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Efficient and reasonable powerflow simulations

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SIAM Student Chapter Seminar
CU Denver
December 8, 2021

SAND2021-DRAFT

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Acknowledgements

Collaborators: D. Sirajuddin, E. G. Phillips, T. D. Pointon, T. C. Powell, P. J. Christenson, K. L. Cartwright

Methods discussed today are implemented in the SNL ASC code EMPIRE. We would like to acknowledge our team for their support.

This work has been supported Sandia National Laboratories Laboratory Directed Research & Development Program.

Plasma Science and Engineering Grand Challenge LDRD Project #209240 (2018-2020)

Coupling Powerflow to Target Simulations LDRD Project #210398 (2021)

Outline

- Intro to Pulsed Power
- EMTL Coupling
- V&V
- Closure

Intro to Pulsed Power

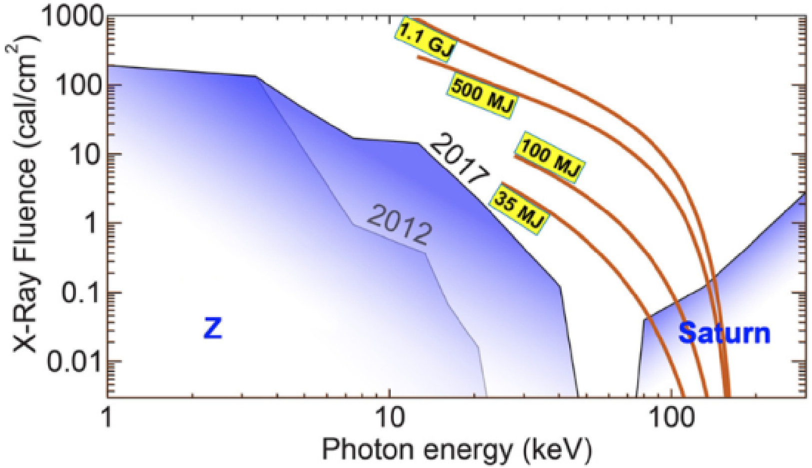
Pulsed Power Drivers

Large scale experimental facilities at SNL

- Radiation sources
- High energy density material science
- Create astrophysical conditions in a laboratory setting

Magnetically Insulated Transmission Lines

Legacy development of these platforms has been experimental/empirical. **Programmatic desire for science based design (i.e. computation) for future systems.**



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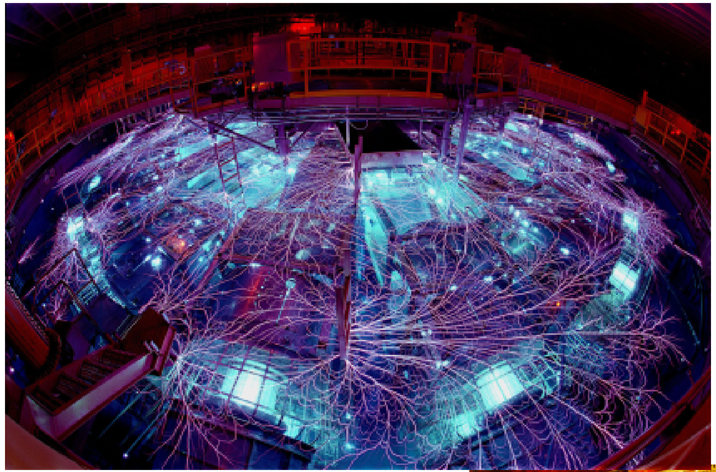
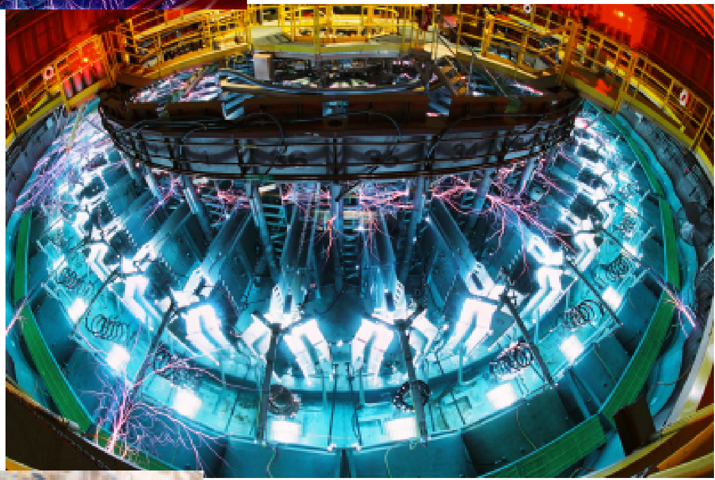
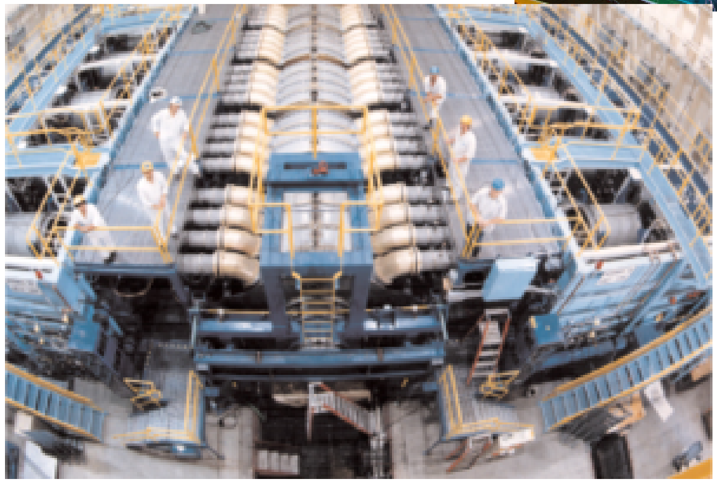


Z Machine

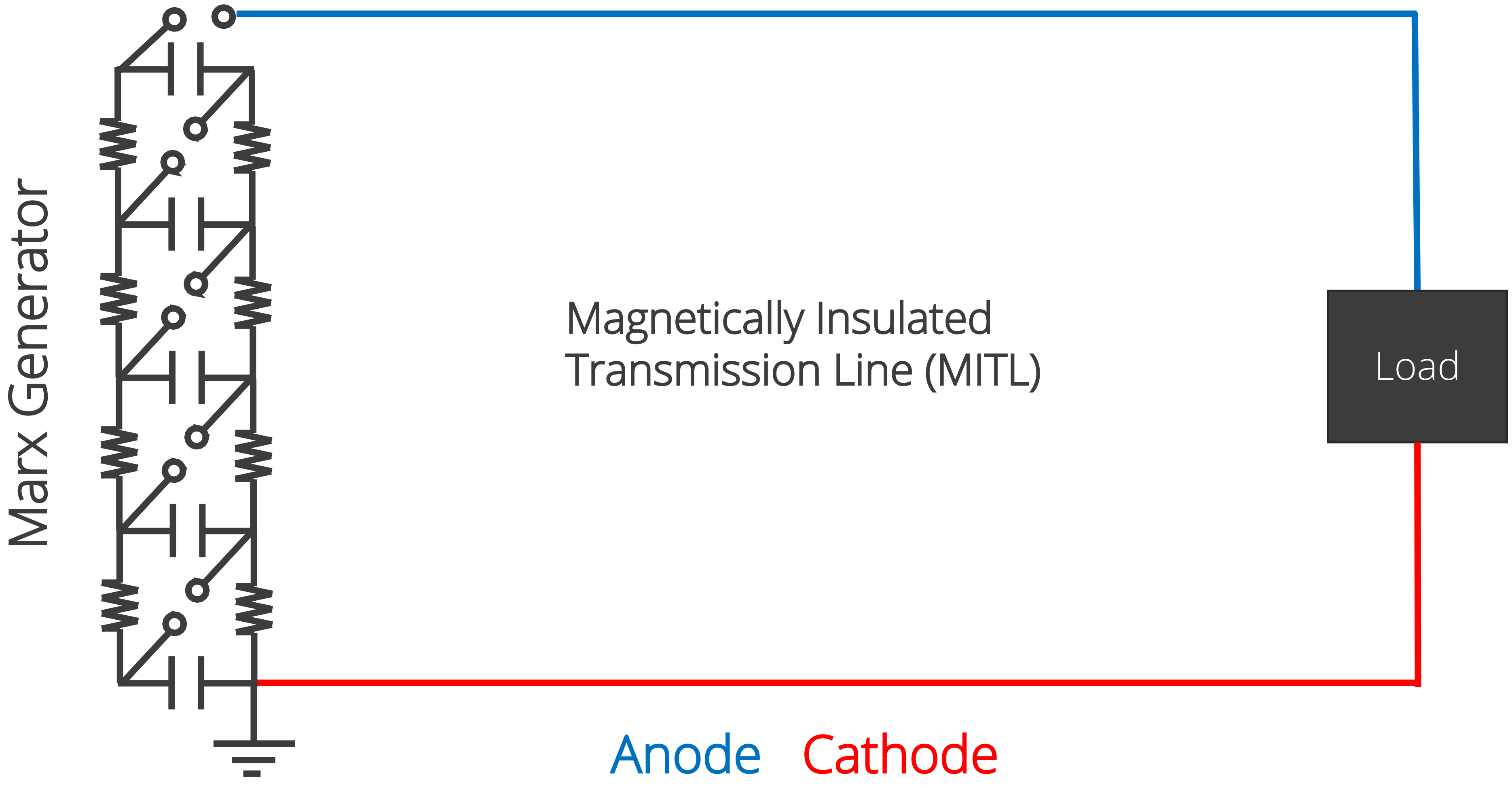
SATURN



HERMES-3



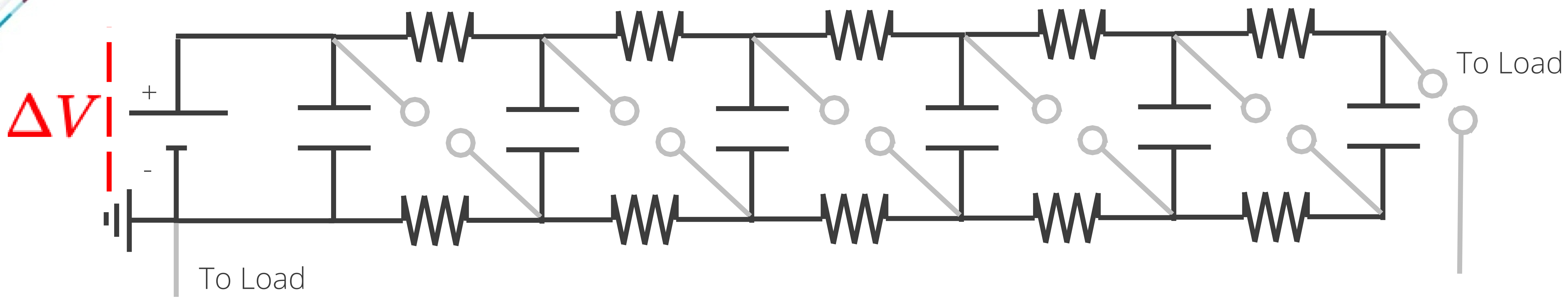
Simplified abstract pulsed power system



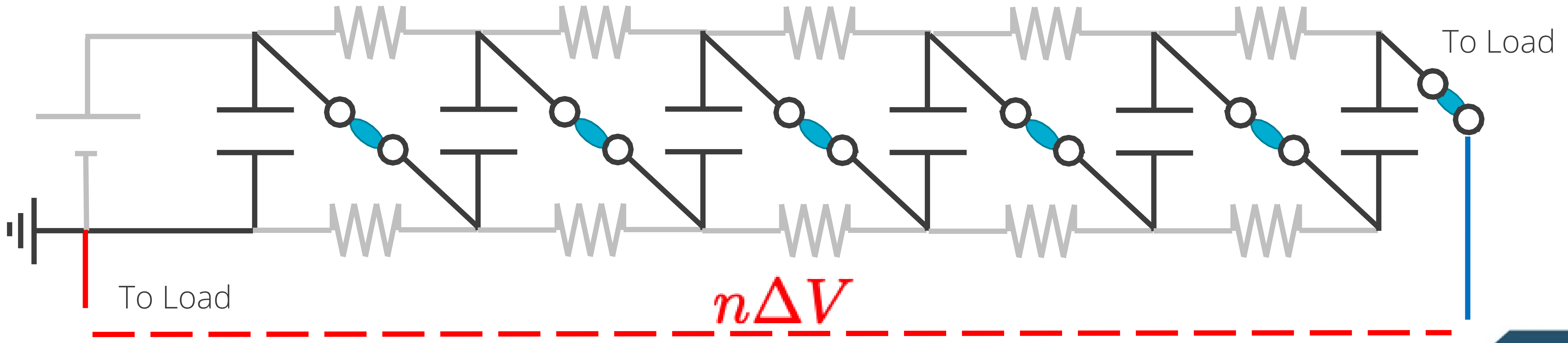
How the pulse gets from the generator to the target is called Powerflow

Marx Generator

Charging Mode



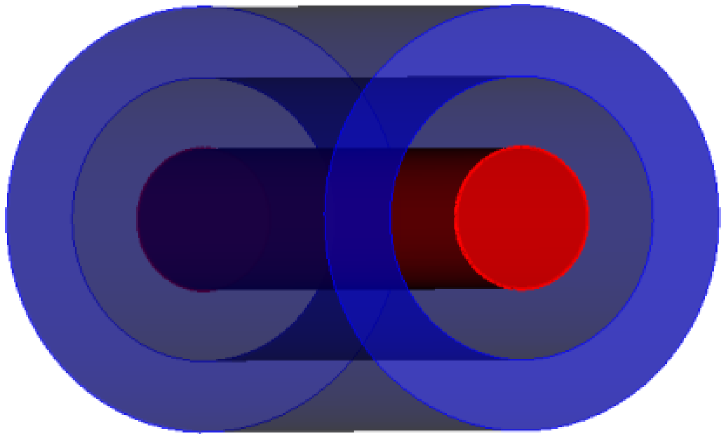
Discharge Mode



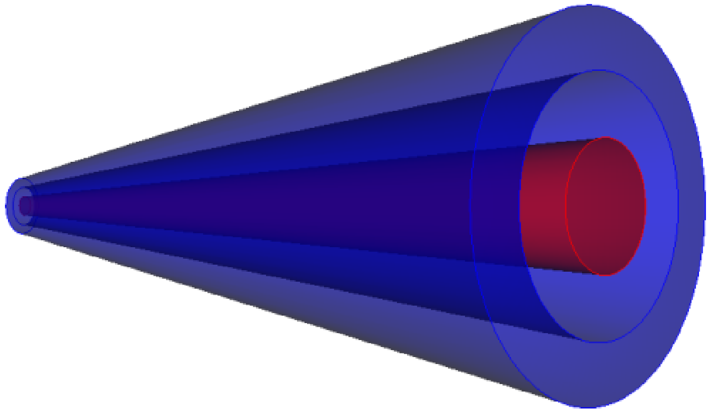
Transmission Lines

We'll discuss modes in more detail later– but enclosed geometries with disjoint conductors support **Transverse Electromagnetic Modes** (TEM).

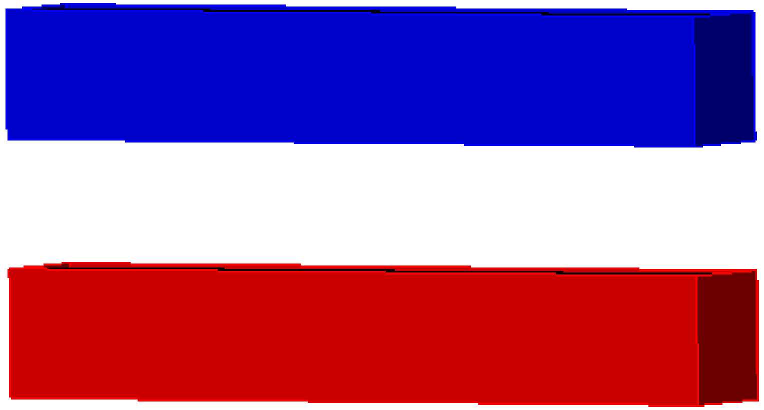
TEM waves obey free space dispersion relationship and therefore can transmit arbitrary frequencies. Pulsed wave forms have large spectra (recall uncertainty principle for Fourier Transforms).



Coaxial



Conical



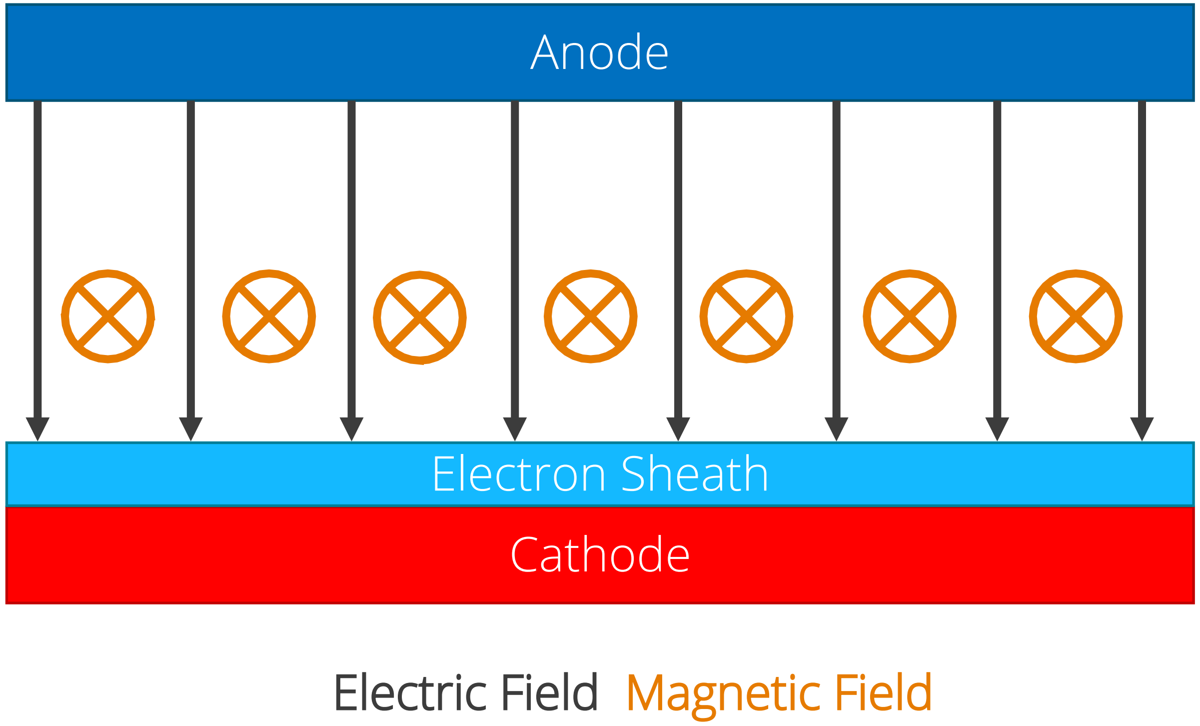
Parallel Plate

Magnetically Insulated Transmission Line (MITL)

Electric field is preferential for electron emission from the cathode – higher power systems more likely to cause breakdown.

Electrons crossing anode cathode (AK) gap results in power loss.

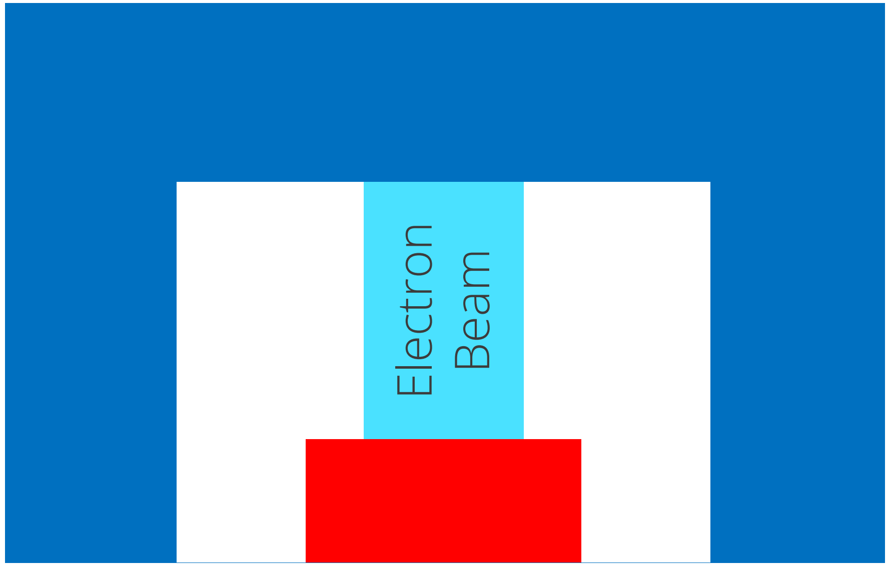
In certain regimes the presence of the **magnetic field limits this effect** insulating the cathode and forming a sheath.



$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Example targets

Bremsstrahlung diode

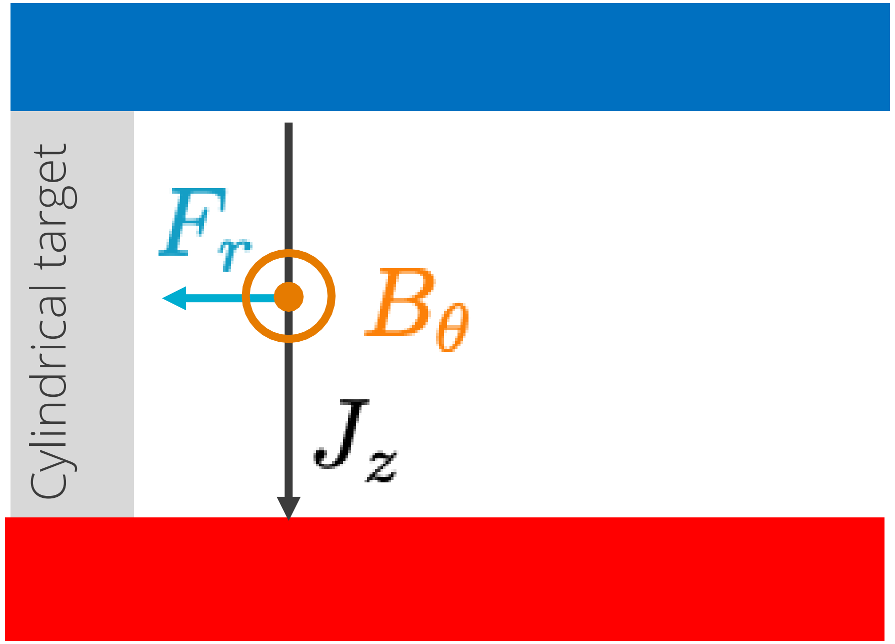


Truncate the transmission line with an open cavity

Configuration preferential for electron emission from cathode.

Relativistic electrons enter converter plate and create high energy x-rays

Z-pinch targets



Short the transmission line with a cylinder.

Currents induce ohmic heating

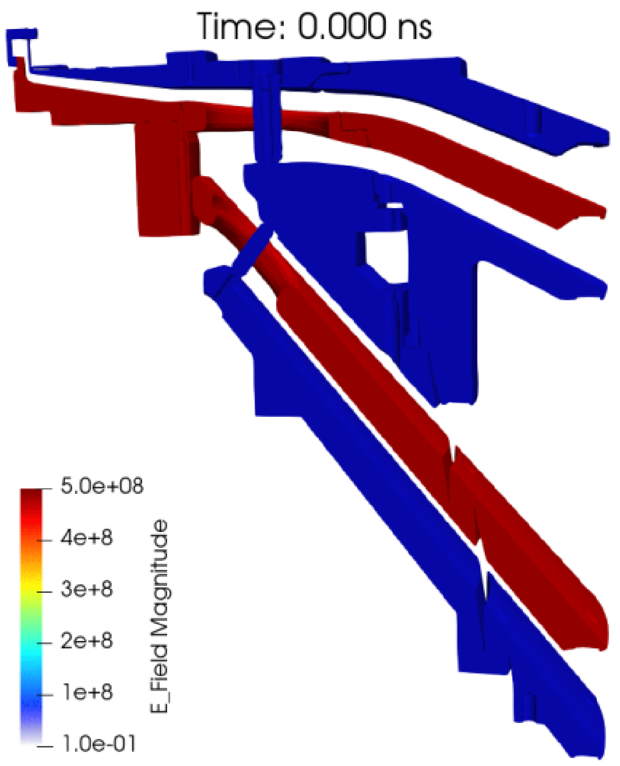
Currents and magnetic fields induce inward Lorentz force.

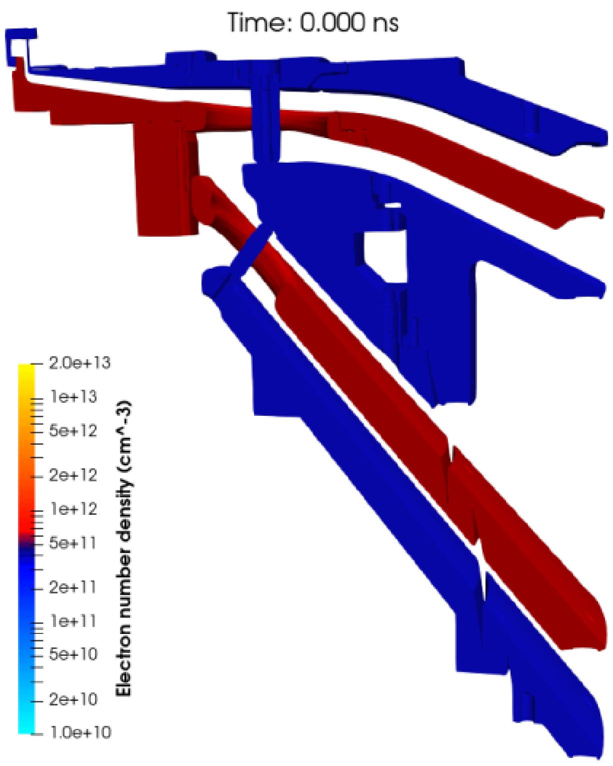
Work is done on the target

Powerflow: How the pulse gets to the target

Questions to answer with powerflow simulations:

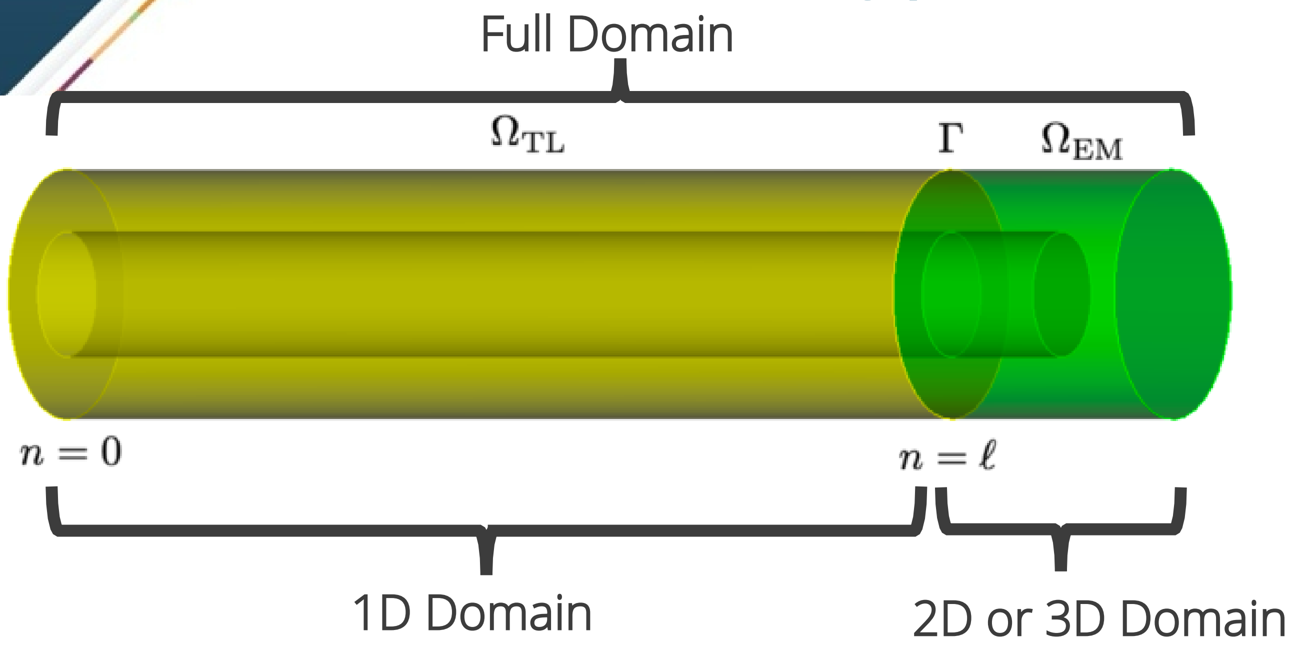
- How much energy is delivered to the target?
- How regular are the fields in the “target region”?
- How much loss can be attributed to different features?



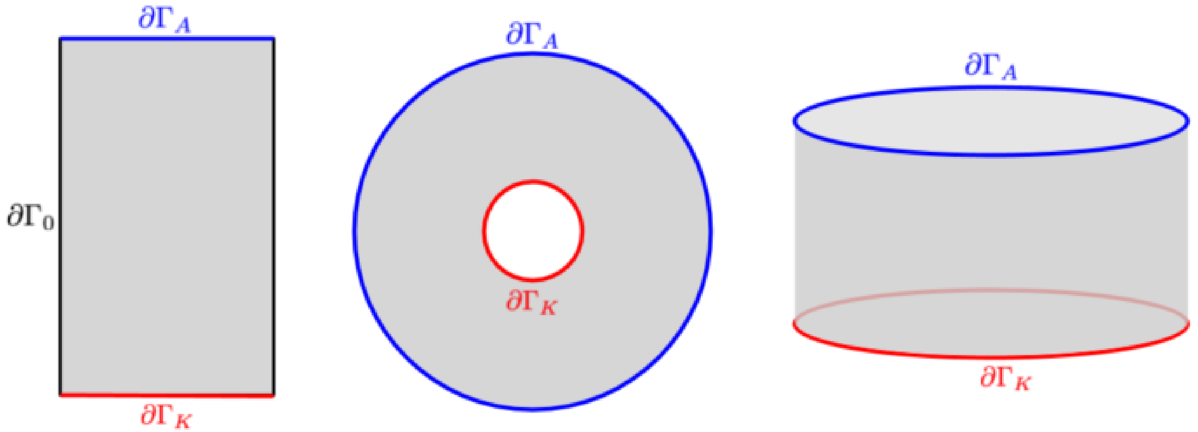


EM-TL Coupling

Abstract modelling problem



Example EM-TL Coupling Interfaces



Maxwell's Equations

$$\begin{cases} \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} - \mathbf{curl}\ \mathbf{H} = \mathbf{0} \\ \frac{\partial \mathbf{B}}{\partial t} + \mathbf{curl}\ \mathbf{E} = \mathbf{0} \\ \mathrm{div}\ \mathbf{D} = \rho \\ \mathrm{div}\ \mathbf{B} = 0 \end{cases}$$

Simple Dielectric

$$\begin{cases} \mathbf{D} = \epsilon \mathbf{E} \\ \mathbf{B} = \mu \mathbf{H} \end{cases}$$

Homogeneous BCs

$$\begin{cases} \mathbf{E} \times \mathbf{n} = \mathbf{0} & \text{on conductors} \\ \mathbf{H} \times \mathbf{n} = \mathbf{0} & \text{on symmetry} \end{cases}$$

$\mathbf{J}$ is data, we'll assume its zero in TL domain

Analogy Fourier Series

A domain $\Omega = [0,1]$.
Represent a function f with eigenfunctions of $-\partial^2_{xx}$.
There are three cases.

$$\begin{cases} -u_0'' = 0 \\ u(0) = u(1) = 1 \end{cases} \qquad k_0 = 0 : u_0(x) = 1$$

$$\begin{cases} -u_{o,n}'' = k_n^2 u \\ u_{o,n}(0) = u_{o,n}(1) = 0 \end{cases} \qquad k_n = n\pi : u_{o,n}(x) = \sin(n\pi x)$$

$$\begin{cases} -u_{e,n}'' = k_n^2 u \\ u_{e,n}'(0) = u_{e,n}'(1) = 0 \end{cases} \qquad k_n = n\pi : u_{e,n}(x) = \cos(n\pi x)$$

Represent f as a series:

$$f = \langle f, u_0 \rangle u_0 + \sum_{n=1}^{\infty} \frac{\langle f, u_{o,n} \rangle}{\langle u_{o,n}, u_{o,n} \rangle} u_{o,n} + \frac{\langle f, u_{e,n} \rangle}{\langle u_{e,n}, u_{e,n} \rangle} u_{e,n}$$

$$\langle f, g \rangle = \int_0^1 f(x) g(x) dx$$

These eigen-functions are dense in L2.

All these modes
This can be shown easily from variational form.

If f is smooth and [0,1]-periodic convergence is "rapid"

Maxwell's Equations in the TL domain

$\Omega_{\mathrm{TL}} = \Gamma \times [0, \ell]$ equipped with coordinates $(\boldsymbol{\tau}, n)$

$$\mathbf{curl} = \begin{pmatrix} \frac{\partial}{\partial n}\mathbf{n}\times & \mathbf{curl}_\tau \\ \mathrm{rot}_\tau & 0 \end{pmatrix}, \qquad \nabla = \begin{pmatrix} \nabla_\tau \\ \frac{\partial}{\partial n} \end{pmatrix}, \qquad \mathrm{div} = (\mathrm{div}_\tau, \frac{\partial}{\partial n})$$

$$\mathbf{curl}_\tau = -\mathbf{n} \times \nabla_\tau, \quad \mathrm{rot}_\tau = \mathrm{div}_\tau \mathbf{n}\times$$

Maxwell's equations in these coordinates can be written as follows:

$$\begin{cases} \epsilon \frac{\partial}{\partial t}\mathbf{E}_\tau - \frac{\partial}{\partial n}\mathbf{n} \times \mathbf{H}_\tau - \mathbf{curl}_\tau H_n = 0 \\ \epsilon \frac{\partial}{\partial t} E_n - \mathrm{rot}_\tau \mathbf{H}_\tau = 0 \\ \mu \frac{\partial}{\partial t}\mathbf{H}_\tau + \frac{\partial}{\partial n}\mathbf{n} \times \mathbf{E}_\tau + \mathbf{curl}_\tau E_n = 0 \\ \mu \frac{\partial}{\partial t} H_n + \mathrm{rot}_\tau \mathbf{E}_\tau = 0 \\ \mathrm{div}_\tau \epsilon \mathbf{E}_\tau + \frac{\partial}{\partial n}\epsilon E_n = 0 \\ \mathrm{div}_\tau \mu \mathbf{H}_\tau + \frac{\partial}{\partial n}\mu H_n = 0 \end{cases}$$

Transverse Modes

Separate Variables:

$$\mathbf{E} = \Big(V_\tau(n,t)\mathbf{E}_\tau(\boldsymbol{\tau}), V_n(n,t)E_n(\boldsymbol{\tau})\Big)$$

$$\mathbf{H} = \Big(I_\tau(n,t)\mathbf{H}_\tau(\boldsymbol{\tau}), I_n(n,t)H_n(\boldsymbol{\tau})\Big)$$

TEM

$E_n = H_n = 0$

$$\begin{cases} \epsilon\mathbf{E}_\tau\frac{\partial V_\tau}{\partial t} - \mathbf{n}\times\mathbf{H}_\tau\frac{\partial I_\tau}{\partial n} = 0 \\ \mu\mathbf{H}_\tau\frac{\partial I_\tau}{\partial t} + \mathbf{n}\times\mathbf{E}_\tau\frac{\partial V_\tau}{\partial n} = 0 \\ \mathrm{rot}_\tau\mathbf{E}_\tau = 0 \\ \mathrm{div}_\tau\epsilon\mathbf{E}_\tau = 0 \\ \mathrm{rot}_\tau\mathbf{H}_\tau = 0 \\ \mathrm{div}_\tau\mu\mathbf{H}_\tau = 0 \end{cases}$$

$$\begin{cases} -u_0'' = 0 \\ u(0) = u(1) = 1 \end{cases}$$

TM

$H_n = 0$

$$\begin{cases} \epsilon\mathbf{E}_\tau\frac{\partial V_\tau}{\partial t} - \mathbf{n}\times\mathbf{H}_\tau\frac{\partial I_\tau}{\partial n} \\ \epsilon E_n\frac{\partial V_n}{\partial t} - I_\tau\mathrm{rot}_\tau\mathbf{H}_\tau = 0 \\ \mu\mathbf{H}_\tau\frac{\partial I_\tau}{\partial t} + \mathbf{n}\times\mathbf{E}_\tau\frac{\partial V_\tau}{\partial n} + V_n\mathbf{curl}_\tau E_n = 0 \\ \mathrm{div}_\tau\epsilon\mathbf{E}_\tau + \frac{\partial}{\partial n}\epsilon E_n = 0 \\ \mathrm{rot}_\tau\mathbf{E}_\tau = 0 \\ \mathrm{div}_\tau\mu\mathbf{H}_\tau = 0 \end{cases}$$

$$\begin{cases} -u_{o,n}'' = k_n^2 u \\ u_{o,n}(0) = u_{o,n}(1) = 0 \end{cases}$$

TE

$E_n = 0$

$$\begin{cases} \epsilon\mathbf{E}_\tau\frac{\partial V_\tau}{\partial t} - \mathbf{n}\times\mathbf{H}_\tau\frac{\partial I_\tau}{\partial n} - I_n\mathbf{curl}_\tau H_n = 0 \\ \mu\mathbf{H}_\tau\frac{\partial I_\tau}{\partial t} + \mathbf{n}\times\mathbf{E}_\tau\frac{\partial V_\tau}{\partial n} = 0 \\ \mu H_n\frac{\partial I_n}{\partial t} + V_\tau\mathrm{rot}_\tau\mathbf{E}_\tau = 0 \\ \mathrm{div}_\tau\epsilon\mathbf{E}_\tau = 0 \\ \mathrm{div}_\tau\mu\mathbf{H}_\tau + \frac{\partial}{\partial n}\mu H_n = 0 \\ \mathrm{rot}_\tau\mathbf{H}_\tau = 0 \end{cases}$$

$$\begin{cases} -u_{e,n}'' = k_n^2 u \\ u_{e,n}'(0) = u_{e,n}'(1) = 0 \end{cases}$$

$$\epsilon E_n\frac{\partial^2 V_n}{\partial t^2} + V_n\triangle E_n + \frac{\partial V_\tau}{\partial n}\mathrm{div}_\tau\mathbf{E}_\tau = 0$$

TEM Mode

IF ϵ, μ constant $\mathbf{J} \equiv \mathbf{0}$

Surface Potential

$$\begin{cases} -\Delta_\tau \varphi_{\text{TEM}} = 0 \\ \varphi_{\text{TEM}}|_{\Gamma_A} = 1 \\ \varphi_{\text{TEM}}|_{\Gamma_K} = 0 \\ \nabla_\tau \varphi_{\text{TEM}} \cdot \mathbf{m}|_{\Gamma_S} = 0 \end{cases}$$

Field Profiles

$$\mathbf{E}_{\text{TEM},\tau} = -\nabla_\tau \varphi_{\text{TEM}}$$

$$\mathbf{H}_{\text{TEM},\tau} = \frac{\mathbf{curl}_\tau \varphi_{\text{TEM}}}{\|\mathbf{E}_{\text{TEM},\tau}\|^2_{L^2(\Gamma)}}$$

TEM Telegrapher's Equations

$$\begin{cases} C_{\text{TEM}} \dot{V}_{\text{TEM},\tau} + I'_{\text{TEM},\tau} = 0 \\ L_{\text{TEM}} \dot{I}_{\text{TEM},\tau} + V'_{\text{TEM},\tau} = 0 \end{cases}$$

TEM Capacitance and Inductance per Length

$$C_{\text{TEM}} = \int_\Gamma \epsilon |\mathbf{E}_{\text{TEM},\tau}|^2 \, dA$$

$$L_{\text{TEM}} = \int_\Gamma \mu |\mathbf{H}_{\text{TEM},\tau}|^2 \, dA$$

THEN

$$\mathbf{E}_\tau = V(n,t)\mathbf{E}_0(\tau_1, \tau_2)$$

$$\mathbf{H}_\tau = I(n,t)\mathbf{H}_0(\tau_1, \tau_2)$$

ARE A SOLUTION TO MAXWELL'S EQAUTIONS ON

$$\Omega_{\text{TL}} = \Gamma \times [0, \ell]$$

WITH DISPERSION RELATIONSHIP

$$\epsilon \mu \omega^2 = k^2$$

TM Modes

IF ϵ, μ constant $\mathbf{J} \equiv \mathbf{0}$

Eigenvalue problem

$$\begin{cases} -\Delta_\tau \varphi_{\mathrm{TM},j} = k_j^2 \varphi_{\mathrm{TM},j} \\ \left(\frac{1}{|\Gamma|} \int_\Gamma |\varphi_{\mathrm{TM},j}|^2 \, dA \right)^{1/2} = 1 \\ \varphi_{\mathrm{TM},j}|_{\Gamma_A} = 0 \\ \varphi_{\mathrm{TM},j}|_{\Gamma_K} = 0 \\ \nabla_\tau \varphi_{\mathrm{TM},j} \cdot \mathbf{m}|_{\Gamma_S} = 0 \end{cases}$$

Field Profiles

$$\mathbf{E}_{\mathrm{TM},\tau,j} = -\nabla_\tau \varphi_{\mathrm{TM},j}$$

$$E_{\mathrm{TM},n,j} = k_j \varphi_{\mathrm{TM},j}$$

$$\mathbf{H}_{\mathrm{TM},\tau,j} = \frac{\mathbf{curl}_\tau \varphi_{\mathrm{TM},j}}{k_j^2 |\Gamma|}$$

TM Telegrapher's Equations

$$\begin{cases} C_{\mathrm{TM},j} \dot{V}_{\mathrm{TM},\tau,j} + I'_{\mathrm{TM},\tau,j} = 0 \\ C_{\mathrm{TM},j} \dot{V}_{\mathrm{TM},n,j} - k_j I_{\mathrm{TM},\tau,j} = 0 \\ L_{\mathrm{TM},j} \dot{I}_{\mathrm{TM},\tau,j} + k_j V_{\mathrm{TEM},n,j} + V'_{\mathrm{TM},\tau,j} = 0 \end{cases}$$

TM Capacitance and Inductance per Length

$$C_{\mathrm{TM},j} = \int_\Gamma \epsilon |\mathbf{E}_{\mathrm{TM},\tau,j}|^2 \, dA = \epsilon k_j^2 |\Gamma|$$

$$L_{\mathrm{TM},j} = \int_\Gamma \mu |\mathbf{H}_{\mathrm{TM},\tau,j}|^2 \, dA = \mu (k_j^2 |\Gamma|)^{-1}$$

THEN

$$\mathbf{E} = (V_{\mathrm{TM},\tau,j} \mathbf{E}_{\mathrm{TM},\tau,j}, V_{\mathrm{TM},n,j} E_{\mathrm{TM},n,j})$$

$$\mathbf{H} = (I_{\mathrm{TM},\tau,j} \mathbf{H}_{\mathrm{TM},\tau,j}, 0)$$

ARE A SOLUTION TO MAXWELL'S EQUATIONS ON

$$\Omega_{\mathrm{TL}} = \Gamma \times [0, \ell]$$

WITH DISPERSION RELATIONSHIP

$$\epsilon \mu \omega^2 = k^2 + k_j^2$$

TL Modal Decomposition

An approximation of an arbitrary TM mode can be written as

$$\begin{cases} \mathbf{E}_\tau = V_{\mathrm{TEM},\tau}\mathbf{E}_{\mathrm{TEM},\tau} + \sum_{j=1}^{M} V_{\mathrm{TM},\tau,j}\mathbf{E}_{\mathrm{TM},\tau,j} \\ E_n = \sum_{j=1}^{M} V_{\mathrm{TM},n,j}\mathbf{E}_{\mathrm{TM},n,j} \\ \mathbf{H}_\tau = I_{\mathrm{TEM},\tau}\mathbf{H}_{\mathrm{TEM},\tau} + \sum_{j=1}^{M} I_{\mathrm{TM},\tau,j}\mathbf{H}_{\mathrm{TM},\tau,j} \\ H_n = 0 \end{cases}$$

These modes are orthogonal

We can evolve each of the modes independently from initial data using their respective telegrapher equations

Initial voltages and currents computed with projections onto the modes

$$V_{\mathrm{TEM},\tau}(n, t=0) = \frac{\int_\Gamma \mathbf{E}_0(\boldsymbol{\tau}, n), \mathbf{E}_{\mathrm{TEM},\tau}\ dA}{\|\mathbf{E}_{\mathrm{TEM},\tau}\|_\Gamma^2} \qquad I_{\mathrm{TEM},\tau}(n, t=0) = \frac{\int_\Gamma \mathbf{H}_0(\boldsymbol{\tau}, n) \cdot \mathbf{H}_{\mathrm{TEM},\tau}\ dA}{\|\mathbf{H}_{\mathrm{TEM},\tau}\|_\Gamma^2}$$

$$V_{\mathrm{TM},\tau,j}(n, t=0) = \frac{\int_\Gamma \mathbf{E}_0(\boldsymbol{\tau}, n) \cdot \mathbf{E}_{\mathrm{TM},\tau,j}\ dA}{\|\mathbf{E}_{\mathrm{TM},\tau,j}\|_\Gamma^2} \qquad I_{\mathrm{TM},\tau,j}(n, t=0) = \frac{\int_\Gamma \mathbf{H}_0(\boldsymbol{\tau}, n) \cdot \mathbf{H}_{\mathrm{TM},\tau,j}\ dA}{\|\mathbf{H}_{\mathrm{TM},\tau,j}\|_\Gamma^2}$$

$$V_{\mathrm{TM},n,j}(n, t=0) = \frac{\int_\Gamma \mathbf{E}_0(\boldsymbol{\tau}, n) \cdot \mathbf{n}E_{\mathrm{TM},n,j}\ dA}{\|E_{\mathrm{TM},n,j}\|_\Gamma^2}$$

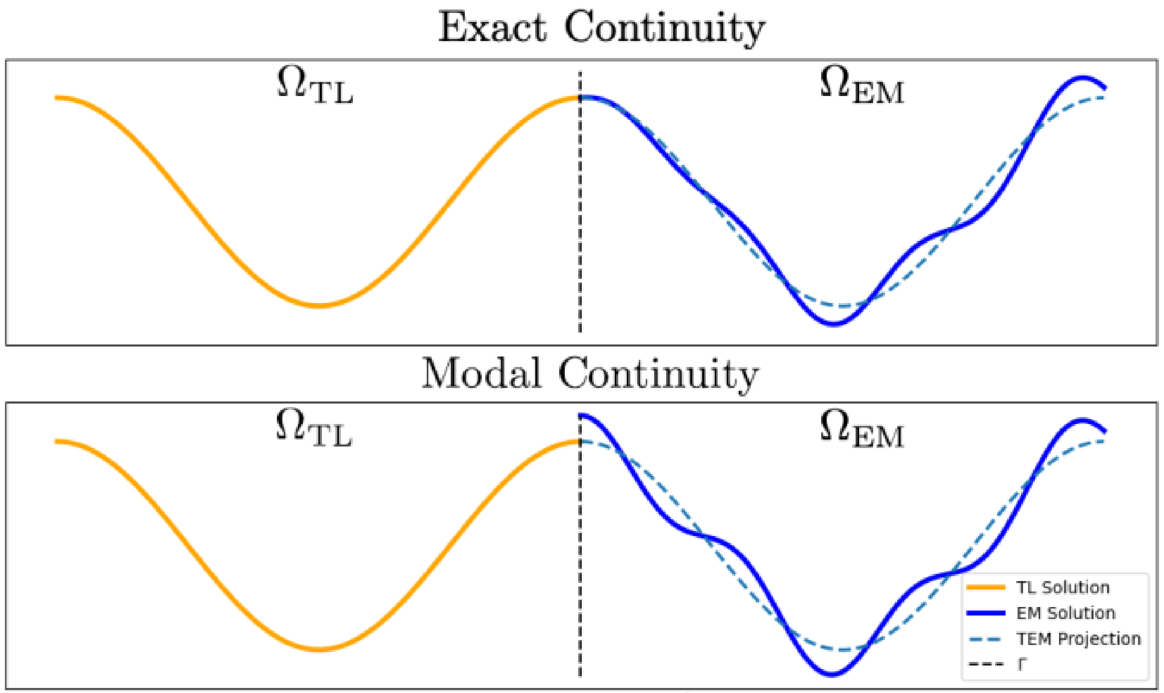
Coupling Strategy

For the coupling between EM and TL domains to be consistent we need tangent fields to "somehow agree" at the coupling interface.

The notion we impose is weaker than exact continuity and we call it "Modal Continuity"

$$\int_\Gamma \mathbf{E}(t) \cdot \mathbf{E}_{\mathrm{TM},\tau,j} \; dA = \|\mathbf{E}_{\mathrm{TM},\tau,j}\|_\Gamma^2 V_{\mathrm{TM},\tau,j}(\ell, t)$$

$$\int_\Gamma \mathbf{H}(t) \cdot \mathbf{H}_{\mathrm{TM},\tau,j} \; dA = \|\mathbf{H}_{\mathrm{TM},\tau,j}\|_\Gamma^2 I_{\mathrm{TM},\tau,j}(\ell, t)$$



$$n = \ell$$

The L2 projection of the EM fields onto the TEM+TM space at the coupling interface recovers the TL solution

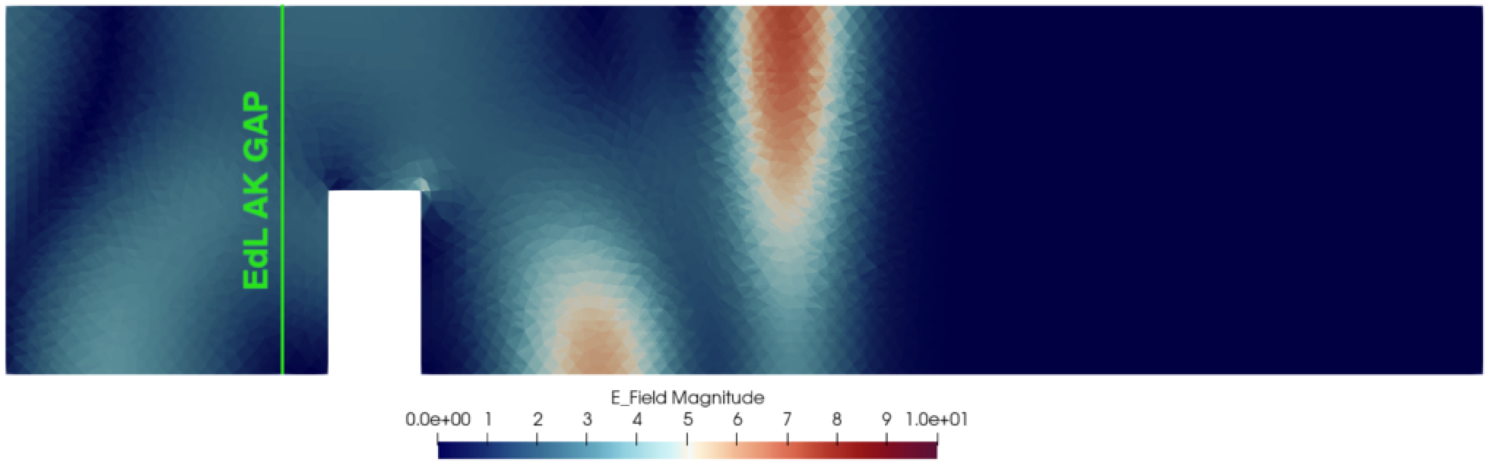
Philosophical difference: The solution is TEM+TM in the TL domain vs. we don't track other components parts in the TL domain

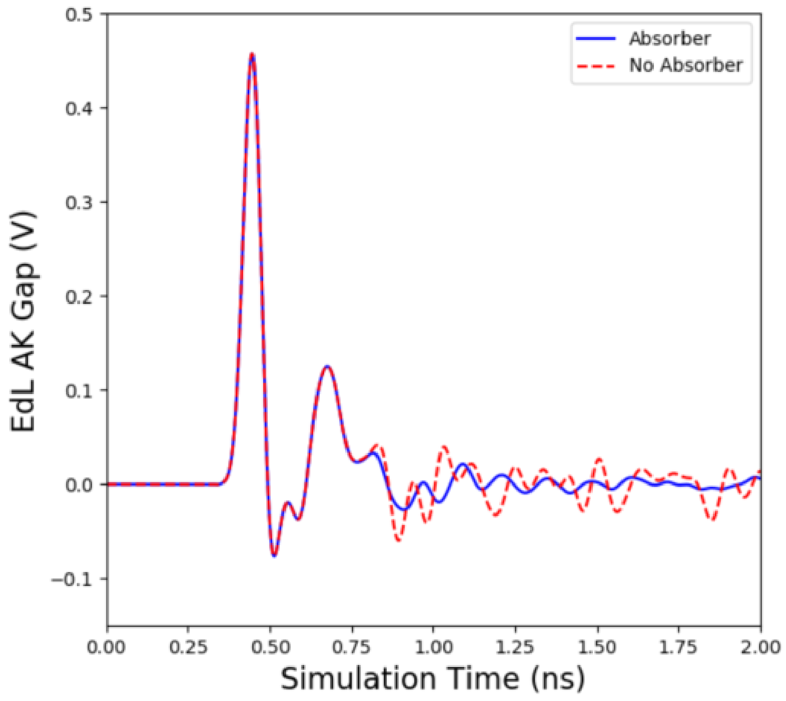
Exact Continuity

$$\begin{cases} \mathbf{E} \times \mathbf{n}|_\Gamma = V(t,\ell)\mathbf{E}_0 \times \mathbf{n} \\ LI(\ell) = \int_\Gamma \mu \mathbf{H} \cdot \mathbf{H}_0 \, dA \end{cases} \qquad \begin{cases} \mathbf{H} \times \mathbf{n}|_\Gamma = I(t,\ell)\mathbf{H}_0 \times \mathbf{n} \\ CV(\ell) = \int_\Gamma \epsilon \mathbf{E} \cdot \mathbf{E}_0 \, dA \end{cases}$$

Both methods have the problem of reflecting non-TEM components off coupling interface in the EM domain. Problematic for plasmas!

Red lineout is Neumann Maxwell/ Dirichlet Telegrapher solution





Modal coupling enables non-TL absorber

$$\Pi_{\mathrm{TEM}} : \mathbf{L}^2(\Gamma) \to \mathbf{L}^2(\Gamma) :$$

$$\Pi_{\mathrm{TEM}}(\mathbf{E}) := \mathbf{E}_0 \frac{\int_\Gamma \epsilon \mathbf{E} \cdot \mathbf{E}_0 \; dA}{\int_\Gamma \epsilon \mathbf{E}_0 \cdot \mathbf{E}_0 \; dA}$$

We can use this projection to define a first order outgoing wave condition
for non-TL Waves! Add this to variational Ampere's law

$$\int_\Gamma Z^{-1}(\mathbf{E} - \Pi_{\mathrm{TEM}}\mathbf{E}) \times \mathbf{n} \cdot \mathbf{\Psi} \times \mathbf{n} \; dA$$

How do we impose 2 boundary conditions at the same time?

Impose voltage coupling as a constraint and relax with a Lagrange multiplier

$$\begin{cases} \int_{\Omega_{\mathrm{EM}}} \epsilon \frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{\Psi} + \mathbf{J} \cdot \mathbf{\Psi} - \mu^{-1}\mathbf{B} \cdot \mathbf{curl} \; \mathbf{\Psi} \; dV \\ + \int_\Gamma \left(Z^{-1}(\mathrm{I} - \Pi_{\mathrm{TEM}})(\mathbf{E}) + \lambda \epsilon \mathbf{E}_0 \right) \times \mathbf{n} \cdot \mathbf{\Psi} \times \mathbf{n} & \forall \mathbf{\Psi} \in \mathbf{H}(\mathbf{curl}, \Omega_{\mathrm{EM}}) \\ \theta \int_\Gamma \epsilon \mathbf{E} \times \mathbf{n} \cdot \mathbf{E}_0 \times \mathbf{n} \; dA = \theta C V & \forall \theta \in \mathbb{R} \end{cases}$$

Variational Formulation

$$(\mathbf{E},\mathbf{B},\boldsymbol{\lambda},\mathbf{V}_\tau,\mathbf{V}_n,\mathbf{I}_\tau)\in\mathbf{H}(\mathbf{curl},\Omega_{\mathrm{EM}})\times\mathbf{H}(\mathrm{div},\Omega_{\mathrm{EM}})\times\mathbb{R}^{M+1}\times[H^1(0,\ell)]^{M+1}\times[L^2(0,\ell)]^M\times[L^2(0,\ell)]^{M+1}:$$

Maxwell's Equations

$$\int_{\Omega_{\mathrm{EM}}}\epsilon\frac{\partial}{\partial t}\mathbf{E}\cdot\boldsymbol{\Psi}+\mathbf{J}\cdot\boldsymbol{\Psi}-\mu^{-1}\mathbf{B}\cdot\mathbf{curl}\ \boldsymbol{\Psi}\ dV$$

$$+\int_\Gamma\left(Z^{-1}\left(\mathrm{I}-\Pi_{\mathrm{TEM}}-\textstyle\sum_{j=1}^{M}\Pi_{\mathrm{TM},j}\right)(\mathbf{E})+\lambda_0\mathbf{E}_{\mathrm{TEM},\tau}+\textstyle\sum_{j=1}^{M}\lambda_i\mathbf{E}_{\mathrm{TM},\tau,j}\right)\times\mathbf{n}\cdot\boldsymbol{\Psi}\times\mathbf{n}dA=0\quad\forall\boldsymbol{\Psi}\in\mathbf{H}(\mathbf{curl},\Omega_{\mathrm{EM}})$$

$$\int_{\Omega_{\mathrm{EM}}}\frac{\partial}{\partial t}\mathbf{B}\cdot\boldsymbol{\Phi}+\mathbf{curl}\ \mathbf{E}\cdot\boldsymbol{\Phi}\ dV=0\quad\forall\boldsymbol{\Phi}\in\mathbf{H}(\mathrm{div},\Omega_{\mathrm{EM}})$$

Voltage Continuity

$$\theta\int_\Gamma\mathbf{E}_{\mathrm{TEM},\tau}\times\mathbf{n}\cdot\mathbf{E}\times\mathbf{n}\ dA=\theta\|\mathbf{E}_{\mathrm{TEM},\tau}\|_\Gamma^2V_{\mathrm{TEM},\tau}(\ell)\quad\forall\theta\in\mathbb{R}$$

$$\theta\int_\Gamma\mathbf{E}_{\mathrm{TM},\tau,j}\times\mathbf{n}\cdot\mathbf{E}\times\mathbf{n}\ dA=\theta\|\mathbf{E}_{\mathrm{TM},\tau,j}\|_\Gamma^2V_{\mathrm{TM},\tau,j}(\ell)\quad\forall\theta\in\mathbb{R}\qquad1\le j\le M$$

Telegrapher's Equations

$$\int_0^\ell C_{\mathrm{TEM}}\frac{\partial}{\partial t}V_{\mathrm{TEM},\tau}\varphi-I_{\mathrm{TEM},\tau}\frac{\partial}{\partial n}\varphi\ dS+I_{\text{EM-TEM}}\varphi(\ell)=0\quad\forall\varphi\in H^1(0,\ell)$$

$$\int_0^\ell L_{\mathrm{TEM}}\frac{\partial}{\partial t}I_{\mathrm{TEM},\tau}\psi+\frac{\partial}{\partial n}V_{\mathrm{TEM},\tau}\psi\ dS=0\quad\forall\psi\in L^2(0,\ell)$$

$$\int_0^\ell C_{\mathrm{TM},j}\frac{\partial}{\partial t}V_{\mathrm{TM},\tau,j}\varphi-I_{\mathrm{TM},\tau,j}\frac{\partial}{\partial n}\varphi\ dS+I_{\text{EM-TM},j}\varphi(\ell)=0\quad\forall\varphi\in H^1(0,\ell)$$

$$\int_0^\ell C_{\mathrm{TM},j}\frac{\partial}{\partial t}V_{\mathrm{TM},n,j}-k_jI_{\mathrm{TM},\tau,j}=0\quad\forall\psi\in L^2(0,\ell)\qquad1\le j\le M$$

$$\int_0^\ell L_{\mathrm{TM},j}\frac{\partial}{\partial t}I_{\mathrm{TM},\tau,j}\psi+k_jV_{\mathrm{TM},n,j}+\frac{\partial}{\partial n}V_{\mathrm{TM},\tau,j}\psi\ dS=0\quad\forall\psi\in L^2(0,\ell)$$

Non-TL Absorber

$$\Pi_{\mathrm{TEM},j}(\mathbf{E})=\frac{\langle\mathbf{E}_{\mathrm{TEM},\tau},\mathbf{E}\rangle_\Gamma}{\|E_{\mathrm{TEM},\tau}\|_\Gamma^2}\mathbf{E}_{\mathrm{TEM},\tau}$$

$$\Pi_{\mathrm{TM},j}(\mathbf{E})=\frac{\langle\mathbf{E}_{\mathrm{TM},\tau,j},\mathbf{E}\rangle_\Gamma}{\|E_{\mathrm{TM},\tau,j}\|_\Gamma^2}\mathbf{E}_{\mathrm{TM},\tau,j}\qquad1\le j\le M$$

Current Continuity

$$I_{\text{EM-TEM}}=\int_{\Omega_{\mathrm{EM}}}\epsilon\frac{\partial}{\partial t}\mathbf{E}\cdot\overline{\mathbf{E}}_{\mathrm{TEM},\tau}+\mathbf{J}\cdot\overline{\mathbf{E}}_{\mathrm{TEM},\tau}-\mu^{-1}\mathbf{B}\cdot\mathbf{curl}\ \overline{\mathbf{E}}_{\mathrm{TEM},\tau}\ dV$$

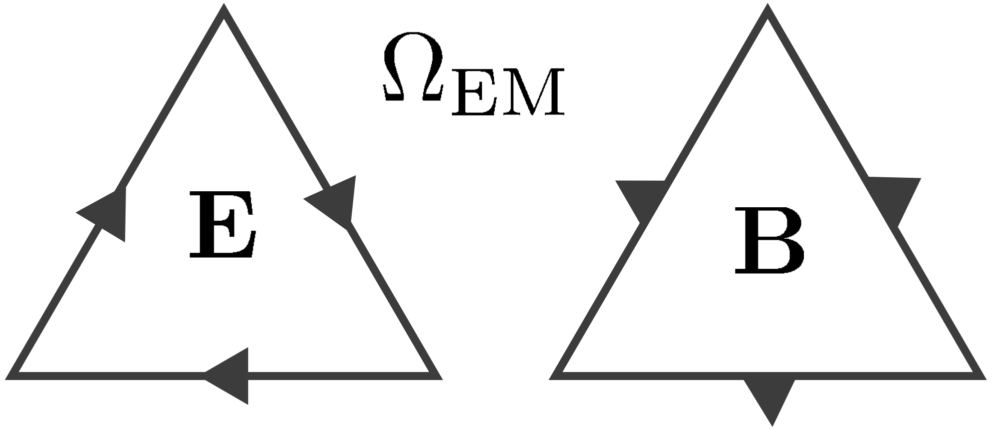
$$I_{\text{EM-TM},j}=\int_{\Omega_{\mathrm{EM}}}\epsilon\frac{\partial}{\partial t}\mathbf{E}\cdot\overline{\mathbf{E}}_{\mathrm{TEM},\tau}+\mathbf{J}\cdot\overline{\mathbf{E}}_{\mathrm{TEM},\tau}-\mu^{-1}\mathbf{B}\cdot\mathbf{curl}\ \overline{\mathbf{E}}_{\mathrm{TEM},\tau}\ dV\qquad1\le j\le M$$

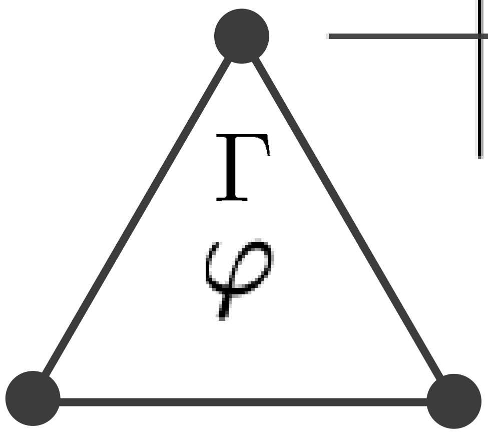
Discretization

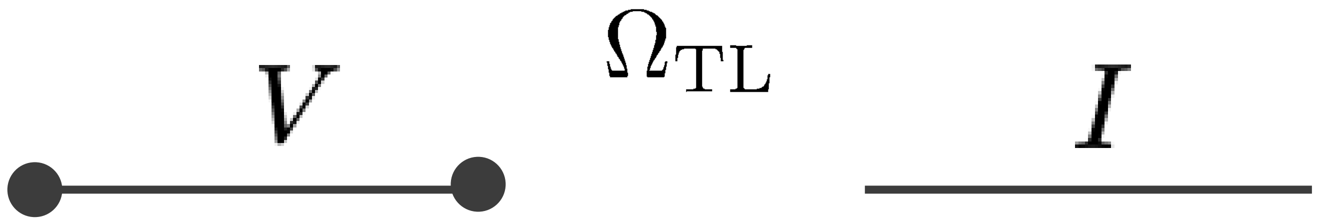
DIRK Time Integration

0	0	0
1	0	1
	1/2	1/2

Lowest Order Compatible Finite Elements







V&V

What is Verification and Validation?

Verification:

Answers question, “Does the code produce solutions to the mathematical model?”

- Code Verification: Usually problems with exact solutions and calculation of errors and rates of convergence
- Solution Verification: Usually problems without exact solution and inference of errors and rates (rates of Cauchy convergence)

Validation:
Answers question, “Is the mathematical model reasonable?”

- Comparison to experimental data
- Often focus on synthetic diagnostics – “How do I simulate what the experiment is measuring?”
- Strives to answer questions of the form ”How accurate are our predictions?”, “Can we bound the uncertainties of the system with uncertainties of the model?”, “Is their bias in our predictions and can we quantify it?”

Uncertainty Quantification:

Statistical/probabilistic interpretation of even deterministic models.
Key to both verification (of probabilistic algorithms) and validation (propagation of uncertainties).

Credit: DM

Verification: O-Wave

- Unmagnetized Plasma wave
- J is data in the EM domain
- Adjust C to capture effect of J in TL domains
- Designed to be extended to fluid and PIC

$$E_z = \widetilde{E}\cos(kx - \omega t), \quad B_y = -\frac{k}{\omega}\cos(kx - \omega t),$$

$$J_z = -\frac{\epsilon_0 \omega_p^2}{\omega}\sin(kx - \omega t)$$

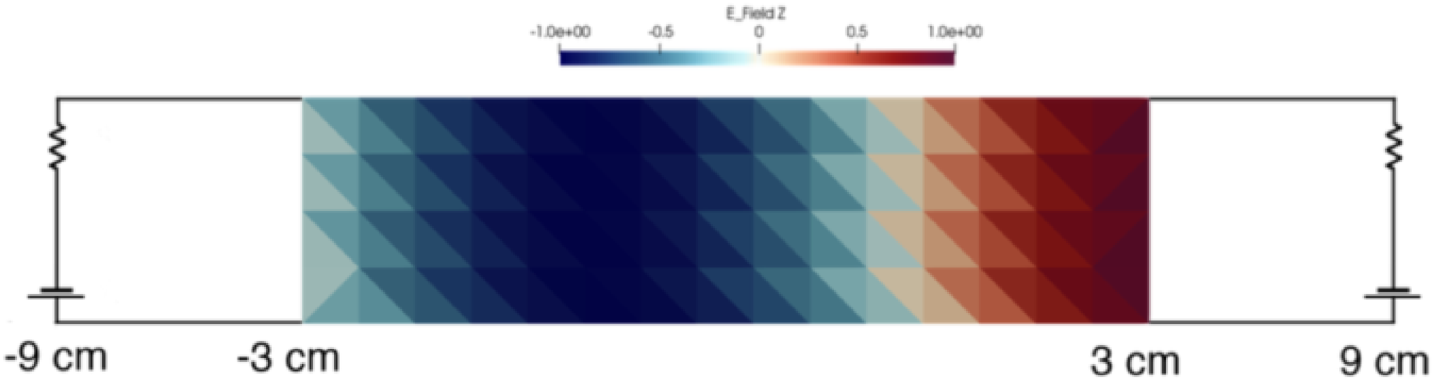
$$E_x = E_y = B_x = B_z = J_x = J_y = 0$$

$$V = \ell_z \widetilde{E}\cos(kx - \omega t), \quad I = -\frac{k}{\omega}\frac{\ell_z \widetilde{E}}{\mu_0}\cos(kx - \omega t)$$

$$C(\omega) = \epsilon_0 \left(1 - \frac{\omega_p^2}{\omega}\right) \|\mathbf{E}_{\mathrm{TEM}}\|^2_{L^2(\Gamma)}$$

27

Verification: O-Wave



(a) CFL varies from (4.35, 7.53)

p.p.w.	p.p.p.	$\mathscr{E}_E$	rate	$\mathscr{E}_B$	rate
21.8	2.5	1.43	–	1.64	–
43.6	5	8.24e-1	0.79	1.10	0.69
87.2	10	1.99e-1	2.04	2.56e-1	1.98
174.5	20	4.99e-2	1.99	6.11e-2	2.06

(b) CFL varies from (0.54, 0.94)

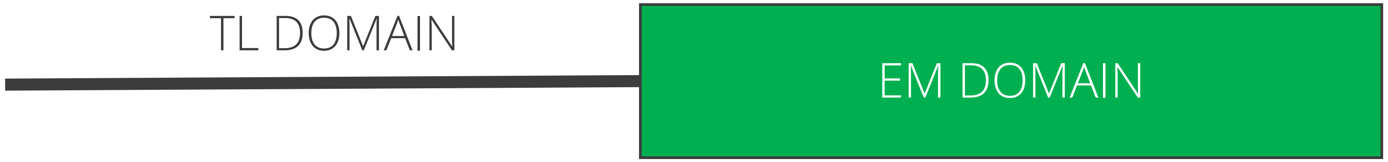
p.p.w.	p.p.p.	$\mathscr{E}_E$	rate	$\mathscr{E}_B$	rate
21.8	10	7.84e-2	–	7.06e-2	–
43.6	20	3.37e-2	1.21	2.41e-2	1.55
87.2	40	1.61e-2	1.07	1.03e-2	1.22
174.5	80	7.97e-3	1.01	5.16e-3	0.99

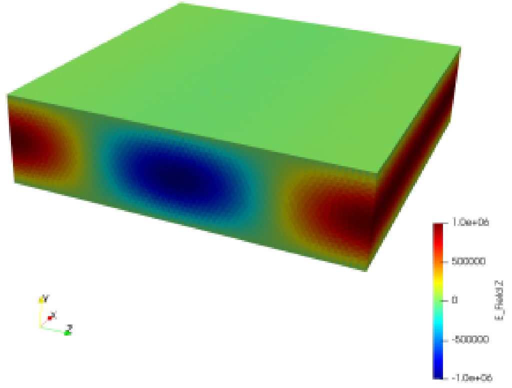
- Using implicit midpoint time-stepping (second order) and first order conforming finite elements for the EM
- Large CFL – error is dominated by time discretization – obtain expected second order convergence
- Small CFL – error is dominated by spatial discretization – obtain expected first order convergence

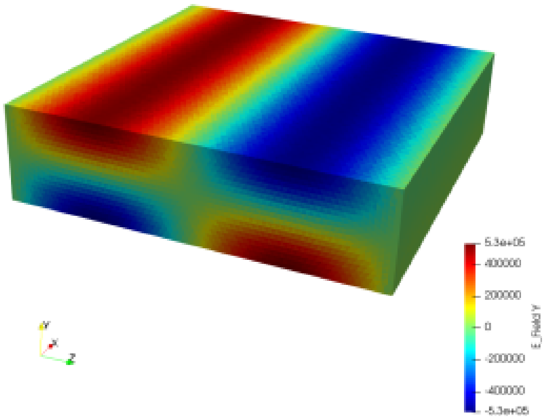
Credit: DM, D. Sirajuddin

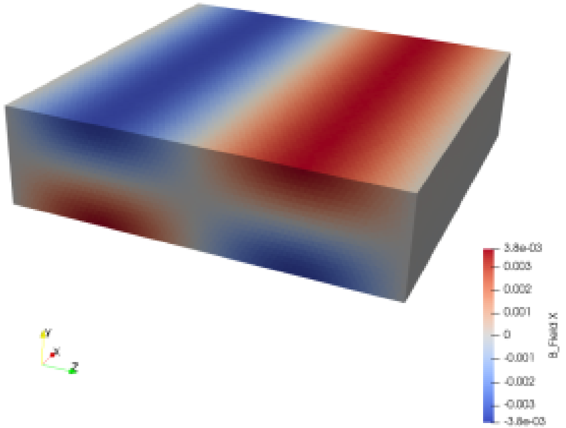
Verification: TM Wave

3D TM Wave on a TL-EM domain









$$E_x = 0$$

$$E_y = \left(\frac{k_z^{[m]}}{k_c^{[m]}} E_{0,z}^{[m]} \right) \cos\left(k_c^{[m]} y\right) \sin(\omega t - k_z^{[m]} z)$$

$$E_z = E_{0,z}^{[m]} \sin\left(k_c^{[m]} y\right) \cos(\omega t - k_z^{[m]} z)$$

$$H_x = -\left(\frac{\omega \epsilon}{k_c^{[m]}} E_{0,z}^{[m]} \right) \cos\left(k_c^{[m]} y\right) \sin(\omega t - k_z^{[m]} z)$$

$$H_y = 0$$

$$H_z = 0$$

$$k_z^{[m]} \approx 2 k_c^{[m]}$$

Credit: DM, D. Sirajuddin

Verification: TM Wave

Theoretical Order of Accuracy (Simplex Mesh)

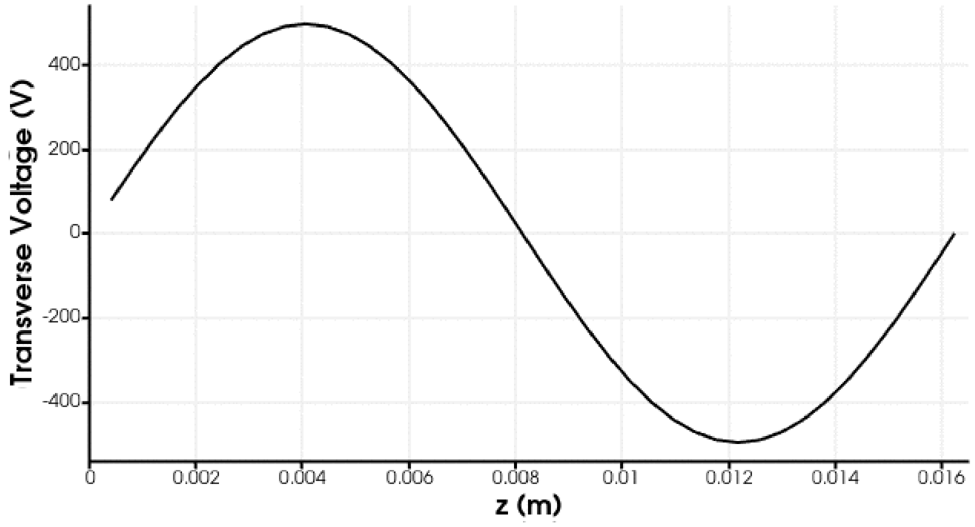
$$\mathcal{O}(\underbrace{\Delta n^2}_{\text{TL}} + \underbrace{h}_{\text{EM}} + \underbrace{k_c^2 h}_{\text{eigen}} + \underbrace{\Delta t^2}_{\text{Time}})$$

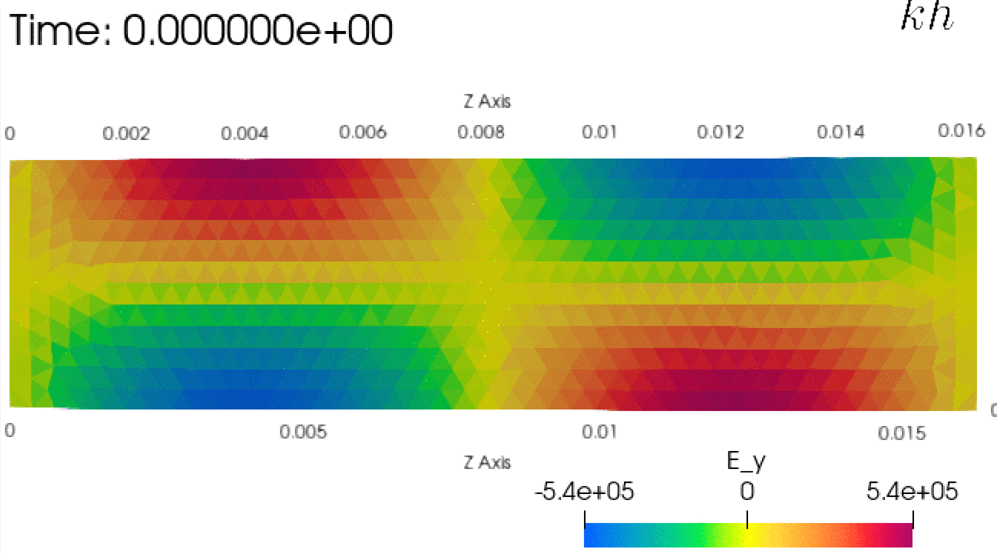
Error denotes a relative L^2 error for functions.

cell size [m]	PPP z	PPP c	E_z error	E_z rate	E_y error	E_y rate	B_x error	B_x rate
1.00E-03	16.2	8.5	3.15E-02	–	5.96E-02	–	4.73E-02	–
6.67E-04	24.3	12.7	1.41E-02	1.98	2.88E-02	1.79	2.58E-02	1.5
5.00E-04	32.5	16.9	9.86E-03	1.24	1.93E-02	1.4	1.85E-02	1.16
4.00E-04	40.6	21.1	7.50E-03	1.23	1.46E-02	1.24	1.43E-02	1.14

Rates are computed using a pairwise fit.

$$\text{PPP} = \frac{2\pi}{kh}$$

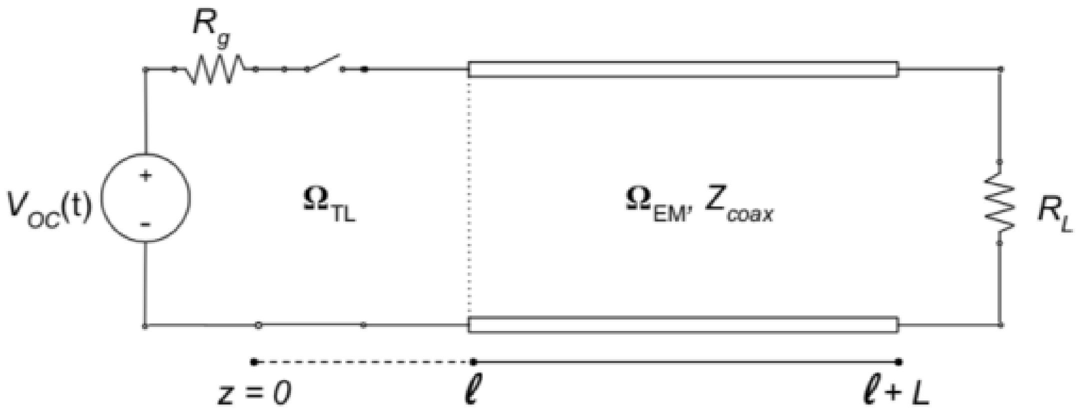




Credit: D. Sirajuddin

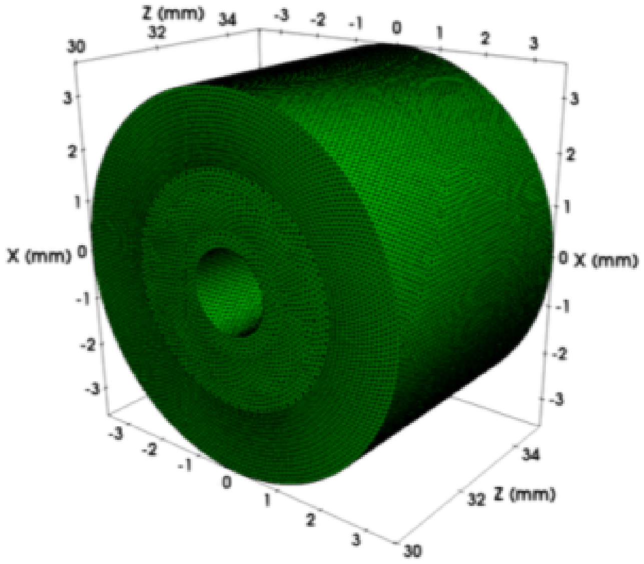
Verification: Steady State

- Coaxial TL driven by an equivalent circuit
- Smooth ramp to steady state
- Obtained first order convergence to analytic steady-state solution



(a) 1D circuit representation of $\Omega_{TL} \cup \Omega_{EM}$

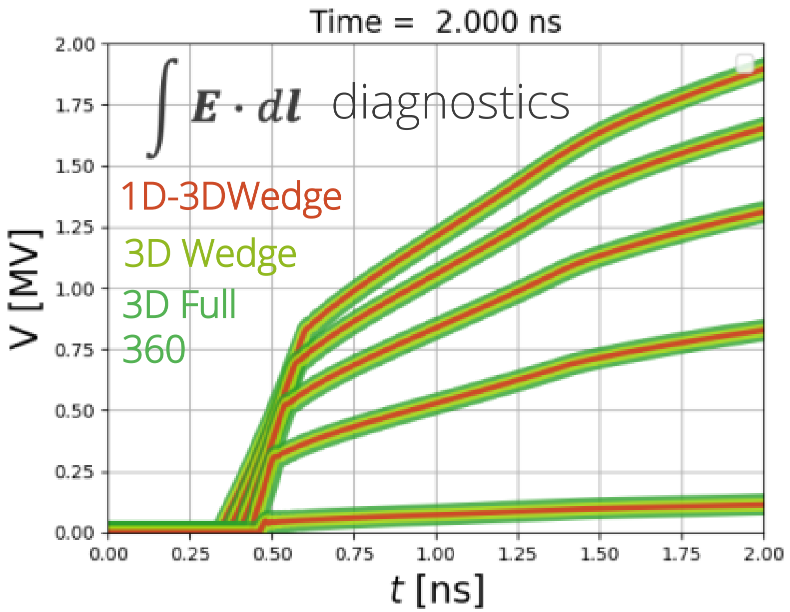
h (m)	$\frac{\text{Gap}}{h}$	$\mathscr{E}_E$	rate	$\mathscr{E}_B$	rate
$1.00e-03$	5.70	$7.89e-02$	–	$5.01e-02$	–
$5.99e-04$	9.70	$7.30e-02$	0.15	$4.70e-02$	0.12
$4.64e-04$	12.3	$6.77e-02$	0.29	$4.49e-02$	0.17
$3.59e-04$	15.9	$5.57e-02$	0.76	$3.64e-02$	0.82
$2.78e-04$	20.5	$4.21e-02$	1.09	$2.67e-02$	1.20
$2.15e-04$	26.5	$3.25e-02$	1.00	$2.03e-02$	1.07
$1.67e-04$	34.2	$2.52e-02$	1.00	$1.55e-02$	1.05
$1.29e-04$	44.3	$1.91e-02$	1.08	$1.17e-02$	1.07
$1.00e-04$	57.0	$1.45e-02$	1.06	$8.96e-03$	1.06

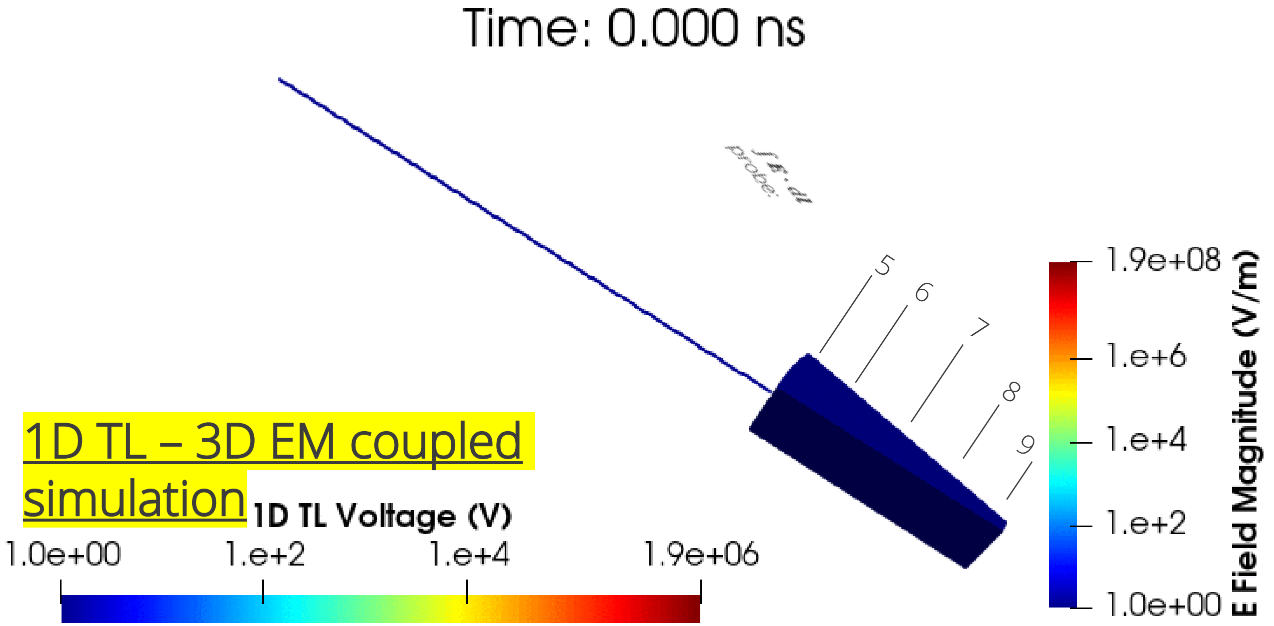


(b) 3D domain Ω_{EM}, 0.1 mm mesh

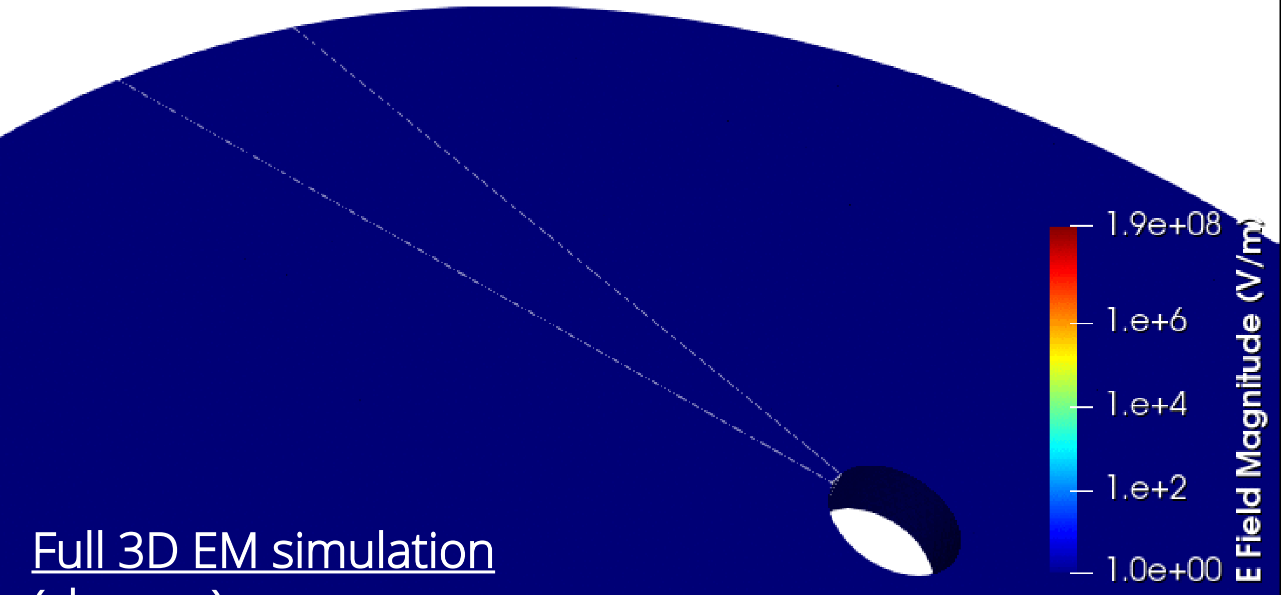
Verification: Non-constant Impedance

Credit: D. Sirajuddin

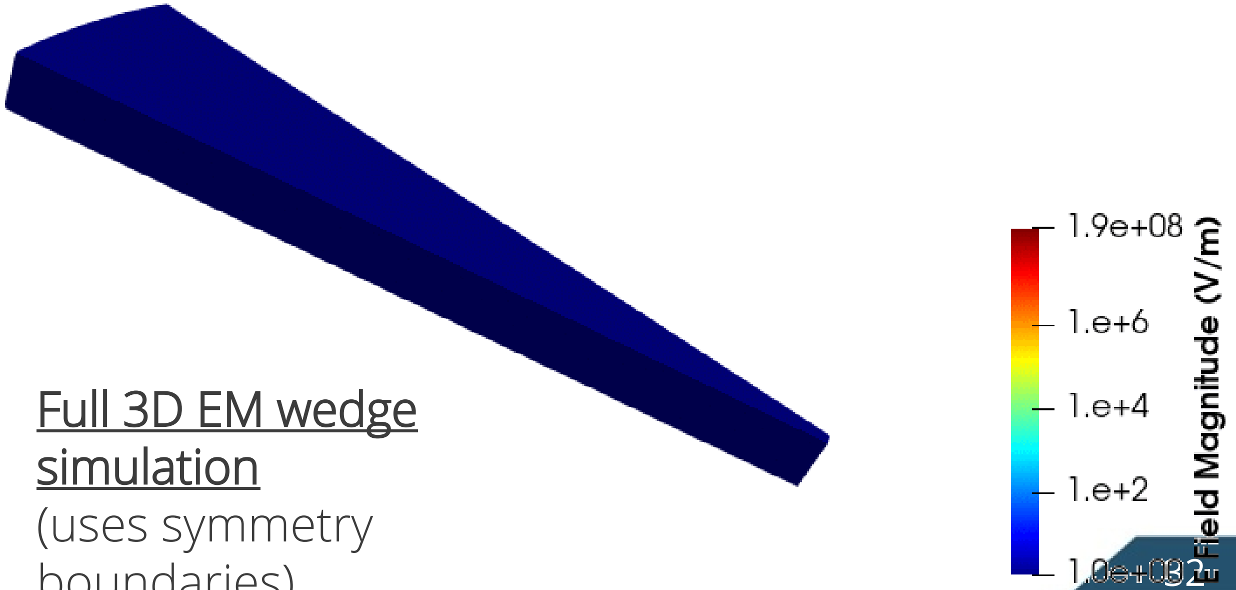




1D TL – 3D EM coupled simulation

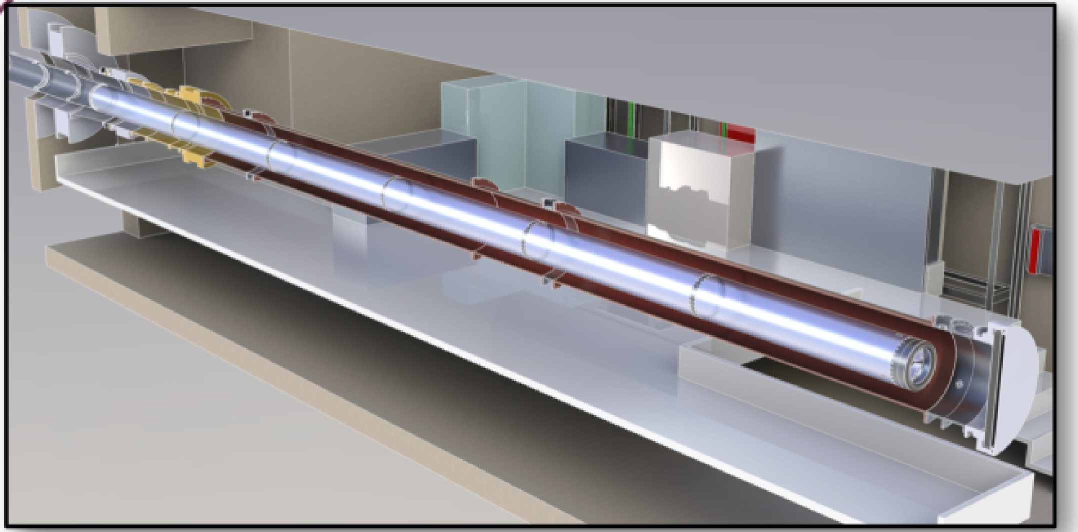


Full 3D EM simulation



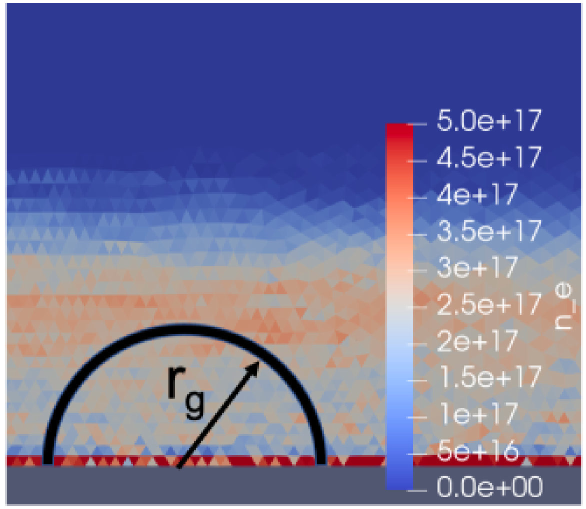
Full 3D EM wedge simulation
(uses symmetry boundaries)

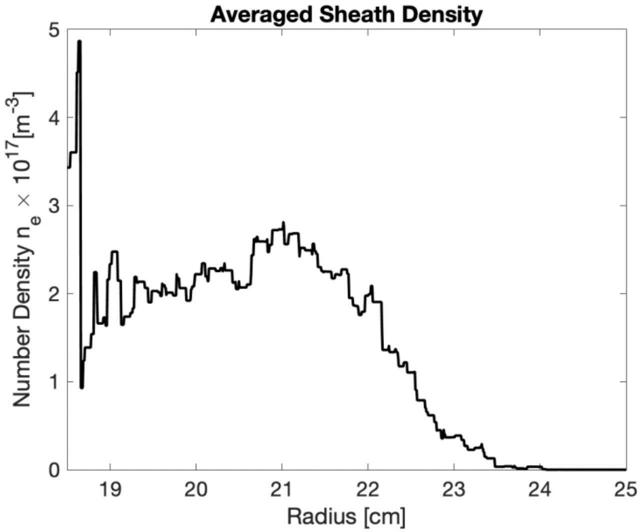
Initial Validation: HERMES 3 MITL

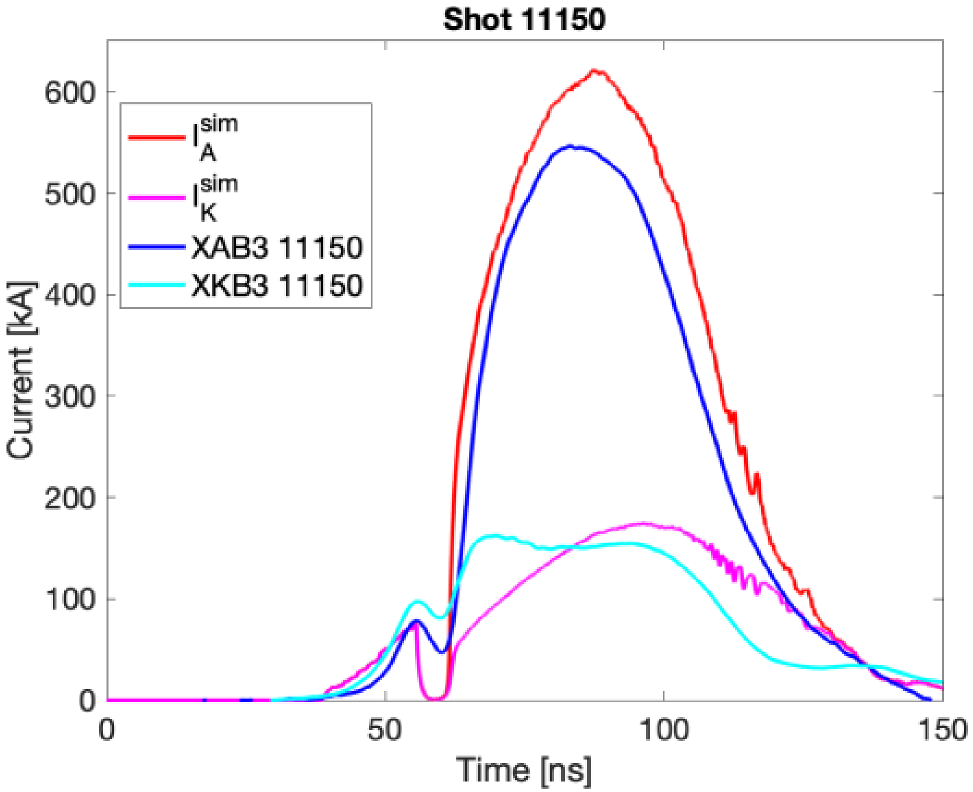


Electromagnetic PIC simulations of the HERMES3 Conical MITL and Diode. Electron emission described by **Space charge limited (SCL) emission.**

Transmission line coupling provides the drive





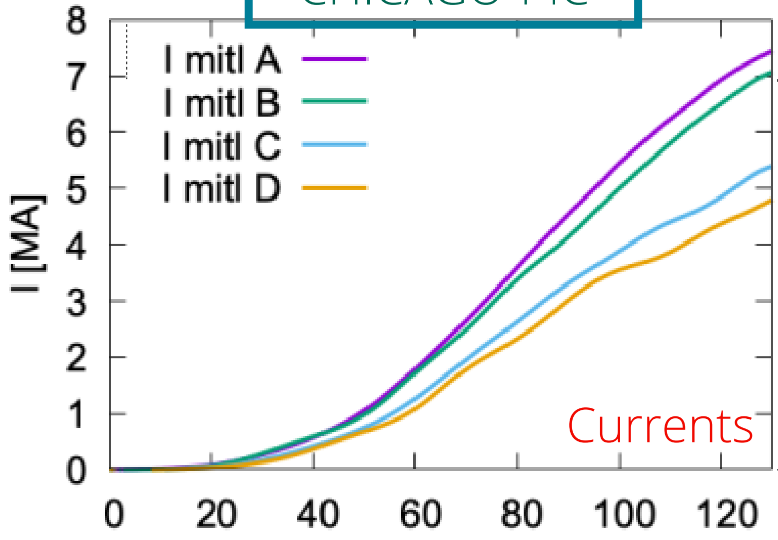


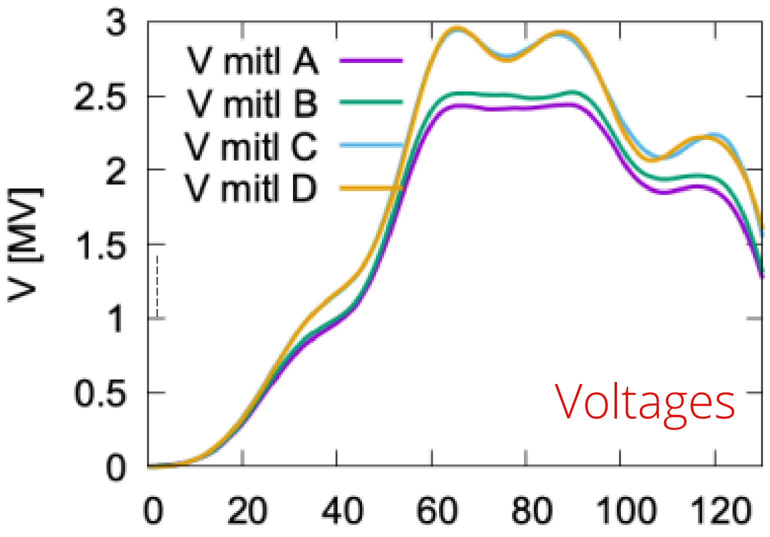
Plasma sheath formation and insulation captured by EMPIC

Credit: D. Sirajuddin

Cross Code Comparisons: EMPIRE and CHICAGO

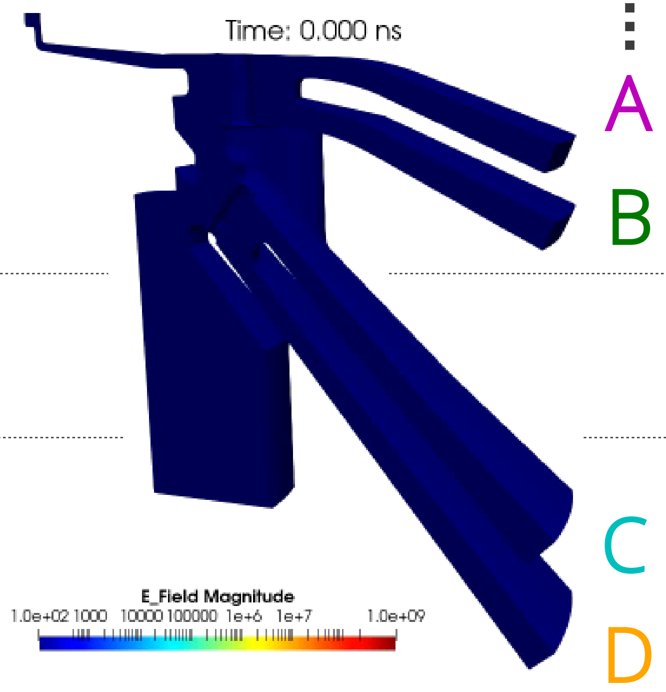
CHICAGO-PIC





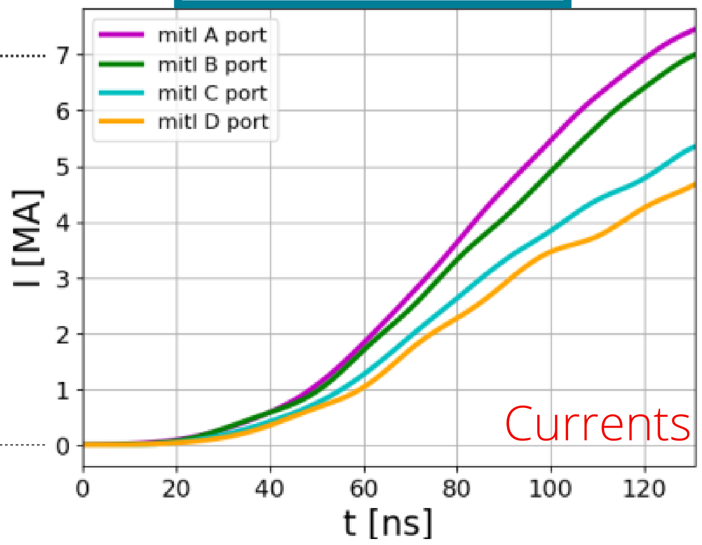
7 MA

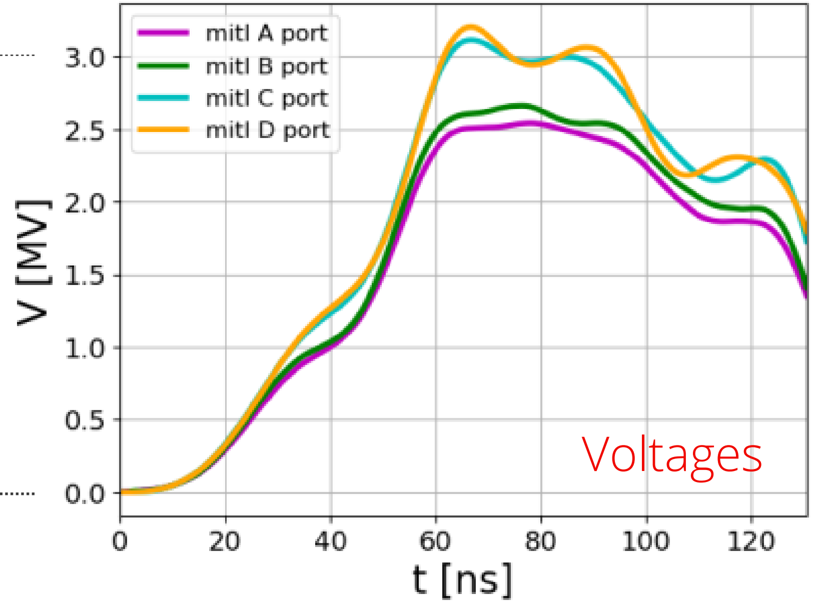
Diagnostics measured at ports:



0 MV

EMPIRE-PIC

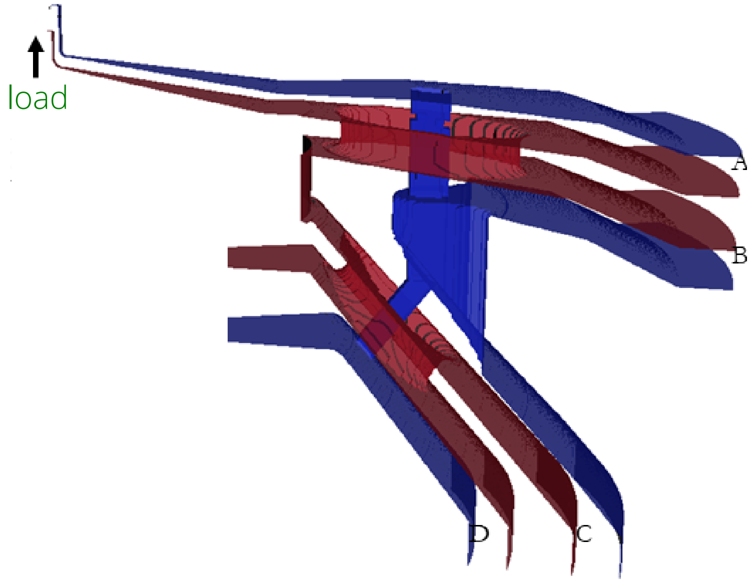


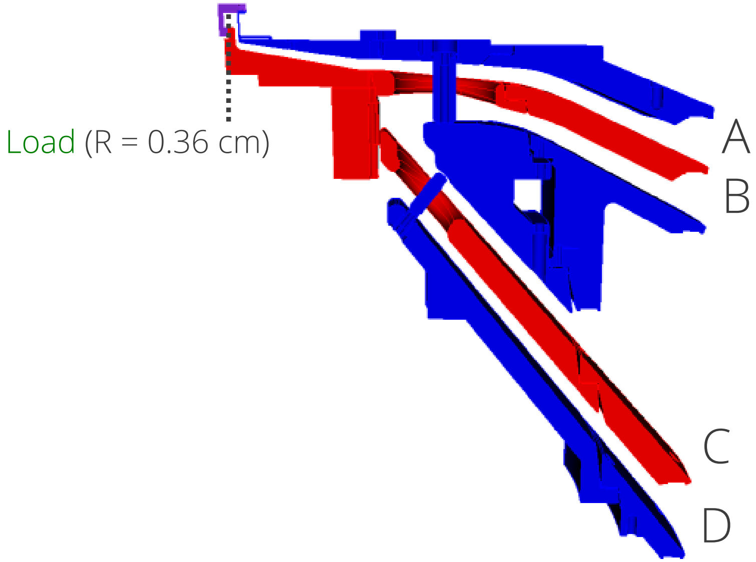


N. Bennett, D. R. Welch, C. A. Jennings, E. Yu, M. H. Hess, B. T. Hutsel, G. Laity, J. K. Moore, D. V. Rose, K. Peterson, and M. E. Cuneo Phys. Rev. Accel. Beams 22, 120401

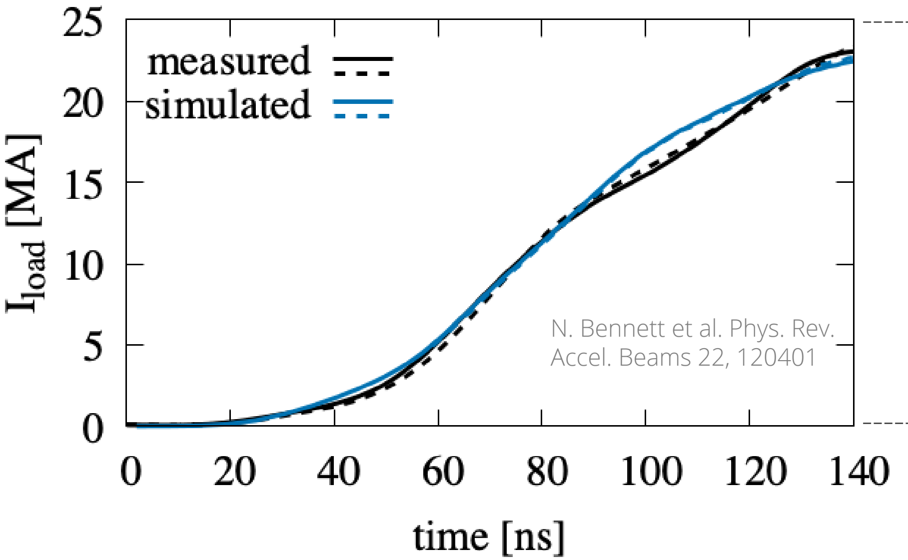
Credit: D. Sirajuddin

Initial Validation: Z Machine Powerflow 18A

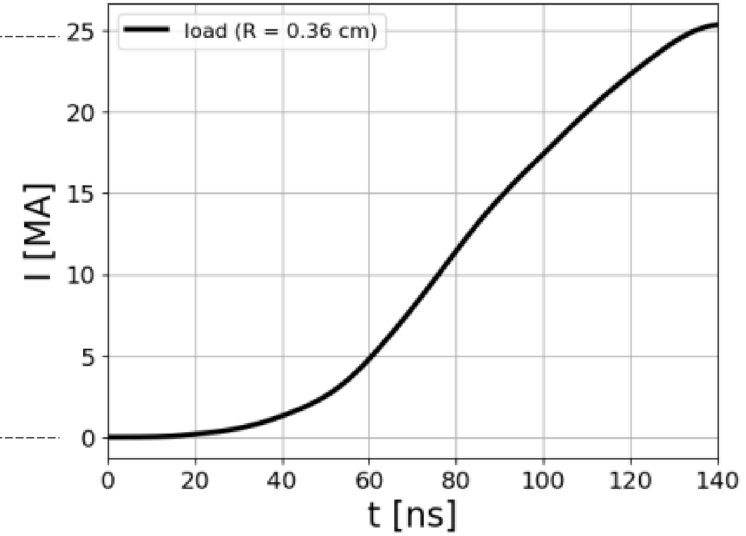




CHICAGO-PIC: full physics



EMPIRE-PIC: EM+SCL electrons



Closure

Closure

We have developed a method a multiscale method for solving electromagnetics for powerflow simulations.

In addition to pulsed power need to simulate more general transmission lines. Spin off capability **EMPIRE-Cable.**

Completed Extensions of this work
- TM Mode coupling
- Mode diagnostics
- Multi-Conductor TLs
- Simple branching
- Cross-section parameterization

EMPIRE-Cable (FY22 goals)
Standalone and coupled TL capability
- General networks
- Circuit Coupling

Research directions and outyears
- Other EM coupling modalities
- TE Modes
- Non-linear TL models (i.e. MITL models)
- Outflow conditions for EM
- Adjoints for coupled problems

Tons of V&V

References

[1] D. A. McGregor, et al. *Variational, stable, and self-consistent coupling of 3D electromagnetics to 1D transmission lines in the time domain*, J. Comput. Phys, (2021, accepted)

[2] M. T. Bettencourt, et al. *EMPIRE-PIC: A Performance Portable Unstructured Particle-in-Cell Code.* Comm. in Comput. Phys. *30* (4). 1232-1268. (Aug. 2021)

backups

Example Analysis

Saturn Pulse Regularization

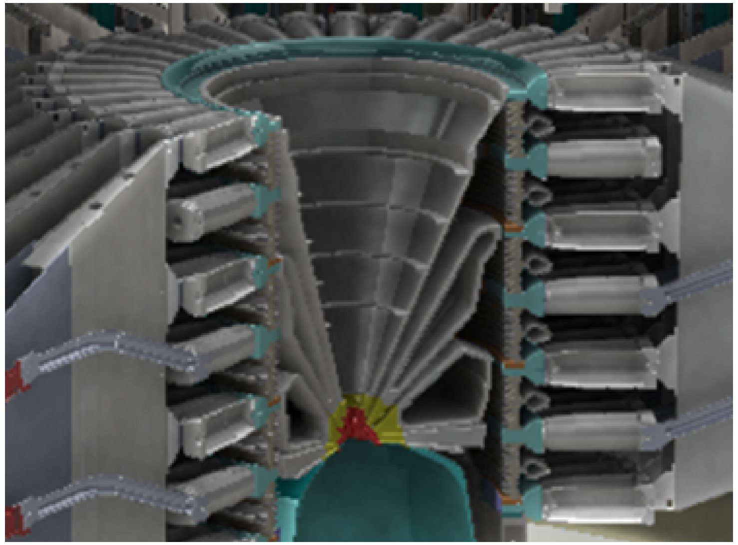
- Saturn's water convolute may introduce jitter to the EM Drive

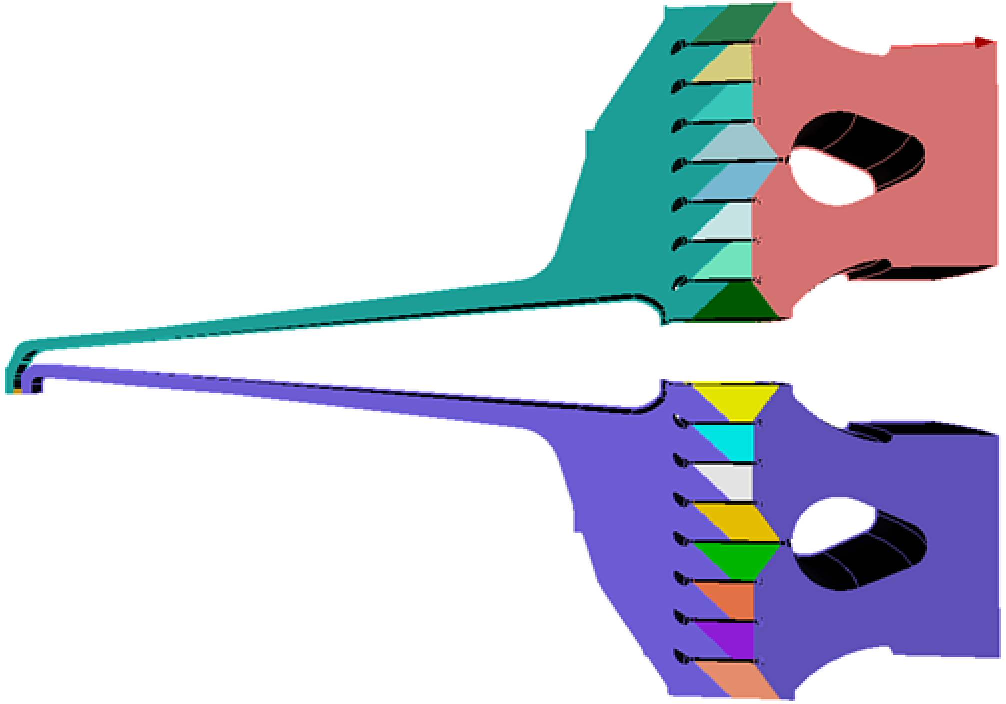
Assuming a cylindrical cross-section

$$\omega^2 - \omega^2_{\mathrm{TM},n,m} = c^2 k^2$$

$$\omega_{\mathrm{TM},n,m} = c\sqrt{\left(\frac{2\pi n}{|AK|}\right)^2 + \left(\frac{m}{r}\right)^2}$$

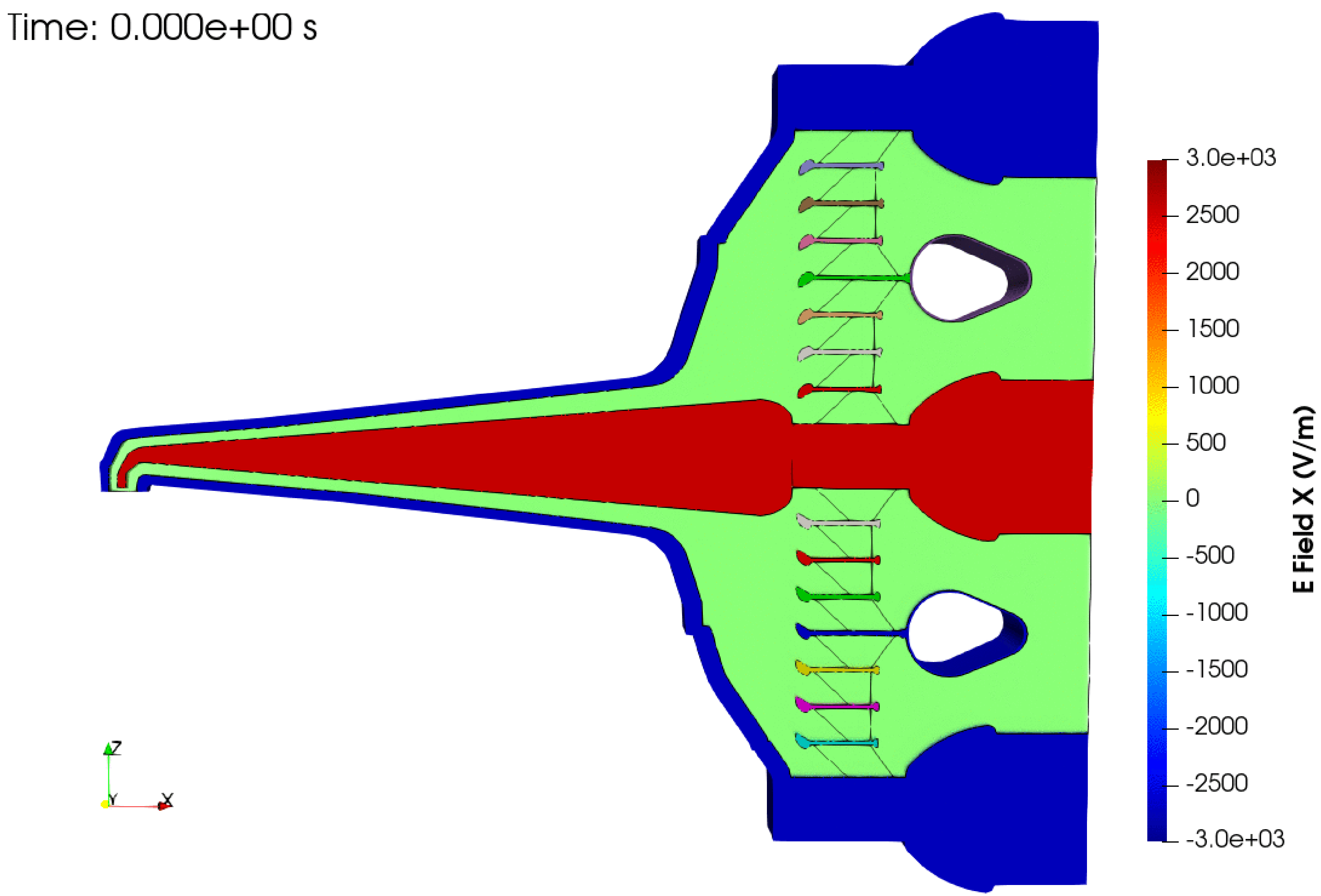
- Decrease in AK gap should regularize z directed jitter
- Radial convergence should regularize θ directed jitter
- Hypothesis: energy in TM modes should decrease as r->0





Credit: D. Sirajuddin

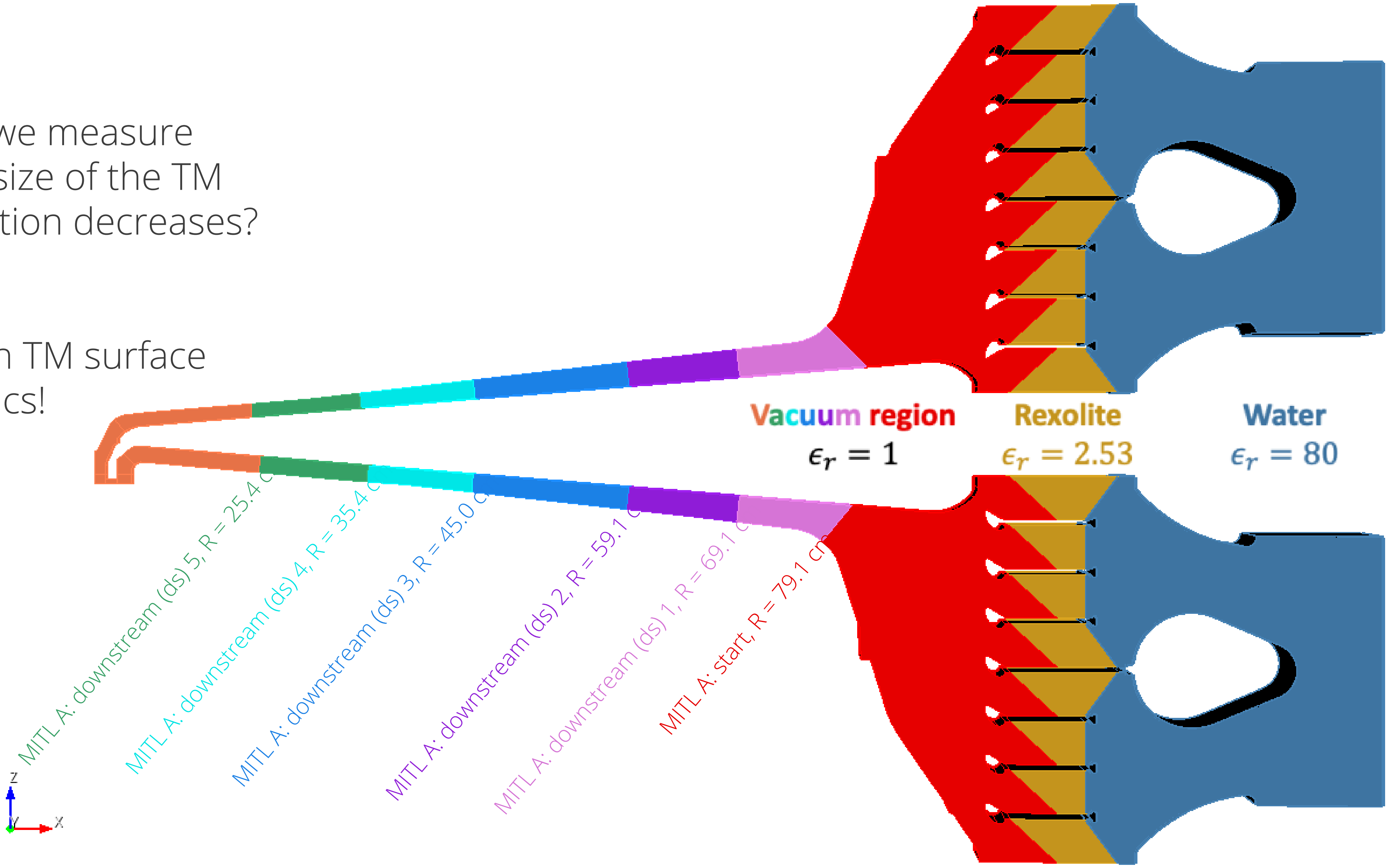
Saturn Pulse Regularization



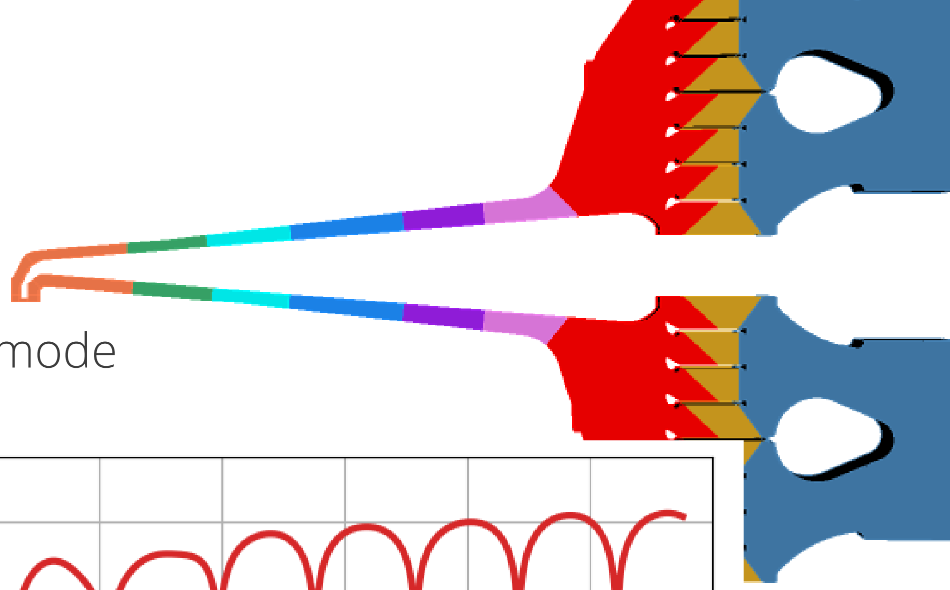
Saturn Pulse Regularization

How do we measure that the size of the TM perturbation decreases?

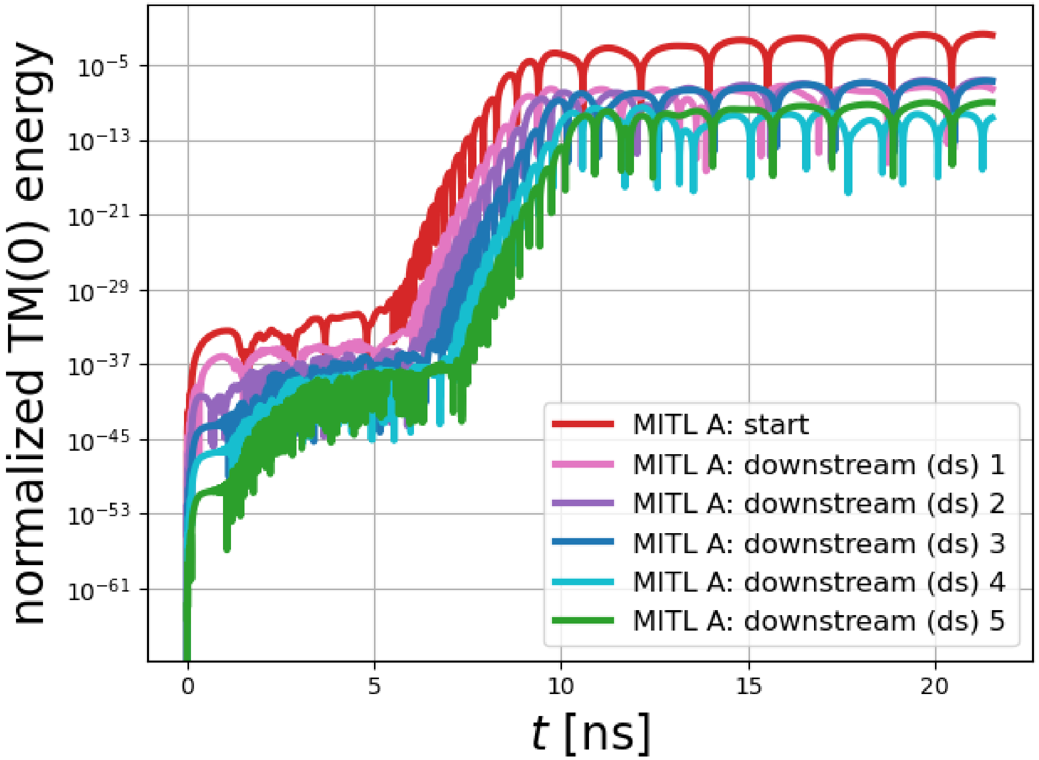
Approach TM surface diagnostics!

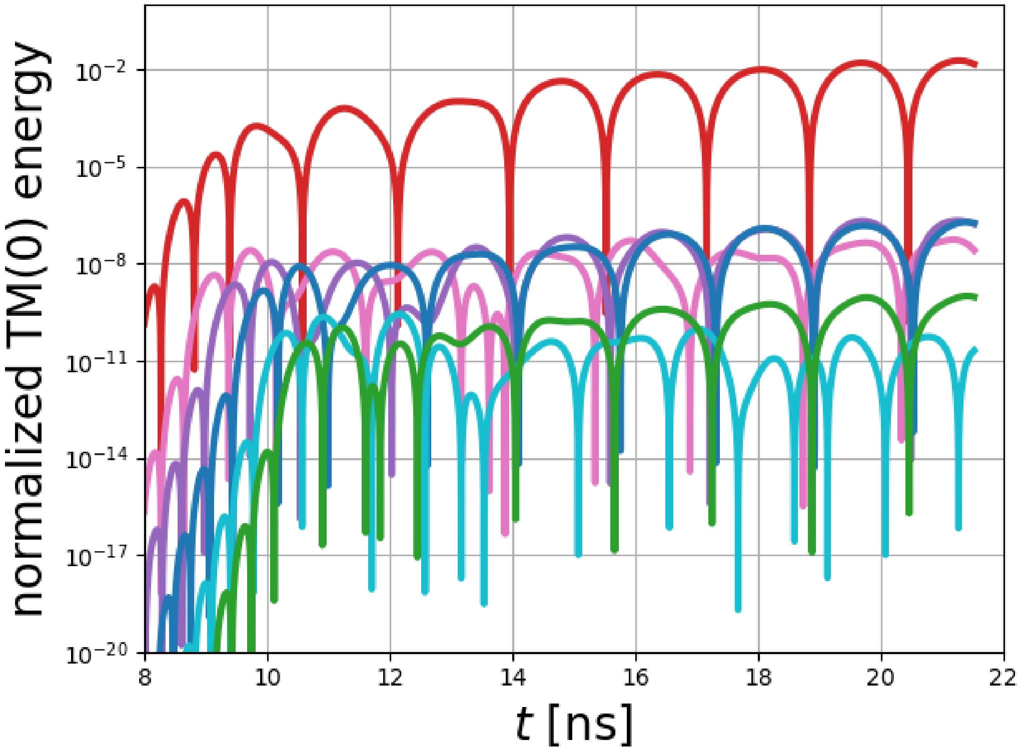


Saturn Pulse Regularization



Electric Energy per unit length normalized by drive energy for the mode



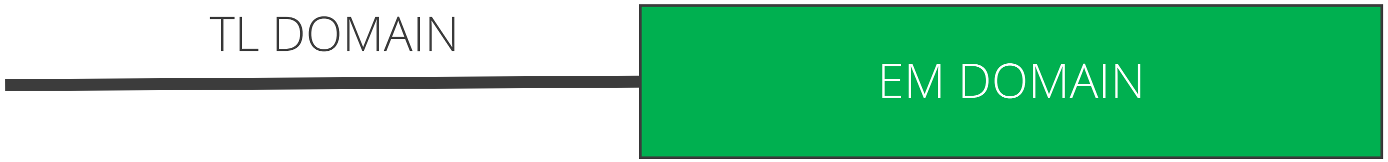


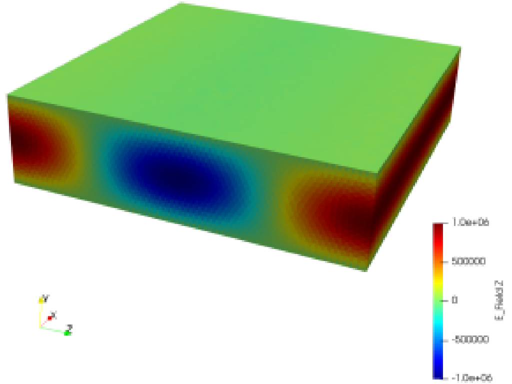
We observe energy content of the TM wave perturbation decreases along the length of the vacuum mitls

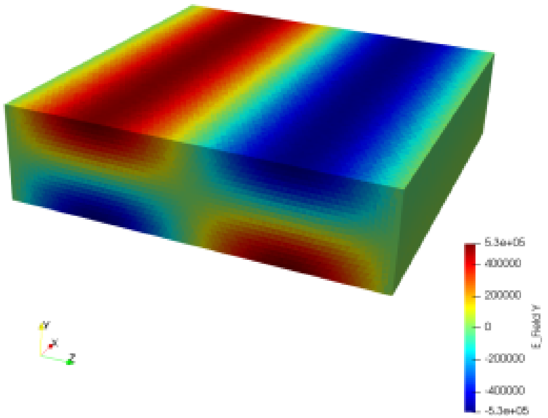
Credit: DM, D. Sirajuddin

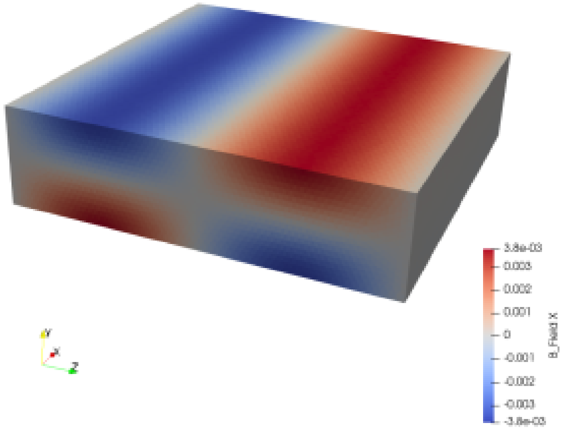
Verification: TM Wave

3D TM Wave on a TL-EM domain









$$E_x = 0$$

$$E_y = \left(\frac{k_z^{[m]}}{k_c^{[m]}} E_{0,z}^{[m]}\right) \cos\left(k_c^{[m]} y\right) \sin(\omega t - k_z^{[m]} z)$$

$$E_z = E_{0,z}^{[m]} \sin\left(k_c^{[m]} y\right) \cos(\omega t - k_z^{[m]} z)$$

$$H_x = -\left(\frac{\omega\epsilon}{k_c^{[m]}} E_{0,z}^{[m]}\right) \cos\left(k_c^{[m]} y\right) \sin(\omega t - k_z^{[m]} z)$$

$$H_y = 0$$

$$H_z = 0$$

$$k_z^{[m]} \approx 2k_c^{[m]}$$

Credit: DM, D. Sirajuddin

Verification: TM Wave

Theoretical Order of Accuracy (Simplex Mesh)

$$\mathcal{O}(\underbrace{\Delta n^2}_{\text{TL}} + \underbrace{h}_{\text{EM}} + \underbrace{k_c^2 h}_{\text{eigen}} + \underbrace{\Delta t^2}_{\text{Time}})$$

Error denotes a relative L^2 error for functions.

cell size [m]	PPP z	PPP c	E_z error	E_z rate	E_y error	E_y rate	B_x error	B_x rate
1.00E-03	16.2	8.5	3.15E-02	–	5.96E-02	–	4.73E-02	–
6.67E-04	24.3	12.7	1.41E-02	1.98	2.88E-02	1.79	2.58E-02	1.5
5.00E-04	32.5	16.9	9.86E-03	1.24	1.93E-02	1.4	1.85E-02	1.16
4.00E-04	40.6	21.1	7.50E-03	1.23	1.46E-02	1.24	1.43E-02	1.14

Rates are computed using a pairwise fit.

$$\text{PPP} = \frac{2\pi}{kh}$$