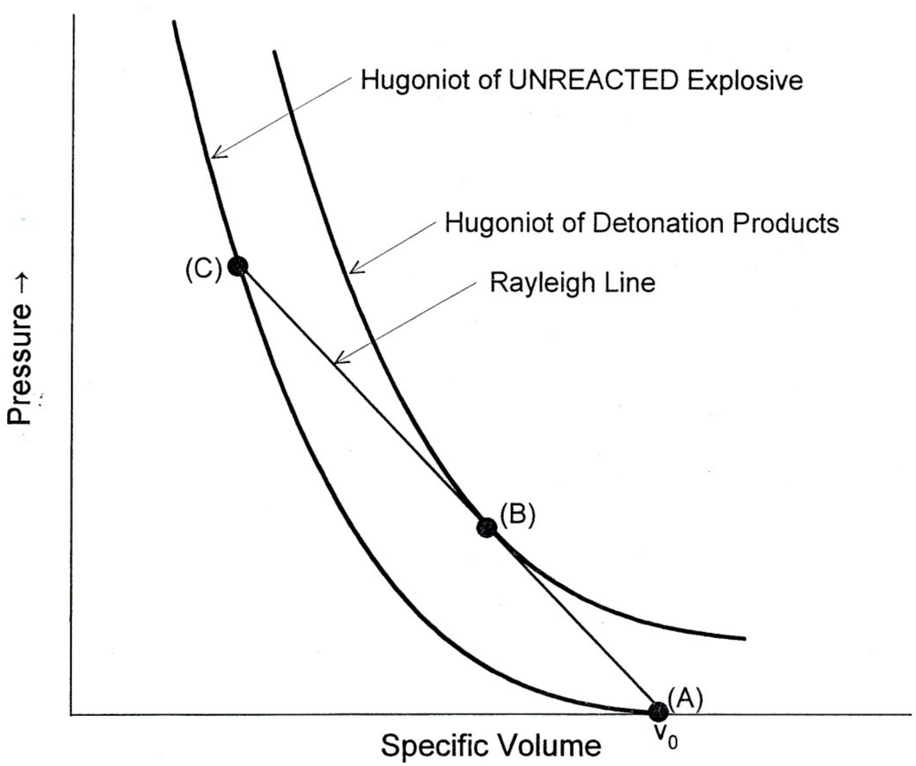
- Theory proposed independently by
  - Zeldovoich
  - Von Neumann
  - Deering
- Considerably simpler explanation that what is actually happening at the detonation front
  - This makes the math bearable

- Shock wave moving through the un-reacted explosive material
- Shock compresses and heats the explosive, which initiates a chemical reaction
- Energy released by the reaction feeds the shock front and drives it forward
- Shock front, chemical reaction, and leading edge of the rarefaction all in equilibrium

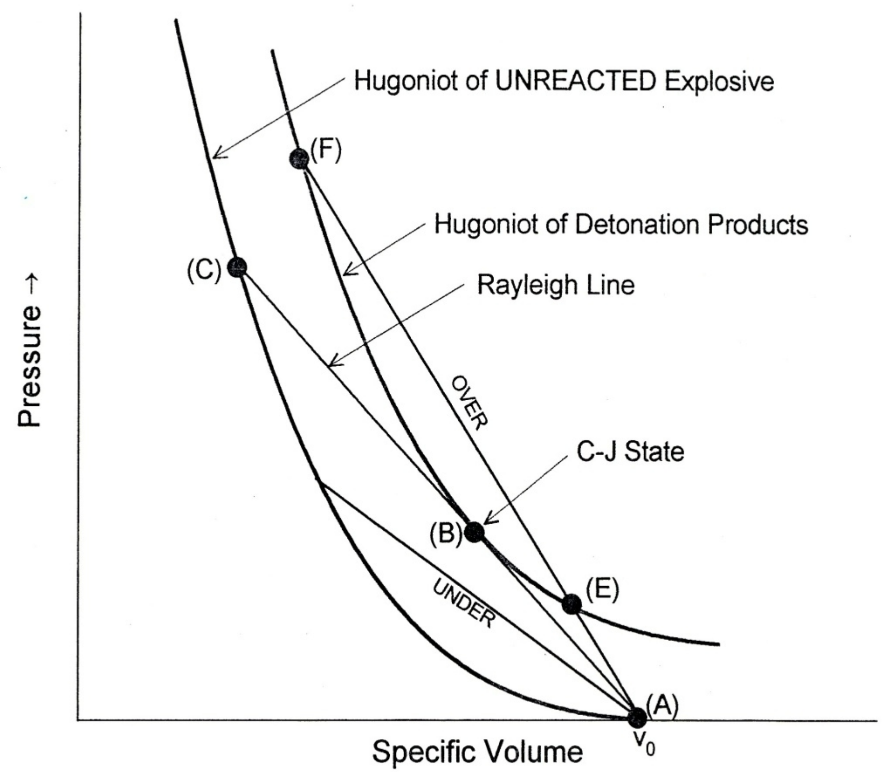
ZND Assumptions

- Flow is one-dimensional, uniaxial
- The front is a jump discontinuity and therefore can be handled in the same manner as the one we used with non-reactive shock waves
- The reaction-product gasses leaving the detonation front are in chemical and thermodynamic equilibrium and the chemical reaction is completed
- The chemical reaction zone length is zero
- The detonation velocity is constant; this is a steady state process; the products leaving the detonation remain at the same state independent of time.
- The gaseous reaction products, after leaving the detonation front may be time dependent and are affected by the surrounding system or boundary conditions

Explosive P-v Hugoniot



P-v plane Representation of a detonation



Other potential Raleigh lines

Interaction Between Explosive and Material

- We have seen that shock pressures are directly a function of material properties, particle velocities, densities, etc.
- We have also seen that much like solids, detonation product gasses at high pressure also have Hugoniot relationships
- So, how do we make the jump from the generated pressure within a detonating explosive and materials in contact?

- Estimation of the P-u Hugoniot for detonation products
  - Needed to find pressure at an explosive/material interface
  - We will see that, much like with inert solids, the interface pressure will be higher or lower than the CJ pressure depending on the impedance mismatch between the detonation products and material
  - Shock impedance is defined as
    - $Z_{det} = \rho_0 D$
    - $Z_{mat} = \rho_0 U$

P-u Hugoniot for Detonation Products

- Through experimentation
  - Where explosives were detonated next to various density materials
  - P-u Hugoniot curves were developed

For $\frac{P}{P_{CJ}} > 0.08$

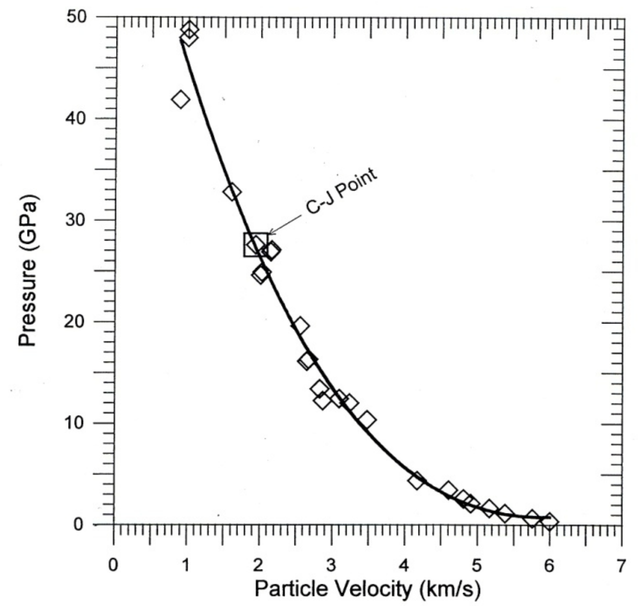
$$\frac{P}{P_{CJ}} = 2.412 - 1.7315\left(\frac{u}{u_{CJ}}\right) + 0.3195\left(\frac{u}{u_{CJ}}\right)^2$$

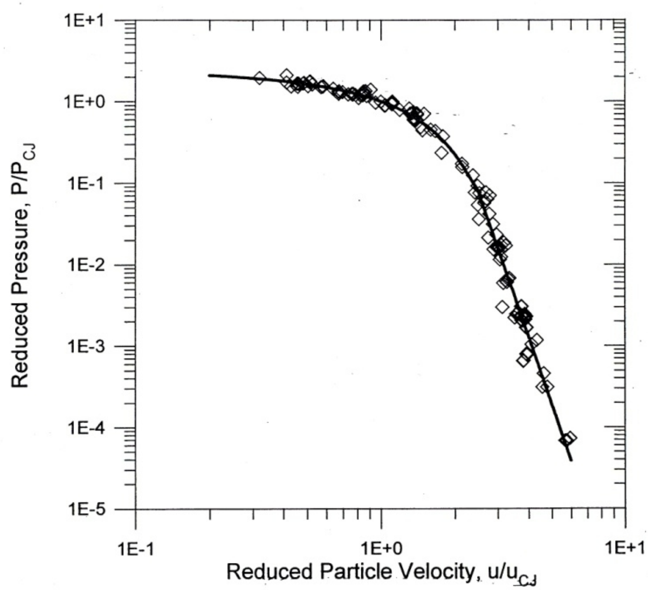
$$P = 2.412P_{CJ} - 1.7315\left(\frac{P_{CJ}}{u_{CJ}}\right)u + 0.3195\left(\frac{P_{CJ}}{u_{CJ}^2}\right)u^2$$

For $\frac{P}{P_{CJ}} < 0.08$

$$\frac{P}{P_{CJ}} = 235\left(\frac{u}{u_{CJ}}\right)^{-8.71}$$

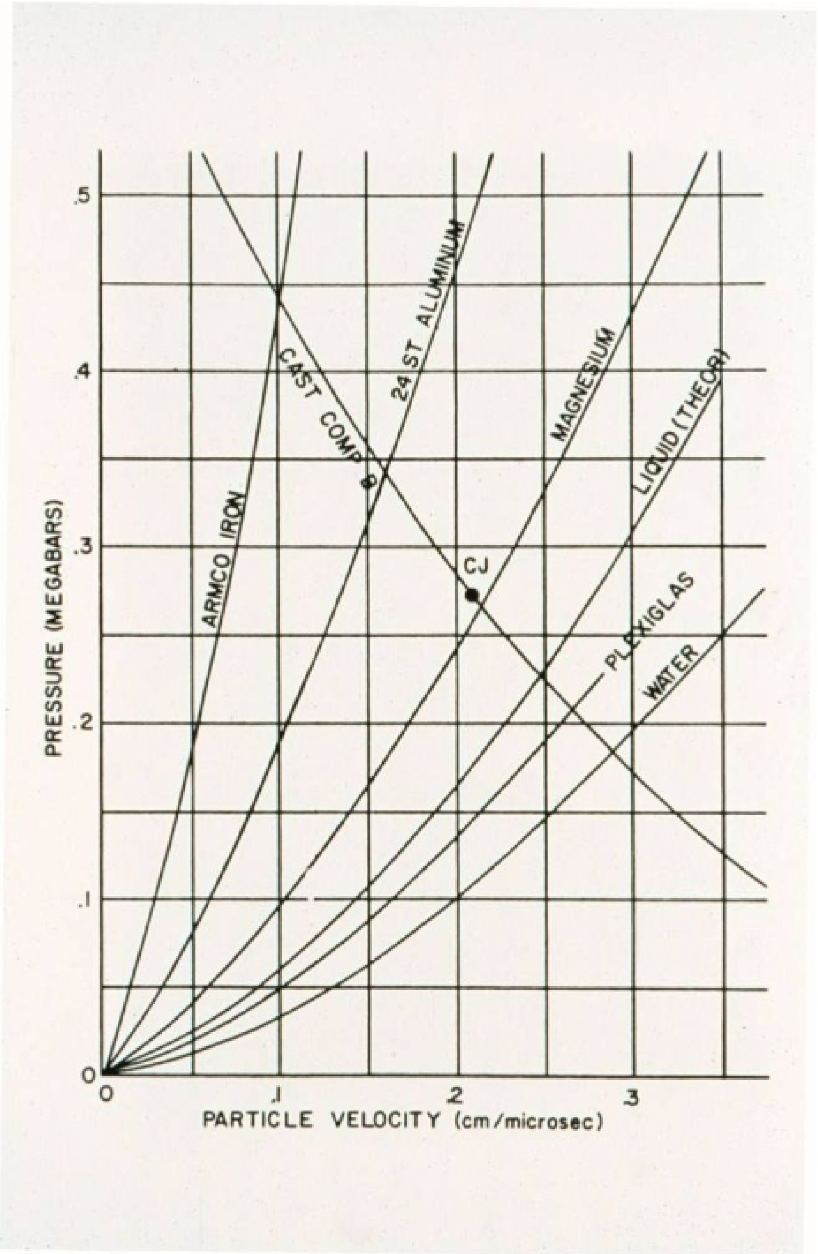
$$P = \left(235 P_{CJ} u_{CJ}^{8.71}\right)u^{-8.71}$$





How Hugoniots are created

- Explosive is detonated against many different materials.
- Pressure is measured at each interface
- Each experiment yields a single point on the curve
- The data is then curve fit to form the Hugoniot equation



P-u Hugoniot for Detonation Products: Example Problem

If we have explosive CJ properties where ρ_0=1.43 g/cm^3, D=6.95 km/s, and P_{CJ}=18.52 GPa. Assuming a detonation from left to right, what is the equation of the left going shock wave P-u Hugoniot of its detonation products?

Solution

First, we'll need the CJ particle velocity, which we can calculate from the momentum equation

$$P_{CJ} = \rho_0 u_{CJ} D$$

$$u_{CJ} = \frac{P_{CJ}}{\rho_0 D} = \frac{18.52}{(1.43)(6.95)} = 1.342 \text{ km/s}$$

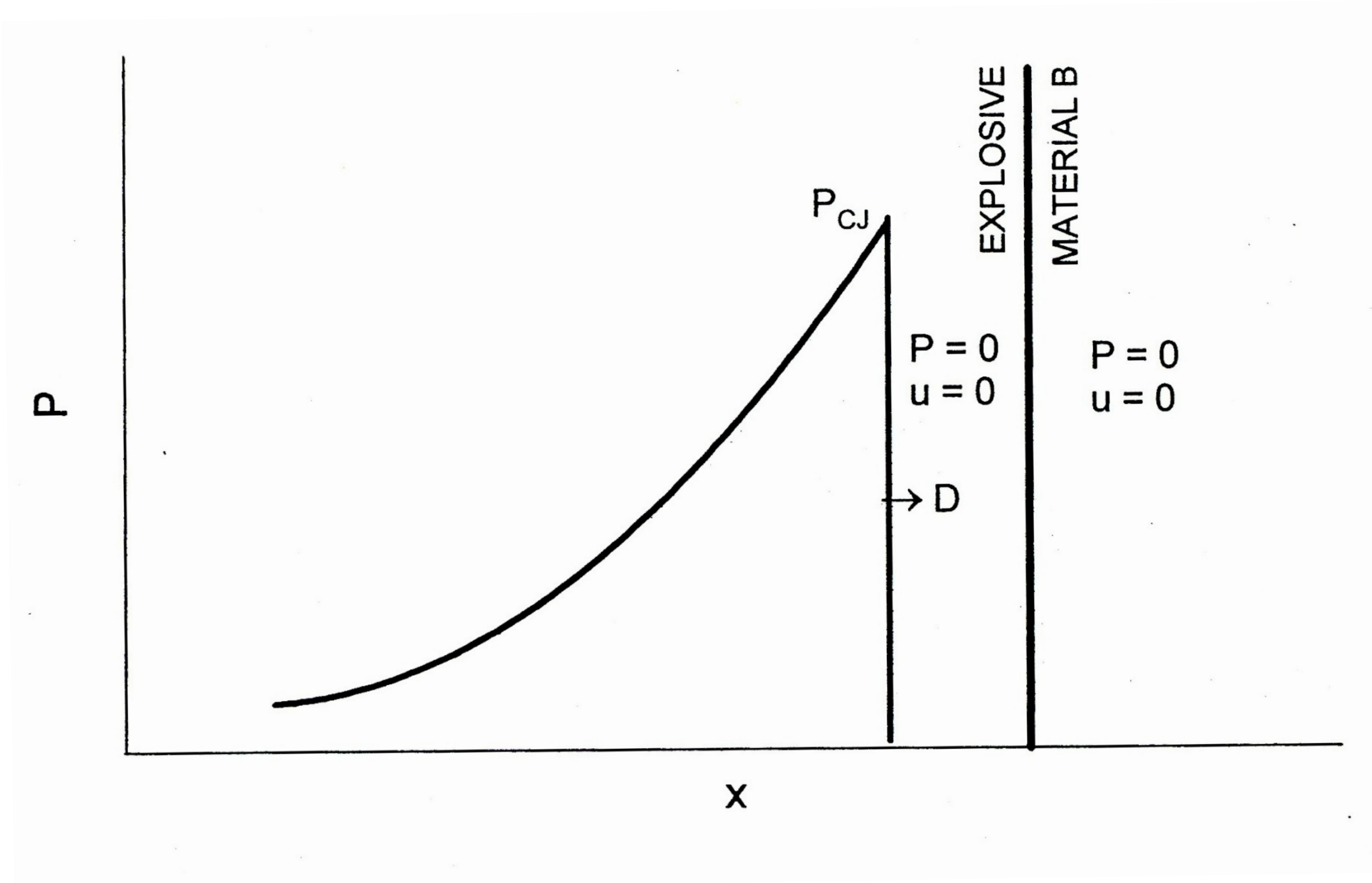
Assuming a reduced pressure greater than 0.08

$$P = 2.412 P_{CJ} - 1.7315\left(\frac{P_{CJ}}{u_{CJ}}\right)u + 0.3195\left(\frac{P_{CJ}}{u_{CJ}^2}\right)u^2$$

$$P = 2.412(18.52) - 1.7315\left(\frac{18.52}{1.342}\right)u + 0.3195\left(\frac{18.52}{1.342^2}\right)u^2$$

$$\boxed{P = 44.67 - 23.93 \cdot u + 3.295 \cdot u^2}$$

Detonation Approaches a Material Interface



Explosive Interface Example Problem $Z_{mat} > Z_{det}$

There is a slab of Comp-B explosive (ρ_0 = 1.733 g/cm^3) in contact with a brass plate. When the Comp-B detonates, what is the shock pressure at the surface of the brass plate?

Solution

First, we'll need the material properties for the brass (table 17.1) and Comp-B (table 20.1)

Brass: ρ_0 = 8.45 g/cm^3, C_0 = 3.726 km/s, s = 1.434

Comp-B: ρ_0 = 1.733 g/cm^3, P_{CJ} = 30 GPa, D = 8.0 km/s

Next, we can construct the resultant right-going Hugoniot in the brass

$$P_{Brass} = \rho_0 C_0 u + \rho_0 s u^2 = (8.45)(3.762)u + (8.45)(1.434)u^2$$

$$P_{Brass} = 31.485 \cdot u + 12.117 \cdot u^2$$

Next, calculate the CJ particle velocity from the momentum equation

$$u_{CJ} = \frac{P_{CJ}}{\rho_0 D} = \frac{30}{(1.733)(8)} = 2.16 \text{ km/s}$$

Explosive Interface Example Problem $Z_{mat} > Z_{det}$

Now construct the resultant left-going Hugoniot in the Comp-B

$$P = 2.412 P_{CJ} - 1.7315\left(\frac{P_{CJ}}{u_{CJ}}\right)u + 0.3195\left(\frac{P_{CJ}}{u_{CJ}^2}\right)u^2$$

$$P = 2.412(30) - 1.7315\left(\frac{30}{2.16}\right)u + 0.3195\left(\frac{30}{2.16^2}\right)u^2$$

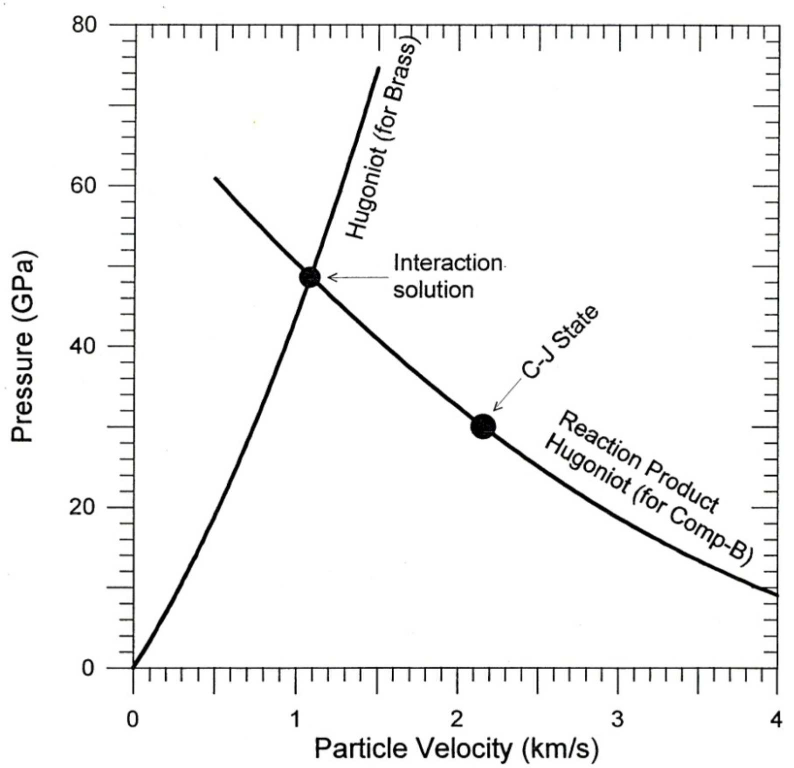
$$P_{Comp-B} = 72.36 - 24.05 \cdot u + 2.054 \cdot u^2$$

Equating P_{Brass} with $P_{Comp\text{-}B}$

$$31.485 \cdot u + 12.117 \cdot u^2 = 72.36 - 24.05 \cdot u + 2.054 \cdot u^2$$

$$10.063 \cdot u^2 + 55.535 \cdot u - 72.36 = 0$$

$$u = 1.088 \,{}^{km}\!/_{s} \quad \text{or} \quad u = -6.607 \,{}^{km}\!/_{s}$$

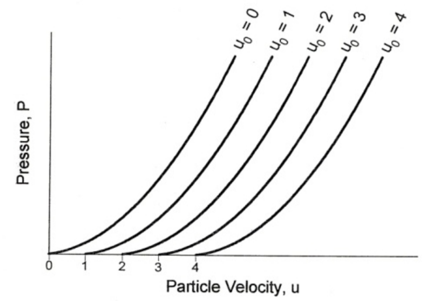


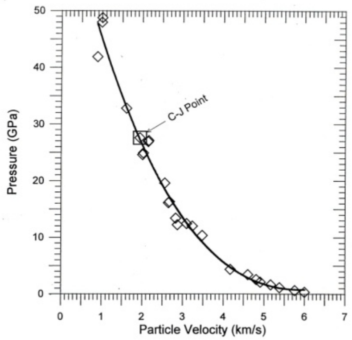
$u = 1.088 \,{}^{km}\!/_{s}$

Finally, plugging u into either pressure equation

$P = 48.6\ \text{GPa}$

Explosives / Material Interface





1. Diagram the problem
2. Find material properties
3. Find explosives properties
4. Construct right going Hugoniot in <u>inert</u> material

$$P_{inert} = \rho_0 C_0 u + \rho_0 s u^2$$

5. Calculate CJ particle velocity

$$u_{CJ} = \frac{P_{CJ}}{\rho_0 D} \qquad u_{CJ} = x.xxx \ \mathrm{km/s}$$

6. Construct left going Hugoniot for explosive products

$$P/P_{CJ} > 0.08$$

$$P_{exp} = 2.412 P_{CJ} - 1.7315 \left(\frac{P_{CJ}}{u_{CJ}}\right) u + 0.3195 \left(\frac{P_{CJ}}{u_{CJ}^2}\right) u^2$$

$$P/P_{CJ} < 0.08$$

$$P_{exp} = \left(235 P_{CJ} u_{CJ}^{8.71}\right) u^{-8.71}$$

7. Set Pressures equal to each other

$$P_{inert} = P_{explosive}$$

8. Solve for particle velocity (u) at collision

$$u = x.xxx \ \mathrm{km/s}$$

9. Solve for pressure at collision

$$P_{inert} = \rho_0 C_0 u + \rho_0 s u^2$$

Lecture Wrap-up

- Shock interactions
- Simple theory detonation
- Estimating Detonation properties (again)
- Estimating explosive product Hugoniots
- Detonation interactions
  - $Z_{mat} > Z_{det}$
  - $Z_{mat} < Z_{det}$