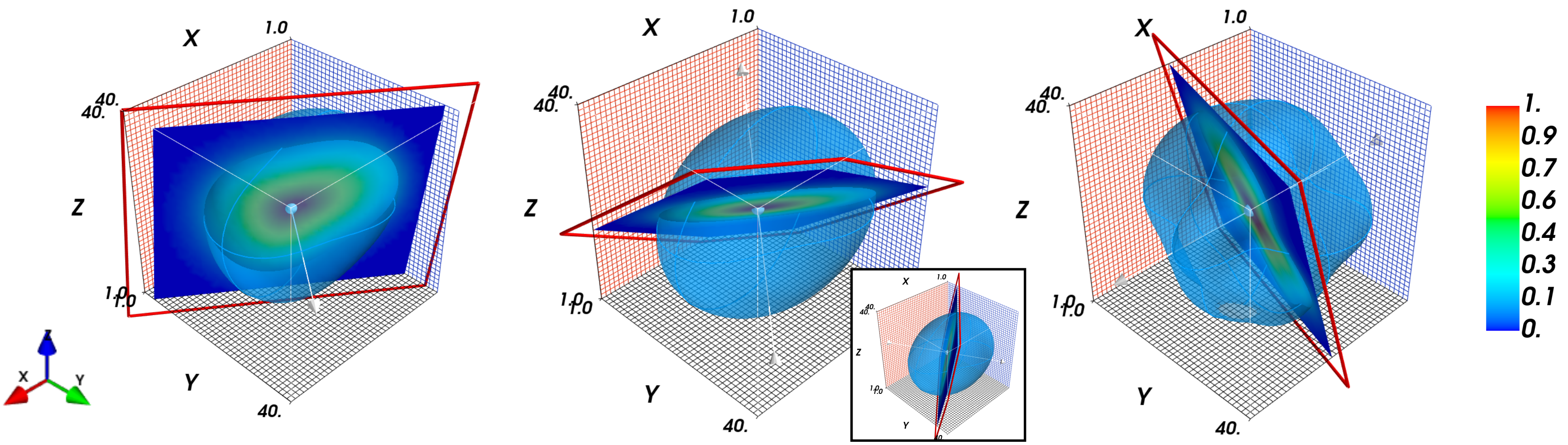
SAND2021-13960C

Three-Dimensional Hot-Spot Reconstruction in Inertial Fusion Implosions



GMXI : $(\theta, \phi) = (96°, 54°)$ TRXI : $(\theta, \phi) = (45°, 234°)$ KBXI : $(\theta, \phi) = (106°, 342°)$

SRTE : $(\theta, \phi) = (101°, 134°)$

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Pittsburgh, Pennsylvania
8 — 12 November 2021

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Summary

A platform of three-dimensional (3-D) hot-spot reconstruction procedures has been developed to quantify 3-D effects in ICF implosion experiments

- A tomography method is developed to reconstruct the 3-D plasma emissivity.
- Procedures of 3-D analysis are developed by integrating 3-D hot-spot shape asymmetries with nuclear measurements including ion-temperature (T_i), flow, and areal-density (ρR) asymmetries.
- Residual kinetic energies (RKE’s) are shown to be a driving factor causing implosion asymmetries.

The 3-D analysis will be applied to minimize the low-mode implosion asymmetry in our future work.

See Jim’s talk in NO04.00009.
See Kristen’s talk in ZO04.00007.

Collaborators

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** Currently at Sandia National Laboratories

A 3-D spherical-harmonic Gaussian function is used to reconstruct the 3-D plasma emissivity

3-D hot-spot emissivity ε_ν at a given spectral frequency ν

$$\ln \varepsilon_\nu(r,\theta,\phi) = \sum_{n=0}^{\infty} \sigma_n R^n \left[1 + \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} \sum_{k=0}^{\infty} A_{\ell m k} R^k Y_{\ell m}(\theta,\phi)\right]^n$$

Unfold electron temperature T_e and density n_e

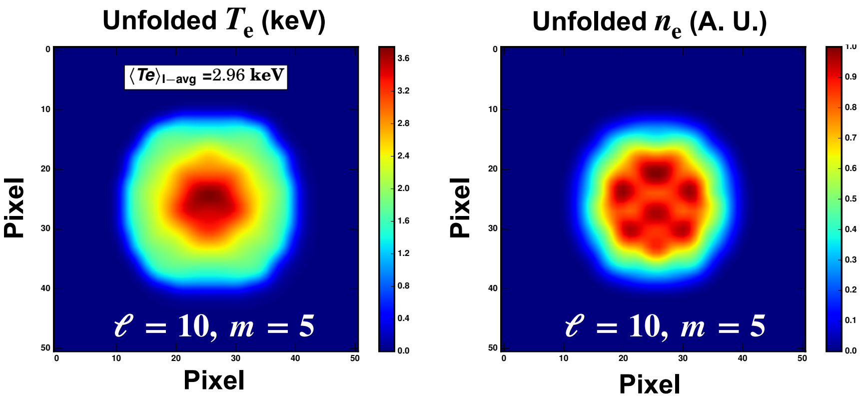
$$T_e = -\varepsilon_\nu / \left[\frac{\partial \varepsilon_\nu}{\partial h\nu}\right] \longrightarrow$$

Unfold n_e from emissivity $\varepsilon_\nu(n_e, T_e)$

For 1-D implosions, the 3-D emissivity model is reduced to a super-Gaussian model with an exponent of 4 and zero mode amplitudes.

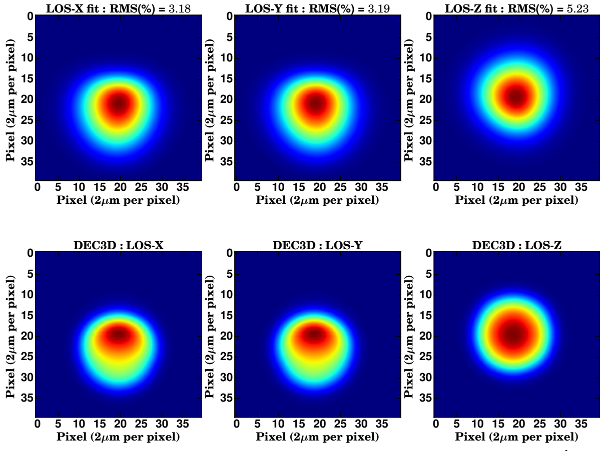
$$\ln I_{1D} = \sum_{n=0}^{4} \sigma_n R^n \rightarrow \boxed{I_{1D} = I_0 e^{-(R/\sigma)^4}}$$

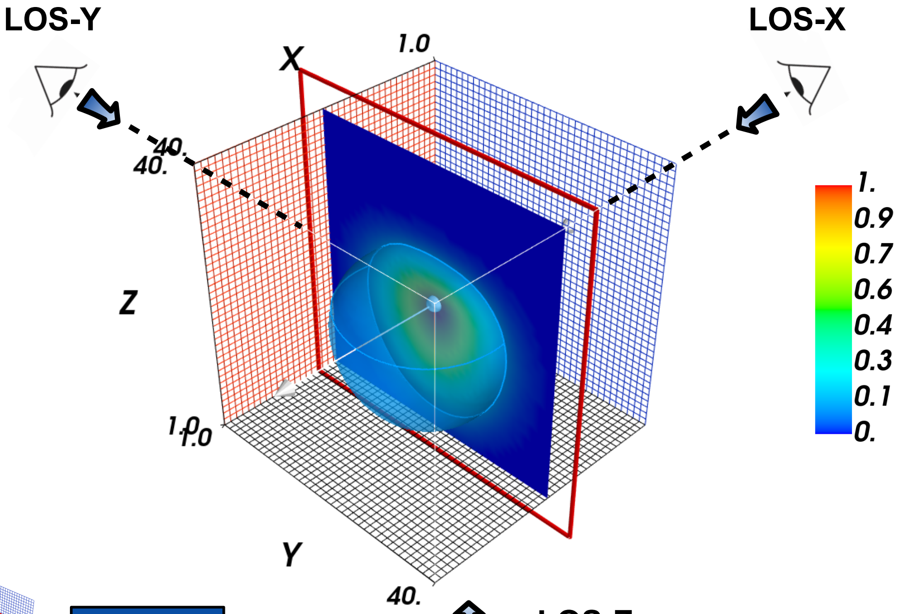
$$I_0 = e^{a_0},\ \sigma_4 = -1/\sigma^4$$



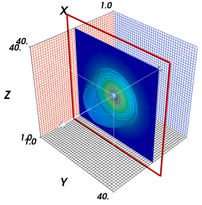
See Ref. [S. Eck et. al. Medical Image Analysis 32, 18-31 (2016)] for applying spherical-harmonic Gaussian functions.
See Ref. [G. Aubert, AIP Advances 3, 062121 (2013)] for rotating spherical harmonics using Wigner d-matrices.
See Kristen's talk in ZO04.00007.

The 3-D plasma emissivity model is optimized by a dynamic learning algorithm* to fit its 2-D projections with x-ray images measured at different lines of sight

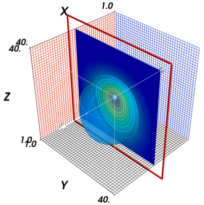




Initial guess



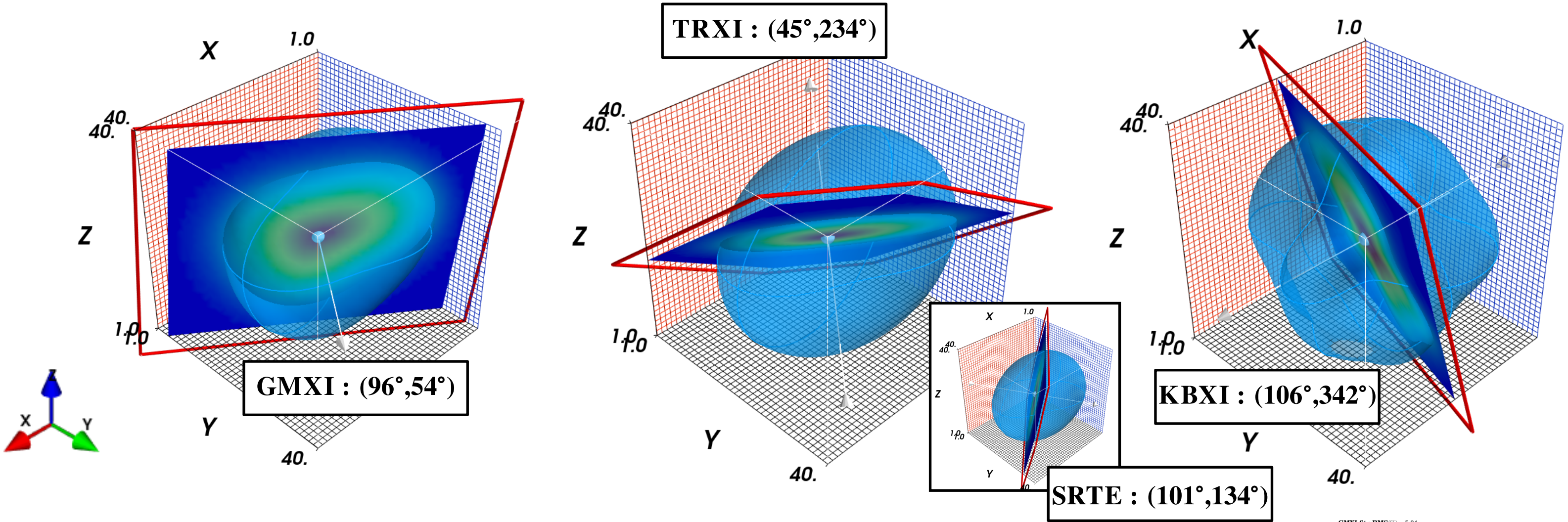
After 5 steps

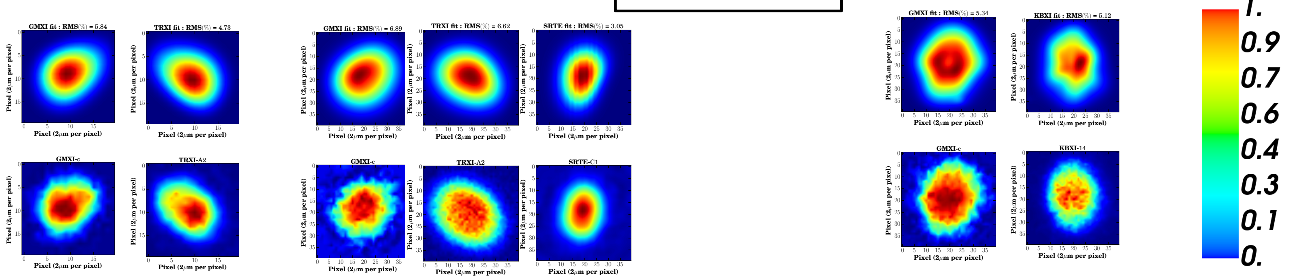


After 20 steps

* Zhenyu Liao et al. arXiv:1805.11917 [stat.ML]

The 3-D plasma emissivity model is used to reconstruct ICF implosions with mode ℓ = 1, 2, and 6 hot-spot shapes





Definitions

Polar angles : (θ, ϕ)
GMXI : gated-monochromatic X-ray imager
TRXI : temporal-resolved X-ray Imager
SRTE : spatially-resolved X-ray imager
KBXI : Kirkpatrick-Baez X-ray imager

Residual kinetic energies are the driving factor for T_i and hot-spot flow asymmetries

Apparent ion temperatures

$$T_{\text{apparent}}^{\text{Brysk**}} = T_{\text{th}} + M_0 \cdot \text{Var}\left[\vec{v} \cdot \hat{d}\right]$$

v **is the flow velocity in the laboratory frame.**
d **is the line of sight (LOS) unit vector.**
M_0 **is the total nuclear reactant mass.**
T_{th} **is the ion thermal temperature.**
$\sigma_{ij} = \langle (v_i - \langle v_i \rangle) \cdot (v_j - \langle v_j \rangle) \rangle$ **is the element for** $\hat{\sigma} = \text{Var}[\vec{v} \cdot \hat{d}]$

Since the matrix elements $\sigma_{ij} = \sigma_{ji}$ commute, the velocity-variance matrix* is Hermitian; hence, it is diagonalizable.

$$T_{\text{apparent}}^{\text{Brysk}} = T_{\text{th}} + M_0 \cdot \left(\sigma'_{xx} \sin^2\theta' \cos^2\phi' + \sigma'_{yy} \sin^2\theta' \sin^2\phi' + \sigma'_{zz} \cos^2\theta'\right)$$

The eigenvalues σ' are hot-spot residual kinetic energies (RKE_{HS}) along three rotated orthogonal axes.

Definition

$$\text{RKE}_{\text{HS}} = M_{\text{HS}}\sigma'_{xx}/2 + M_{\text{HS}}\sigma'_{yy}/2 + M_{\text{HS}}\sigma'_{zz}/2$$

* K. M. Woo et. al., Phys. Plasmas 25, 102710 (2018).
** H. Brysk, Plasma Physics 15 611 (1973); the velocity variance term can be obtained by removing the isotropic flow assumption in Brysk's analysis.

Ion-temperature asymmetries in OMEGA experiments are mostly driven by mode 1

$\mathrm{RKE}_{\mathrm{HS}}^{\perp}$ $\mathrm{RKE}_{\mathrm{HS}}^{\mathrm{Jet}}$

Mode $\ell = 1 : \sigma'_{xx} = \sigma'_{yy} \ll \sigma'_{zz}$

$$T_{\mathrm{i}} = T_{\mathrm{th}} + M_0 \cdot \left(\sigma_{\mathrm{iso}} + \sigma'_{zz} \cos^2\theta'\right)$$

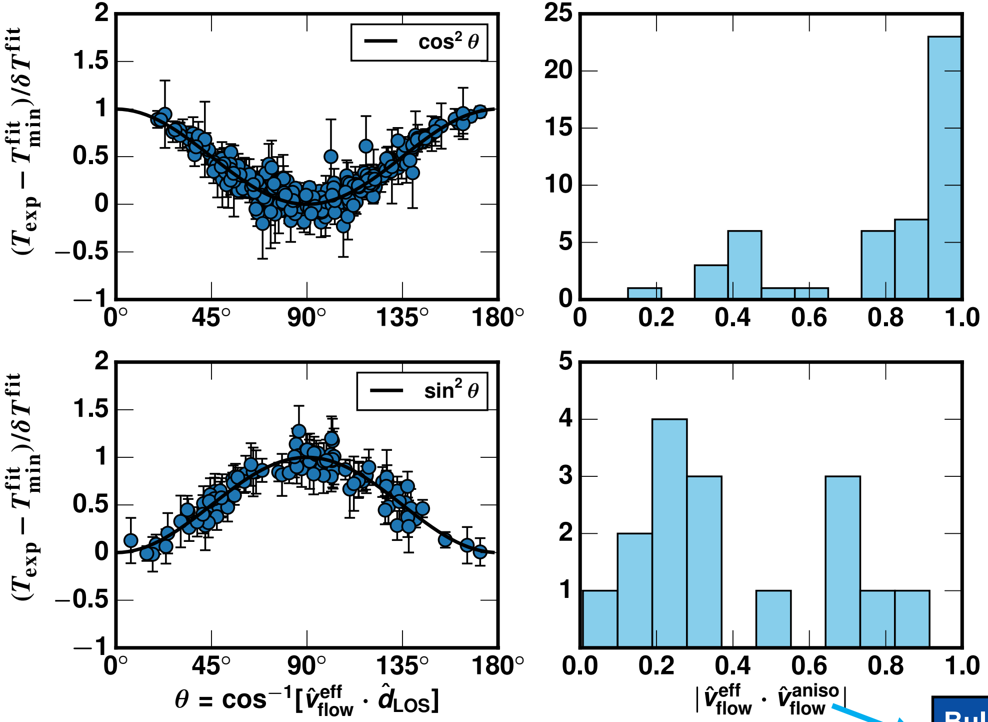
$\mathrm{RKE}_{\mathrm{HS}}^{\mathrm{Bubble/Spike}}$ $\mathrm{RKE}_{\mathrm{HS}}^{\mathrm{Spike/Bubble}}$

Mode $\ell = 2 : \sigma'_{xx} = \sigma'_{yy} \gg \sigma'_{zz}$

$$T_{\mathrm{i}} = T_{\mathrm{th}} + M_0 \cdot \left(\sigma_{\mathrm{iso}} + \Delta\sigma'_{xx} \sin^2\theta'\right)$$

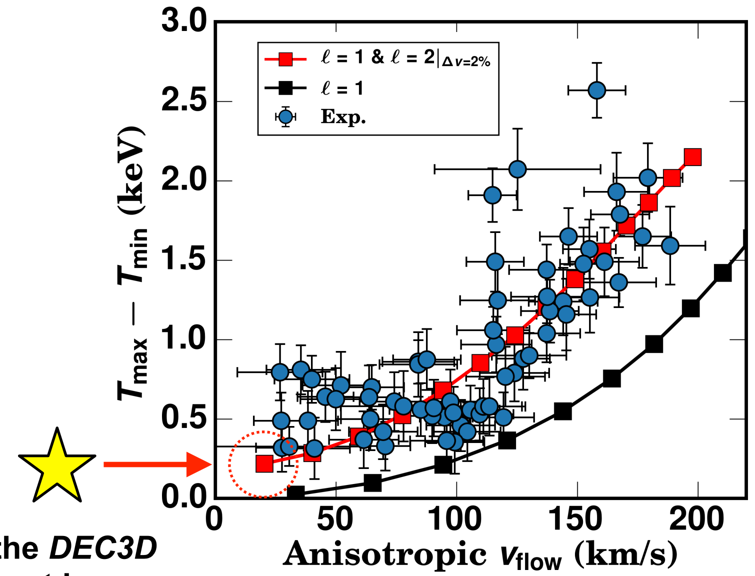
$$\Delta\sigma'_{xx} = \sigma'_{xx} - \sigma_{\mathrm{iso}}$$

Definition : $\sigma_{\mathrm{iso}} = \min[\sigma'_{xx}, \sigma'_{yy}, \sigma'_{zz}]$



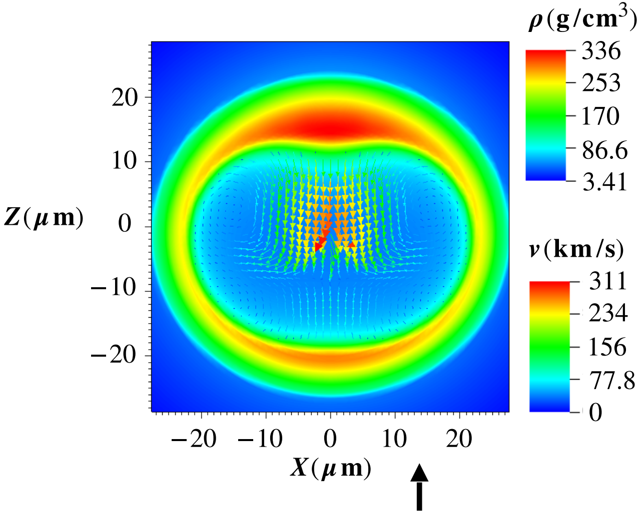
Bulk flow

The presence of quasi-isotropic flows from even-L modes provides additional ion-temperature asymmetries



See the *DEC3D* hot-spot image on the right.

Indication of mode $\ell = 2$



The flow structure in a mode-1 and mode-2 *DEC3D* simulation.

The $v.d$ term in the mode-1 areal-density model* captures the ρR asymmetry**

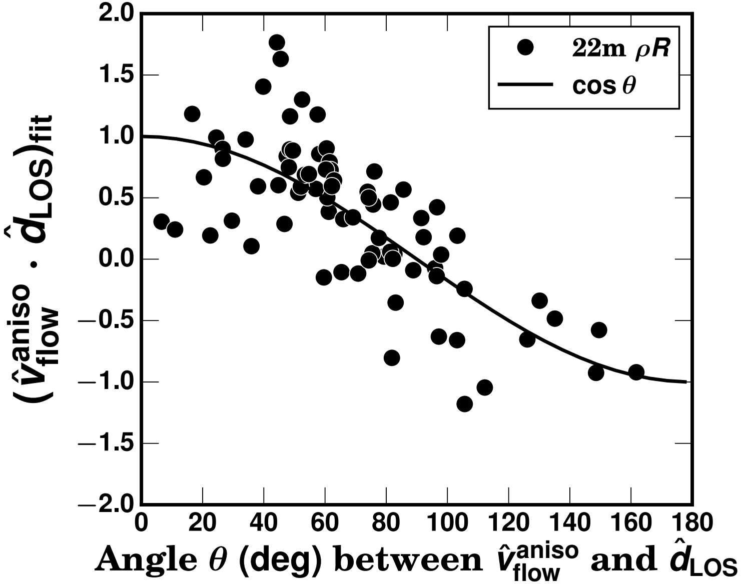
Mode-1 ρR Model* : $\rho R_{\mathrm{LOS}}^{\mathrm{apparent}} = \langle \rho R_{\mathrm{3D}} \rangle_{\mathrm{AM}} - \Delta\rho R(\hat{v}_{\mathrm{flow}}^{\mathrm{aniso}} \cdot \hat{d}_{\mathrm{LOS}})$

The arithmetic-mean (AM) and harmonic-mean (HM) ρR are related.

$$\langle \rho R \rangle_{\mathrm{HM}} = \sqrt{\langle \rho R_{\mathrm{3D}} \rangle_{\mathrm{AM}}^2 - (\Delta \rho R)^2}$$
$$\Delta \rho R = (\rho R_{\mathrm{max}} - \rho R_{\mathrm{min}})/2$$

$$\langle \rho R_{\mathrm{3D}} \rangle_{\mathrm{AM}} = \rho R_{\mathrm{1D}} R_T^{\alpha}, \quad \langle \rho R_{\mathrm{3D}} \rangle_{\mathrm{HM}} = \rho R_{\mathrm{1D}} R_T^{\beta}$$
$$R_{\mathrm{T}} = T_{\mathrm{max}}/T_{\mathrm{min}}, \quad \alpha = -0.3, \quad \beta = -0.47$$

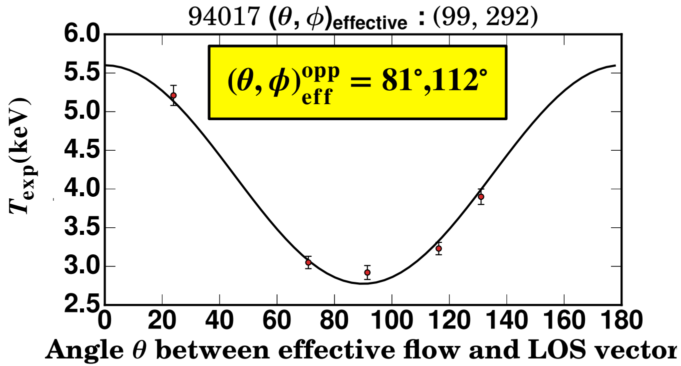
$$\rho R_{\mathrm{model}}(\theta, \phi, R_T) = \rho R_{\mathrm{1D}} \cdot \left[R_T^{\alpha} - \sqrt{R_T^{2\alpha} - R_T^{2\beta}} \times \hat{v}_{\mathrm{flow}}^{\mathrm{head}} \cdot \hat{d}_{\mathrm{LOS}}(\theta, \phi) \right]$$

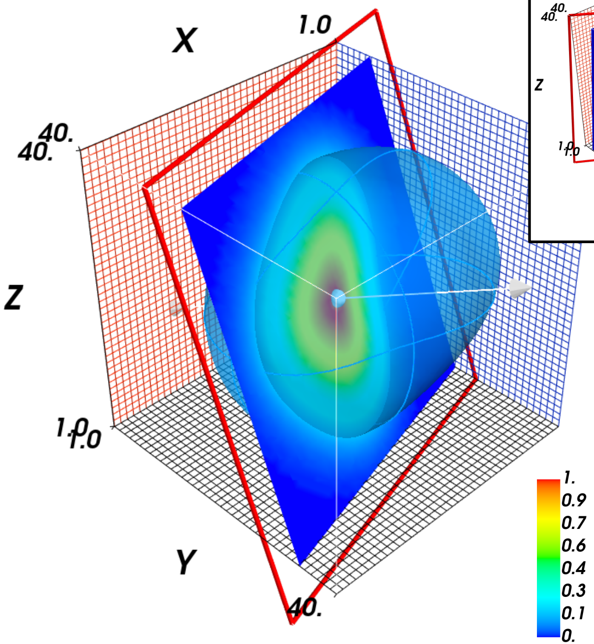


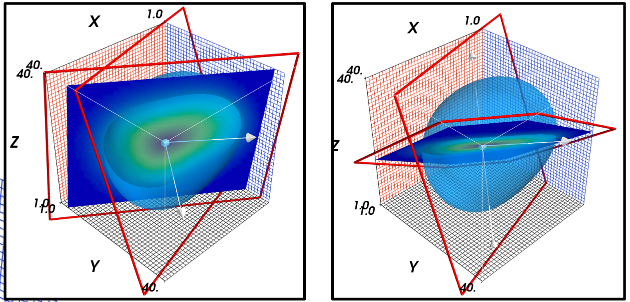
* K. M. Woo *et al.*, Phys. Plasmas 28, 054503 (2021)
** Z. L. Mohamed *et al.* Rev. Sci. Instrum. 92, 043546 (2021)
See Jim's talk in NO04.00009.

The mode information is quantified by T_i, flows, ρR, and hot-spot shape asymmetries

Effective flow reconstruction







$\theta_{sol} = 68°$ Z

$\phi_{sol} = 106°$ X

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See Jim's talk in NO04.00009.
See Kristen's talk in ZO04.00007.

Appendix

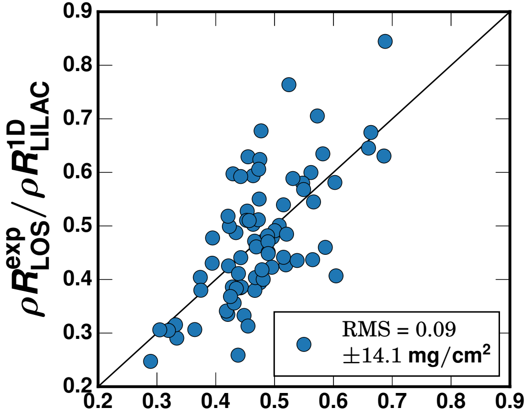
The *v.d* term in the mode-1 areal-density (ρR) model is shown to capture the ρR variations in OMEGA experiments

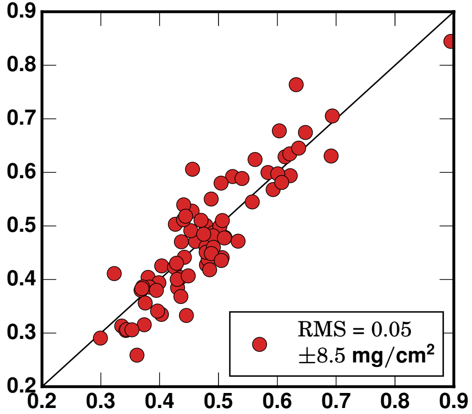
UR LLE

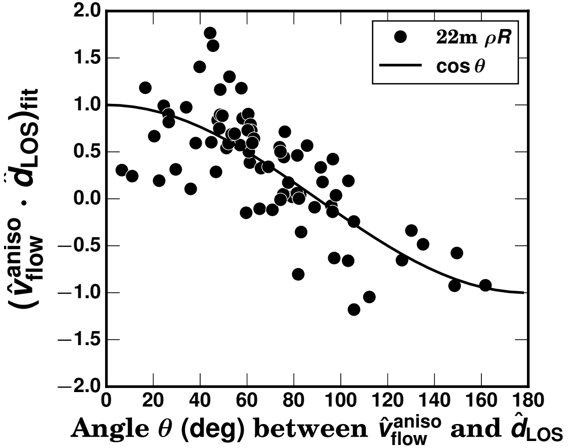
Mode 1 RhoR Model

$$\rho R_{\text{model}}(R_T) = \rho R_{\text{1D}} \cdot \left[R_T^{\alpha} - \sqrt{R_T^{2\alpha} - R_T^{2\beta}} \times \hat{v}_{\text{flow}}^{\text{head}} \cdot \hat{d}_{\text{LOS}}(\theta, \phi) \right]$$

$$\alpha = -0.3, \quad \beta = -0.47$$







$\text{CR}^{-1.9}\text{IFAR}^{-0.4}R_{\text{B}}^{1.6}$

$\text{CR}^{-1.4}\text{IFAR}^{-0.5}R_{\text{B}}^{0.8} \times$

$$\left(R_T^{\alpha_{\text{fit}}} - \sqrt{R_T^{2\alpha_{\text{fit}}} - R_T^{2\beta}} \hat{v}_{\text{aniso}}^{\text{flow}} \cdot \hat{d}_{\text{LOS}} \right)^{\gamma_{\text{fit}}}$$

1-D kernel to account for systematic 1-D coding errors.

3-D kernel to account for mode-1 RhoR asymmetry.

R_T : $T_{\max}/T_{\min}$

R_B : beam/target radii

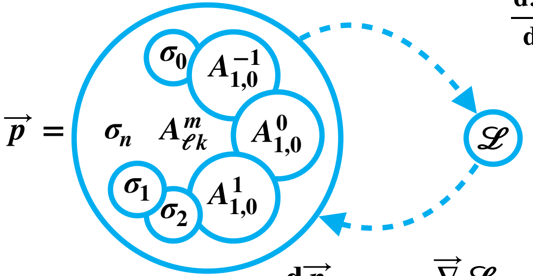
$\alpha_{\text{fit}} = -0.37, \quad \gamma_{\text{fit}} = 1.55$

The 3-D plasma emissivity model is optimized* by minimizing the fitting error between its 2-D projections and experimental x-ray images

Loss function $\mathscr{L}$

$$\mathscr{L}(\vec{p}) = \frac{1}{N}\sum_{i=1}^{N}\sqrt{\left\langle\left(\mathscr{P}_{\mathrm{LOS}_i}\cdot I^{3\mathrm{D}}(\vec{p}) - I^{\mathrm{exp}}_{\mathrm{LOS}_i}\right)^2\right\rangle}$$

Parameters P

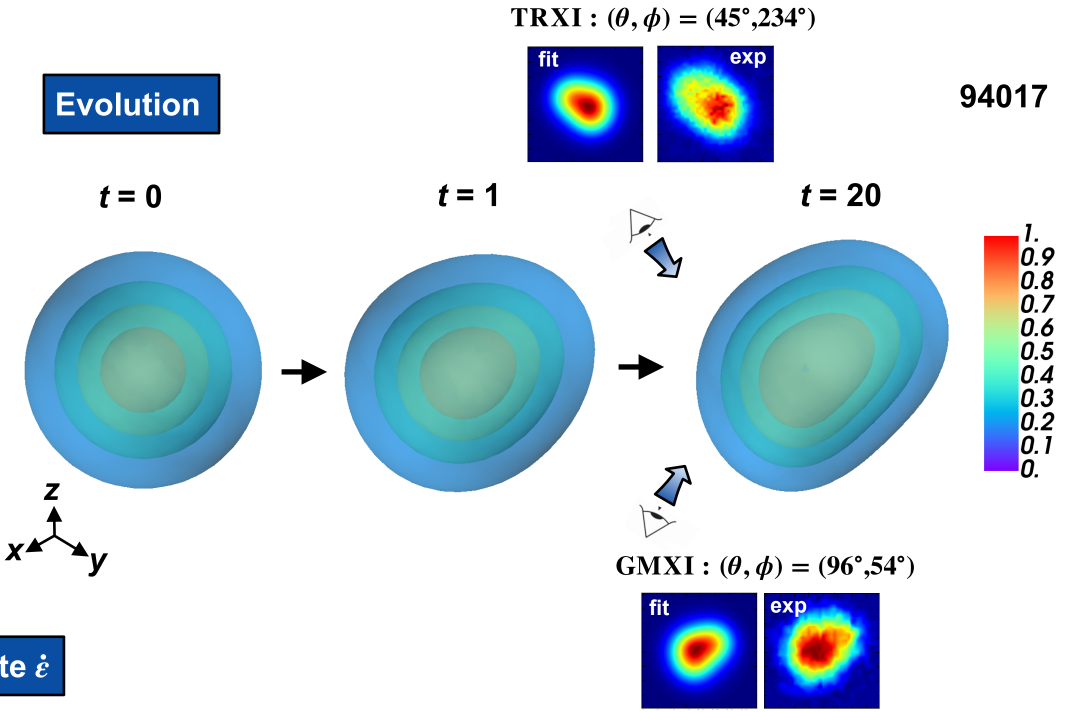


Steepest descent

$$\frac{\mathrm{d}\mathscr{L}}{\mathrm{d}t} = \vec{\nabla}\mathscr{L}\cdot\frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

$$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = -\frac{\vec{\nabla}\mathscr{L}}{|\vec{\nabla}\mathscr{L}|}\dot{\varepsilon}$$

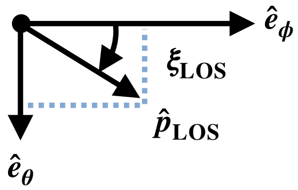
Learning rate $\dot{\varepsilon}$



* Zhenyu Liao *et al.* arXiv:1805.11917

The major axis for mode 1 and prolate mode 2 can be reconstructed using its projection measured at two lines of sight

Major axis reconstruction



$$\begin{bmatrix} \hat{p}_1 \cdot \hat{z}\cos(\xi_1) - \hat{e}_{\phi,1} \cdot \hat{z} \\ \hat{p}_2 \cdot \hat{z}\sin(\xi_2) - \hat{e}_{\theta,2} \cdot \hat{z} \end{bmatrix} = \begin{bmatrix} \hat{e}_{\phi,1} \cdot \hat{x} - \hat{p}_1 \cdot \hat{x}\cos(\xi_1) & \hat{e}_{\phi,1} \cdot \hat{y} - \hat{p}_1 \cdot \hat{y}\cos(\xi_1) \\ \hat{e}_{\theta,2} \cdot \hat{x} - \hat{p}_2 \cdot \hat{x}\sin(\xi_2) & \hat{e}_{\theta,2} \cdot \hat{y} - \hat{p}_2 \cdot \hat{y}\sin(\xi_2) \end{bmatrix} \begin{bmatrix} \tan\theta\cos\phi \\ \tan\theta\sin\phi \end{bmatrix}$$

Major axis vector : $\hat{v} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{x} + \cos\theta\,\hat{z}$

Projection vector : $\hat{p}_{\mathrm{LOS}} = \cos\xi_{\mathrm{LOS}}\hat{e}_{\phi,\mathrm{LOS}} + \sin\xi_{\mathrm{LOS}}\hat{e}_{\theta,\mathrm{LOS}}$