SAND2021-14096C

Dissipativity-based Voltage Control in Distribution Grids

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***Abstract*—We consider the problem of decentralized control of reactive power provisioned by distributed energy resources for voltage support in the distribution grid. We assume that the reactance matrix of the grid is unknown and potentially time-varying. We present conditions for stability of the system when the reactive power at each inverter is set using a potentially heterogeneous droop curve. These conditions utilize energy dissipation requirements and can be naturally satisfied even when the reactance matrix is unknown by using an adaptive controller and when the reactance matrix is time-varying.**

***Index Terms*—Decentralized control, dissipativity, energy storage, power distribution systems, volt/VAr control.**

I. Introduction

As the incidence of distributed energy resources (DERs), such as residential photovoltaic (PV) grids, has increased in the distribution grid, new challenges and opportunities have arisen. A major challenge is that due to rapid fluctuations in voltages and supply due to the presence of DERs, standard techniques, such as switching capacitor banks or controlling on-load tap changers, are no longer sufficient for guaranteeing that power quality specifications are met. The opportunity arises from the fact that the inverters used by DERs to share their power with the distribution network (DN) can easily achieve prescribed reactive output powers. Thus, they provide a new control knob to stabilize the voltage and meet power quality constraints through controlling the reactive/active power flows.

Indeed, several control algorithms to this end have now been proposed. Rather than centralized algorithms with a single decision point that computes and prescribes set points for each inverter, local or distributed algorithms are expected to be more scalable and robust [1], [2] [3]. Distributed controllers require neighboring inverters to communicate with each other, while local algorithms do not require any information exchange [4] [5], [6].Given the lack of communication infrastructure at the moment, local controllers are preferred over distributed approaches. Local Volt/VAR droop curves implemented at each inverter are an example of such controllers and have been extensively studied [7]–[14].

A known problem with these controllers is that multiple local controllers when connected to the distribution network can result in stability loss [15]. This problem manifests itself as oscillations in the voltage or reactive power in the network in such a way that the power quality constraints are not met. This problem has been considered in previous work. As some representative examples, a continuous time approach was proposed in [16]; however, as [17] argues, a discrete time formulation is more appropriate for the problem. Some works have considered a small signal analysis by considering only the linear region of the droop curves [7], [14], [18]. A reverse engineering based optimization framework was proposed in [19]; however, assuming the same droop curve at every inverter. An algorithm that utilizes a first-order filter along with a droop curve was proposed in [17]. An interesting observation in these works is that the stability conditions are a function of the reactance matrix of the distribution network. If the matrix is unknown, looser sufficient conditions can be derived based on some property such as spectral radius or norm of the matrix. However, these conditions can be conservative, especially when the reactance matrix may be varying over time due to factors such as changing topology of the DN.

In this paper, we present a dissipativity-based approach to guarantee stability of a DN with multiple inverters and local voltage control. We consider a discrete-time formulation, allow for heterogeneous droop curves, and present conditions for stability with saturation regions included in the droop curves at the inverters. The stability proof requires the knowledge of the reactance matrix to design the droop curves. For the case when the reactance matrix is unknown but constant, we present an adaptive controller building on extremum seeking optimization that does not require this knowledge. One advantage of our dissipativity-based approach is that it allows us to derive stability conditions even in the case when the reactance matrix is time-varying. These properties may become increasingly important as DER penetration increases.

We begin by defining the notation and the problem in Section II. We present our dissipativity based controller in

This work was supported by the U.S. Department of Energy, Office Electricity, Energy Storage program. This paper describes objective technical results and analysis. Any subjective views or opinions that might be expressed in the paper do not necessarily represent the views of the U.S. Department of Energy or the United States Government. Sandia National Laboratories is a multi-mission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy National Nuclear Security Administration under contract DE-NA0003525. SAND2021-XXXX X.

Section III-A. For the case when the reactance matrix is constant but unknown, we present an extremum seeking approach in Section III-B to converge to the correct controller. We show how stability can be guaranteed when energy dissipation does not hold at every time step due to a time-varying reactance matrix in Section III-C. Some numerical case studies are presented in Section IV.

II. Problem formulation

Consider a radial distribution network with $n+1$ buses numbered as 0, 1, $\cdots$, n. Without loss of generality, bus 0 is the substation bus assumed to be at a fixed voltage. Define the set of buses by $\mathcal{N} \triangleq \{0, 1, \ldots, n\}$. Denote the set of lines connecting the buses by $\mathcal{L}$ with the line $(i, j) \in \mathcal{L}$ connecting buses i and j with $i, j \in \mathcal{N}$. For each bus $i \in \mathcal{N}$, denote the complex voltage at this bus by v_i, the real power injection by p_i and the reactive power injection by q_i (with injection in either case denoted by a positive value and consumption by a negative sign). Denote the stacked vectors of these quantities at all the buses by v for the complex voltages, p for the real power injections, and q for the reactive power injections.

In a distribution grid, the voltage magnitude v can be observed and the reactive power injections q can be controlled. We follow the development of [17] to describe the dynamics of the system. Specifically, we consider a single-phase grid and assume that the dynamics of the grid are considered in a discrete-time fashion with the discretization time T_s that is sufficiently large so that the power system dynamics (grid, load, and inverter dynamics) reach a steady state between the discrete time steps. In other words, if the reactive power injections $q(k)$ are specified at time step k, then the actual injections will reach these values at time step $k+1$. Further, the corresponding voltages (given by the power flow equations) have also reached a steady state at time step $k+1$. Finally, we assume that the voltages are obtained through a linearization of the nonlinear power flow equations at 1pu.

If we denote the *prescribed values* of the reactive power injections at iteration k by the control input $u(k)$, and assume that the voltages $v(k)$ can be observed, then the above discussion can be summarized in the system model of the form

$$\Sigma_l : \quad \begin{aligned} q(k+1) &= u(k) \\ y(k) &= v(k), \end{aligned} \tag{1}$$

with the voltage $v(k)$ and reactive power $q(k)$ satisfying $v(k) = Xq(k) + \bar{v}$, where X is a positive definite matrix that characterizes the reactance of the network and $\bar{v}$ is a vector that depends on the real power injections and the resistances in the network and is not controllable. Note that X is a constant matrix; however, the precise value of X is often unknown. This linearized model is widely accepted, see, e.g., [17], [19]–[22].

The control objective is to design the control input $u(k)$ so that the voltage $v(k)$ locally asymptotically stabilizes to a desired set point $v^\star$. The control input $u(k)$ should be a causal function of the output $y(0), \cdots, y(k)$. Further, it should be a local controller in the sense that each input $u_i(k)$ depends only on the local outputs (or voltages) at bus i. Finally, the control input $u(k)$ must satisfy the physical limits of the reactive power that can be supplied or absorbed by the inverter, so that $u(k) \in [q_{\min}, q_{\max}]$.

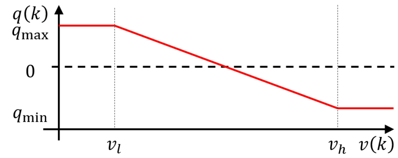

Figure 1. Typical droop controller.

For Σ_l, denote the set of all feasible operating points by

$$\mathcal{C} = \{(q, v) \in \mathbb{R}^n \times \mathbb{R}^n | v = Xq + \tilde{v}\}. \tag{2}$$

Let $(q^\star, v^\star) \in \mathcal{C}$ denote the desired operating point of the system Σ_l corresponding to the voltage set point $v^\star$ and the reactive power $v^\star = Xq^\star + \tilde{v}$,. with $q^\star \in [q_{\min}, q_{\max}]$. Finally, denote the incremental quantities $\Delta q(k) = q(k) - q^\star$, $\Delta u(k) = u(k) - u^\star$, and $\Delta v(k) = v(k) - v^\star$.

III. Dissipativity based controller

We begin by presenting a dissipativity property for the linearized system Σ_l. We provide the basic definitions of dissipativity in the Appendix and refer the reader to books such as [23] for more information. Using that, we design a new dissipativity-based controller to ensure that the voltage set point is asymptotically stable. The dissipativity analysis of the system Σ_l is slightly complicated by the fact that the input $u(k)$ is related to $q(k+1)$ and hence the output $y(k+1)$ at time $k+1$. We use the well known tool of scattering transformations used in the theory of dissipativity of time-delay systems [24]. Specifically, we use the scattering transform defined by

$$\nu(k) = \Delta v(k) + X \Delta u(k), \tag{3a}$$

$$\omega(k) = -\Delta v(k) + X \Delta u(k). \tag{3b}$$

We can then prove the following result.

Lemma 1: The linearized system Σ_l is passive with respect to the input $\nu(k)$ and output $\omega(k)$ irrespective of how $u(k)$ is designed.

Proof: It follows readily using the storage function $S(k) = \|X\Delta q(k)\|_2^2$ and noting that $S(k+1) - S(k) = \nu(k)^\top \omega(k)$. ■

We emphasize that the dissipativity above has been proven with respect to a 'dummy' input $\nu(k)$ and output $\omega(k)$ and holds irrespective of the controller used to design $u(k)$.

A. Control design

We now show that the dissipativity property proved above can be utilized to design a controller that stabilizes the system around the desired set point. Specifically, consider the following controller inspired by the popular droop controllers and shown in Fig. 1:

$$u(k) = \begin{cases} q_{\max} & v(k) < v_l \\ u^\star - \overline{K}\,(v(k) - v^\star) & v_l \le v(k) \le v_h \\ q_{\min} & v(k) > v_h, \end{cases} \tag{4}$$

where for the system not to have a trivial equilibrium in the saturated regime, we assume that the parameters v_l and v_h are chosen to satisfy

$$v_h \geq Xq_{\max} + \tilde{v} \geq v_l \quad (5)$$

$$v_l \leq Xq_{\min} + \tilde{v} \leq v_h. \quad (6)$$

These relations can be interpreted as imposing constraints on the allowed voltage range as a function of the reactive power capacity of the DERs so that stability can still be guaranteed. We can show the following result.

Theorem 2: Consider the system Σ_l with the controller (4). Let $\overline{K}$ be a diagonal matrix that satisfies the condition

$$K := (I + X\overline{K})^{-1}\left(X\overline{K} - I\right) < 0 \quad (7)$$

in the sense that $K + K^T$ is negative-definite. Then:

(i) The closed loop system is dissipative with respect to the supply-rate $w(\omega) := \omega^\top K\omega$.

(ii) The closed loop system is asymptotically stabilized to the desired operating point $(q^\star, v^\star)$.

Proof: If the initial condition $v(0)$ satisfies $v(0) < v_l$, then the controller (4) implies that $u(0) = q_{\max}$. The state $q(1) = u(0) = q_{\max}$, which, in turn, implies from (5) that $v_l \leq v(1) \leq v_h$. Similarly, if $v(0) > v_h$, $v_l \leq v(1) \leq v_h$. Thus, for asymptotic behavior of the system, we can assume without loss of generality that the initial condition of the system satisfies $v_l \leq v(0) \leq v_h$ and the control input is given by

$$u(k) = u^\star - \overline{K}\left(v(k) - v^\star\right). \quad (8)$$

To prove part (i), we begin by simplifying (3) using the controller (8) to write

$$\begin{aligned} \nu(k) &= (I - X\overline{K})\Delta v(k) \\ \omega(k) &= -(I + X\overline{K})\Delta v(k). \end{aligned} \quad (9)$$

Further, we can simplify (7) to write

$$X\overline{K} = (I + K)(I - K)^{-1}. \quad (10)$$

We can use (10) to rewrite (9) as

$$\nu(k) = K\omega(k). \quad (11)$$

Using Lemma 1 and (11), we have

$$S(k+1) - S(k) = \nu(k)^\top\omega(k) = \omega(k)^\top K\omega(k). \quad (12)$$

This concludes the proof of part (i).

For part (ii), we note that since $K < 0$, (12) implies:

- If $w(\omega) = 0$, then $\omega(k) = 0$. From (11), we then obtain $\nu(k) = 0$. Thus, the relation (3) yields $v(k) = v^\star$ and consequently $q(k) = q^\star$.
- If $w(\omega) \neq 0$, then $S(k+1) < S(k)$. Since $X > 0$, $S(k) \geq 0$, with $S(k) = 0$ if and only if $\Delta q(k) = 0$, or in other words, $q(k) = q^\star$ and $v(k) = v^\star$.

These two observations imply that the system Σ_l with (4) is asymptotically stabilized to the operating point $(q^\star, v^\star)$. ■

Theorem 2 proves the stability of a controller inspired by the droop controller, under some conditions on the range of the linear portion of the controller (given by (5) and (6)) and restrictions on the slope of the linear portion of the controller with respect to the matrix X of the distribution grid.

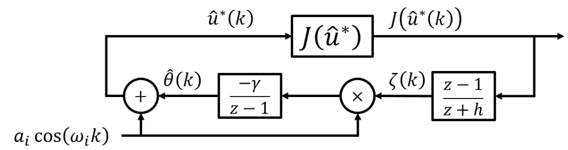

Figure 2. Proposed algorithm for identifying $\hat{u}^\star(k)$.

B. Adaptive Controller

Although the controller (4) is sufficient to stabilize the system Σ_l, it requires the knowledge of the desired set point $u^\star$, which, in turn, requires the value of the matrix X. Assuming the knowledge of X in a distribution grid with DERs that are not under the direct control of the operator can be restrictive. We now propose an adaptive controller that does not require any knowledge of $u^\star$.

Following the theory of extremum seeking controllers (ESCs) [25] the controller implemented at time k is of the form:

$$u(k) = \hat{u^\star}(k) - \overline{K}\left(v(k) - v^\star\right), \quad (13)$$

where $\hat{u^\star}(k) \in \mathbb{R}^n$ denotes the current estimate of the unknown desired reactive power $u^\star$. To update $\hat{u^\star}(k)$, we follow the design in Figure 2. Since $u^\star$ corresponds to the desired set point $q^\star$ of the reactive power, we define a cost function $J(\hat{u^\star}(k))$ associated with any choice of $\hat{u^\star}(k)$ as

$$J(\hat{u^\star}(k)) = \|Xu(k) + \tilde{v} - v^\star\|_2^2. \quad (14)$$

Then, the estimate is updated as

$$\hat{u^\star}_i(k+1) = \hat{\theta}_i(k+1) + a_i\cos(\omega_i(k+1)) \quad (15)$$

$$\hat{\theta}_i(k+1) = \hat{\theta}_i(k) - \gamma_i a_i\cos(\omega_i k)\left(J(\hat{u^\star}(k)) - (1+h)\zeta(k)\right) \quad (16)$$

$$\zeta(k) = -h\zeta(k-1) + J(\hat{u^\star}(k-1)), \quad (17)$$

where $\zeta(k)$ is a scalar and the subscript i indicates the i-th vector entry. The signal $a_i\cos(\omega_i k)$ is a dither signal with a small amplitude a_i and frequency $\omega_i = \alpha^i\pi$, with $0 < \alpha < 1$. γ_i is the adaptation gain. The highpass filter $\frac{z-1}{z+h}$ is designed with $0 < h < 1$ and a cutoff frequency well below ω_i.

The ESC is a gradient based controller and thus any convergence result is necessarily local. Thus, we assume that the the system Σ_l remains in the linear range of the controller (4).

Theorem 3: Consider the system Σ_l in closed loop with the controller of the form (13) such that $v_l \leq v(k) \leq v_h$ at every step. Let the parameter $\hat{u^\star}(k)$ in the controller be chosen according to the equations (15)-(17). Then, it holds that the parameter $\hat{u^\star}(k)$ locally exponentially converges to an $O(\alpha_i)$ neighborhood of the correct value $u^\star$.

Proof: The proof follows that of [25, Theorem 2] by defining $F_0(z) = F_i(z) = 1$ and noting that the condition in [25, Theorem 2] on the positive realness is met if $0 < h < 1$. A detailed proof is omitted for space constraints. ■

C. Time-varying reactance Matrix

The condition (7) guarantees stability with the controller (4) for the system Σ_l. In a distribution grid with multiple DERs that are not under the control of the operator, the value of X

may be time-varying. In this case, while we may satisfy (7) for a nominal value of X, this relation may be violated at the time steps when X deviates from this value. The stability proof in Theorem 2 (as in other works in the literature) will be violated in this case. In this section, we show how our dissipativity based proof can be extended to this case.

Specifically, we assume that the relation (7) is satisfied for a nominal value X_0 of X. However, X can also take values from the set $\{X_1, \cdots, X_n\}$, such that (7) is not guaranteed for X_i. We obtain below a sufficient condition on the frequency with which $X = X_0$ should be ensured for stability. Assume that the matrix $\bar{K}$ is designed such that

$$K_0 := (I + X_0\bar{K})^{-1}(X_0\bar{K} - I) > 0, \tag{18}$$

so that $K_0 + K_0^T$ is positive definite, while the matrices

$$K_i := (I + X_i\bar{K})^{-1}(X_i\bar{K} - I), \qquad i = \{1, \cdots, n\} \tag{19}$$

may not satisfy this constraint. Denote

$$\min \frac{\omega(k)^\top K_i \omega(k)}{\omega(k)^\top \omega(k)} := \lambda_i \tag{20}$$

$$\lambda_{\min} = \min_{i=1,\cdots,n} \lambda_i. \tag{21}$$

If at time k, X_i was the correct matrix, we have

$$S(k+1) - S(k) = -\omega(k)^\top K_i \omega(k). \tag{22}$$

For notational ease, denote the matrices active at time k by $X(k)$ and $K(k)$, with the smallest eigenvalue $\lambda(k)$. Adding the relations (22) for every N steps, we obtain

$$S(k+N) - S(k) \geq -\sum_{i=0}^{N-1} \omega(k+i)^\top \omega(k+i)\lambda(k+i). \tag{23}$$

Thus, a sufficient condition for $S(k+N) \leq S(k)$ is that $-\sum_{i=0}^{N-1} \lambda(k+i) < 0$. The most conservative case is when X_0 was active for exactly m steps out of N and all other λ_i's are negative for $i = 1, \cdots, n$ and equal to $\lambda_{\min}$. In this case, this condition can be rewritten as

$$-m\lambda_0 - (N-m)\lambda_{\min} < 0. \tag{24}$$

Thus, we can state the following result.

Theorem 4: Consider the system Σ_l with the controller (4), where the matrix $\bar{K}$ is designed to satisfy (7) for a nominal matrix X_0. Let X take matrices from a set $\{X_0, \cdots, X_n$, such that the quantities λ_0 and $\lambda_{\min}$ can be defined as in (20) and (21) respectively. A sufficient condition for the stability of the closed loop system is for the operator to ensure that for every block of N steps, the nominal matrix X_0 is active at least m times such that (24) is satisfied.

Proof: If (24) is satisfied for every N steps, we can consider the equivalent system evolving at times kN, for $k = 0, 1, \cdots$. This system is dissipative with the same storage function as in Theorem 2. The rest of the proof mirrors that of Theorem 2. ■

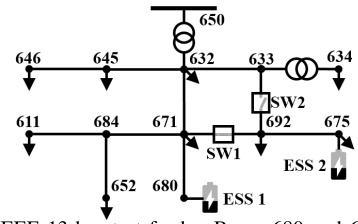

Figure 3. IEEE 13-bus test feeder. Buses 680 and 675 have ESS.

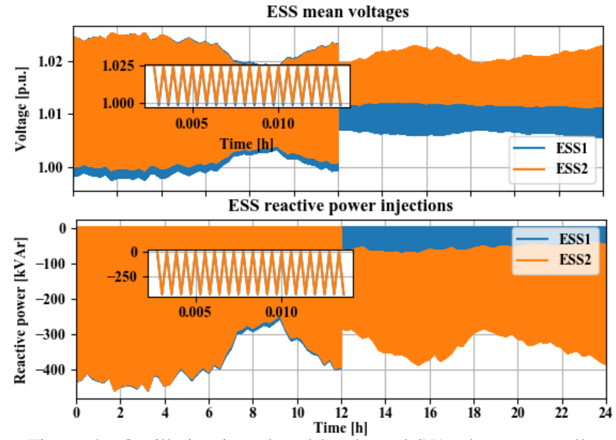

Figure 4. Oscillation introduced by the volt/VAr droop controller.

IV. Case Study

The voltage controller was validated in a simulation implemented in OpenDSS with a timestep of 1 second over 24 hours. A normally open switch (SW2) was added between buses 633 and 692. Halfway through the simulation SW1 opens and SW2 closes, which changes the reactance of the system. Two three-phase 600 kW/600kWh energy storage systems (ESS) were added to the unbalanced IEEE 13-bus test feeder's buses 680 and 675, as shown in Fig. 3. These ESS provide volt/VAr regulation to the feeder, so that voltage fluctuations caused by time-varying loads are mitigated.

The voltage-reactive power controller settings were chosen within the range of allowable settings of standard IEEE 1547-2018. Similarly to what is shown in the motivational example of [17], an oscillation in the output of the volt/VAr droop controller is found, shown in Fig. 4. The controllers saturate at every time step and introduce an oscillation in power flows and voltage of the distribution feeder. The detail plot in Fig. 4 provides a clearer idea of how those oscillations occur.

The proposed controller (8) was designed using $\bar{K} = diag\{10000, 1000\}$, which resulted in $K < 0$. The parameters of the adaptive controller were chosen as $a_i = 0.1$, $\omega_i = \pi/2$, $\gamma_i = 0.027$, $h = 0.99$. The controller stabilizes the voltages, with the power injections also shown in Fig. 5.

The ESC provides the desired controller outputs, $u^\star$ shown in Fig. 5. Following a brief transient, when these controllers obtain a stable $u^\star$, which provides the first term at the right hand side of (8). When added to the second term of the same equation, we obtain the controller outputs $\hat{u}^\star$. We notice that those act to counter the voltage fluctuations in the feeder, providing good voltage regulation by maintaining voltage very close to the target of $1pu$. It is also important to highlight that the parameters chosen plus the actuation of the adaptive

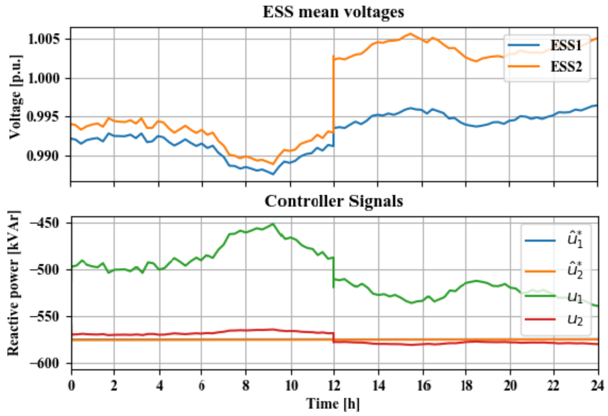

Figure 5. ESS voltages and power injections from the proposed controller.

controller avoid saturation of the actuator, i.e., the reactive power absorption capability of the two ESS units.

V. Conclusion

We provided a dissipativity based adaptive controller for decentralized control of reactive power from DERs in the distribution grid, where the reactance matrix of the grid is unknown and may even be time-varying. The controller effectiveness was demonstrated through simulations. The effectiveness of the controller is currently limited by the saturation of the actuators. Therefore, more ESS capacity would be necessary to further reduce voltage fluctuations. Future work includes extending this approach to distributed control of DERs.

Appendix

Consider the discrete-time nonlinear state-space system

$$\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= h(x(k), u(k)) \end{aligned} \tag{25}$$

Let the nonlinear functions f and h be real analytic about the equilibrium point for (25), $(x = 0, u = 0)$. x, u, and y are the state, input, and output vectors, respectively.

Definition (dissipativity [23]): (25) is dissipative with respect to the supply rate $w(u(k), y(k))$, if there exists a non-negative storage function $S(x)$ satisfying $S(0) = 0$ such that

$$S(x(k+1)) - S(x(k)) \leq w(u(k), y(k)). \tag{26}$$

In particular, we call the system passive if $w(u(k), y(k))$ is of the form $u^\top(k)y(k)$.

Acknowledgment

The authors would like to thank Dr. Imre Gyuk, Director of the Energy Storage Program, for his continued support. The authors would also like to thank Dr. Tu Nguyen and Dr. Felipe Wilches-Bernal for their technical contributions to the paper.

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