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Constraining subglacial hydrology with ice surface velocity observations by solving an inverse ice dynamics-subglacial hydrology problem

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The First Order (FO) model

For the ice dynamics, we consider the usual First-Order model (Blatter 1995, Pattyn 2003) for the horizontal velocity $\mathbf{u} = (u_x, u_y)$

$$-\nabla \cdot \underline{\underline{\sigma}} = -\rho_i g \nabla s$$

where s is the surface elevation, and

$$\underline{\underline{\sigma}} = 2\mu(\mathbf{u})\underline{\underline{\varepsilon}}, \quad \underline{\underline{\varepsilon}} = \begin{bmatrix} 2\varepsilon_{xx} + \varepsilon_{yy} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{xx} + 2\varepsilon_{yy} & \varepsilon_{xz} \end{bmatrix}, \quad \mu(\mathbf{u}) = \frac{1}{2} A^{-\frac{1}{n}}(T) \|\underline{\underline{\varepsilon}}\|^{\frac{1}{n}-1}$$

At the ice-bedrock interface, we consider the sliding condition

$$\underline{\underline{\sigma}}\mathbf{n} + \beta \mathbf{u} = \mathbf{0}, \qquad \beta > 0.$$

Constant field $\beta = \beta(\mathbf{x})$

- Simple mathematical model, but pay price of high-dimension parameter space

Functional form $\beta = \beta(\mathbf{u}, \mathbf{p})$

- Complex physically-informed model, but (hopefully) get a low-dimension parameter space

Subglacial hydrology

We consider a steady distributed model (Hewitt 2013) for subglacial hydrology.

$$\begin{cases} \cancel{\dfrac{\partial h}{\partial t}} + \nabla \cdot \mathbf{q} = \dfrac{m}{\rho_w} & \text{mass conservation} \\ \cancel{\dfrac{\partial h}{\partial t}} = \dfrac{m}{\rho_i} + \dfrac{h_r - h}{l_r}|u| - c_c A h N^3 & \text{cavities evolution} \end{cases}$$

with melting $m = (G + \beta|u|^2)/L$, water discharge $\mathbf{q} = -k h^{\alpha_1} |\nabla \Phi|^{\alpha_2} \nabla \Phi$, ice thickness H, transmissivity k, source geothermal flux G, ice sliding velocity u, bed bumps height/length h_r, l_r are given, effective pressure $N = \rho_i g H - \Phi$.

The unknowns are water hydropotential Φ, and water thickness h.

Inverse FO-Hydrology

Idea: perform data assimilation of surface velocity measures to identify uncertain parameters in *both* hydrology and sliding law, by solving a *coupled* FO-Hydrology problem.

Method: solve a deterministic inverse problem, by minimizing mismatch between computed and observerd surface velocity. Use μ_f and k as control variables.

$$[\mu, k] = \arg\min \int_{surf} |\mathbf{u} - \mathbf{u}_{obs}|^2 dx + R(\mu, k)$$

$$\text{s.t.} \begin{cases} -\nabla \cdot \underline{\underline{\sigma}} = -\rho_i g \nabla s \\ \nabla \cdot \mathbf{q} = \dfrac{m}{\rho_w} \\ \dfrac{m}{\rho_i} + \dfrac{h_r - h}{l_r}|u| - c_c A h N^3 = 0 \\ \underline{\underline{\sigma}}\mathbf{n} + \beta \mathbf{u} = \mathbf{0} \end{cases}$$

Numerical experiments

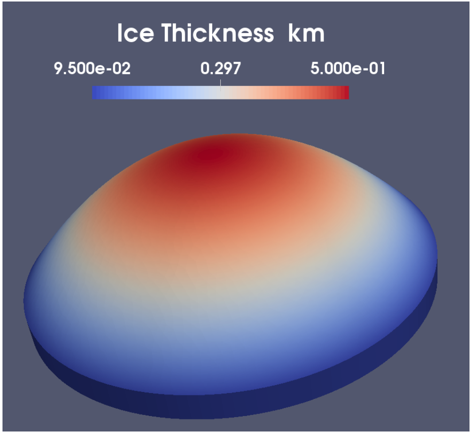
- Pre-processing: solve FO+Hydro for given (μ_f, k) to generate surface velocity measures. Add fudge factor to measures, to emulate for measurements errors.
- Algorithm: Quasi-Newton (BFGS) with backtrack line search.

Software

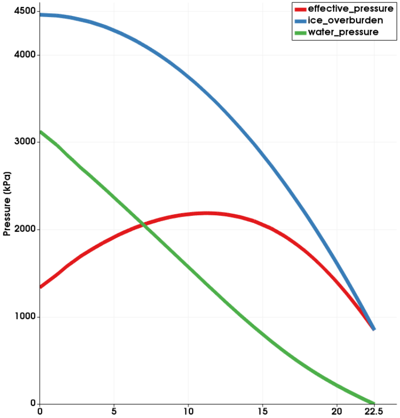
- **Trilinos**: general-purpose scientific library for high-performance distributed computing. Provides linear/nonlinear algebra, solvers, discretization tools.
- **Albany**: parallel finite element library (built on Trilinos), with analysis/sensitivities capabilities, timestepping, continuation, optimization, and more.
- **LandIce**: a package in Albany which implements ice-sheet and subglacial hydrology models.

Idealized Dome: problem specs

- radially simmetric geometry: $H(r) = s(r) = 0.5\left(1 - r^2\right)/R^2$ km
- horizontal resolution: 1km
- prescribed uniform melting in mass eqn: $m/\rho_w = 5.4$ mm/day
- no melting in cavities eqn
- power-law sliding: $\beta = \mu_f N u^{q-1}$, $q = 4/3$
- linear water flux: $\mathbf{q} = -kh\nabla\Phi$.



Idealized Dome: forward solve



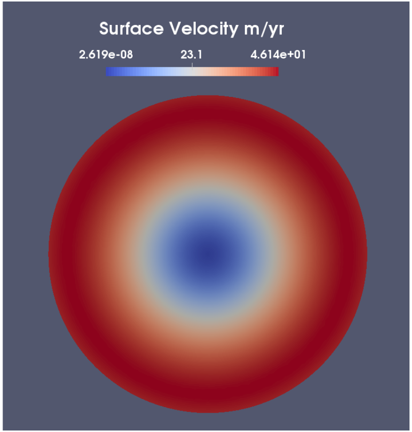


Figure: Water/effective pressures (left) and surface velocity (right) obtained solving the coupled FO+Hydrology problem for $k = 1e-5$ and $\mu_f - 0.01$.

Idealized dome: inverse problem

	Fwd	$\eta = 0$	$\eta = 5\%$	$\eta = 10\%$	$\eta = 20\%$
μ_f	1e-2	0.994e-2	0.994e-2	0.995e-2	0.993e-2
k	1e-5	0.994e-5	0.994e-5	0.995e-5	0.993e-5
# BFGS it	-	39	39	40	35

Table: Estimated parameters for different choices of noise level.

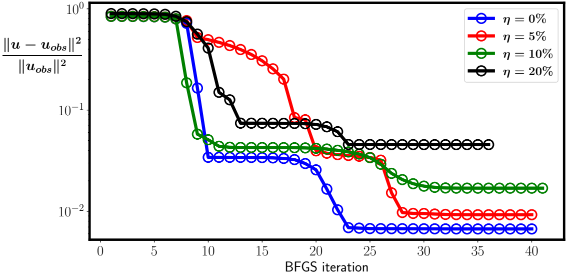


Figure: Convergence history of BFGS for different choices of noise level.

Idealized dome: inverse problem

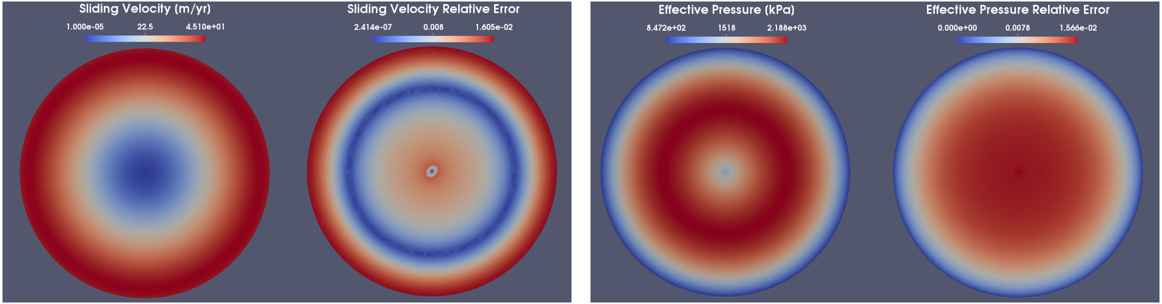


Figure: Exact solution and relative error plots for sliding velocity (left) and effective pressure (right), with 20% relative noise in the surface velocity measures.

Considerations and future directions

- FO+Hydro solver in LandIce is robust on idealized geometry.
- Small parameter space makes inversion feasible.
- Next: synthetic problem on a real geometry (Humboldt).
- Next: assimilate real measures on a real geometry.