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Fluid-shell interactions using non-intrusive coupling based on the Immersed Finite Element Method

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Fluid-structure involving thin solid

In Nature: insect flapping wings **[1]**

Engineering : thin-shell roof structure **[2]**

[1] https://en.wikipedia.org/wiki/Dragonfly
[2] https://en.wikipedia.org/wiki/Thin-shell_structure

Introduction

FSI simulation for thin structures

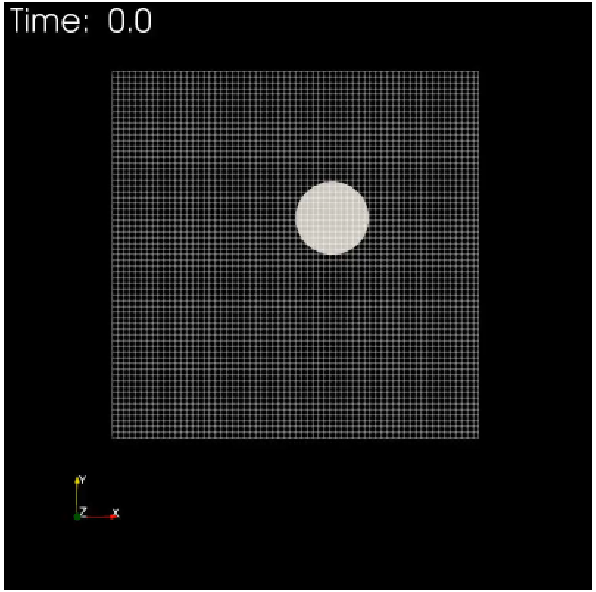
Boundary conforming mesh

- **Arbitrary Lagrangian-Eulerian**
  - Sawada et al. (2007) [1]
  - Expensive mesh update for large deformation

Boundary non-conforming mesh

- **Immersed Boundary Method**
  - Peskin (2002) [2]
  - Solid and fluid domain: independent meshes
  - Cirak et al. (2007) [3], Gilmanov et al. (2005) [4], Tian et al. [5]
    - Lacks volume representation of solid
    - Smeared interface
    - Inaccurate solid interface loading conditions accounting for both forces and moments

Objective



- **Objective**:
  - Develop a **robust fluid-shell coupling** strategy using immersed approach.
- **Modified Immersed Finite Element Method (mIFEM) [1]:**
  - **Volume based interpolation** and **sharp interface**.
- **OpenIFEM [2], [3]:** Open source software based on mIFEM
  - Modularly couple OpenIFEM with external shell solver.

[1] Xingshi Wang and Lucy T. Zhang. Modified immersed finite element (2013)
[2] Cheng J et al., OpenIFEM: A high performance modular open-source software fluid -structure interactions. (2019)
[3] **https://github.com/OpenIFEM**

virtual work done by the real fluid | virtual work done by the solid

$$\int_{\Omega^{\mathrm{f}}} \delta v_i \left(\rho^{\mathrm{f}} \frac{\mathrm{D}v_i^{\mathrm{f}}}{\mathrm{D}t} - \sigma^{\mathrm{f}}_{ij,j} - \rho^{\mathrm{f}} g_i\right) d\Omega + \int_{\Omega^{\mathrm{s}}} \delta v_i \left(\rho^{\mathrm{s}} \frac{\mathrm{D}v_i^{\mathrm{s}}}{\mathrm{D}t} - \sigma^{\mathrm{s}}_{ij,j} - \rho^{\mathrm{s}} g_i\right) d\Omega = 0$$

$$\int_{\bar{\Omega}} \delta v_i \left(\rho^{\mathrm{s}} \frac{\mathrm{D}v_i^{\mathrm{f}}}{\mathrm{D}t} - \sigma^{\mathrm{f}}_{ij,j} - \rho^{\mathrm{s}} g_i\right) d\Omega + \int_{\Omega^{\mathrm{s}}} \delta v_i \left(\rho^{\mathrm{s}} \left(\frac{\mathrm{D}v_i^{\mathrm{s}}}{\mathrm{D}t} - \frac{\mathrm{D}v_i^{\mathrm{f}}}{\mathrm{D}t}\right) - \left(\sigma^{\mathrm{s}}_{ij,j} - \sigma^{\mathrm{f}}_{ij,j}\right)\right) d\Omega$$

$$\bar{\rho} = \begin{cases} \rho^{\mathrm{s}} & \text{in} \quad \Omega^{\mathrm{s}} \\ \rho^{\mathrm{f}} & \text{in} \quad \Omega^{\mathrm{f}} \end{cases}$$

$$-F_i^{\mathrm{FSI}}$$

$$\int_{\Omega} \delta v_i \left(\bar{\rho} \frac{\partial v_i^{\mathrm{f}}}{\partial t} + \bar{\rho} v_j^{\mathrm{f}} v^{\mathrm{f}}_{i,j} - \sigma^{\mathrm{f}}_{ij,j} - \bar{\rho} g_i\right) d\Omega = \int_{\Omega^{\mathrm{s}}} \delta v_i F_i^{\mathrm{FSI}} d\Omega$$

mIFEM Governing Equations

solid

Lagrangian

$$\rho^s u_{,tt} = \sigma^s_{ij} \quad \text{in } \Omega^s$$

$$u_i = q_i = v_i^j \Delta t \quad \text{on } \Gamma^{sq}$$

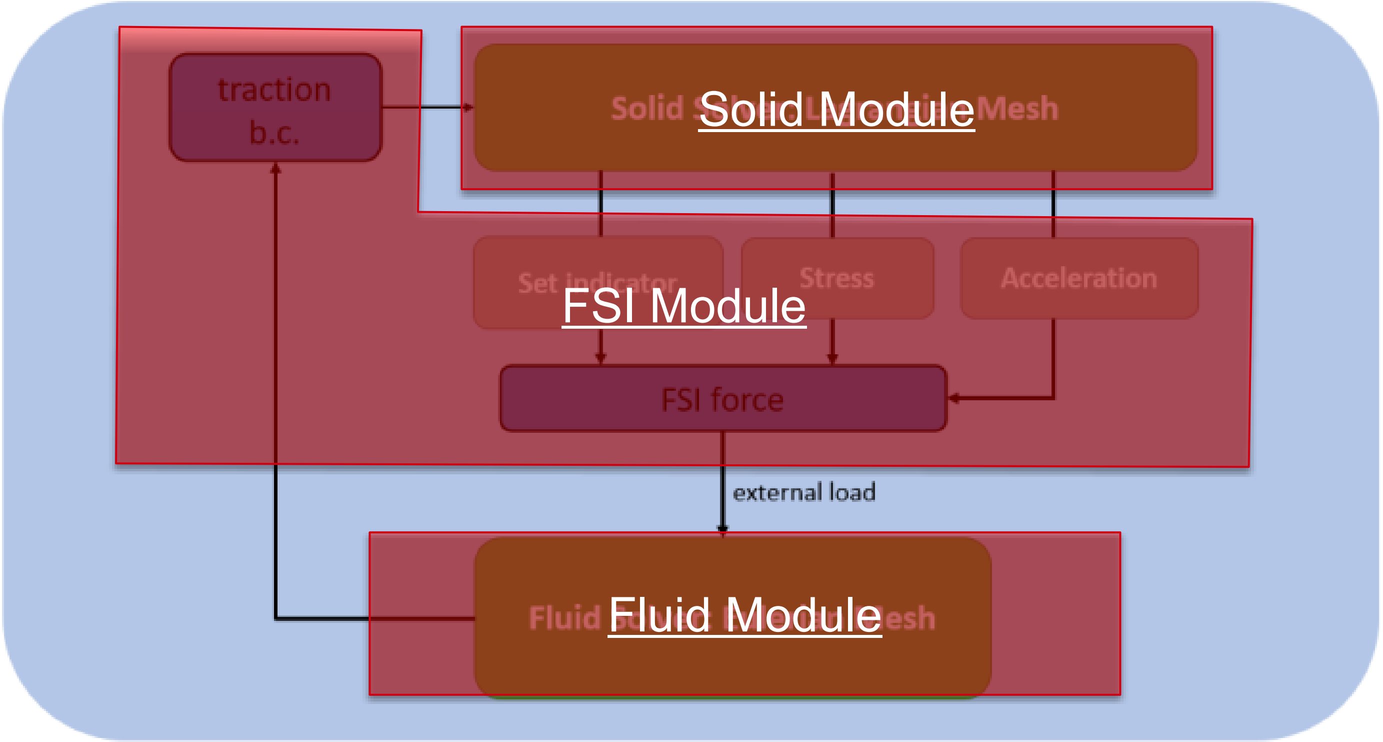
$$\sigma^s_{ij} n_j = h_i = -\sigma^f_{ij} n_j \quad \text{on } \Gamma^{sh}$$

fluid

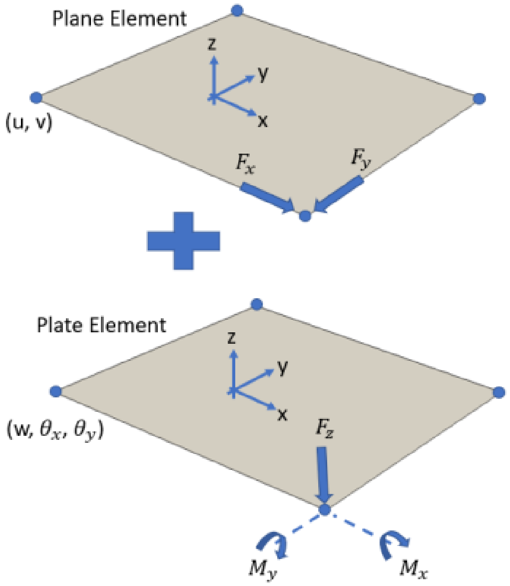
Eulerian

$$\bar{\rho}\left(\frac{\partial v_i^{f}}{\partial t} + v_j^{f} v_{i,j}\right) = \sigma^{f}_{ij,j} + f_i^{FSI}(\mathbf{x}, t)$$

mIFEM Workflow

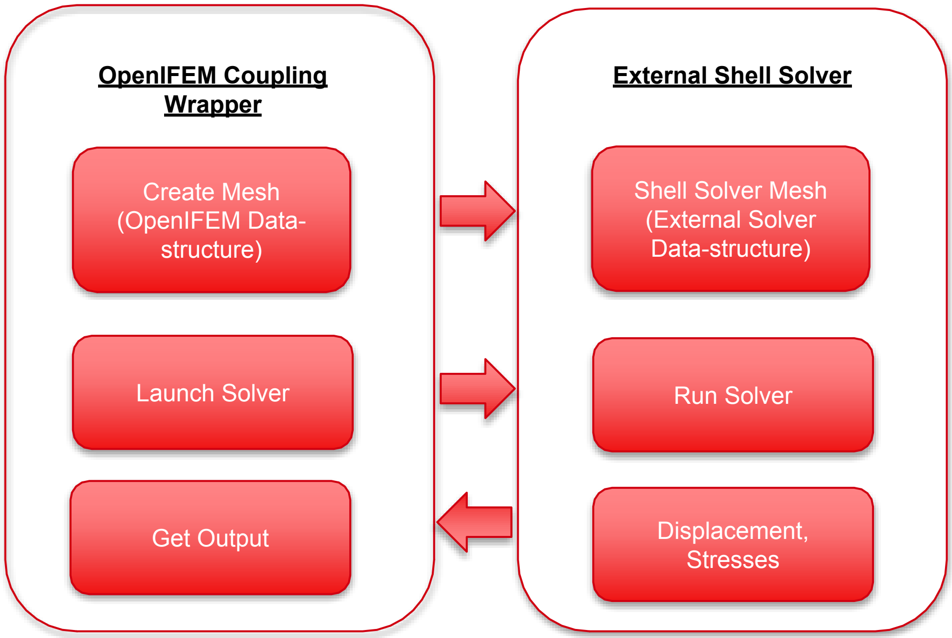


Shell Solver



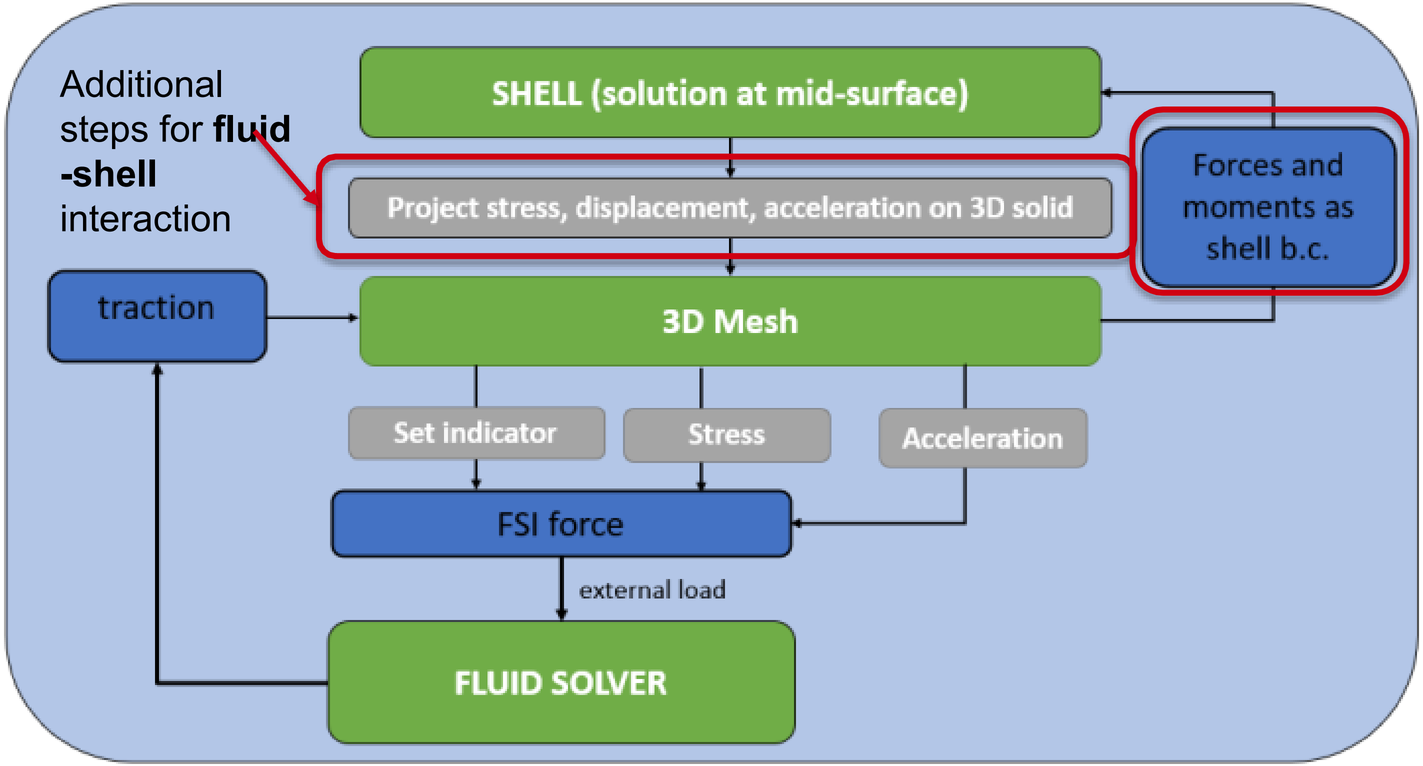
- External shell solver [4]: **flat shell theory**
- Shell Element: **6 d.o.f.** : (u, v, w, θ_x, θ_y, θ_z)

Coupling Wrapper Workflow

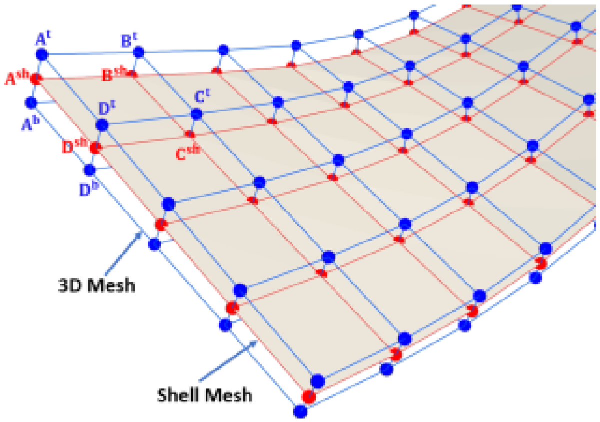


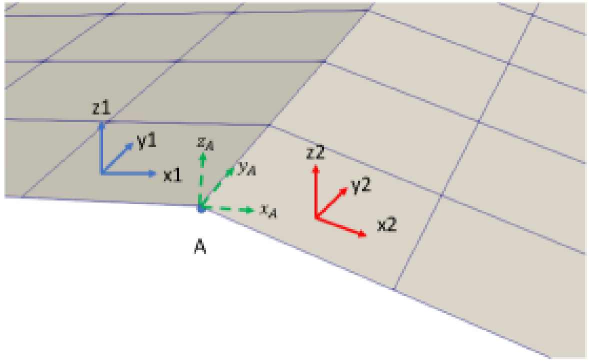
[4] https://github.com/precice/fem-shell

mIFEM for shell



Project solution on 3D solid





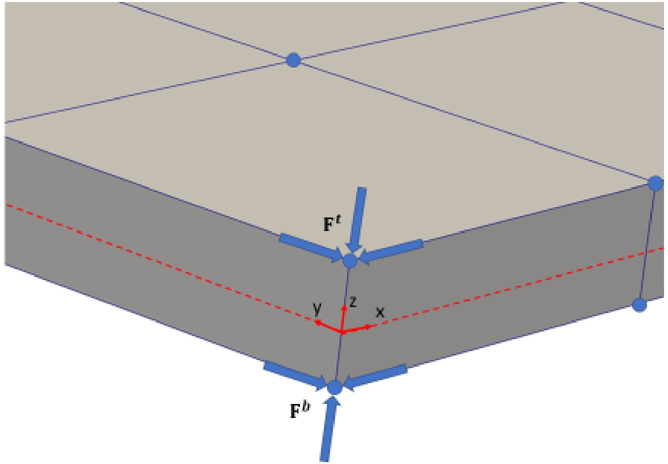
Local coordinate system

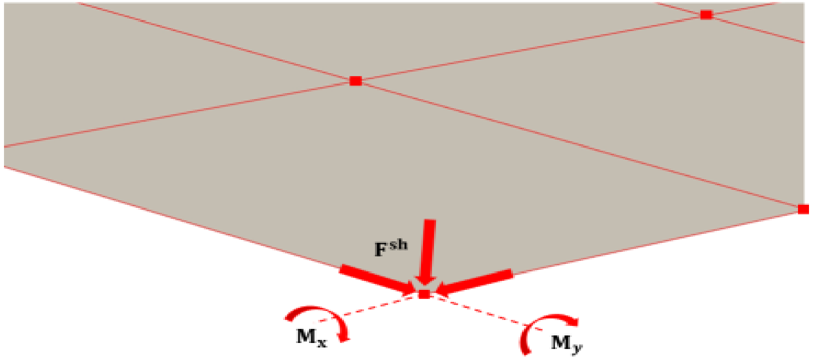
- Define Local coordinate system for each node.
- Local coordinate system at node A:
- $x_A = \frac{x1+x2}{||x1+x2||} \quad y_A = \frac{y1+y2}{||y1+y2||} \quad z_A = \frac{z1+z2}{||z1+z2||}$
- Transformation matrix: $T_A = \begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix}$

Project solution to 3D mesh

- Create 3D solid mesh and extrapolate solution from shell to 3D mesh to appropriate nodes:
  - $(u, v, w, \theta_x, \theta_y, \theta_z)_{global} \rightarrow (u, v, w, \theta_x, \theta_y, \theta_z)_{local}$
  - $U_{local} = u_{local} - z * (\theta_y)_{local}$ $-t/2 \le z \le t/2$
  - $V_{local} = v_{local} - z * (\theta_x)_{local}$
  - $W_{local} = w_{local}$
  - $(U, V, W)_{local} \rightarrow (U, V, W)_{global}$
- Similarly project velocity, acceleration and stress.

Evaluate forces and moments onto shell surface

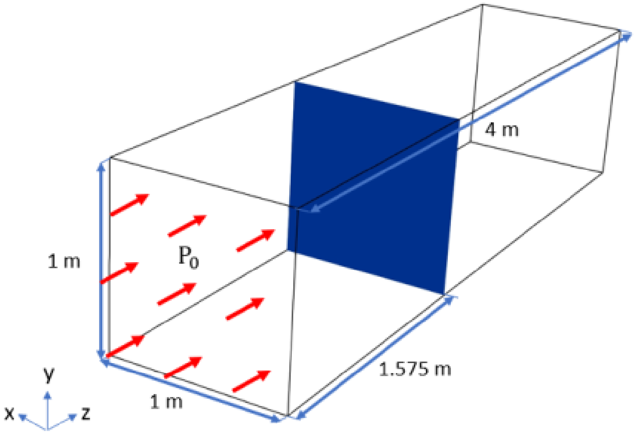




- Evaluate solid traction from fluid stress: $t = [-p\mathbf{I}+\mu(\nabla \mathbf{v}+\nabla \mathbf{v}^T)].\mathbf{n}$
- Calculate nodal forces on volume (two sides)

- **Equivalent forces on shell node: $F^{sh} = F^b + F^t$**
- **Equivalent Moments on shell node:**
  - Transfer force into local coordinates
  - $M_y = F_x^t * \frac{t}{2} - F_x^b * \frac{t}{2}$
  - $M_x = -F_y^t * \frac{t}{2} + F_y^b * \frac{t}{2}$
  - Transfer moments into global coordinates

Test Cases 1: Setup



- Shell Dimensions: 1 m x 1m
- **Shell thickness: 0.05 m**

- **Objective:** Verify that FSI coupling**, no numerical leaking**

Material Properties:

- Fluid Properties:
  - Viscosity: $1.8 * 10^{-5}$ Pa.s
  - Density : $1.3\ \mathrm{kg/m^3}$
- Solid Properties:
  - Young's Modulus : 10^4 Pa
  - Poisson's Ration: 0.3
  - Density : $1000.0\ \mathrm{kg/m^3}$

Boundary Conditions:

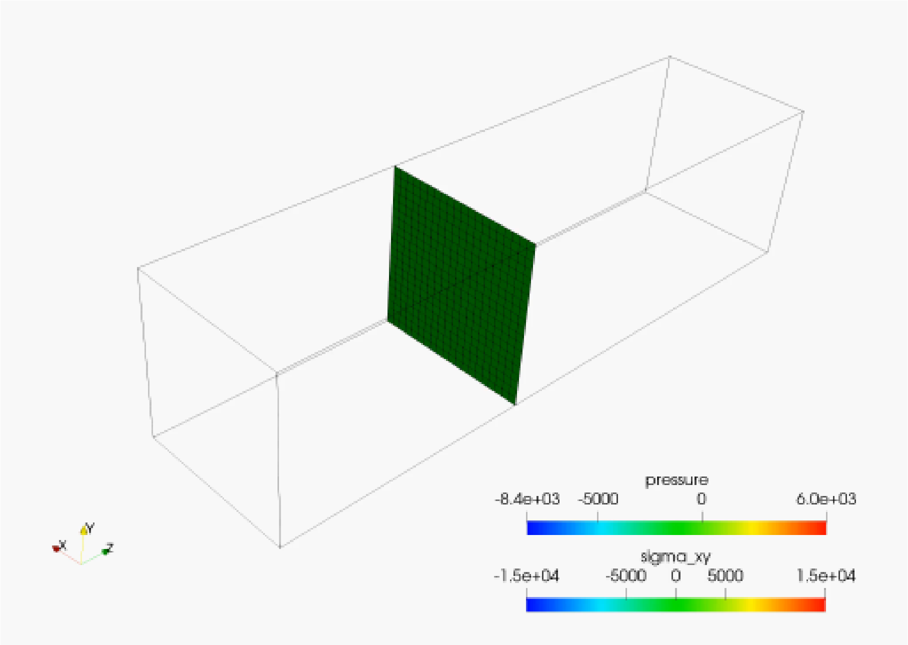
- Fluid:
  - left (inlet) : constant pressure inlet P_0 =500 Pa
  - right boundary : outflow
  - Other boundaries: no penetration
- Solid:
  - All sides clamped.

Time Step:

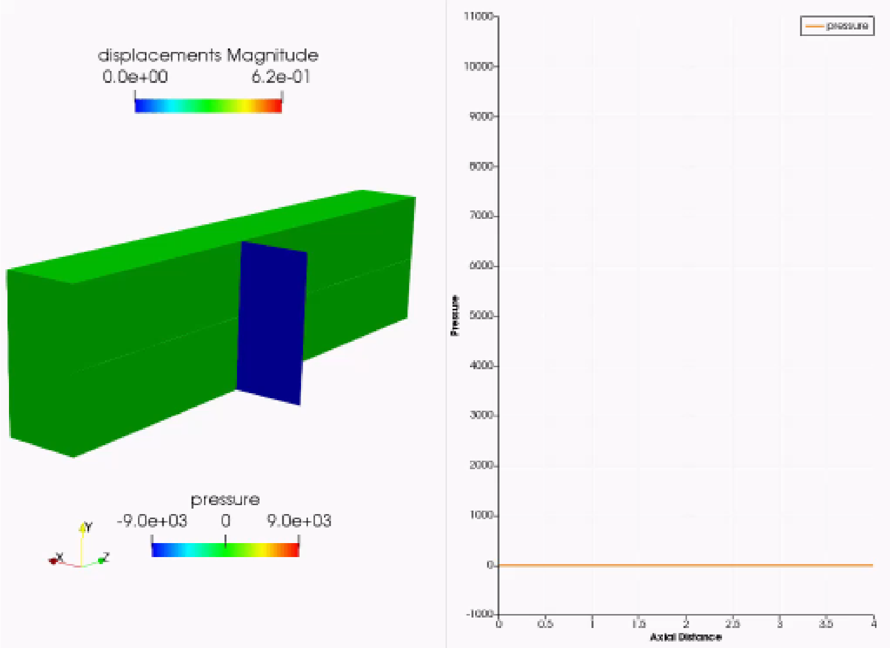
- Time Step: 10^{-6} s
- Final Time: $6 * 10^{-3}$ s

Test Case 1: Results

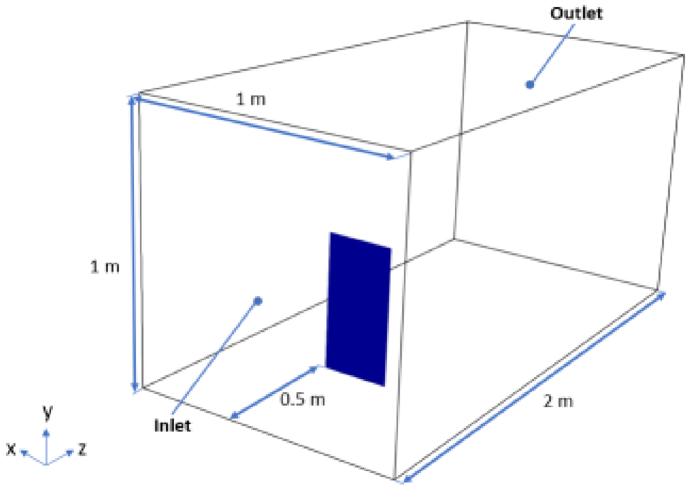
Flow streamlines coloured by pressure (Pa) and solid stress σ_{xy} on top surface (Pa)



Cross section of fluid domain and pressure (Pa) vs. time (s) along centreline



Test Cases 2: Setup



- Shell Dimensions: 0.5 m x 0.25m
- **Shell thickness: 0.0125 m**
- **Shell thickness: 0.00625 m**
- **Shell thickness: 0.003125 m**

- **Objective:** Demonstrate FSI coupling for problem with **large deformation**.

Material Properties:

- Fluid Properties:
  - Viscosity: $1.8 * 10^{-5}$ Pa. s
  - Density : $1.3\ \mathrm{kg/m^3}$
- Solid Properties:
  - Young's Modulus : 10^3 Pa
  - Poisson's Ration: 0.4
  - Density : $5000.0\ \mathrm{kg/m^3}$
- Re: 20

Boundary Conditions:

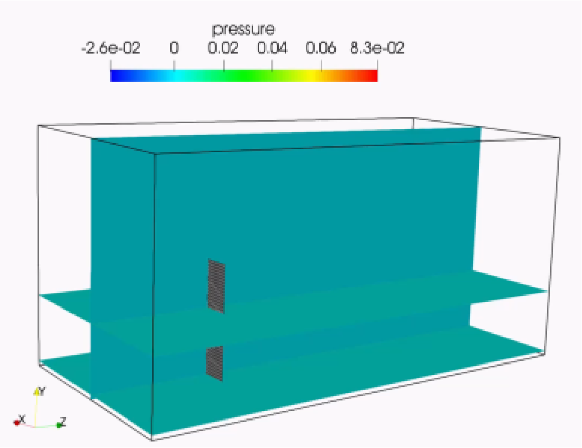
- Fluid:
  - left (inlet) : constant velocity inlet $\boldsymbol{V_0} = \mathbf{0.05}\ \mathrm{m}/s$
  - right boundary : outflow
  - bottom: no slip
  - front, back, top: no penetration
- Solid:
  - Bottom side clamped.

Time Step:

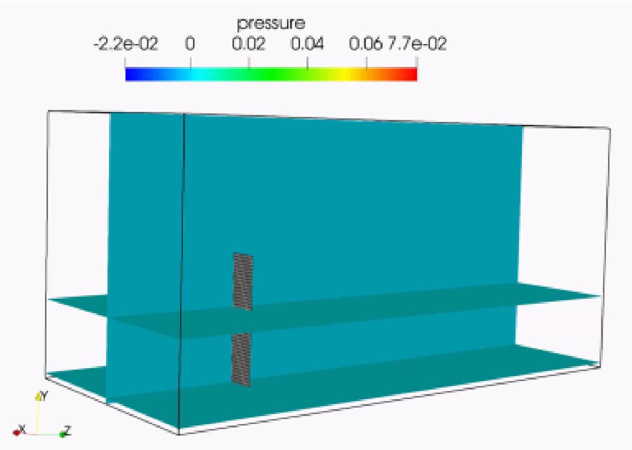
- Time Step: $2 * 10^{-4}$ s
- Final Time: 1 s

Test Cases 2: Result

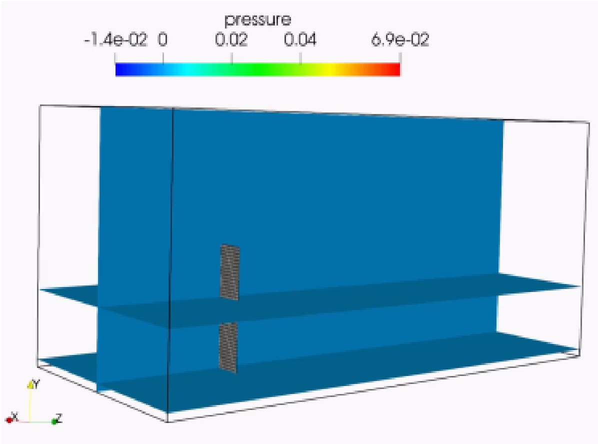
Fluid pressure contours (Pa) at various cross sections



Thickness: 0.0125 m



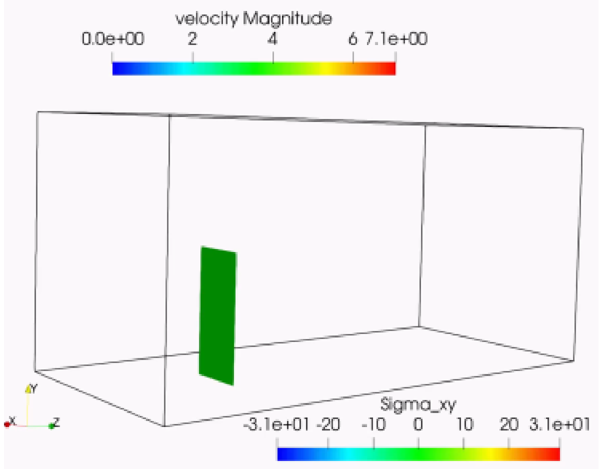
Thickness: 0.00625 m



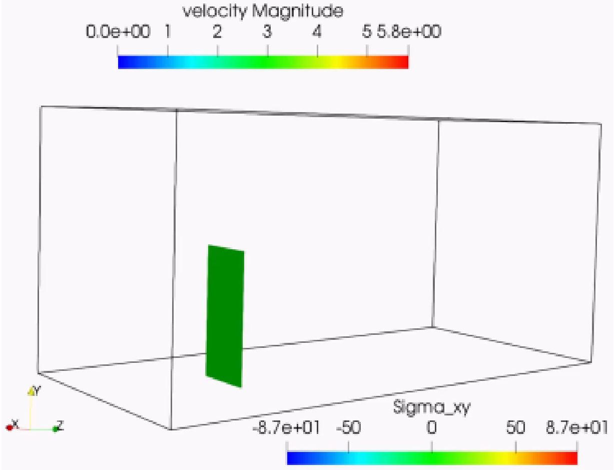
Thickness: 0.003125 m

Test Cases 2: Result

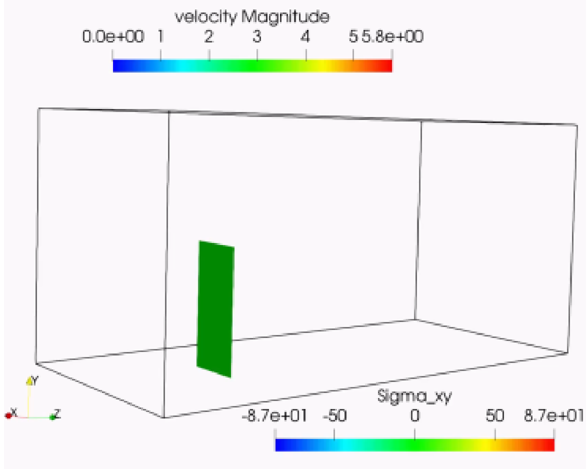
Flow streamlines coloured by velocity (m/s) and and solid stress σ_{xy} on top surface (Pa)



Thickness: 0.0125 m

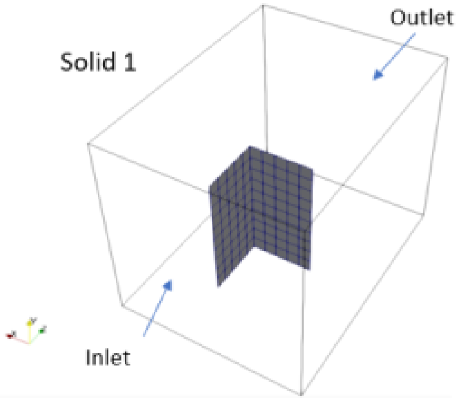


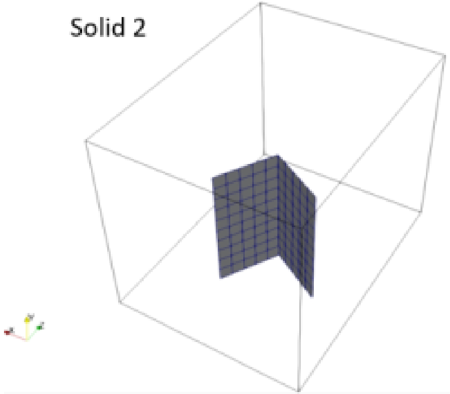
Thickness: 0.00625 m



Thickness: 0.003125 m

Test Cases 3: Setup





- **Objective:** Demonstrate that the solver can handle **shapes with bends** or shells in **different planes**. Demonstrate use of **local coordinate system**.

Material Properties:

- Fluid Properties:
  - Viscosity: $1.8 * 10^{-5}$ Pa.s
  - Density : $1.3\ \mathrm{kg/m^3}$
- Solid Properties:
  - Young's Modulus : 10^3 Pa
  - Poisson's Ration: 0.4
  - Density : $5000.0\ \mathrm{kg/m^3}$

Problem Setup:

- Fluid: 1.5 m x 1.5 m x 2m
- Solid:
  - **0.5 m x 1 m rectangles intersect at right angle**
  - **Thickness: 0.0125 m**

Boundary Conditions:

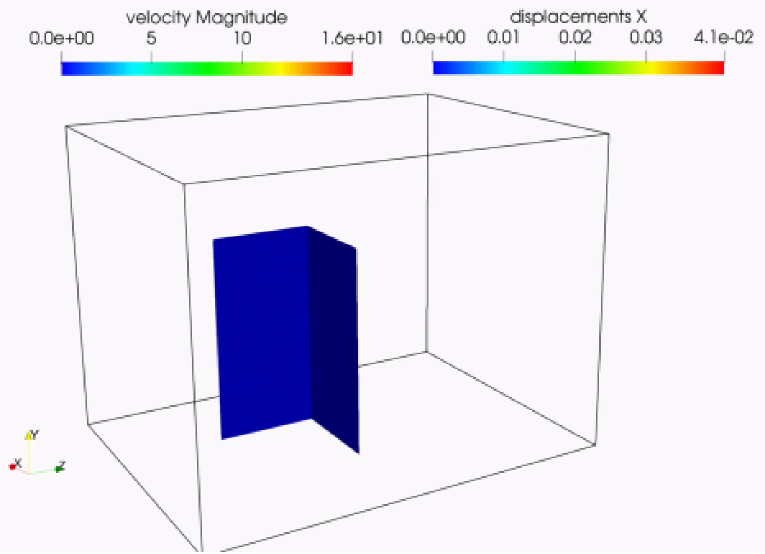
- Fluid:
  - left (inlet) : constant velocity inlet $\boldsymbol{V_0} = \mathbf{0.1}\ \mathrm{m}/s$
  - right boundary : outflow
  - bottom: no slip
  - front, back, top: no penetration
- Solid:
  - Bottom side clamped.

Time Step:

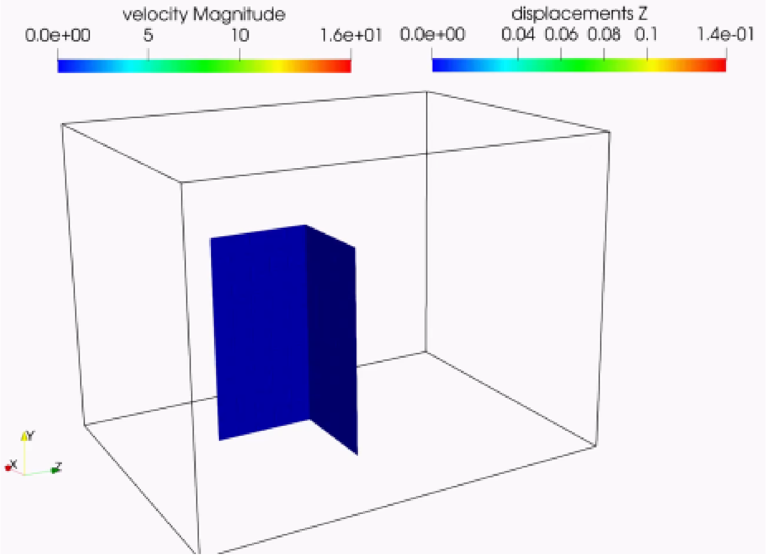
- Time Step: $2 * 10^{-4}$ s
- Final Time: 1 s

Test Cases 3 : Results

Solid 1 (90^0)



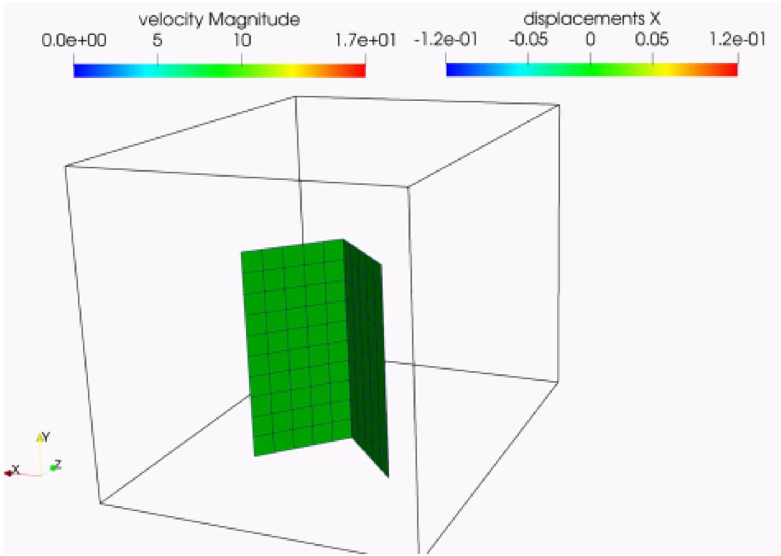
Flow streamlines coloured by velocity (m/s) and solid **displacement in x direction** (m)



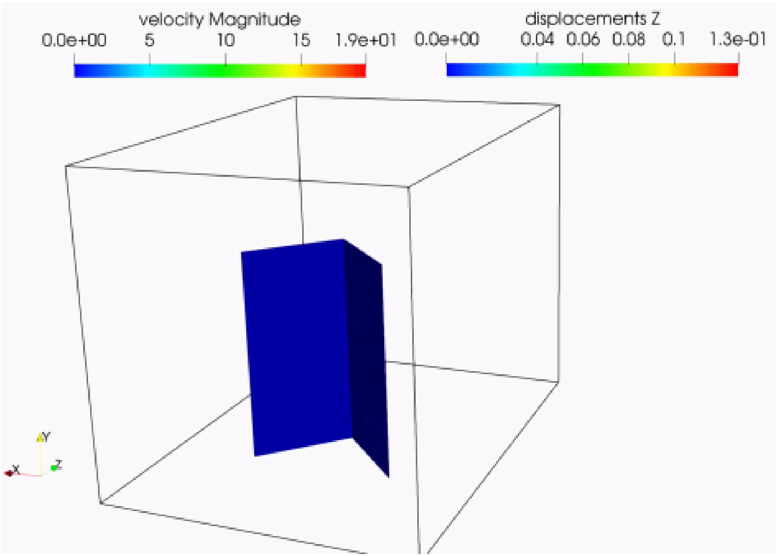
Flow streamlines coloured by velocity (m/s) and solid **displacement in z direction** (m)

Test Cases 3 : Results

Solid 2 (45^0)



Flow streamlines coloured by velocity (m/s) and solid **displacement in x direction** (m)



Flow streamlines coloured by velocity (m/s) and solid **displacement in z direction** (m)

Summary

- **Modularly** couple an external shell solver with OpenIFEM.

- **Extend** OpenIFEM capabilities for shell structures.
  - Represent thin shell with given finite thickness
  - Realistic solid loading which accounts for both forces and moments
  - Adoptable for different shell formulations

- OpenIFEM can handle **both general 3D bodies and different types of shells**.

Acknowledgment

➢ Funding and Resources:

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➢ OpenIFEM :

- Feimi Yu for developing coupling wrapper for shell solver.

Refrences

[1] Tomohiro Sawada and Toshiaki Hisada. Fluid–structure interaction analysis of the two-dimensional flag-in-wind problem by an interface-tracking ale finite element method.Com-puters & Fluids, 36(1):136–146, 2007

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