SAND2021-15315C

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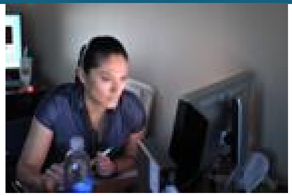
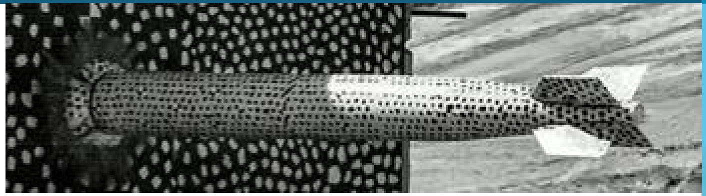
# Continuous conditional generative adversarial networks for data-driven solutions of poroelasticity with heterogeneous material properties





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## AGU 2021 Fall Meeting

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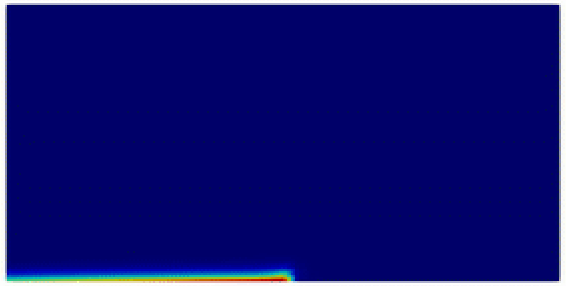
# Why reduced order model?

Full order model (FOM) is computationally demanding



This would take 1-2 hours[1,2]

Imagine if you do 100,000 times of this

FOM is computationally very expensive for large-scale uncertainty quantification, optimization, or inverse modeling

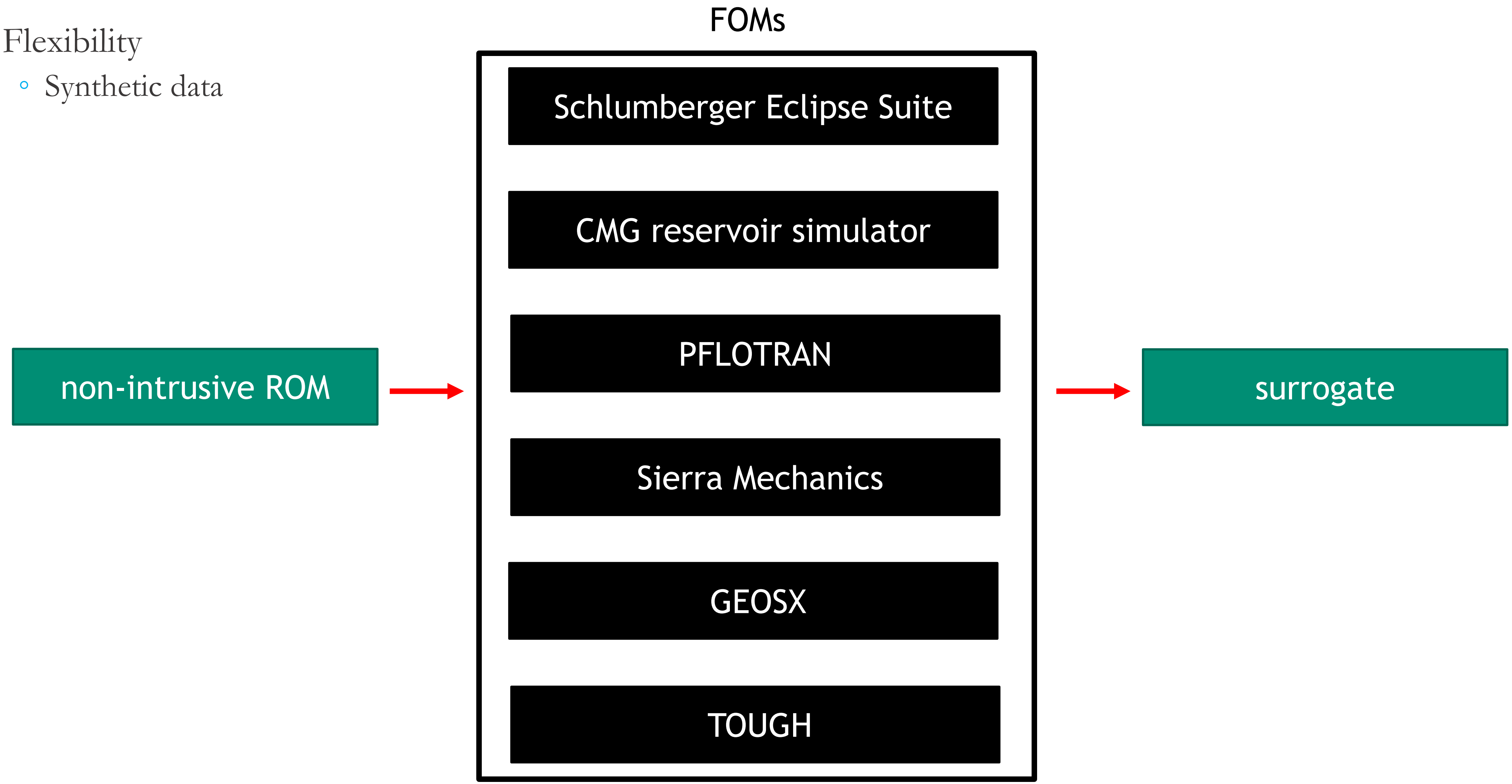
[1]Kadeethum T, Ballarin F, Choi Y, O'Malley D, Yoon H, Bouklas N. Non-intrusive reduced order modeling of natural convection in porous media using convolutional autoencoders: comparison with linear subspace techniques. arXiv preprint arXiv:2107.11460. 2021 Jul 23. [Avd. Water Res. In review after moderate revision]

[2]Kadeethum T, Lee S, Ballarin F, Choo J, Nick HM. A locally conservative mixed finite element framework for coupled hydro-mechanical-chemical processes in heterogeneous porous media. Computers & Geosciences. 2021 Jul 1;152:104774.

# Why non-intrusive approach?

Flexibility

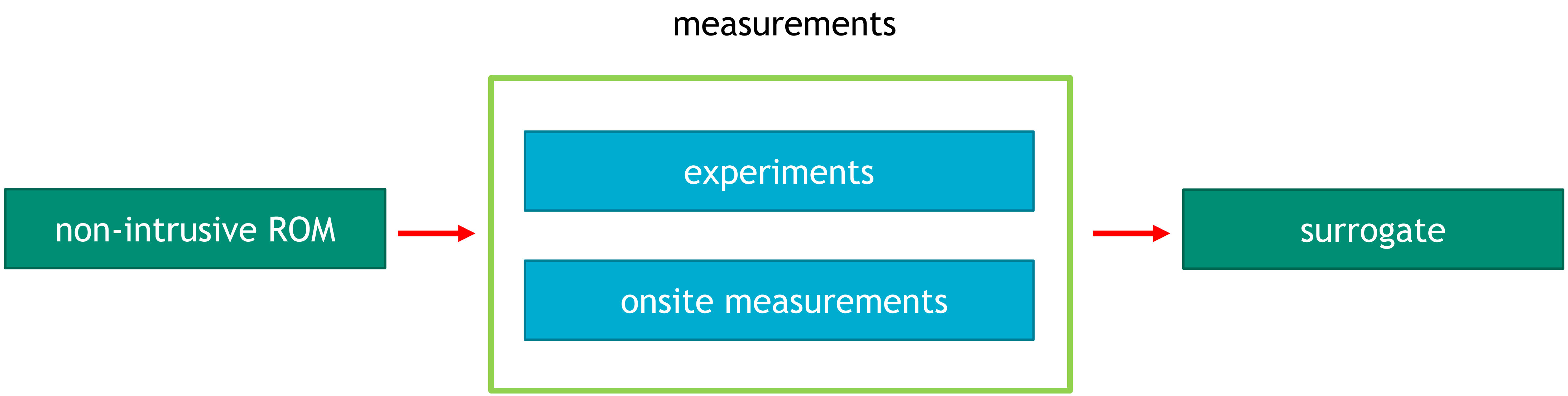
- Synthetic data



# Why non-intrusive approach? - continued

Flexibility

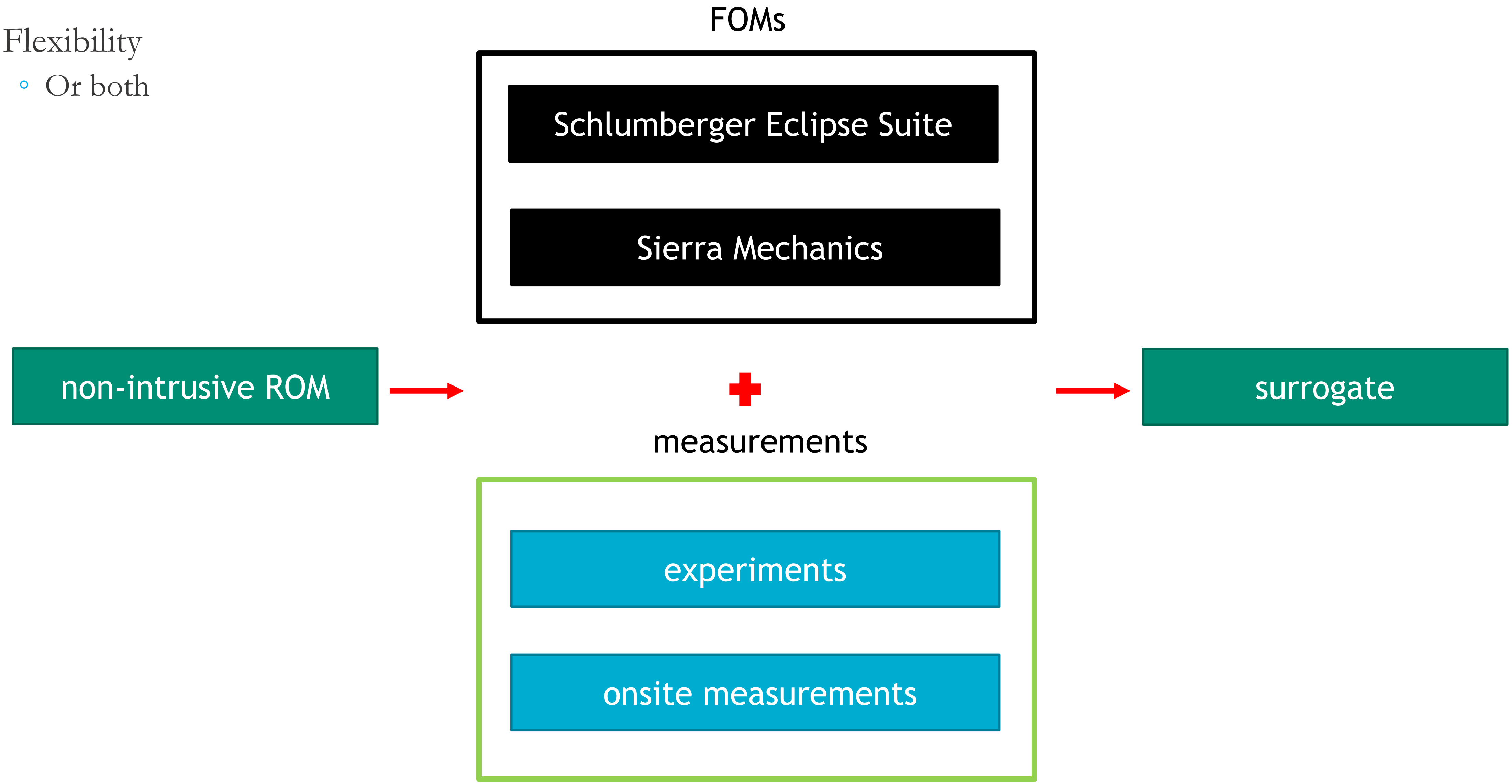
- measurement



# Why non-intrusive approach? - continued

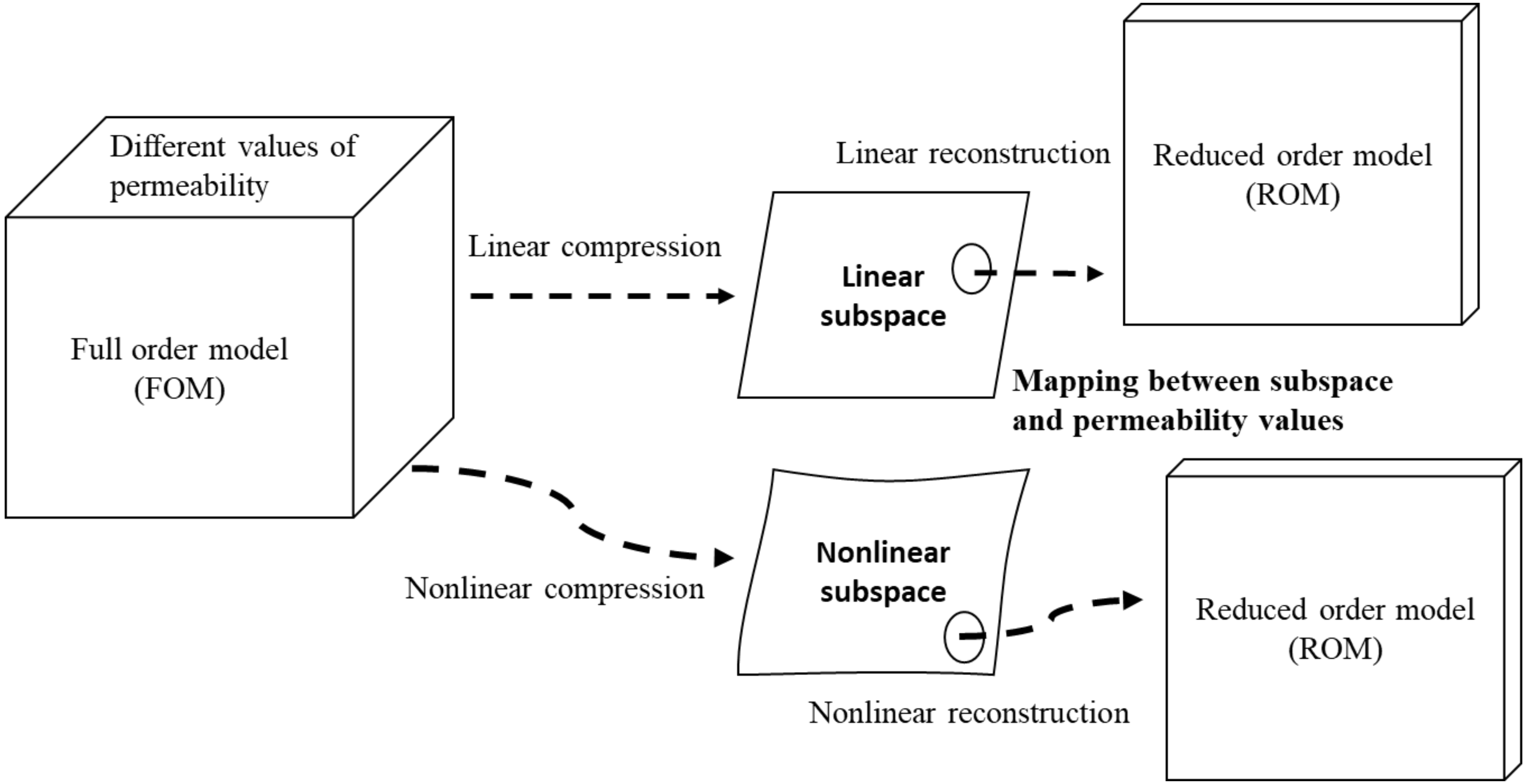
Flexibility

- Or both



# Motivation

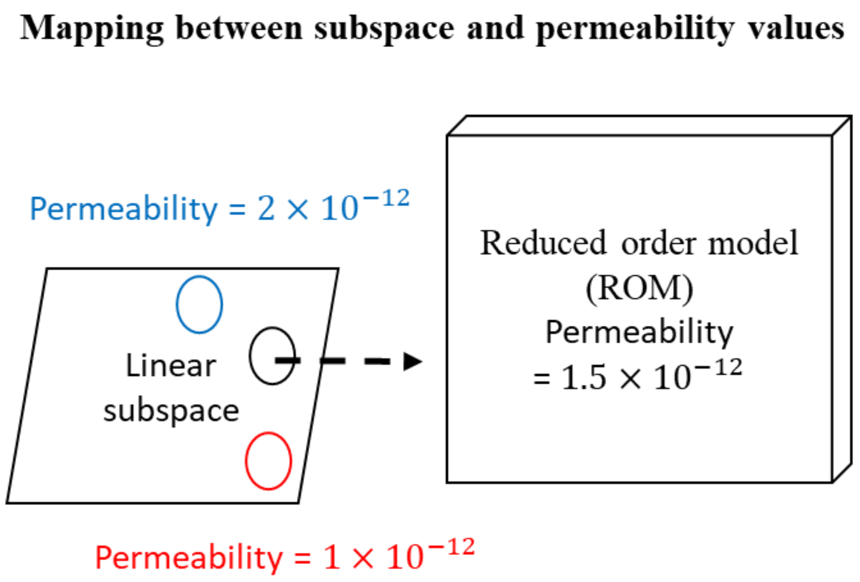
ROM typically works on 'parameterized PDEs' and 'reduced subspace'

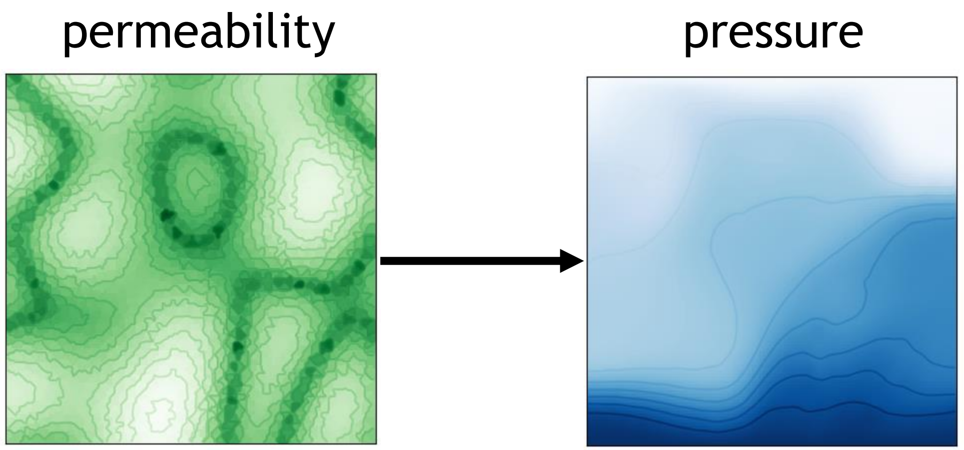


# Motivation - continued

Parameterization for a single value (low dimension) is straightforward

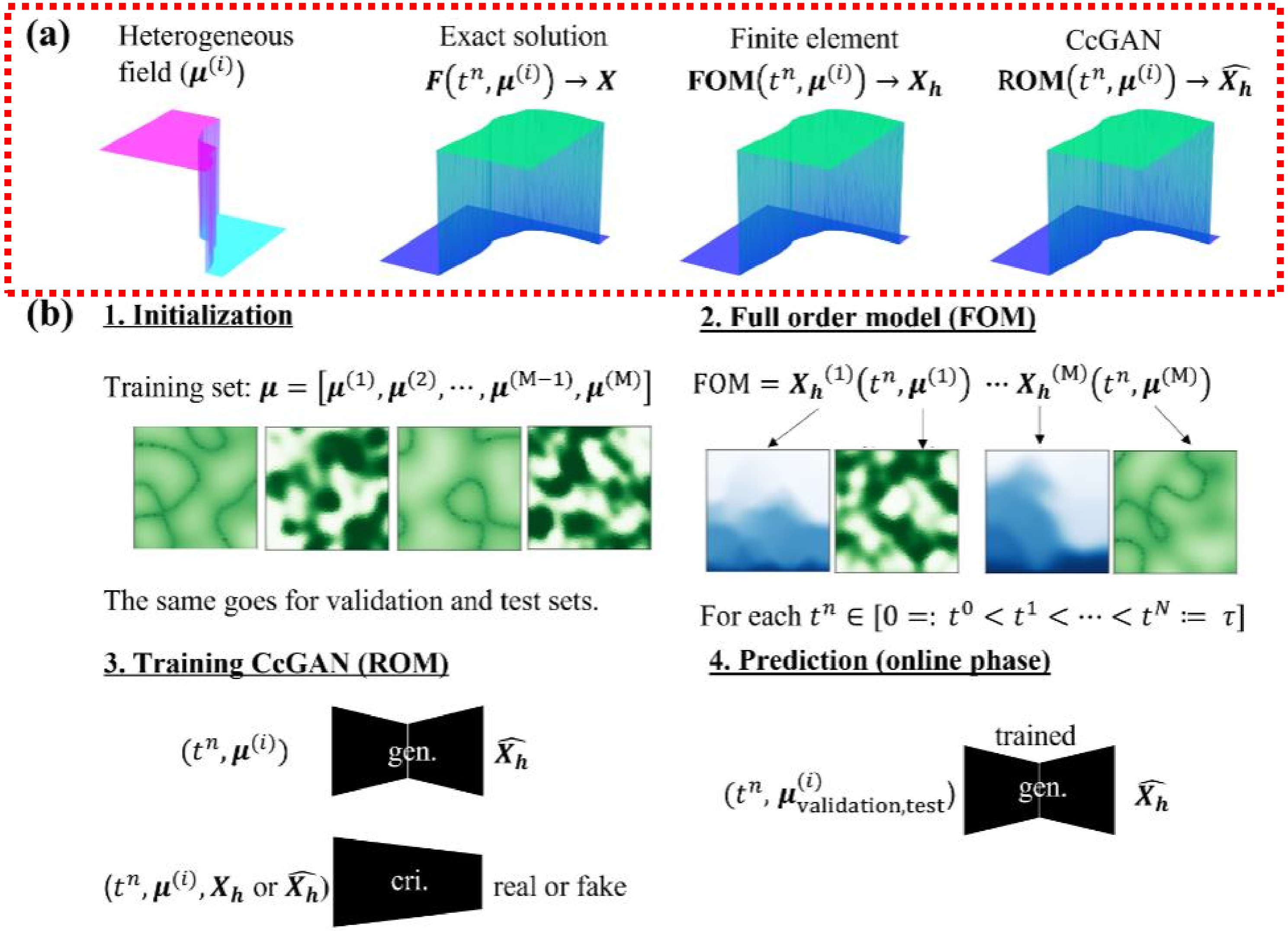
However, how could we parameterize the whole heterogeneous field?



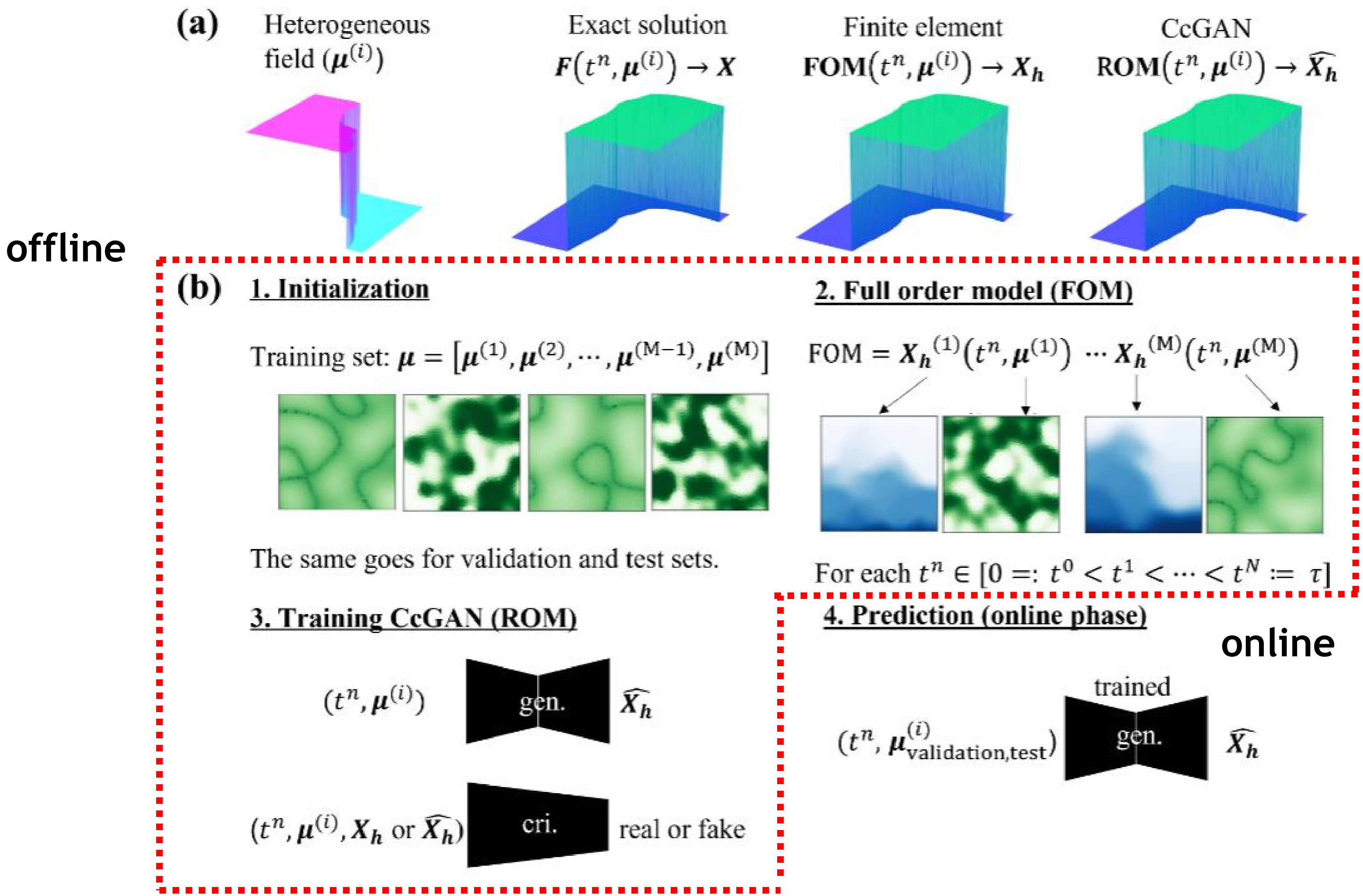
# Methodology

We want to create a **surrogate model** that can provide a good accuracy comparable to that of **finite element model** with a much cheaper computational cost



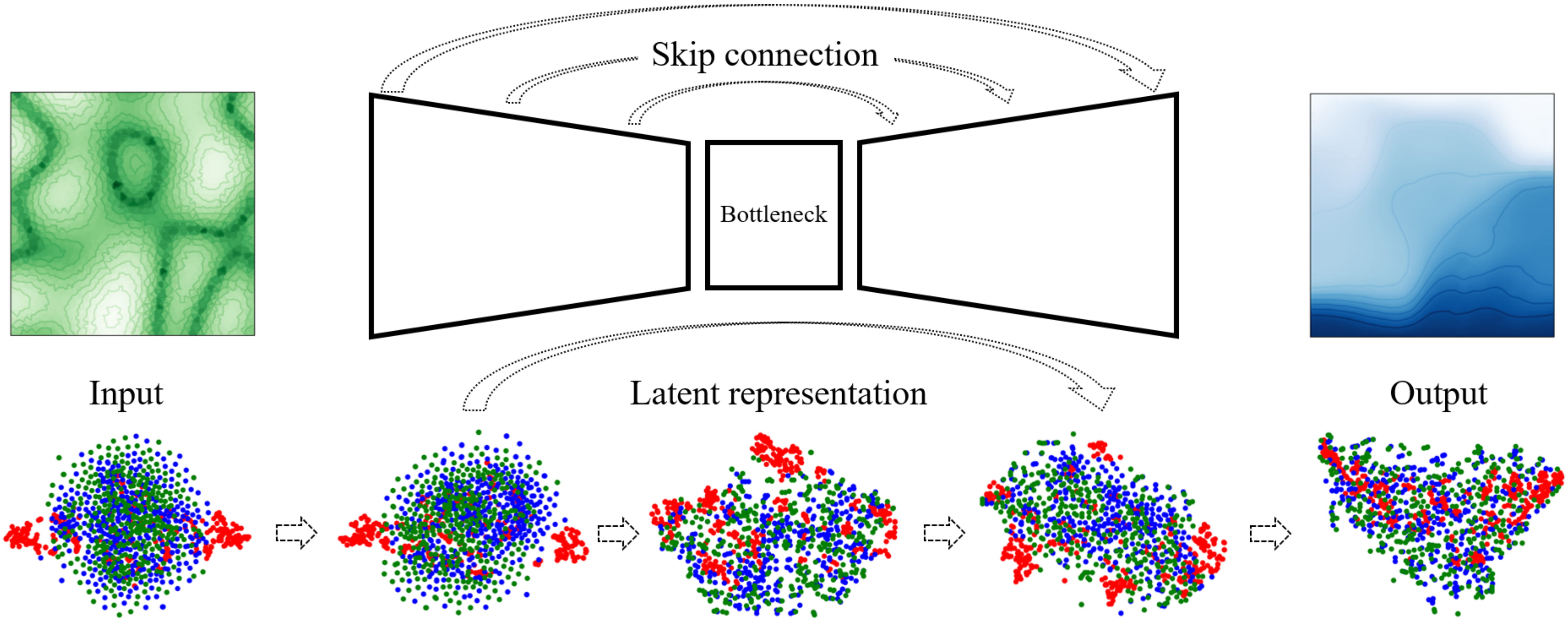
# Methodology - continued

Our framework is developed for **parameterized** coupled **multiphysics** processes based on **offline** and **online** phases



# Methodology - continued

In short, we extend the work done by Kadeethum et al. (2021) to time-dependent problems.




Kadeethum, T., O'Malley, D., Fuhg, J. N., Choi, Y., Lee, J., Viswanathan, H. S., & Bouklas, N. (2021). A framework for data-driven solution and parameter estimation of PDEs using conditional generative adversarial networks. arXiv preprint arXiv:2105.13136.

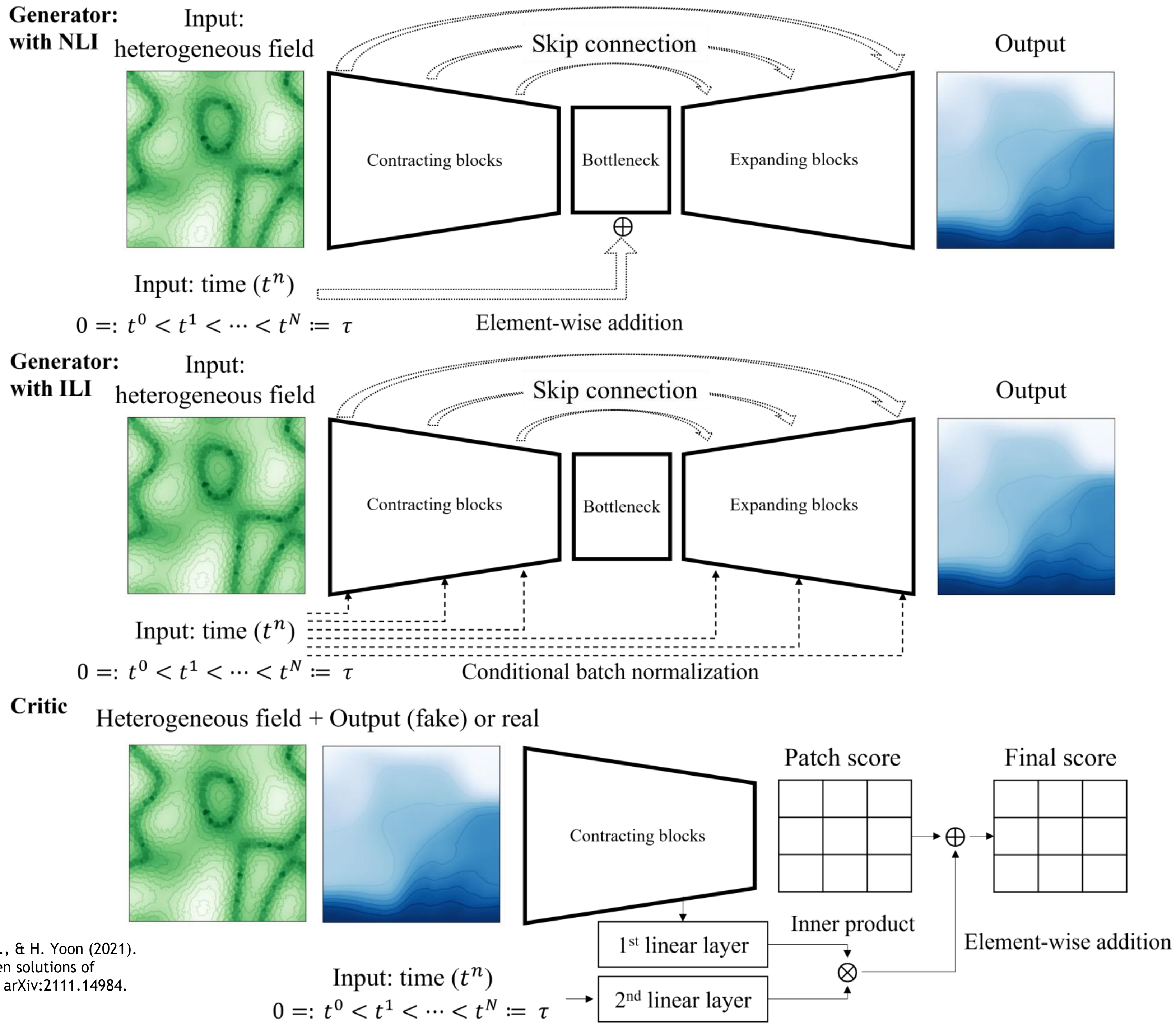
# Methodology - continued

We have two models:

1. naive label input (**NLI**)

2. Improved label input (**ILI**)

Both models use the same critic.




Kadeethum, T., O'Malley, D., Choi, Y., Viswanathan, H. S. Bouklas, N., & H. Yoon (2021). Continuous conditional generative adversarial networks for data-driven solutions of poroelasticity with heterogeneous material properties. arXiv preprint arXiv:2111.14984. [Submitted to GRL]

# Methodology - continued

Generator architecture – Batch normalization is used by NLI, and it is replaced by 'conditional batch normalization' for ILI model

Table S1. Generator: NLI's detail used in this study (input and output sizes are represented by [B, C, $\widetilde{N}_h^p$, $\widetilde{N}_h^p$]. We use hidden layers H = 32). BN refers to batch normalization.

| Block | Input size | Output size | BN | Dropout |
|---|---|---|---|---|
| 1st convolutional layer | [B, C, 128, 128] | [B, 32, 128, 128] | | |
| 1st contracting block | [B, 32, 128, 128] | [B, 64, 64, 64] | ✓ | ✓ |
| 2nd contracting block | [B, 64, 64, 64] | [B, 128, 32, 32] | ✓ | ✓ |
| 3rd contracting block | [B, 128, 32, 32] | [B, 256, 16, 16] | ✓ | ✓ |
| 4th contracting block | [B, 256, 16, 16] | [B, 512, 8, 8] | ✓ | |
| 5th contracting block | [B, 512, 8, 8] | [B, 1024, 4, 4] | ✓ | |
| 6th contracting block | [B, 1024, 4, 4] | [B, 2048, 2, 2] | ✓ | |
| 1st expanding block | [B, 2048, 2, 2] | [B, 1024, 4, 4] | ✓ | |
| 2nd expanding block | [B, 1024, 4, 4] | [B, 512, 8, 8] | ✓ | |
| 3rd expanding block | [B, 512, 8, 8] | [B, 256, 16, 16] | ✓ | |
| 4th expanding block | [B, 256, 16, 16] | [B, 128, 32, 32] | ✓ | |
| 5th expanding block | [B, 128, 32, 32] | [B, 64, 64, 64] | ✓ | |
| 6th expanding block | [B, 64, 64, 64] | [B, 32, 128, 128] | ✓ | |
| 2nd convolutional layer | [B, 32, 128, 128] | [B, C, 128, 128] | | |

# Methodology - continued

Critic architecture

**Table S3.** Critic: NLI and ILI's detail used in this study (input size is represented by [B, C, $\widetilde{N}_h^p$, $\widetilde{N}_h^p$], and output size is represented by [B, C, $\text{PATCH}_\text{X}$, $\text{PATCH}_\text{Y}$]. We use hidden layers H = 8).

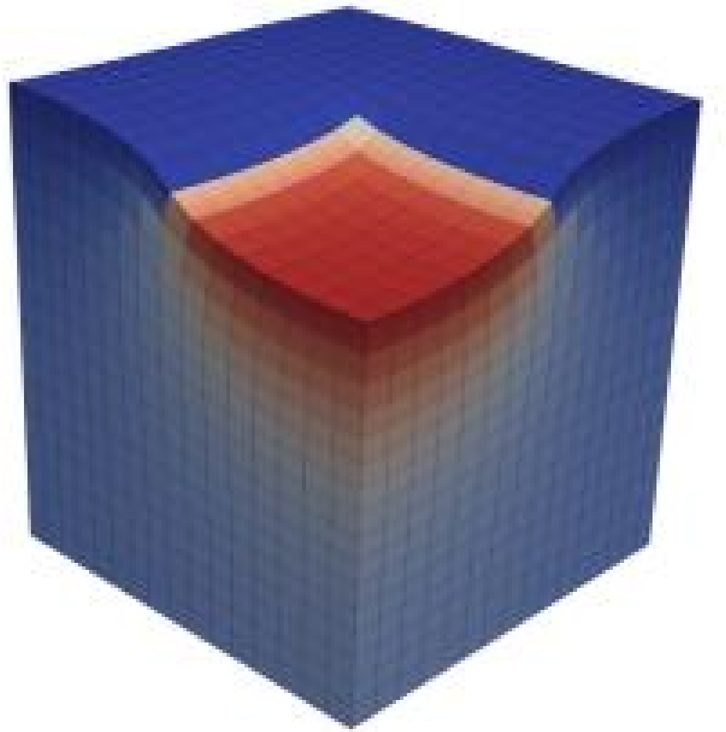
| Block | Input size | Output size | BN |
|---|---|---|---|
| 1st convolutional layer | [B, C+1, 128, 128] | [B, 8, 128, 128] | |
| 1st contracting block | [B, 8, 128, 128] | [B, 16, 64, 64] | |
| 2nd contracting block | [B, 16, 64, 64] | [B, 32, 32, 32] | ✓ |
| 3rd contracting block | [B, 32, 32, 32] | [B, 64, 16, 16] | ✓ |
| 4th contracting block | [B, 64, 16, 16] | [B, 128, 8, 8] | ✓ |
| 2nd convolutional layer | [B, 128, 8, 8] | [B, C, 8, 8] | |

# Governing equations

## Momentum balance equation

$$\begin{aligned} \nabla \cdot \boldsymbol{\sigma}'(\boldsymbol{u}) - \alpha \nabla \cdot (p\mathbf{I}) + \boldsymbol{f} = \mathbf{0} \quad &\text{in} \quad \Omega \times \mathbb{T}, \\ \boldsymbol{u} = \boldsymbol{u}_D \quad &\text{on} \quad \partial\Omega_u \times \mathbb{T}, \\ \boldsymbol{\sigma}(\boldsymbol{u}) \cdot \mathbf{n} = \boldsymbol{t}_D \quad &\text{on} \quad \partial\Omega_t \times \mathbb{T}, \\ \boldsymbol{u} = \boldsymbol{u}_0 \quad &\text{in} \quad \Omega \text{ at } t = 0, \end{aligned}$$

where $\boldsymbol{\sigma}'$ is the effective stress, $p$ is the pore pressure, $\boldsymbol{u}$ is bulk displacement, $\alpha$ is the Biot coefficient, f is the body force.



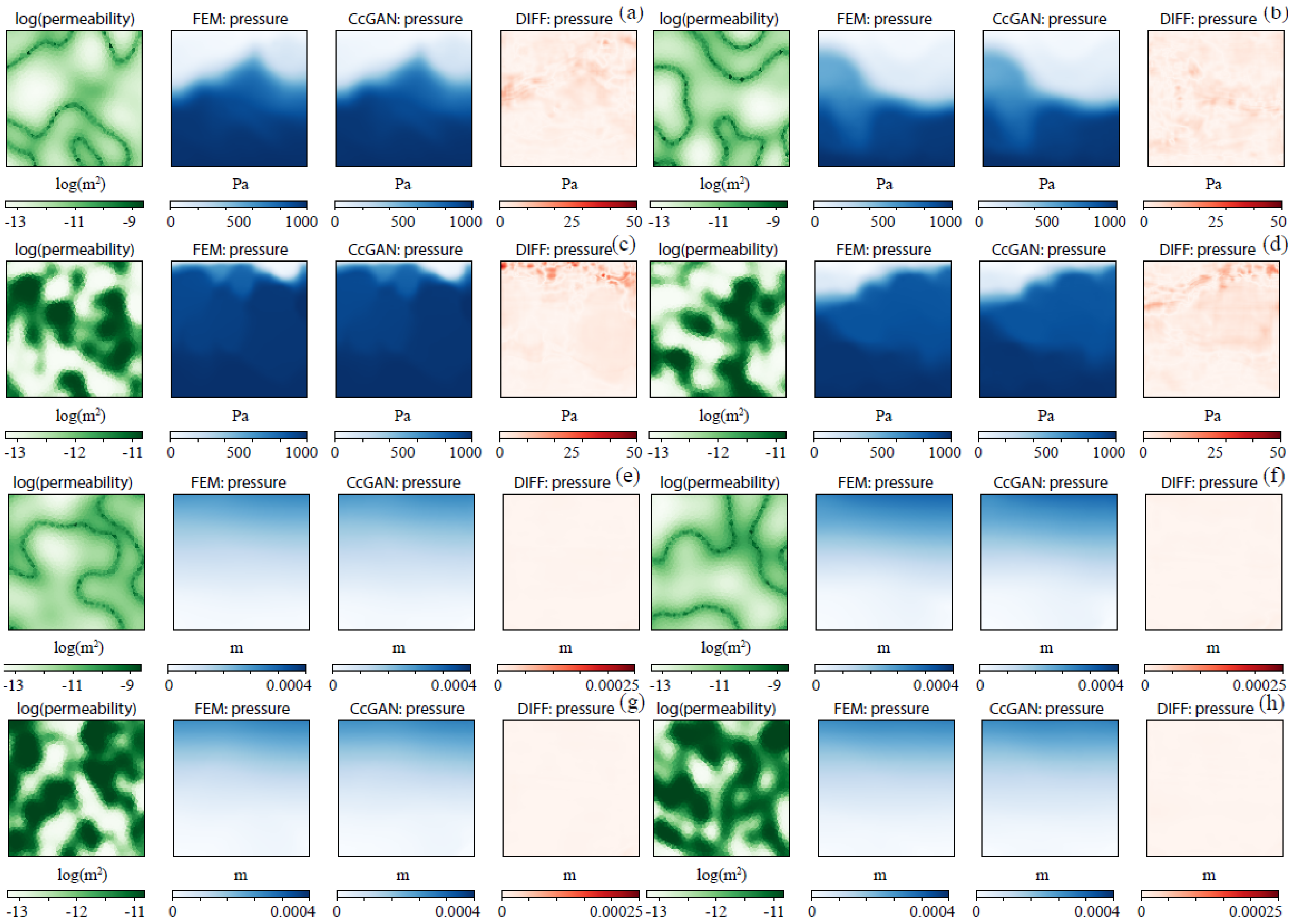
Pic from: J.Choo. Stabilized mixed continuous/enriched Galerkin formulations for locally mass conservative poromechanics. CMAME. 2019.

## Mass balance equation

$$\begin{aligned} \left(\frac{1}{M} + \frac{\alpha^2}{K}\right)\frac{\partial p}{\partial t} + \frac{\alpha}{K}\frac{\partial \sigma_v}{\partial t} - \nabla \cdot (\boldsymbol{\kappa} \nabla p) = g \quad &\text{in} \quad \Omega \times \mathbb{T}, \\ p = p_D \quad &\text{on} \quad \partial\Omega_p \times \mathbb{T}, \\ -\boldsymbol{\kappa} \nabla p \cdot \mathbf{n} = q_D \quad &\text{on} \quad \partial\Omega_q \times \mathbb{T}, \\ p = p_0 \quad &\text{in} \quad \Omega \text{ at } t = 0, \end{aligned}$$

where $M$ is the Biot modulus, $\boldsymbol{\sigma}_v$ is the volumetric stress, $K$ is bulk modulus, $\boldsymbol{\kappa}$ is the porous media conductivity

# Results


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
log(permeability)
FEM: pressure
CcGAN: pressure
DIFF: pressure
log(m²)
Pa
m
-13
-12
-11
-9
0
500
1000
25
50
0.0004
0.00025

# Results - continued

**Table 1.** The relative RMSE (Eq. (9)) results for testing data of three example cases as a function of the number of training data (M) for pressure and magnitude of displacement. Each example is evaluated with both NLI and ILI models.

| | | | | | |
|---|---|---|---|---|---|
| Pressure | Example 1 | M = 1250 | M = 2500 | M = 5000 | M = 10000 |
| | NLI (%) | 4.63 | 3.24 | 2.34 | 1.74 |
| | ILI (%) | 4.55 | 3.15 | 2.30 | 1.67 |
| | Example 2 | M = 1250 | M = 2500 | M = 5000 | M = 10000 |
| | NLI (%) | 3.60 | 2.61 | 1.83 | 1.24 |
| | ILI (%) | 3.73 | 2.55 | 1.67 | 1.22 |
| | Example 3 | M = 2500 | M = 5000 | M = 10000 | M = 20000 |
| | NLI (%) | 3.36 | 2.31 | 1.65 | 1.32 |
| | ILI (%) | 3.07 | 2.24 | 1.63 | 1.29 |
| Displacement | Example 1 | M = 1250 | M = 2500 | M = 5000 | M = 10000 |
| | NLI (%) | 4.32 | 2.98 | 2.14 | 1.57 |
| | ILI (%) | 4.13 | 2.78 | 2.03 | 1.33 |
| | Example 2 | M = 1250 | M = 2500 | M = 5000 | M = 10000 |
| | NLI (%) | 3.60 | 2.51 | 1.47 | 1.10 |
| | ILI (%) | 3.37 | 2.07 | 1.26 | 0.83 |
| | Example 3 | M = 2500 | M = 5000 | M = 10000 | M = 20000 |
| | NLI (%) | 3.28 | 2.28 | 1.27 | 1.27 |
| | ILI (%) | 2.74 | 2.15 | 1.18 | 1.05 |

Example 3: a total number of M is the sum of training data from both Examples 1 and 2.

$x_i$ and $\hat{x}_i$ are the ground truth (FOM result) and approximated values (ROM result)

$$\text{relative RMSE} = \frac{\sqrt{\frac{\sum_{i=1}^{M}(x_i-\hat{x}_i)^2}{M}}}{\sqrt{\frac{\sum_{i=1}^{M} x_i^2}{M}}}$$

# Results - continued

**Table S4.** Comparison of the wall time (seconds) used for each operation presented in Figure 1 (main text). $\boldsymbol{\mu}$ is a set of parameterize spatial fields, and $\boldsymbol{\mu}_i \in \boldsymbol{\mu}$.

| | NLI | ILI | remark |
|---|---|---|---|
| Build FOM snapshots | 40 | 40 | per $\boldsymbol{\mu}_i$ for $N^t = 10$ |
| Train ROM with M = 1250 | 12600 | 12600 | approximately 3.75 hours |
| Train ROM with M = 2500 | 25200 | 25200 | approximately 7.5 hours |
| Train ROM with M = 5000 | 50400 | 50400 | approximately 15 hours |
| Train ROM with M = 10000 | 108000 | 108000 | approximately 30 hours |
| Train ROM with M = 20000 | 216000 | 216000 | approximately 60 hours |
| Prediction | 0.001 | 0.001 | per testing $(t^n, \boldsymbol{\mu}_i)$ |

FOM: 36 cores AMD Ryzen Threadripper 3970X
ROM: a single Quadro RTX 6000,

# Conclusions

1. High dimensional parameterization – the whole heterogeneous field.

2. Reduced order model that could provide much faster calculation and maintain reasonable accuracy.

3. ILI provides a better accuracy than NLI without additional computational cost.

4. Including physics-information (PDEs) is in progress.