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Characterizing Experimental Nonlinear Modal Coupling with 3D Surfaces

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INTRODUCTION

Nonlinearity is often encountered in structural dynamics, and while many system identification techniques have been explored, these existing approaches are ill equipped to properly address many nonlinear behaviors that are observed in practice. Although great progress has been made in developing methods for analyzing isolated nonlinear resonances, these methods typically assume that the modal properties are uncoupled, in that the response of one mode does not have any effect on the behavior of another. This is a suitable assumption when analyzing weakly nonlinear systems in which variations in the stiffness and damping of each mode can be accurately characterized as a univariate function of its response amplitude. However, this approach fails when the effects of modal coupling are no longer negligible, and individual modal properties vary with respect to multiple modal responses. To that end, this work explores a method for efficiently characterizing the coupling effects between two nonlinear resonances in terms of the relative modal response amplitude of each. This is done by simultaneously exciting the relevant structural modes at a variety of amplitude combinations and extracting amplitude dependent natural frequency and damping ratio curves for each mode from the response measurements. By representing each quasilinear modal parameter as a function of both modal amplitudes, a three-dimensional surface can then be formed that describes how each parameter changes as a function of coupling between the response amplitudes. In this paper, this approach is demonstrated utilizing experimentally measured nonlinear response data from a structure containing a large, jointed interface. Coupling behavior is observed by concurrently exciting pairs of modes of the structure that each activate the joint nonlinearity. Representing the measured response data in terms of the three-dimensional modal coupling surface offers an intuitive and concise means of characterizing and visualizing this complex nonlinear structural behavior.

BACKGROUND

Within structural dynamics, linear modal analysis is the most well understood and utilized approach for characterizing the dynamic behavior of a system. Linear methods typically amount to determining constant valued natural frequencies, damping ratios, and mode shapes, $[\omega_\mathrm{n}, \zeta, \phi]$, that collectively describe the dynamic motion of the structure as a superposition of uncoupled and orthogonal modal responses. While linear techniques can be extremely useful, all real systems are nonlinear in some regard and the highly idealized representation produced by linear parameters is likely only accurate at low excitation levels. As a means of extending the concept of linear modal parameters beyond the linear regime, the nonlinear behavior exhibited by the stiffness and energy dissipation of the structure can be modelled in terms of equivalent natural frequencies and damping ratios that are a function of the amplitude of the structural response, $[\omega_\mathrm{n}(A), \zeta(A), \phi]$. These quasilinear parameters present an intuitive extension to their linear counterparts, converging to the underlying constant, linear value at low amplitudes, while diverging at higher response amplitudes to emulate the nonlinear behavior of the structure in an averaged sense. Typically, quasilinear parameters are assumed to be a function purely of their respective modal response amplitude. In certain structures, such as those with bolted joints, some identified quasilinear parameters display inconsistent trends with respect to their modal amplitude [1]. Usually, this anomalous behavior is attributed to being measurement error or noise, or some ambiguous modal coupling effect that cannot be reliably modelled or accounted for, as it does not fit the assumption that the modes are uncoupled.

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In an effort to understand and model observed modal coupling, the quasilinear parameters may be considered to be a function of two modal amplitudes, $[\omega_\mathrm{n}(A_1, A_2), \zeta(A_1, A_2), \phi]$, such that, instead of tracing out a backbone curve in 2D space, a surface is formed in 3D space, or if more than two modal amplitudes are considered, a higher dimensional plane. This approach was investigated by Haslam et. Al. in [2], in which numerical models with known nonlinearity were examined. Two methods for forming the 3D surface were explored, the first was to utilize the Restoring Force Surface method to form coupled nonlinear equations of motion that attempt to emulate the nonlinear behavior illustrated by the modal coupling surface. While this approach showed some promise in those numerical cases, it has subsequently been shown to be very ineffective in modeling experimentally measured modal coupling data [3]. The second method was to directly fit an interpolating plane to a set of identified quasilinear curves that are represented as a function of two modal amplitudes. This is the approach that is demonstrated in the following section on a set of experimentally measured data that displays behavior suspected to be a resultant of modal coupling.

EXPERIMENTAL DEMONSTRATION

The data shown here was collected from the Cylinder-Plate-Beam (CPB) structure located at Sandia National Laboratories and was originally measured for use in [3] in an effort to use the Restoring Force Surface method to model the observed modal coupling behavior. The CPB and test setup is shown below on the left side of Figure 1. Excitation was supplied by the shaker attached to the back of the cylinder via a steel stinger with transmitted force measured by a force transducer. For the data examined here, the applied force was in the form of sine beats, which are windowed sinusoids designed to provide a pulse of energy into a narrow frequency band. The CPB response was recorded by an array of triaxial accelerometers positioned over the surface of the structure. An example of the typical force and response profiles are displayed on the right side of Figure 1. Test cases included sine beats at various magnitudes applied to the first three elastic modes of the CPB individually and in pairs. These modes are essentially the beam moving in three perpendicular directions: first order cantilever modes in the soft and stiff directions and translating axially upon the first order drum mode of the plate.

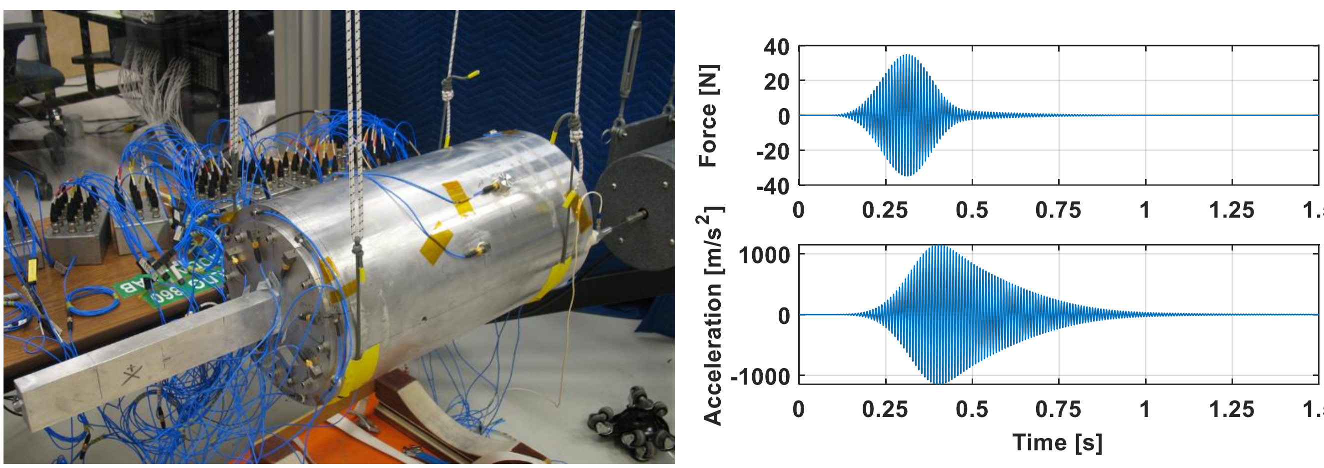

Figure 1: (Left) The Cylinder-Plate-Beam Structure in the test layout utilized to acquire the nonlinear data. (Right) An example of the measured sine beat force and resultant response.

The measured CPB responses were reduced to single-degree-of-freedom modal responses via modal filtering with the linear mode shapes. While the quasilinear parameters are assumed to be coupled, the modal filter operates upon the mode shapes, which should remain roughly constant and independent such that the modally filtered responses are approximately SDOF. The modal responses were reduced to monoharmonic oscillations by bandpass filtering the signals such that the fundamental is retained while noise and higher order harmonics are removed. The amplitude of this oscillation was then determined by numerically approximating their Hilbert Transform. To calculate the quasilinear parameters from these filtered responses, the process from [4], referred to as QL-LSQ, was utilized to accurately process the forced responses examined here. QL-LSQ determines varying stiffness and damping parameters via a linear least-squares solution to a set of equations comprised of the force and response quantities.

Shown below in Figure 2 are identified quasilinear parameters of the third elastic mode as a function of the mode 3 and mode 1 response amplitudes. The underlying linear values for each, as determined from a low-level burst random noise test, were $\omega_n = 544.8$ [Hz] and $\zeta = 0.275$ [%]. In the plots, results from four experiments are shown: one in which forcing was applied only in the frequency range of mode 3 at high level, and three that were defined to excite both the first and third modes at magnitude ratios of 1:1, 2:1, and 1:2 to produce responses that exhibit various levels of coupling. Additional test cases with animated figures will be shown in the IMAC presentation. In the frequency results in Figure 2, the curves begin near the linear value and soften to lower frequencies at higher amplitudes. The result of note is that the frequency decreases with respect to both mode 1 and mode 3 amplitudes. In the damping, the curves begin near the linear value before displaying a sort of bell shape with the increasing amplitude. While the isolated mode 3 result, on the right side of the plot, follows the same trend with increasing and decreasing amplitude, the cases with mode 1 response on the left side ring down with higher damping ratios. These results show that the quasilinear behavior of the third mode is very sensitive to the excitation of the first mode. This is likely a result of both modes heavily involving motion of the beam which warps the plate, distorting the contact between the plate and the cylinder, causing variations in frictional effects and perceived stiffness of the interface.

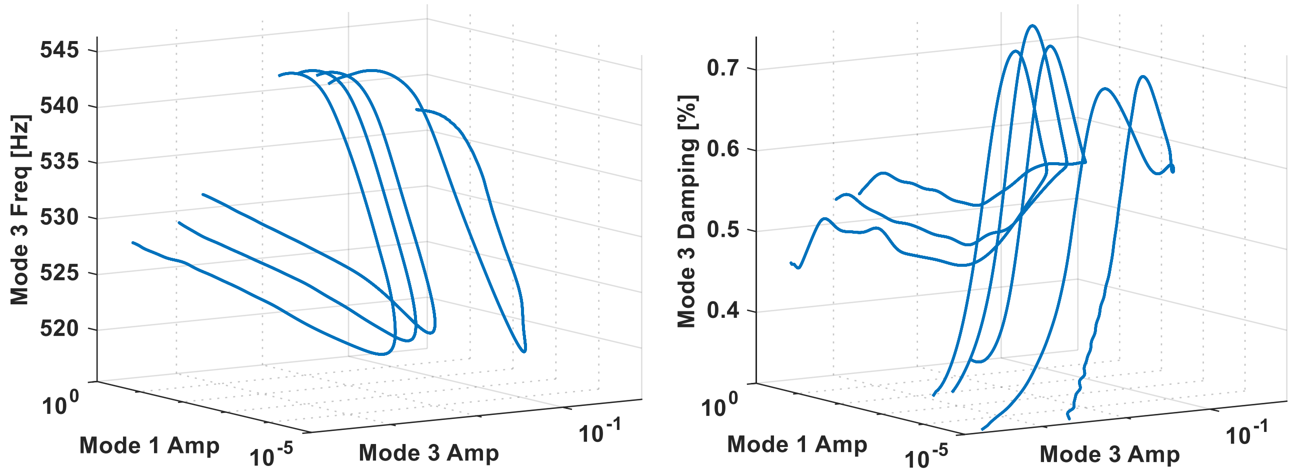

Figure 2: The quasilinear parameters of CPB mode 3 vs. the response amplitudes of mode 1 and mode 3, with Natural Frequency on Left and Damping Ratio on the Right.

CONCLUSIONS

Modal coupling is a complicated dynamic phenomenon that has proved challenging to identify and model, to the extent that coupling is often avoided by simply assuming that it is negligible. In systems where that is not feasible, techniques are needed that present the nonlinear behavior of the modal coupling in an intuitive format. This work demonstrated the use of one such approach on experimentally identified quasilinear parameters that exhibit the effects of modal coupling. By plotting the varying natural frequency and damping ratio in a 3D space with respect to multiple modal responses, the combined amplitude dependence of the parameters can be visualized and more easily interpreted. In the future, extensions to this process include investigating the use of amplitude dependent mode shapes, methods for integrating response from the identified surface, and devising a testing methodology that combines sine beats and dwells to efficiently collect response data that occurs at a wide range of relative amplitudes, filling in more areas of the 3D surface plot.

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