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Dakota and Pyomo for Closed and Open Box Controller Gain Tuning

Tutorial Session: Open Source Software for Control

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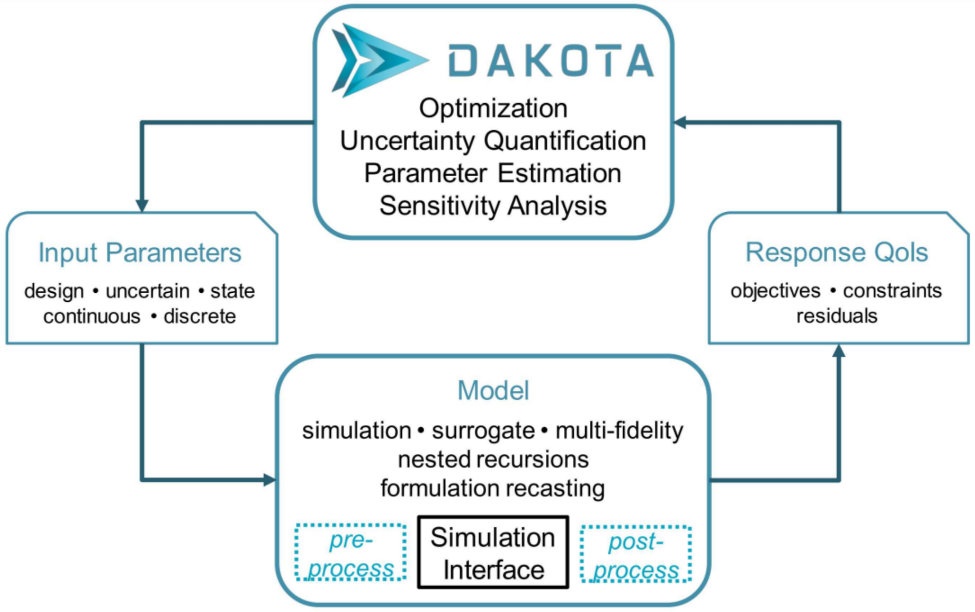
Introduction

- Control engineering: stabilization of dynamic systems
  - Developing mathematical model
  - Synthesizing a control law
  - **Tuning the parameters**
- Many toolboxes exist
  - Tool suites for linear time invariant systems
  - Robust control
  - Multi-objective software
- Dakota and Pyomo for control system tuning
  - Open source, developed at Sandia
  - Dakota: written in C++, can operated in “closed box” form via direct interaction with an input/output model
  - Pyomo: written in Python, requires transparent “open box” model

Background

- Dakota
  - Complex optimization problems
  - Closed box interface: only needs I/O
  - Can interact with MATLAB, Simulink, GNU Octave, Python, etc.
  - Implements a variety of optimization algorithms (genetic algorithms, gradient based)
- Pyomo (**Py**thon **O**ptimization **Mo**deling)
  - Open box: needs modeling equations
  - Supports a wide range of optimization problems (LP, QP, NLP, MIP, SP)
  - Supports differential algebraic equations (DAEs)
  - Transparent parallelization of subproblems using Python parallel communication libraries



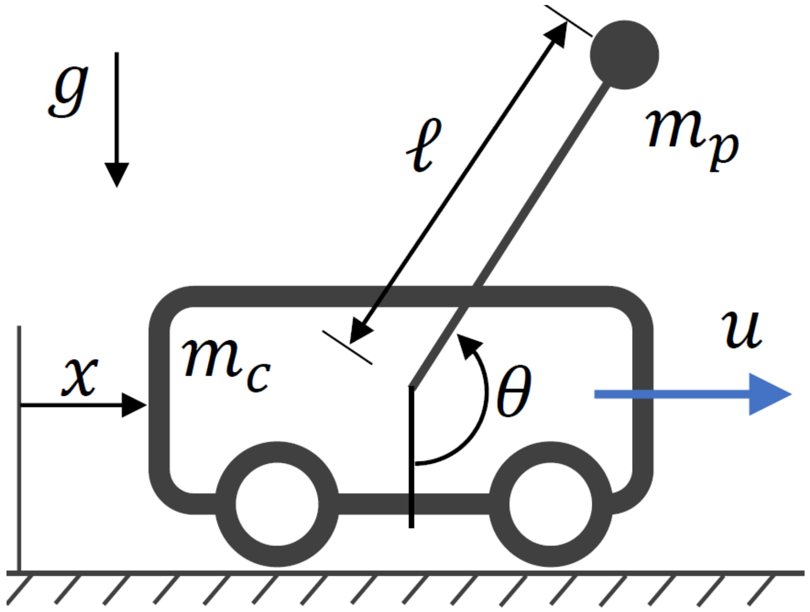
Available at https://dakota.sandia.gov

PYOMO

Available at https://www.pyomo.org

Example problem: cart-pole system

- Nonlinear, underactuated system
- Goal is to balance the pole at the unstable equilibrium (vertical position)
- Controlled in two ways:
  - Linear quadratic regulator (LQR) design
  - Partial feedback linearization (PFL) design



$$\ddot{x} = \frac{1}{m_c + m_p \sin^2\theta}\left[u + m_p \sin\theta(\ell\dot{\theta}^2 + g\cos\theta)\right] \quad (1a)$$

$$\ddot{\theta} = \frac{1}{\ell(m_c + m_p \sin^2\theta)}\left[-u\cos\theta - m_p\ell\dot{\theta}^2\cos\theta\sin\theta - (m_c + m_p)g\sin\theta\right] \quad (1b)$$

LQR optimization: setup

$$\text{minimize} \quad J = \int_0^T \tilde{\mathbf{x}}(t)^T Q \tilde{\mathbf{x}}(t) + \tilde{\mathbf{u}}(t)^T R \tilde{\mathbf{u}}(t) dt$$ ← LQR objective

$$\text{subject to} \quad \text{Nonlinear dynamics given by (1)}$$ ← Full nonlinear dynamics (Pyomo solver taking gradients)

$$\tilde{\mathbf{u}}(t) = -K \tilde{\mathbf{x}}(t)$$ ← Linear control law

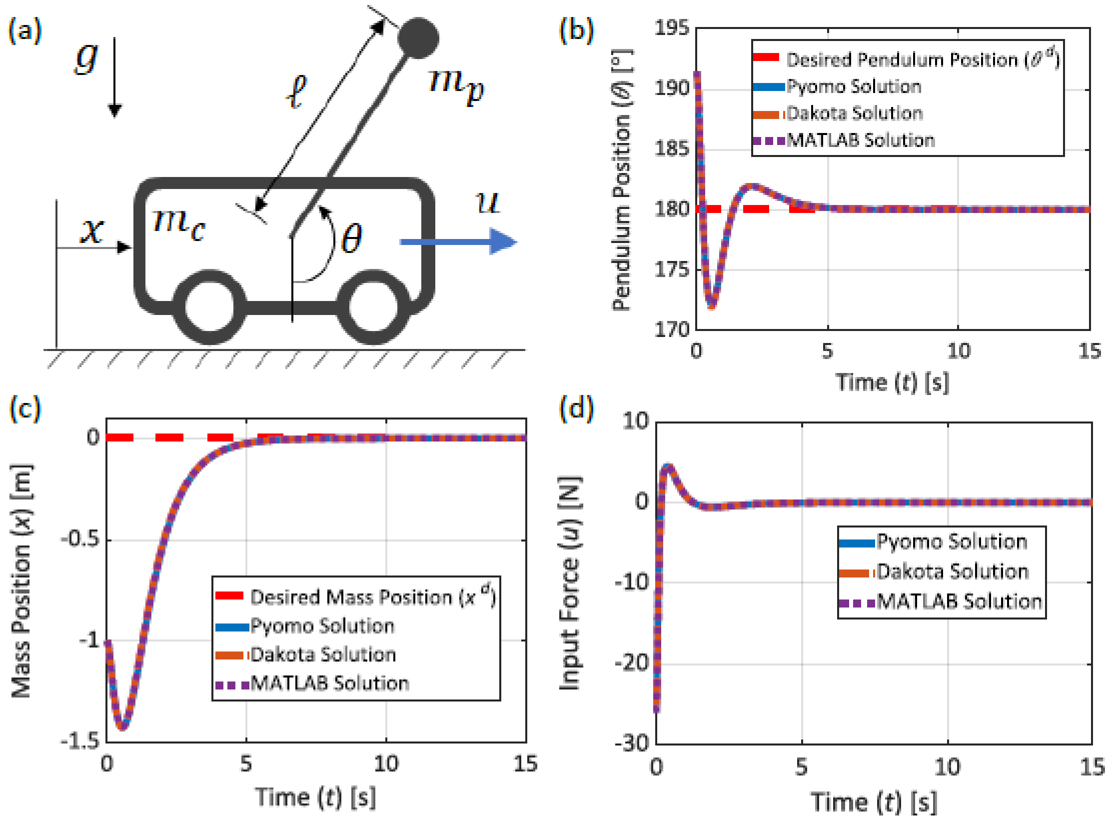
- Pyomo steps
  1. Create Pyomo model: state / control / objective vars, derivative vars, time horizon
  2. Define dynamics constraints "for t in m.T: m.dx1dt[t] = m.x2[t]"
  3. List boundary conditions and initialize guess
  4. Solve: define solver, in our case **IPOPT**

- Dakota steps
  1. *.in file: input file which specifies solver type (**COLINY EA**), ranges and initial guesses
  2. *.sh/*.vbs: opens MATLAB in either Windows or Linux
  3. *_Wrapper.m: a MATLAB file used to specifiy the current parameter choices made by Dakota
  4. *.m: additional MATLAB files which contain dynamics and control laws (can call *.slx files); called from *_Wrapper.m

LQR optimization: results

- Validation
  - Pyomo and Dakota produce controller gains which stabilize $x = 0, \theta = \pi$
  - Pyomo compute time: ~2 seconds
  - Dakota compute time: ~30 minutes

  Fundamental tradeoff: upfront setup time for computation time



- Gain comparison

Method	K_x	$K_{\dot{x}}$	K_θ	$K_{\dot{\theta}}$	J
Pyomo	-6.82	-12.45	92.32	28.68	60.92
Dakota	-6.85	-12.42	91.13	28.10	60.86
MATLAB	-7.07	-12.98	94.94	29.48	61.06

Partial feedback linearization

- Why PFL?
  - Linear control law: LQR design only valid near the chosen equilibrium point
  - Nonlinear control law: Partial feedback linearization demonstrates “swing up” capabilities
- Step 1: prescribe desired dynamics: $\ddot{\theta} = v \equiv k_d(\dot{\theta}^d - \dot{\theta}) + k_p(\theta^d - \theta)$
- Step 2: solve for control law

$$u = -\frac{1}{\cos\theta}\left[v\ell(m_c + m_p \sin^2\theta) + m_p \ell \dot{\theta}^2 \cos\theta \sin\theta + (m_c + m_p) g \sin\theta\right]$$

- Assuming a perfect model with perfect cancelation, we get the following transfer function which is stable for $\mathrm{Re}\{s^2 + k_d s + k_p\} < 0$

$$\theta(s) = \underbrace{\frac{k_p + k_d s}{s^2 + k_d s + k_p}}_{T(s)} \theta^d(s)$$

Additional study:
- Apply time delayed control $u(t - \tau)$ signal
- Closed loop transfer function is lost

Partial feedback linearization

$$\text{minimize}\quad J=\int_0^T\left(\theta^d(t)-\theta(t)\right)^2dt+\boxed{w_1k_p+W_{ST}t_s}$$

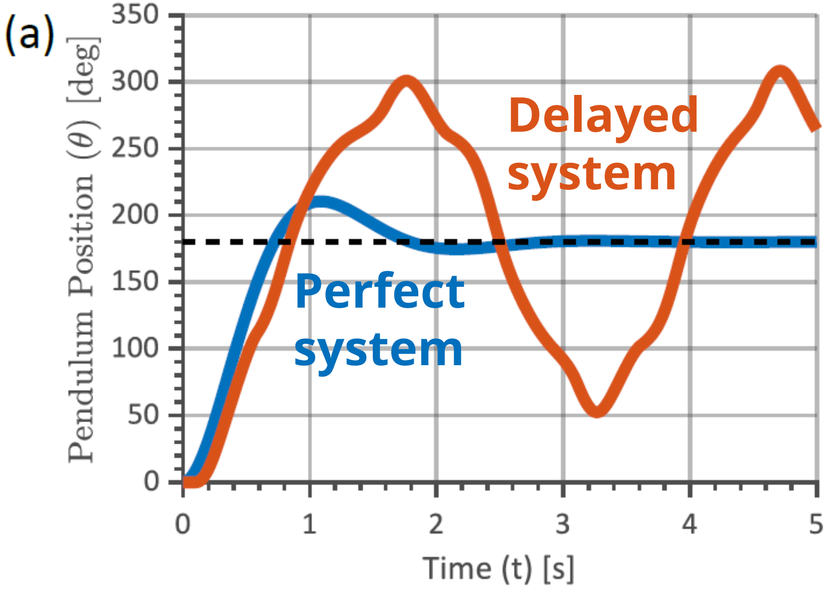
subject to nonlinear dynamics, prev nonlinear control law

Penalty on settling time t_s

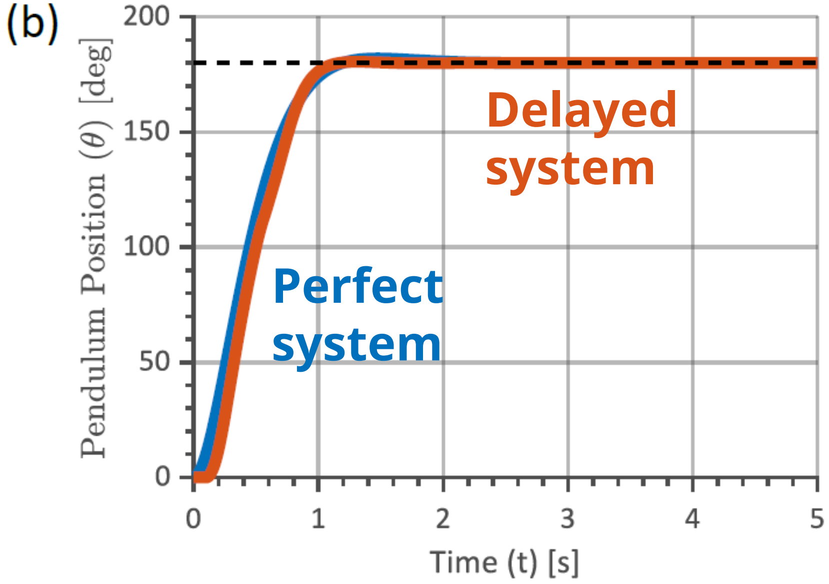
- Pyomo: use w_1=0.02 as surrogate
- Dakota: use t_s directly

Pyomo: with and without 0.1s control input delay

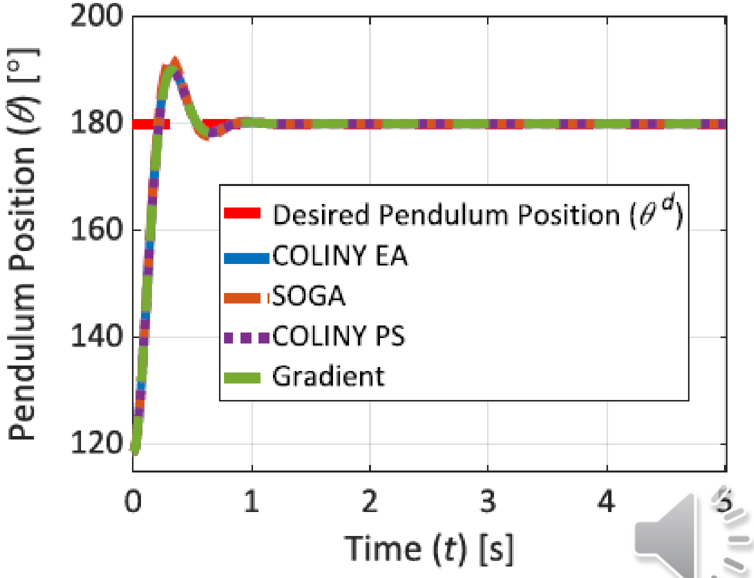
Nominal gains



Improved gains, which consider time delay



Dakota with 0.1s delay



Summary

- Dakota and Pyomo are powerful tools for control design
- Primary tradeoff: setup time vs. optimization time
  - Pyomo enjoys fast computation times, but model setup is non-trivial
  - Dakota computation times can be very long, but enjoys freedom in optimization criteria
- Single input system shown in this work, easily extensible to to multi-input systems (see ref for example)
- Although not shown here, Bayesian Optimization is another optimization framework to be considered
  - In [18], BO is used to tune Q and R matrices of an LQR synthesis to induce some desired behavior