SAND2021-15000C

Velocity Measurements Using Magnetic Pickups

Anders Hansen, John Griffin, Adam Moya
Sandia National Laboratory, Albuquerque, NM, 87123

A method for measuring the velocity of a passing object by recording the voltage induced by a magnet fixed to that object is explored in this article. Both theory and applications are discussed along with the strengths and limitations of the technique.

I. Introduction

Any linear trajectory, such as a rocket sled on a track, can be described simply by measuring the times at which an object crosses discrete points along its path. With groups of measurements placed in close proximity of known separation distances, a set of positional markers can be used to calculate local velocity estimates. In the case of rocket sleds, a traditional method for acquiring these measurements is to use thin wires that trigger a voltage rise or fall when broken. The wire is placed in tension across the track such that it is severed as the sled passes through. Instrumentation connected to the wire records the time at which the wire continuity is broken, thus acquiring the time at which the sled crosses a known position along the track. This method is reliable due to its simplicity and robustness to environmental and test conditions typical of rocket sled tests (e.g. wind, dirt, shock waves, etc.), making a false trigger extremely unlikely. The disadvantage of this method is that for each installment of a break wire only one passage time can be recorded, making it a cumbersome method for acquiring a large quantity of data points and an impossible method for measuring multiple sleds or objects. One obvious alternative to break wires that could be used to record multiple crossing events is a photo gate which would be triggered by the breaking of a beam of light rather than a wire; however, this method is vulnerable to false triggers and maintenance issues caused by dusty conditions.

For these reasons, an alternative method of detection is devised using magnetic fields. This method involves a magnet placed on a rocket sled accompanied by a pickup coil placed along the track such that as the rocket sled crosses a predefined point along the track, the magnet will cross over the pickup coil inducing a voltage that can be recorded in time by instrumentation. Most of the dust and sand present during a rocket sled test is magnetically neutral and does not have a significant effect on magnetic fields present making a false trigger unlikely. Unlike break wires, magnetic pickup coils are not destroyed by the process of recording data and therefore may be used multiple times to track multiple bodies or multiple magnets on a single body during a test. While using the same number of channels as a break wire method, this method was able to collect 6 times more data by simply installing 6 magnets divided between two sleds; however, this is not a limit and the data gathering capacity could be greatly increased beyond this. Finally, placing a magnet on a rocket sled does not add any active power sources to the rocket sled which is a favorable condition for explosive safety considerations.

II. Theory of operation

Magnetic fields follow Faraday's law which is defined as [2]

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \int_s \vec{B} \cdot \hat{n} da \tag{1}$$

relating the electromotive force of the electric field $\vec{E}$ to the rate of change of the magnetic flux, where $\vec{B}$ is the magnetic field. For the method described in this paper, a pickup coil is used to measure the electric field. The left hand side of the equation is evaluated by integrating around the lengths of the coil windings, resulting in an equivalent Voltage V that can be directly measured across the two leads of the pickup coil. The surface integral on the right side of the equation is an integration of the magnetic field that goes through the area of the pickup coil. This term is referred to as magnetic flux, denoted as Φ, and given by

$$\Phi = \int_s \vec{B} \cdot \hat{n} da\,. \tag{2}$$

Equation (1) can then be reduced to

$$V = -\frac{d\Phi}{dt}\,. \tag{3}$$

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

If the source of a magnetic field is a permanent magnet, than there is no explicit time dependence of the magnetic field, and the total derivative of the magnetic flux is solely determined by the change in position of the magnetic field relative to the area of the pickup coil.

For the measurements proposed, the pickup coil is stationary and the magnet is fixed to a sled confined to a linear track (no relative rotation), allowing only for translation in the system. Utilizing both features, the chain rule applied to the right side of the above equation results in the following:[3]

$$V = -(\vec{v} \cdot \nabla)\,\Phi\,. \tag{4}$$

The velocity vector $\vec{v}$ represents the velocity of a permanent magnet relative to a stationary pickup coil, and the resulting voltage change across the coil is given from the motion of a spatially varying magnetic flux. In all positions, the orientation of the magnet is kept constant relative to the pickup coil.

Further understanding of the proposed method is aided by Figure 1, in which the coil and magnet are defined on a Cartesian coordinate system. The pickup coil is placed at the origin of the coordinate system and oriented such that its area normal vector is aligned with the z axis. The cylindrical magnets center is placed at a height h above the x axis with the major axis aligned with the z axis. The motion of the magnet is confined to the line parallel to the x axis separated by a distance h, and the velocity v_x is assumed to be approximately constant. In this scenario, equation (4) reduces to

$$V = -v_x \frac{\partial \Phi}{\partial x}, \tag{5}$$

where $\frac{\partial \Phi}{\partial x}$ is the partial derivative of the magnetic flux with respect to the direction of the relative motion as the magnet is moved in the x direction. The value for $\frac{\partial \Phi}{\partial x}$ is calculated by performing a numerical derivative of $\Phi(x)$ over an array of values x in the locations of interest.

Equation (5) is the fundamental equation after applying the foundational assumptions for the proposed measurement method. To solve for the magnetic flux, the magnetic field $\vec{B}$ is first determined from the curl of the magnetic vector potential $\vec{A}$, or

$$\vec{B} = \nabla \times \vec{A}\,. \tag{6}$$

This equation is substituted into the right integrand of equation (2), and then Stokes theorem is applied to relate the surface integral of the curl to the line integral of the contained $\vec{A}$ field [3]:

$$\Phi = \int_s \left(\nabla \times \vec{A}\right) \cdot \hat{n} da = \oint \vec{A} \cdot dl\ . \tag{7}$$

The nature of $\vec{A}$ depends on the assumed shape of the permanent magnet. Different shapes of magnets produce slightly different magnetic fields, all of which closely approximate that of a perfect dipole. The cylindrical bar magnet uniformly polarized along its major axis in the z direction is used for this paper. Here, $\vec{A}$ is the magnetic vector potential, in this case generated by the bound current of the magnet, and the line integral is evaluated along the windings of the pickup coil. The general equation for magnetic vector potential is defined by [2]

$$\vec{A} = \frac{\mu_o}{4\pi} \int \frac{\vec{j}(r')}{|r - r'|} d\tau \tag{8}$$

where $\vec{j}$ is the volume current originating from the bound current of the magnet, τ is a volume element, r is the observation vector, and r' is the distribution vector, and μ_0 is the permeability of free space. For the case of a uniformly polarized cylinder, the only current is surface current current present is around the curved surface of the magnet and the volume current $\vec{j}$ reduces to a surface current $\vec{K}$, which is given by the following equation [2]:

$$\vec{K} = \vec{M} \times \hat{n}\,. \tag{9}$$

Here, $\vec{M}$ is the magnetization of the bar magnet which is pointed in the z direction and $\hat{n}$ is the unit vector normal to the surface of the bar magnet. Because of this, the only location where surface current exists (non-zero) is around the curved surface of the cylinder in the $\vec{\phi}$ direction. Note that this is only true for a uniformly polarized magnet where $\nabla \times \vec{M} = \vec{0}$. Otherwise there may be additional currents that must be considered $\vec{j} = \nabla \times \vec{M}$. In practice, residual flux

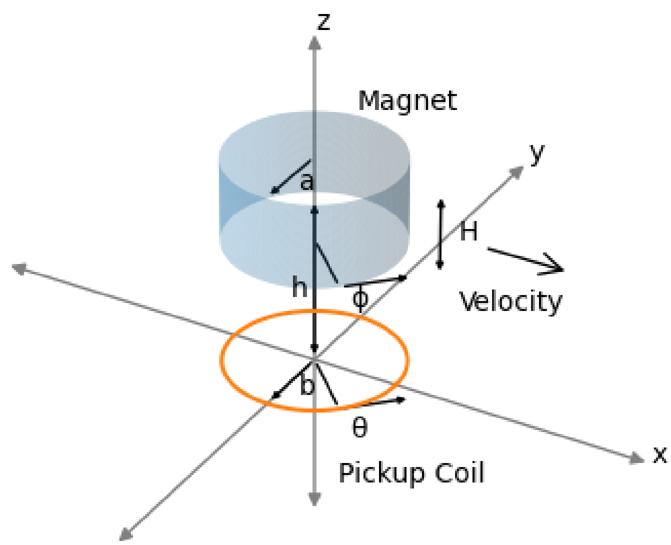

Fig. 1 This is a diagram of the arrangement of the magnet and pickup coil used to set up the integral for finding the magnetic flux. This configuration assumes a single turn pickup coil. If more turns are required than the results can be multiplied by a factor N. For this test, the pickup coils were made out of multiple concentric circles so the total flux was calculated by integrating the flux of each circle and the summing the results together.

density, denoted as B_r, is supplied by magnet vendors [4] as opposed to magnetization. The two are related by the equation $\vec{B_r} = \mu_o \vec{M}$ [5]. The general equation for magnetic vector potential then reduces to [2].

$$\vec{A} = \frac{\mu_o}{4\pi} \int \frac{\vec{K}(r')}{|r - r'|} dS \tag{10}$$

which is the same form as Equation 8 but replacing the volume current $\vec{j}$ with the surface current $\vec{K}$, and the volume element τ with the surface element S taken over the curved surface of the magnet.

Now, $\Phi(x)$ can be solved numerically with the following integral. The variables z' and ϕ are integrated over for obtaining the magnetic vector potential and the variable θ is integrated over for calculating the magnetic flux into the pickup coil from the magnetic vector potential.

$$\Phi(x) = \frac{B_r}{4\pi} \int_0^{2\pi} \int_0^{2\pi} \int_{-\frac{1}{2}H+h}^{\frac{1}{2}H+h} \frac{b * \sin(\theta)\sin(\phi) + b * \cos(\theta)\cos(\phi)}{\sqrt{(a * \cos(\phi) - b * \cos(\theta) + x)^2 (a * \sin(\phi) - b * \sin(\theta))^2 + (z')^2}} dz' d\phi d\theta \tag{11}$$

Here x is the position of the magnet relative to the coil on a line parallel the x axis a distance h above the origin, a is the radius of the magnet, H is the height of the magnet, and b is the radius of the circular pickup coil. An outline of code used for how to numerically solving this integral is shown in the Appendix.

For coils with multiple turns, the results of this integral can be multiplied by the number of turns present in the coil. For this paper, the pickup coil is multiple concentric circles on a printed circuit board. To calculate flux at any given position, this integral is performed around each circle and summed for all circles for each position. Once $\Phi(x)$ is calculated, $\frac{d\Phi}{dx}$ can easily be calculated. Because of the relatively small range in which the pickup coil is able to pick up any flux compared to the length of a track, it is safe to assume that the velocity does not change substantially during the time the rocket sleds path across the pickup and for now we can approximate for this time the velocity is constant, then it will be very easy to convert an array of $\frac{d\phi}{dx}(x)$ values into $\frac{d\phi}{dx}(x(t))$ with the substitution$x = v * t$, this can be used to compare to the $V(t)$ signal that will be recorded on an oscilloscope which should be equal to equation 5.

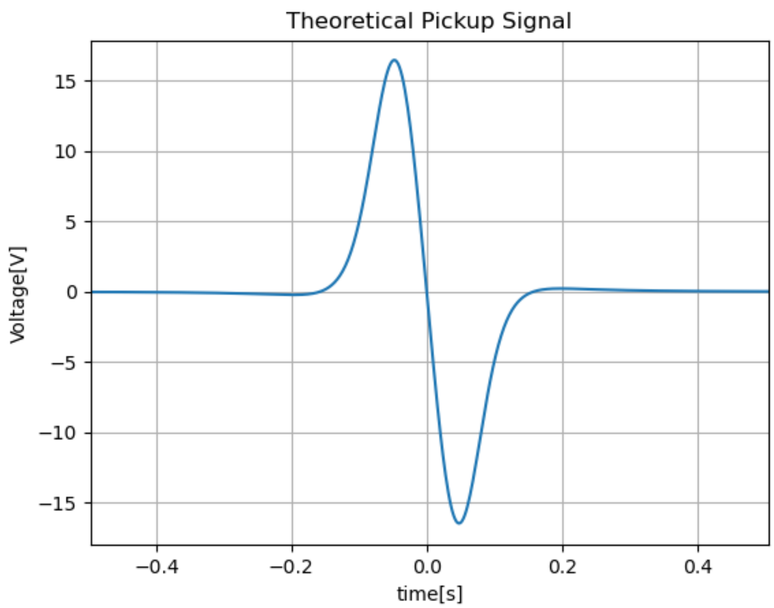

Fig. 2 A theoretical plot of the magnetic pickup voltage generated from the integral derived in this paper. The integral was taken at each point x along the path of the magnet and then differentiated to find the $\frac{d\Phi}{dx}$ then an assumed constant velocity of $x = vt$ was used to map x to t.

III. Measurements

The zero crossing point is the key feature utilized for the passage measurements. This zero crossing occurs when the amount of flux passing through the pickup coil is at its peak. For a symmetrical magnet and coil, this event occurs when the magnet is centered over the pickup coil. This feature makes the precise configuration of the magnet and pickup coil less important, only requiring that they both have symmetry across their center axes and peak magnetic flux at their center position. In addition, in this application, operational amplifiers are used to buffer the pickup coils. Because the amplitude varies significantly with spacing and velocity, the operational amplifier can clip the higher amplitude signals. Because the point of interest is at zero, a signal clipped on the upper bounds will not lose any of its usability.

It may be tempting to use the linear dependence of velocity on peak amplitude; however, small variations in magnet displacement affect the peak value significantly and make this an inaccurate method of measurement. Trajectory measurements are made possible by measuring the time between multiple zero crossing events. This is accomplished by placing coils next to each other with a known distance. Measuring the time between two crossing points along with their distance can be used to compute an average velocity measurement. The coil spacing can be made close enough so that the average velocity measurement closely approximates an instantaneous velocity measurement.

The time between the two zero crossings can be computed by post processing previously recorded analog signals or can be done in real time with hardware. The advantage of a hardware solution is it that it requires recording less data and can supply instantaneous results for systems that require state feedback. The disadvantages are that there is less ability to compensate for errors, such as null offsets in analog electrical components (e.g. operational amplifiers and comparators).[1]

An example of a hardware trigger circuit is shown in Figure 4. The circuits must be triggered by the zero crossing event shown above, but not by noise while the signal is resting at 0 V before the event. This is solved by using a circuit that is armed by the positive voltage of the first part of the signal before the zero crossing and then triggered at the zero

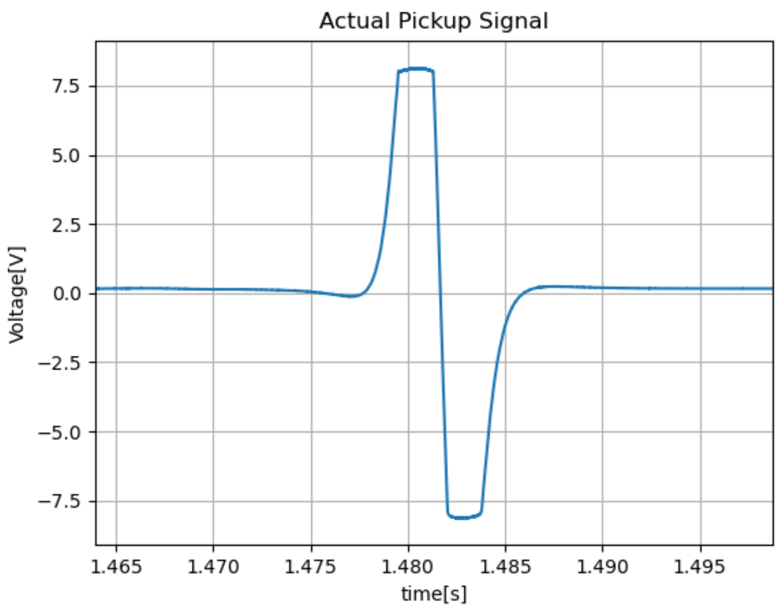

Fig. 3 An actual pickup signal taken in the field. The top amplitudes are clipped, however there is no loss in usability because the only point in question is the zero crossing point which is unaffected by amplitude shifts.

crossing point. In the figure, U1 is triggered when the pickup signal reaches a threshold determined by R1 and R2. When U1's output goes high, it charges C1, holding pin 1 of U3 high. Pin 1 of U3 is held high for the duration that it takes the charge stored on C1 to drain through R2. When the signal goes down and crosses 0 Volts, U2 goes high. At this point both pin 1 and pin 2 of U2 are high, and its output goes high for as long as the signal is negative or the charge of C1 drains to the point where pin 1 of U3 is no longer high. The same process occurs for trigger circuit #2 except it occurs after trigger circuit #1 when the magnet crosses the second pickup coil. When trigger circuit #1 outputs high it sets the flip-flop latch causing its output to go high, and when the trigger circuit #2 goes high it resets the flip-flop latch and makes its output go low. The additional "AND" gate U4 and inverter U9 are there so that if trigger circuit #1 is still high when trigger circuit #2 goes high the flip-flop latch will still go low and not be put into an indeterminate state where both set and reset are high. All the additional components in this circuit contribute to offsets which cause error in the velocity measurement. This is a rough sketch of how circuitry could be used to measure the zero crossing point, more optimized methods could be made, however in every case some sort of comparison of analog voltage levels would be required which would result in some additional error.

For rocket sled tests, the need for measurement accuracy typically outweighs the cost of additional channels and the entire analog output of the pickup coil is recorded for the duration of the test. This added cost takes out additional error caused by the null offset of the buffer amplifiers used on the pickup coils as well as the offsets in the comparators used to trigger the pulse. Furthermore, interpolation can be used for finding zero crossing points that occur between sampling, making it an extremely precise method of velocity and position in time recording.

When coil pairs or triplets are used for velocity or acceleration measurements, they are placed on a single printed circuit board. This gives the advantage that coil spacing and dimensions are precise, limited by the precision and accuracy of the printed circuit board manufacturer.

IV. Error Analysis

The accuracy of all trajectory measurements depends on how accurately position and time are measured. The position of measure is when the magnetic flux through a pickup coil is maximum, which is in turn the point when the derivative of the magnetic flux is zero and the velocity dependence of the signal no longer matters. At this point V = 0 will be recorded. In practice V will have error in it generated from signal noise, imperfections in the data recorder, offset generated by the amplifier etc. Grouping these terms together the zero crossing term can be expanded as the following.

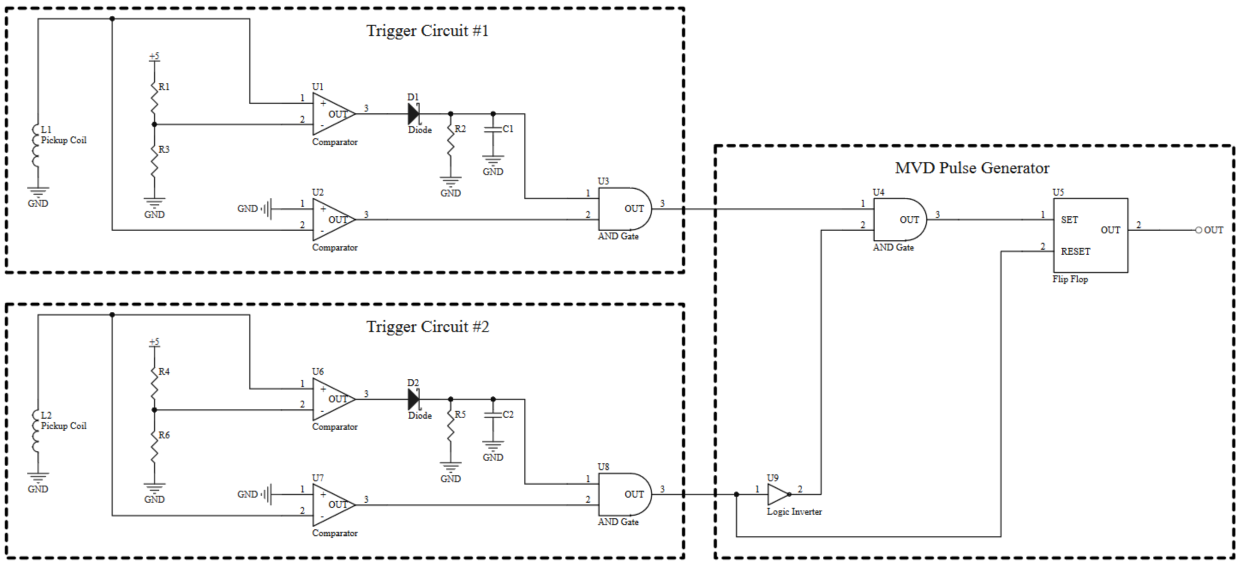

Fig. 4 This is a simplified schematic of circuit meant to measure the time it take for a magnet to cross two pickup coils. As the magnet crosses the first coil the signal goes high and as the magnet cross the second coil the signal goes low. The duration of this signal along with knowledge of the spacing between the two coils will allow for a velocity measurement to be performed. This method reduces the need for entire analog channels to be recorded however lacks some of the necessary values to perform error analysis.

$$0 = V\left(t_{recorded}\right) + \delta V \tag{12}$$

Where, V is the true voltage signal, δV is all of the voltage error in the system, and $t_{recorded}$ is the time the zero crossing is recorded to have occurred. The term $t_{recorded}$ can be expanded as the following.

$$t_{recorded} = t_{cross} + \delta t \tag{13}$$

Where t_{cross} is the true time that the crossing event occurred and δt is the error in time created by the voltage error. Assuming δt is small, $V(t)$ can be Taylor expanded about t_{cross}. Substituting this back in results in the following equation.

$$0 = \frac{dV}{dt} * \delta t + \delta V \tag{14}$$

$$\delta t = -\frac{\delta V}{\left(\frac{dV}{dt}\right)} \tag{15}$$

The error in time is simply the error in voltage divided by the slope of the signal as the zero crossing occurs. The positions of where the pickup coils are placed along the track as well as where the magnets are placed on the rocket sled will have some error. Additionally, imperfections in the magnets and the coils may add an offset to precise position where a zero crossing occurs. All these terms will be combined into an error δp for each position tracked along the rocket sled trajectory. Combining the two over sources of error the total error for the trajectory $x(t)$ can be obtain from the following equation.

$$\delta x = \sqrt{\left(\frac{dx}{dt}\delta t\right)^2 + \delta p^2} \tag{16}$$

In the above equation $\frac{dx}{dt}$ is taken at the time x is recorded. For this method, velocity measurements can be made extremely accurately because they have the additional benefit of the coil spacing being determined by the precision of the printed circuit board manufacturer rather than a measurement taken by hand in the field. velocity is approximated using the equation, $v = \frac{\Delta x}{\Delta t}$ where, $\Delta t = t_{finnal} - t_{initial}$ and $\Delta x = x_{finnal} - x_{initial}$. Feeding the above equations through the propagation of error equation[3] results in the following equation.

$$\delta\Delta t = \sqrt{\left(\frac{\delta V}{\frac{dV}{dt}}\right)^2_{finnal} + \left(\frac{\delta V}{\frac{dV}{dt}}\right)^2_{initial}} \tag{17}$$

Where the two errors in time are calculated the same way they were for trajectory. For calculating the error in Δx the following equation is used.

$$\delta\Delta x = \sqrt{\delta p^2_{finnal} + \delta p^2_{initial}} \tag{18}$$

In the case the coil positioning error is used rather than the trajectory positioning error because the trajectory position accounts for the time error contribution the absolute position of the rocket sled. This contribution is already included in the measurement of $\delta\Delta t$ so using the trajectory position error would over count that error. Combining the above equations, again using the propagation of error equation results in the following, for error in recorded velocity.

$$\delta v = \sqrt{\left(\frac{\delta\Delta x}{\Delta t}\right)^2 + \left(\frac{\Delta x}{(\Delta t)^2}\sqrt{2}\frac{\delta V}{\left(\frac{dV}{dt}\right)}\right)^2} \tag{19}$$

Error from $\delta\Delta t$ results in random error while error in $\delta\Delta x$ will result in a systematic error that is tied to the spacing of the pickup coils. With the opportunity to calibrate the coil spacing with a know velocity it is possible to come put with an effective Δx that will get rid of this uncertainty. In addition, for tests involving multiple measurements on a single coil pair, each measurement taken on a specific pair will have the same error which may provide additional options for data analysis. The analog signals recorded on the coils can be used to calculate the relative error of the velocity recorded. From the analog signal it is possible to obtain the noise δV and the slope $\frac{dV}{dt}$ at the point of zero crossing.

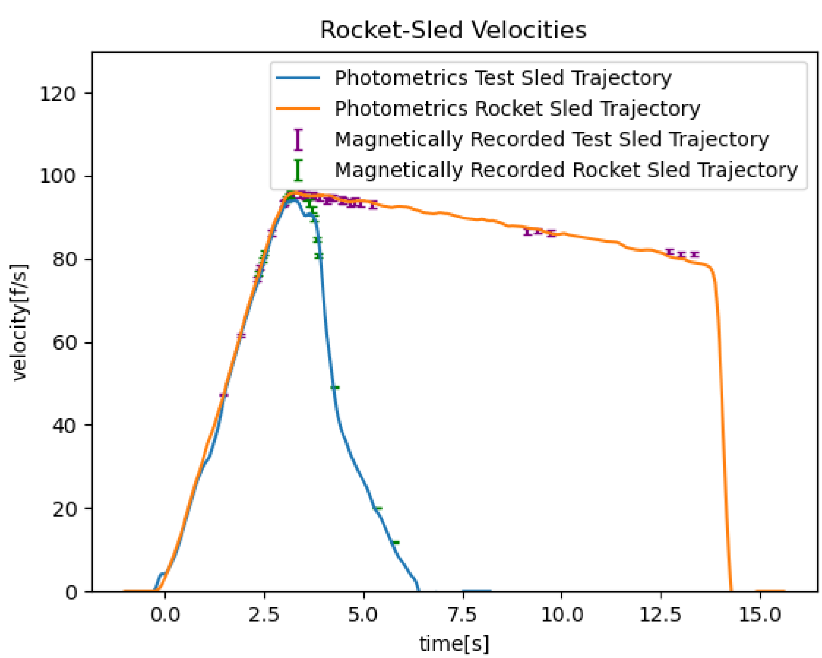

Fig. 5 This is a plot of the final trajectory data comparing the magnetically recorded velocities to photometric data.

V. Conclusion

This method was used during an outdoor rocket sled test where it was validated against conventional means of rocket sled trajectory measurements, including tracking from high speed video and break wire measurements. The test configuration utilized 10 magnetic coil pair circuits using circuitry similar to the example shown above, output a high signal as the magnet crosses between the two pickup coils. The test included two sleds: a test sled that was accelerated to speed by a separate rocket sled. At a specified point along the track, the rocket sled separated from the test sled and came to a stop, while the test sled continued to move forward until it collided with a target. To analyze the separate test sled and rocket sled trajectories separately, their measurements are separated after the separation event. With that, two independent trajectories are able be plotted, shown in Figure 5.

The data collected by these measurement devices have proven the method to be reliable and versatile for rocket sled tests. For this test digital duration pulses were used for measuring the crossing time of the magnet. This was originally used so that the signals would only require one bit of data to be recorded in time compared to an entire analog. For rocket sled tests where error analysis is of critical importance this is pointless because the analog signal must be recorded anyways for error analysis. In addition, a big contribution to the δV error is caused by the null off set in the amplifier. When recording the total analog signal, it is possible to remove this contribution by normalizing it out of the signal. For future rocket sled tests where data values are most critical it is advisable to record the full analog signals of both the pickup coils on a coil pair and performed data analysis in post processing. The digital method of recording still has its merits for applications where data accuracy is not as critical, and the error calculations are not necessary. This test focused on velocity measurements; however, similar methods could be employed for acceleration measurements. In addition, because most of the error had to do with $\delta\Delta x$ this error could be improved by placing the coils farther apart which would make Δt larger. This in combination with normalizing the null offset of the amplifiers should be able to greatly improve the accuracy of this measurement method for future tests.

Appendix

Here is an example of the python code used to the calculate the signal the pickup coils will receive when a magnet crosses them. For ease of computation the integrals were taken form the reference point of the magnet rather than the pickup coil but will result in the same values after integration. This is a computationally intensive process and may take a few minutes to run on a normal computer

```
import numpy as np
from scipy.integrate import nquad #used for tripple integral
import matplotlib.pyplot as plt   #used for plotting
from tqdm import tqdm             #used for creating status bar
import numba                      #precompiles function to make program faster


#paramaters##################################################################
b1=.0196          #coil radius max m
b2=.00127         #coil radius min m
n=30              #number of turns assuming evenly spaced concentric circles
a=.0254           #magnet radius m
H=.01905          #magnet height m
v = .5            #velocity m/s
length = 1        #length of track m
pn = 1000         #number of positions
h=-(.0254+.5*H)  #center separation
R=1.48            #magnetic remanance used to calculate surface current
gain=21           #ampllifier gain of pickup coil
############################################################################

b3=np.linspace(b2,b1,n)
position=np.linspace(-.5*length,.5*length,pn)
y=[]
k=R/(4*np.pi)
```

```
@numba.jit #flags the function to be precompiled
def f(H,phi, theta, point, b, a , k, h ):
    return( k*(b*np.sin(theta)*np.sin(phi)+b*np.cos(theta)*np.cos(phi))/
           np.sqrt((b*np.cos(theta)+point-a*np.cos(phi))**2+
                   (b*np.sin(theta)-a*np.sin(phi))**2+(h-H)**2))
for point in tqdm(position): #integrats for each position
    totalflux=0
    for b in b3: #integrats for each concentric circle
        flux=nquad(f,[[-.5*H,.5*H],[0,2*np.pi],[0,2*np.pi]],
                   args=(point,b,a,k,h))
        totalflux+=flux[0]
    y.append(totalflux)
plot=np.diff(y)/np.diff(position)*v
voltage=gain*plot
x=np.linspace(-.5*length,.5*length,pn-1)
t=x/v
fig=plt.figure()
ax=plt.subplot()
plt.title('Theoretical Pickup Signal')
plt.ylabel('Voltage[V]')
plt.xlabel('time[s]')
plt.grid()
plt.plot(t,voltage)
plt.show()
```

References

[1] P. Horowitz, and W. Hill, "The Art of Electronics, 2nd ed." Cambridge University Press 1989.

[2] A. Zangwell, "Modern Electrodynamics" Cambridge University Press 2013.

[3] M. Boas, "Mathematical Methods in the Physical Sciences, 3rd ed." John Wiley and Sons, inc. 2006.

[4] "K&J Magnetics - Glossary". www.kjmagnetics.com.

[5] Wang, Wei-Chih "http://depts.edu/mictech/optics/sensors/week2.pdf" lecture notes, University of Washington