LA-UR-22-32772

Approved for public release; distribution is unlimited.

Title: Speed Limits: From Thermodynamics to Annealing

Author(s): Garcia Pintos Barcia, Luis Pedro

Intended for: Talk

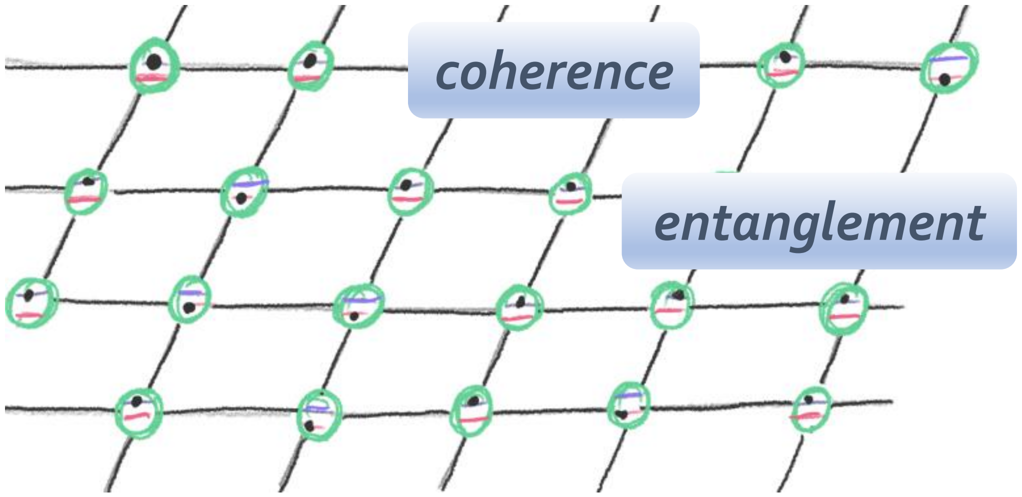
Issued: 2022-12-09

Speed Limits: From Thermodynamics to Annealing

Luis Pedro García-Pintos
Los Alamos National Laboratory

Isolated Quantum Systems *vs* Open Quantum Systems

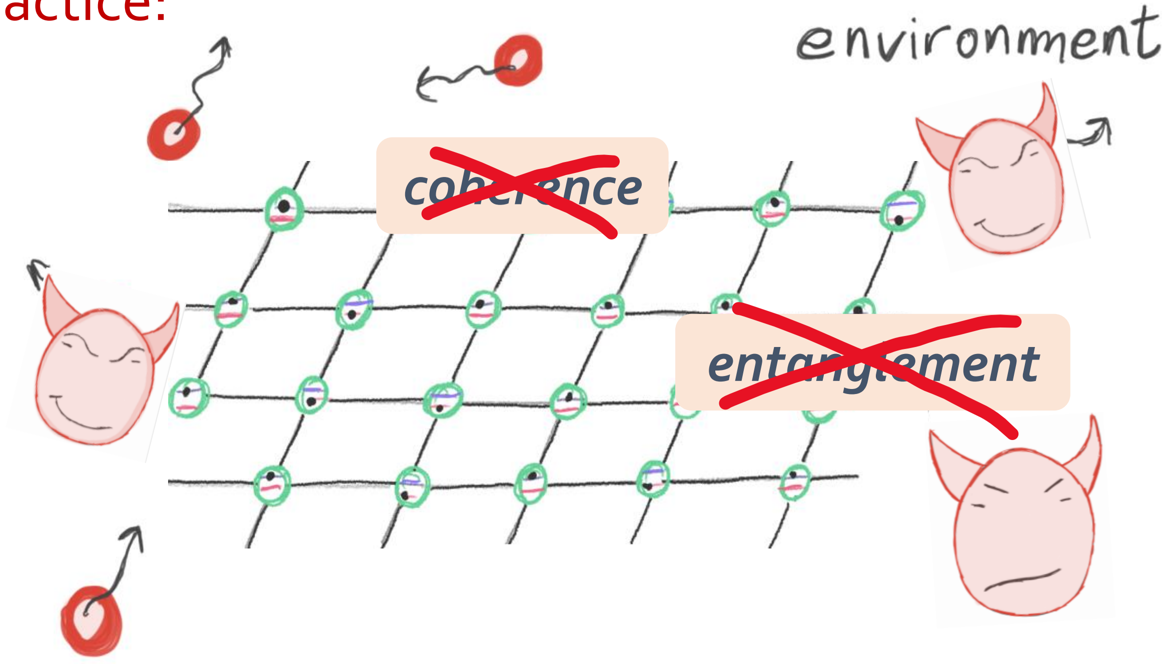
Ideally:



$$\frac{d\,|\psi_t\rangle}{dt} = -iH\,|\psi_t\rangle$$

Quantum advantages over classical processes

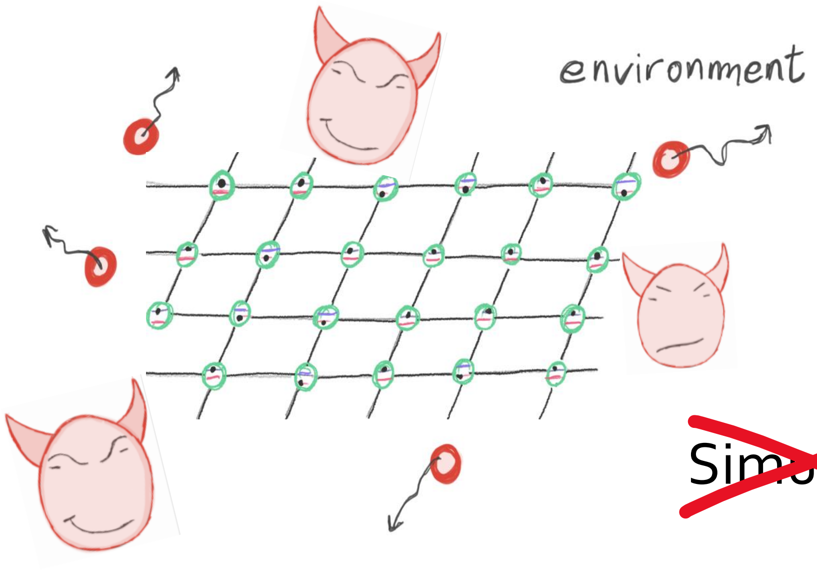
In practice:



$$\frac{d\rho_t}{dt} = -i\left[H', \rho_t\right] + \mathcal{D}(\rho_t)$$

~~Quantum advantages over classical processes~~

Dynamics of a Realistic Quantum System: *How do we usually solve it?*



~~Simulations~~ of ~~master equations~~

memory constraints

$$\frac{d\rho_t}{dt} = -i[H, \rho_t] + \mathcal{D}(\rho_t)$$
$$\approx -i[H, \rho_t] + \sum_n \gamma_n [L_n[L_n, \rho_t]]$$

approximations
(weak coupling, Markov)

Today – alternative first-principles analysis:

general traits of dynamics from minimal assumptions

$$\frac{d\langle A \rangle}{dt} = ?$$

Insight?

I. **Introduction: Quantum Speed Limits**

II. Speed Limits on Observables

III. An application : Quantum Annealing

Quantum Speed Limits

Bounds to the speed of
evolution of a quantum system

J. Phys. USSR 9, 249–254 (1945)

The Uncertainty Relation Between Energy and Time in Non-relativistic Quantum Mechanics

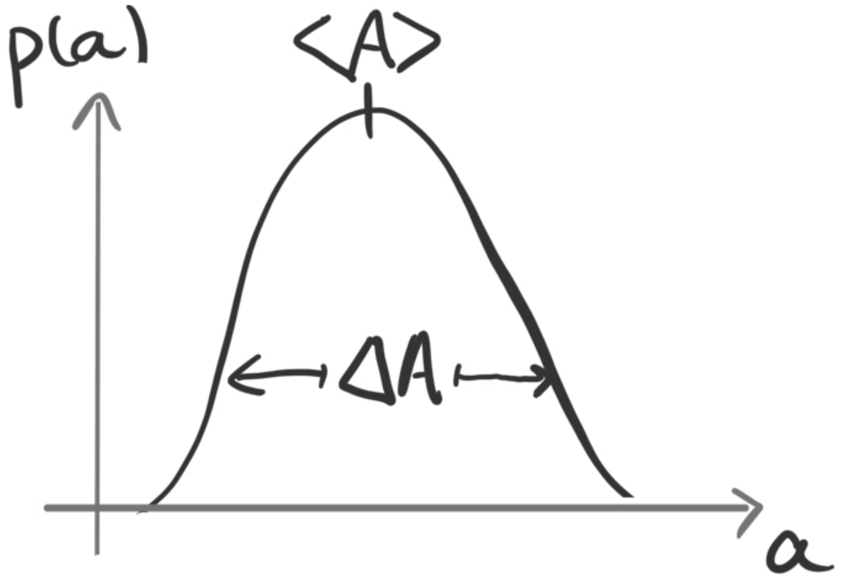
By *L. Mandelstam*[1] and *Ig. Tamm*

For isolated systems:

$$\left|\frac{d\langle A\rangle}{dt}\right| \leq 2\,\Delta A\,\Delta H$$

expectation value ⟹ $\langle A\rangle = \mathrm{Tr}\left[A\rho_t\right]$

standard deviations ⟹ $\begin{cases} \Delta A = \sqrt{\langle A^2\rangle - \langle A\rangle^2} \\ \Delta H = \sqrt{\langle H^2\rangle - \langle H\rangle^2} \end{cases}$



Mandelstam-Tamm's Uncertainty Relation:
limits to the speed of any physical process on isolated **quantum** systems

Quantum Speed Limits

For isolated systems:

$$\left|\frac{d\langle A\rangle}{dt}\right| \leq 2\,\Delta A\,\Delta H$$

J. Phys. USSR 9, 249–254 (1945)

The Uncertainty Relation Between Energy and Time in Non-relativistic Quantum Mechanics

By *L. Mandelstam*[1] and *Ig. Tamm*

Lots of posterior work:
speed limits on the state $\dot{\rho}_t$
of an open quantum system

TOPICAL REVIEW

Quantum speed limits: from Heisenberg's uncertainty principle to optimal quantum control

Sebastian Deffner[1] and Steve Campbell[2]

Published 10 October 2017 •

Shortcoming: speed of observables can be very different!

$$|\psi_0\rangle = |\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\rangle$$

$$|\psi_t\rangle = |\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\rangle$$

Aim: **speed limits on observables for open quantum systems**

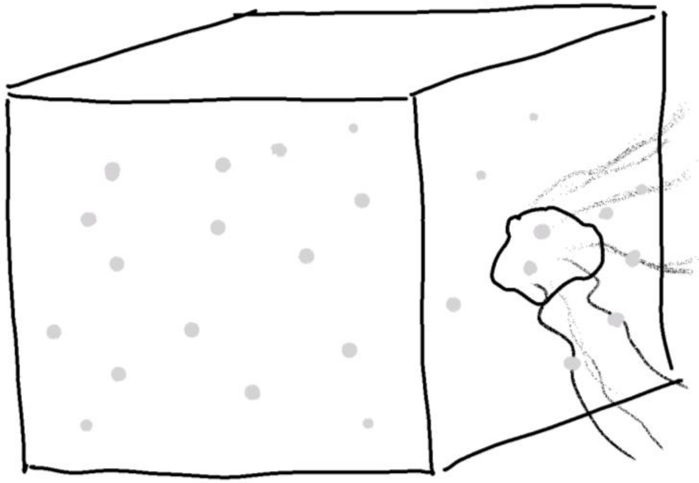
I. Introduction: Quantum Speed Limits

II. Speed Limits on Observables $\frac{d\langle A\rangle}{dt} = ?$

III. An application : Quantum Annealing

Classical Stochastic Systems

Classical system with 'states' j that
occur with probabilities p_j



(e.g., j denotes a point in phase space)

The distribution $\mathbf{p} = \{p_{x_1}, p_{x_2}, \dots\}$ represents the state of the system

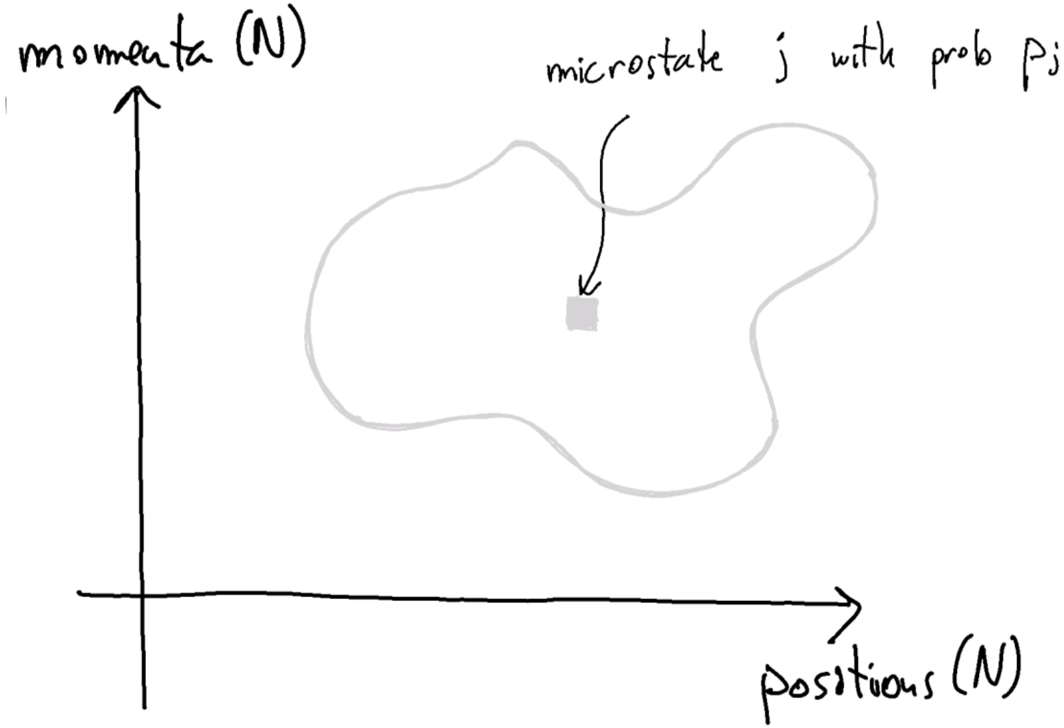
Dynamics: $\mathbf{p}(t) = \{p_1(t), p_2(t), \dots\}$

Article | Published: 21 September 2020

Time–information uncertainty relations in thermodynamics

Schuyler B. Nicholson, Luis Pedro García-Pintos, Adolfo del Campo & Jason R. Green

Nature Physics **16**, 1211–1215(2020) | Cite this article



Classical observable A
takes values a_j for state j
(e.g., particle density, energy)

$$\langle A \rangle = \sum_j p_j(t) a_j$$

Classical Speed Limits

Article | Published: 21 September 2020

Time–information uncertainty relations in thermodynamics

Schuyler B. Nicholson, Luis Pedro García-Pintos, Adolfo del Campo & Jason R. Green

Nature Physics **16**, 1211–1215(2020) | Cite this article

Classical speed limit on observables

$$\text{surprisal} \longleftarrow I_j := \ln \frac{1}{p_j} \qquad \text{cov}(A,B) := \langle AB\rangle - \langle A\rangle\langle B\rangle \qquad \text{assumptions: } p_j \neq 0 \quad \& \quad \exists\, \dot{p}_j$$

$$\left|\frac{d\langle A\rangle}{dt}\right| = \left|\text{cov}(A,\dot{I})\right| \leq \Delta A \; \Delta\dot{I}$$

speed — classical uncertainty — dynamics

(Classical) Fisher Information (~ measure of speed of p_j)

$$\mathcal{I}_F := (\Delta\dot{I})^2 = \sum_j p_j \left(\frac{d}{dt}\ln p_j\right)^2$$

Uncertainty relation bounds the speed of classical stochastic processes

Compare: quantum bound $\left|\frac{d\langle\hat{A}\rangle}{dt}\right| \leq 2\,\Delta\hat{A}\,\Delta\hat{H}$

Applications to stochastic thermodynamics:

- heat flow $|\dot{Q}|$
- entropy rates $|\dot{S}|$

Dynamics of Open Quantum Systems

PHYSICAL REVIEW X 12, 011038 (2022)

Unifying Quantum and Classical Speed Limits on Observables

Luis Pedro García-Pintos[1,*] Schuyler B. Nicholson,[2] Jason R. Green[3,4,5]
Adolfo del Campo[6,7,5] and Alexey V. Gorshkov[1]

Any open quantum system evolves following

$$\frac{d}{dt}\rho_t = -i[H,\rho_t] + \mathcal{D}(\rho_t)$$

$$\rho_t = U_t \sum_j p_j(t)\,|j\rangle_0\langle j|\,U_t^\dagger; \quad H_t \equiv i\dot{U}_t U_t^\dagger$$

$\rho_t = \sum_j p_j(t)\,|j\rangle_t\langle j|$ ⟹ density matrix

$-i[H,\rho_t]$ ⟹ coherent evolution

$\mathcal{D}(\rho_t)$ ⟹ incoherent evolution (dephasing, errors, environment)

coherent "quantum" → ~ *bound similarly to Mandelstam-Tamm 1945*

incoherent "classical" (*changes in probabilities*) → ~ *bound similarly to Nat Phys 2020 (classical speed limits)*

If $\mathcal{D}(\rho_t) = 0 \implies$ 'Ideal' isolated quantum dynamics

If $H = 0 \implies$ *Classical* stochastic dynamics

Speed Limits for Open Systems

PHYSICAL REVIEW X **12,** 011038 (2022)

Unifying Quantum and Classical Speed Limits on Observables

Luis Pedro García-Pintos,[1,*] Schuyler B. Nicholson,[2] Jason R. Green,[3,4,5]
Adolfo del Campo,[6,7,5] and Alexey V. Gorshkov[1]

$$\frac{d}{dt}\rho_t = -i[H,\rho_t] + \mathcal{D}(\rho_t)$$

coherent "quantum"

incoherent "classical"

$$\left|\frac{d\langle A\rangle}{dt}\right| = \left|\mathrm{cov}\left(A_C, L_C\right) + \mathrm{cov}\left(A_I, L_I\right)\right|$$

$$\leq \Delta A_C\,\Delta L_C + \Delta A_I\,\Delta L_I$$

$$\rho_t = \sum_j p_j(t)\,|j\rangle\langle j|$$

$$A_C = \sum_{j\neq k} A_{jk}\,|j\rangle\langle k|$$

$$A_I = \sum_j A_{jj}\,|j\rangle\langle j|$$

$$L_C := -2i\sum_{j\neq k}\frac{\langle j|\,[H_t,\rho_t]\,|k\rangle}{(p_j+p_k)}\,|j\rangle\langle k|$$

$$L_I := \sum_j \frac{d\ln p_j}{dt}\,|j\rangle\langle j|$$

Coherent Fisher Information $\mathcal{I}_F^C := \left(\Delta L_C\right)^2$

Incoherent Fisher Information $\mathcal{I}_F^I := \left(\Delta L_I\right)^2$

Speed Limits for Open Systems

$$\frac{d}{dt}\rho_t = -i[H, \rho_t] + \mathcal{D}(\rho_t)$$

coherent
"quantum"

incoherent
"classical"

$$\left|\frac{d\langle A\rangle}{dt}\right| = \left|\mathrm{cov}\left(A_C, L_C\right) + \mathrm{cov}\left(A_I, L_I\right)\right|$$

$$\leq \Delta A_C \, \Delta L_C + \Delta A_I \, \Delta L_I$$

PHYSICAL REVIEW X **12**, 011038 (2022)

Unifying Quantum and Classical Speed Limits on Observables

Luis Pedro García-Pintos[1,*] Schuyler B. Nicholson,[2] Jason R. Green,[3,4,5] Adolfo del Campo,[6,7,5] and Alexey V. Gorshkov[1]

$$\rho_t = \sum_j p_j(t)\,|j\rangle\langle j|$$

$$A_C = \sum_{j\neq k} A_{jk}\,|j\rangle\langle k|$$

$$A_I = \sum_{j} A_{jj}\,|j\rangle\langle j|$$

$$L_C := -2i\sum_{j\neq k}\frac{\langle j|\,[H_t, \rho_t]\,|k\rangle}{(p_j + p_k)}\,|j\rangle\langle k|$$

J. Phys. USSR 9, 249–254 (1945)

The Uncertainty Relation Between Energy and Time in Non-relativistic Quantum Mechanics

By *L. Mandelstam*[1] and *Ig. Tamm*

Lebedev Physical Institute, Academy of Sciences of the USSR

Article | Published: 21 September 2020

Time–information uncertainty relations in thermodynamics

Schuyler B. Nicholson, Luis Pedro García-Pintos, Adolfo del Campo & Jason R. Green

Nature Physics **16**, 1211–1215(2020) | Cite this article

Speed Limits for Open Systems

PHYSICAL REVIEW X 12, 011038 (2022)

Unifying Quantum and Classical Speed Limits on Observables

Luis Pedro García-Pintos[1,*] Schuyler B. Nicholson,[2] Jason R. Green,[3,4,5]
Adolfo del Campo,[6,7,5] and Alexey V. Gorshkov[1]

$$\frac{d}{dt}\rho_t = -i[H, \rho_t] + \mathcal{D}(\rho_t)$$

coherent "quantum" | incoherent "classical"

$$\left|\frac{d\langle A\rangle}{dt}\right| = \left|\mathrm{cov}\left(A_C, L_C\right) + \mathrm{cov}\left(A_I, L_I\right)\right|$$

$$\leq \; \Delta A_C\, \Delta L_C \; + \; \Delta A_I\, \Delta L_I$$

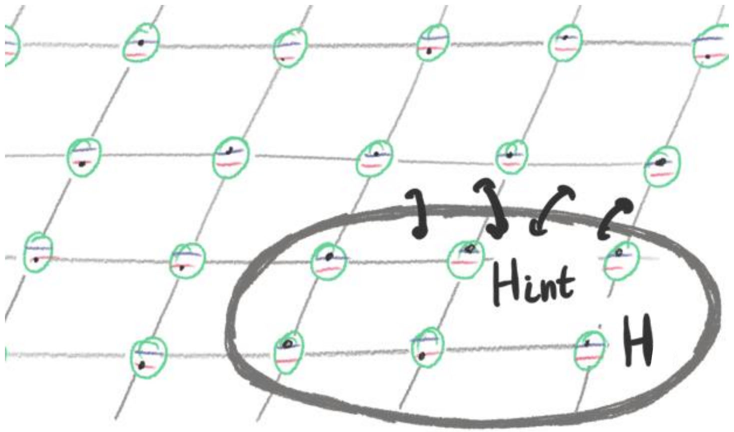
$$\rho_t = \sum_j p_j(t)\, |j\rangle\langle j|$$

$$A_C = \sum_{j\neq k} A_{jk}\, |j\rangle\langle k|$$

$$A_I = \sum_j A_{jj}\, |j\rangle\langle j|$$

$$L_C := -2i \sum_{j\neq k} \frac{\langle j|\,[H_t, \rho_t]\,|k\rangle}{(p_j + p_k)}\, |j\rangle\langle k|$$

$$L_I := \sum_j \frac{d \ln p_j}{dt}\, |j\rangle\langle j|$$

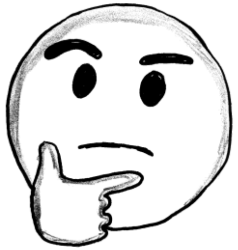


$$\mathcal{I}_F^I \leq 4(\Delta H_{\mathrm{int}})^2$$

Coherent Fisher Information $\mathcal{I}_F^C := (\Delta L_C)^2$

Incoherent Fisher Information $\mathcal{I}_F^I := (\Delta L_I)^2$

So, how are these bounds useful anyways?



Can they accurately reflect timescales of evolution?

$$\left|\frac{d\langle A\rangle}{dt}\right| \leq \Delta A_C\,\Delta L_C \;+\; \Delta A_I\,\Delta L_I$$

Speed limits are saturated in a range of problems!

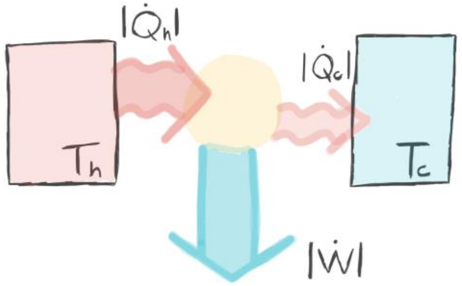
Classical and quantum thermodynamics

Article | Published: 21 September 2020

Time–information uncertainty relations in thermodynamics

Schuyler B. Nicholson, Luis Pedro García-Pintos, Adolfo del Campo & Jason R. Green

Nature Physics **16**, 1211–1215(2020) | Cite this article



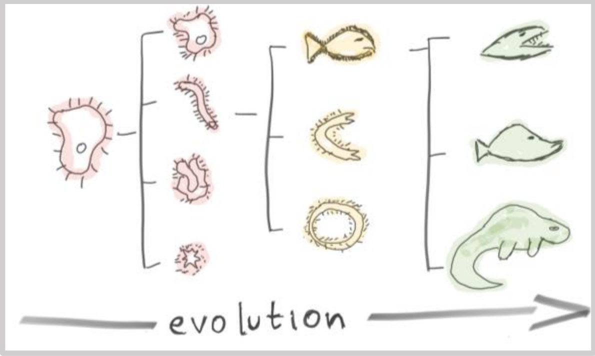
Evolutionary biology*

Diversity and Fitness Uncertainty Allow for Faster Evolutionary Rates

Luis Pedro García-Pintos[1,*]

[1] *Joint Center for Quantum Information and Computer Science and Joint Quantum Institute, University of Maryland, College Park, Maryland 20742, USA*

(Dated: February 22, 2022)

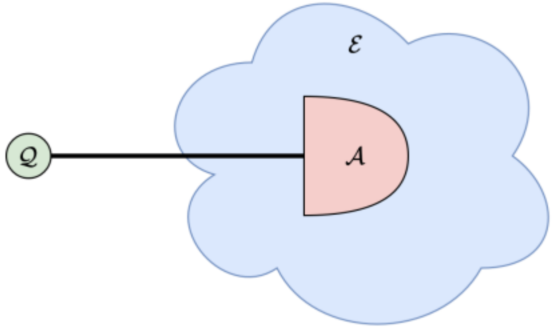


$$\left|\frac{d\langle A\rangle}{dt}\right| \leq \Delta A_C\,\Delta L_C + \Delta A_I\,\Delta L_I$$

Quantum-to-classical transition

Bounding the Minimum Time of a Quantum Measurement

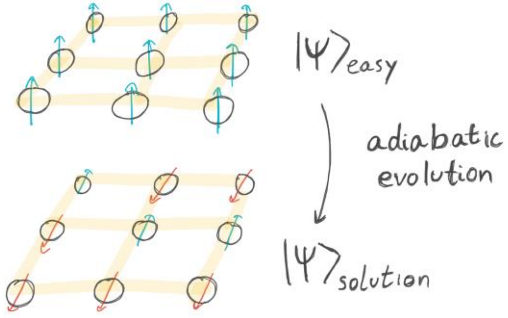
Nathan Shettell,[1] Federico Centrone,[2] and Luis Pedro García-Pintos[3]



Quantum annealing*

Lower Bounds on Quantum Annealing Times

Luis Pedro García-Pintos,[1,2,*] Lucas T. Brady,[3,4] Jacob Bringewatt,[1,2] and Yi-Kai Liu[1,5]

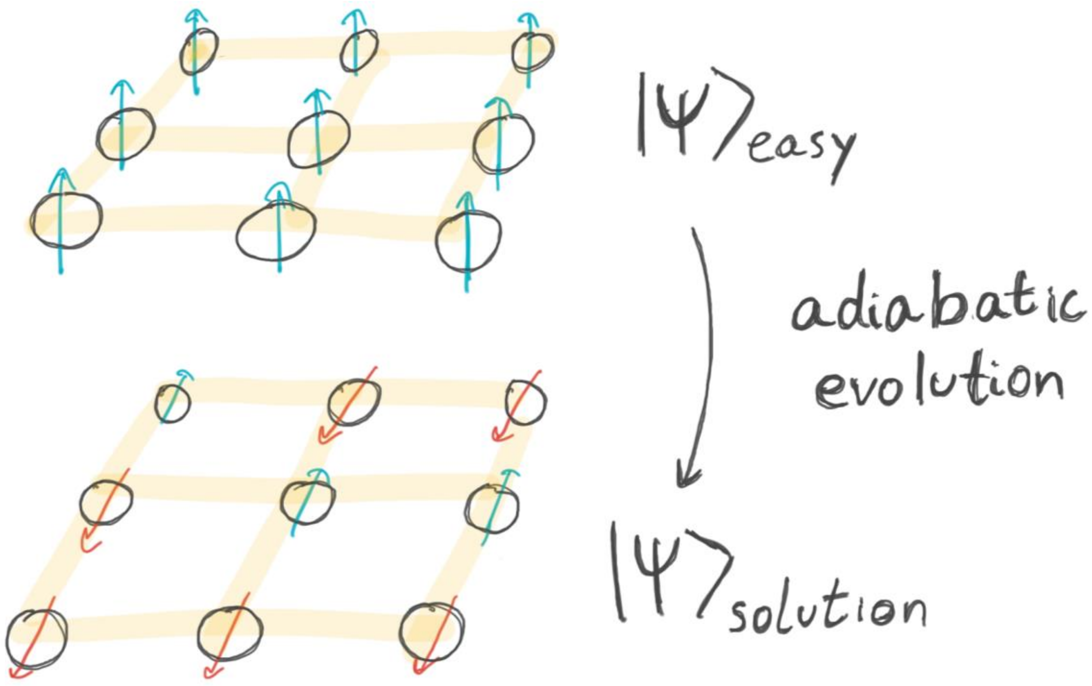


I. Introduction: Quantum Speed Limits

II. Speed Limits on Observables

III. **An application : Quantum Annealing**

Adiabatic quantum annealing



Annealing:

$$H_t = (1 - g_t)H_0 + g_t H_1 \qquad g_0 = 0; \quad g_1 = 1$$

$$|\psi\rangle_0 = |\mathrm{gs}\rangle_0 \qquad \text{aim:} \quad |\psi\rangle_{t_f} = |\mathrm{gs}\rangle_{t_f}$$

Adiabatic theorem: if the process is slow enough the system ends close to the target state

if $\quad t_f \geq T_{\mathrm{adiab}} \sim \frac{\|t_f \dot{H}_t\|}{\Delta^2} \quad \Longrightarrow \quad |\psi\rangle_{t_f} \approx |\mathrm{gs}\rangle_{t_f}$

sufficient *condition for annealing*

But, annealing need not be adiabatic! How fast can it be?

Jansen, Ruskai, Seiler, "Bounds for the adiabatic approximation with applications to quantum computation", J. Math. Phys. 2007

Lower bounds on annealing times

$$\|A\|_1 = \mathrm{Tr}\left[\sqrt{AA^\dagger}\right] \qquad \|A\| = \max \mathrm{eigs}(A)$$

How fast can annealing be?

Lower Bounds on Quantum Annealing Times

Luis Pedro García-Pintos,[1,2,*] Lucas T. Brady,[3,4] Jacob Bringewatt,[1,2] and Yi-Kai Liu[1,5]

Defining $\langle H_1\rangle_0 = \langle\psi_0| H_1 |\psi_0\rangle$ $\quad \langle H_0\rangle_{t_f} = \langle\psi_{t_f}| H_0 |\psi_{t_f}\rangle$ $\quad \langle H_1\rangle_{t_f} = \langle\psi_{t_f}| H_1 |\psi_{t_f}\rangle$

$$C_1(|\psi_t\rangle) = \min_{\sigma_t:[\sigma_t, H_t]=0} \left\| |\psi_t\rangle\langle\psi_t| - \sigma_t \right\|_1$$

measure of coherence in eigenbasis of H_t

Quantifying Coherence

T. Baumgratz, M. Cramer, and M. B. Plenio

Phys. Rev. Lett. **113**, 140401 – Published 29 September 2014

Necessary *conditions for annealing:* $t_f \geq \tau_1 \geq \tau_2$

$$\tau_1 \equiv 2\frac{\langle H_0\rangle_{t_f} + \langle H_0\rangle_0 - \langle H_1\rangle_{t_f}}{\left\|[H_1, H_0]\right\| \frac{1}{t_f}\int_0^{t_f} C_1(|\psi_t\rangle)dt} \qquad \tau_2 \equiv \frac{\langle H_0\rangle_{t_f} + \langle H_0\rangle_0 - \langle H_1\rangle_{t_f}}{\left\|[H_1, H_0]\right\|}$$

Lower bounds on annealing times

$\|A\|_1 = \mathrm{Tr}\left[\sqrt{AA^\dagger}\right]$ $\|A\| = \max \mathrm{eigs}(A)$

How fast can annealing be?

Lower Bounds on Quantum Annealing Times

Luis Pedro García-Pintos,[1,2,*] Lucas T. Brady,[3,4] Jacob Bringewatt,[1,2] and Yi-Kai Liu[1,5]

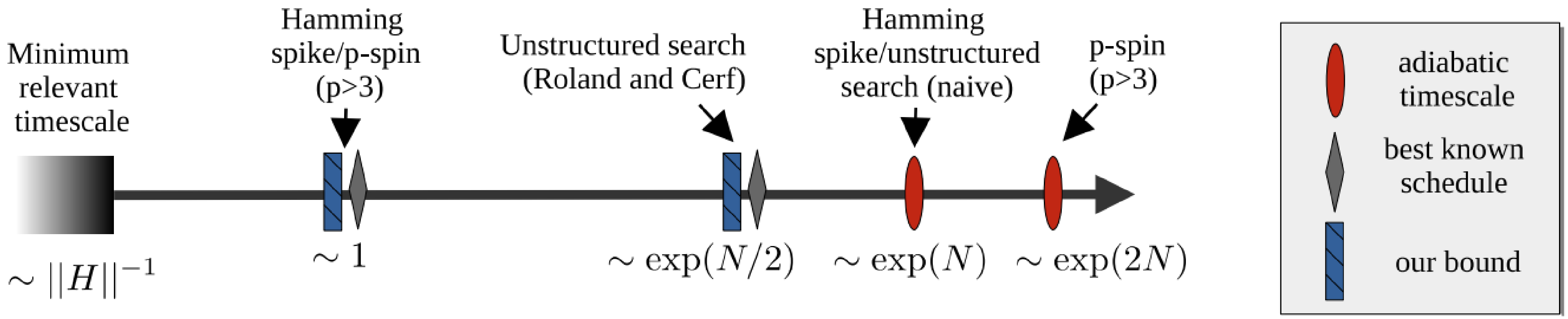


FIG. 1. **Annealing timescales.** An illustration of the range of possible timescales in annealing problems and how our bounds and the adiabatic timescales fit in the picture.

***Necessary** conditions for annealing:* $t_f \geq \tau_1 \geq \tau_2$

$$\tau_1 \equiv 2\frac{\langle H_0\rangle_{t_f} + \langle H_0\rangle_0 - \langle H_1\rangle_{t_f}}{\big\|[H_1, H_0]\big\|\frac{1}{t_f}\int_0^{t_f} C_1(|\psi_t\rangle)dt} \qquad \tau_2 \equiv \frac{\langle H_0\rangle_{t_f} + \langle H_0\rangle_0 - \langle H_1\rangle_{t_f}}{\big\|[H_1, H_0]\big\|}$$

Summary

I. General framework to derive bounds on rates $\frac{d\langle A\rangle}{dt}$ and when they are saturated

II. Speed limits reflect actual dynamics in a range of problems

III. Saturable lower bounds on annealing times

Luis Pedro García-Pintos
Los Alamos National Laboratory

Thank you!