

Risk-Averse Investment Optimization for Power System Resilience to Winter Storms

Manuel Garcia[†], Brent Austgen^{*}, Brian Pierre[†], John Hasenbein^{*}, and Erhan Kutanoglu^{*}

^{*}*Operations Research & Industrial Engineering
Cockrell School of Engineering
The University of Texas at Austin
Austin, USA
brent.austgen@utexas.edu*

[†]*Electric Power Systems Research
Sandia National Laboratories
Albuquerque, USA
mgarc19@sandia.gov*

Abstract—We propose a two-stage scenario-based stochastic optimization problem to determine investments that enhance power system resilience. The proposed optimization problem minimizes the Conditional Value at Risk (CVaR) of load loss to target low-probability high-impact events. We provide results in the context of generator winterization investments in Texas using winter storm scenarios generated from historical data collected from Winter Storm Uri. Results illustrate how the CVaR metric can be used to minimize the tail of the distribution of load loss and illustrate how risk-aversity impacts investment decisions.

Index Terms—Stochastic Optimization, Risk Metrics, Conditional Value at Risk, Power System Resilience

I. INTRODUCTION

Natural disasters such as hurricanes, fires, and winter storms are increasing in frequency, motivating investment in power system resilience. Winter Storm Uri, which occurred in February 2021, was particularly devastating to the state of Texas and, at its peak, resulted in approximately 10 million Texans losing access to electricity [1]. Texas now faces the task of overseeing winterization investments. These low-probability high-impact events are difficult to assess in risk-averse decision-making. This two-part paper addresses this problem in the context of winter storm planning. Part I quantifies the uncertainty of a winter storm by creating probabilistic scenarios based on historical data [2]. This paper represents Part II and uses the scenarios to formulate and solve a risk-averse scenario-based two-stage stochastic optimization problem.

Although the definition of power system resilience is not established precisely, previous work suggests that it should include the ability to prepare for and withstand low-probability high-impact events [3–5]. Furthermore, various risk metrics exist that specifically target low-probability high-impact events [6]. For example, reference [5] suggests possible risk measures to characterize resilience in a probabilistic

This work was supported by the Laboratory Directed Research and Development program at Sandia National Laboratories and the Energy Institute at the University of Texas at Austin. Sandia National Laboratories is a multimission laboratory managed, and operated by the National Technology and Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under Contract DE-NA0003525.

setting such as Value at Risk, Conditional Value at Risk (CVaR), and maximum value. However, no previous works in investment optimization for power system resilience have utilized these risk metrics.

Previous works on the topic of power system investment optimization to improve resilience use an objective function that represents the expected value of some loss quantity, which usually represents load loss. Indeed the expected value metric is often referred to as risk-neutral and does not focus on the low-probability high-impact events [6]. Reference [7] provides a scalable stochastic optimization method that considers proactive redispatch and transmission line hardening to mitigate resilience. Reference [8] analyzes the impact of considering the AC power flow equations in the restoration stage of the investment optimization problem. Reference [9] accounts for initial system transients as well as long term restoration in response to an event.

This paper proposes a risk-averse investment optimization problem that minimizes the CVaR metric. Intuitively, the ϵ -CVaR of the loss represents the expected value of the loss given that the loss lies above the $1 - \epsilon$ quantile [10]. As compared to other risk metrics we choose to incorporate CVaR because it has been demonstrated to exhibit favorable computational and probabilistic properties [11]. CVaR is also easily adjustable by varying the parameter ϵ , which intuitively specifies the size of the tail to be optimized.

Accurately representing the CVaR metric requires thorough sampling of the tails of the loss distribution and so our proposed problem must be scalable to accommodate a large number of scenarios. To facilitate efficient computation we do not include the novel features suggested in the aforementioned references, which are computationally burdensome, allowing us to accommodate thousands of scenarios. This will be done in future work while maintaining scalability using progressive hedging methods similar to those used in [7] and using importance sampling techniques to reduce the number of scenarios required to accurately represent the CVaR metric.

This paper is organized as follows. Section II explains how to incorporate the CVaR metric into a general scenario-based two-stage stochastic investment optimization problem. Section

III provides a specific formulation that decides winterization investments to target winter storms using scenarios sampled from [2]. Section IV provides results illustrating the risk-aversity of our formulation using out-of-sample scenarios.

II. GENERAL FORMULATION

This section presents a general risk-averse two-stage stochastic investment optimization problem that incorporates the CVaR metric into the objective function. A practical reformulation of this optimization problem is also provided.

Notation is established separately from the companion paper [2]. In this paper we denote the interval of integers between a and b as $[a, b]$. The n -dimensional set of real numbers and non-negative real numbers are denoted \mathbb{R}^n and \mathbb{R}_+^n . A random variable that represents a general disturbance to the system is denoted \mathbf{e} and a specific realization of this random variable is denoted e . The expected value taken with respect to the distribution of \mathbf{e} is denoted $\mathbb{E}_{\mathbf{e}}$ and the ϵ -CVaR taken with respect to the distribution of \mathbf{e} is denoted $\text{CVaR}_{\mathbf{e}}^{\epsilon}$. The i^{th} element of a vector v is v_i and the transpose is v^{\top} .

A. General Two Stage Investment Optimization

The first stage represents the investment stage and determines the investment decisions years in advance of a winter storm. The second stage represents the restoration stage and represents the response to a winter storm.

1) *Investment Stage*: The investment stage chooses investment decisions denoted $x \in \mathcal{X}$, where \mathcal{X} represents constraints on the investments such as budget constraints. The investments depend on a random variable \mathbf{e} that represents a general disturbance to the system and is distributed over a discrete support set of $[1, E]$. We denote the associated probability mass function as $\phi(e)$, which is a scalar value that represents the probability that $\mathbf{e} = e$.

To focus on low-probability high-impact events, we choose to minimize the ϵ -Conditional Value at Risk metric of a loss function $\tilde{\ell}(x, \mathbf{e})$ taken with respect to the distribution of \mathbf{e} . We denote this metric as $\text{CVaR}_{\mathbf{e}}^{\epsilon}[\tilde{\ell}(x, \mathbf{e})]$. Intuitively, this metric represents the expected value of the loss function given that the loss lies above the $1 - \epsilon$ quantile. This metric focuses on the right tail of the distribution of losses and is further explained in [10], [11]. Furthermore, if we choose $\epsilon = 1$, then the resulting $\text{CVaR}_{\mathbf{e}}^1$ metric is equivalent to the expected value. The investment stage optimization problem is as follows:

$$\min_{x \in \mathcal{X}} \text{CVaR}_{\mathbf{e}}^{\epsilon}[\tilde{\ell}(x, \mathbf{e})], \quad (1)$$

where the *first stage loss function* $\tilde{\ell}(x, \mathbf{e})$ represents the optimal objective value of an optimization problem in the restoration stage.

2) *Restoration Stage*: The restoration stage is modeled as a general optimal power flow problem. During this stage the random disturbance has already been realized. For this reason the restoration stage is deterministic and is as follows:

$$\tilde{\ell}(x, e) = \min_{y \in \mathcal{Y}(x, e)} \ell(y, e). \quad (2)$$

The *second stage loss function* is denoted $\ell(y, e)$ and is a function of the restoration decision variables y and the realized random variable e . The restoration decision variables y fall in a feasible set denoted $\mathcal{Y}(x, e)$, which is a function of the investment decision x and the realized random variable e .

B. Practical Reformulation

Reference [10] provides an expression for CVaR that allows the objective function in (1) to be expressed as follows:

$$\text{CVaR}_{\mathbf{e}}^{\epsilon}[\tilde{\ell}(x, \mathbf{e})] = \inf_{\beta \in \mathbb{R}} \beta + \frac{1}{\epsilon} \mathbb{E}_{\mathbf{e}}[\max\{\tilde{\ell}(x, \mathbf{e}) - \beta, 0\}]. \quad (3)$$

Since the random variable \mathbf{e} is discretely distributed among the integers $[1, E]$, we can write out the expectation in (3) explicitly. Furthermore, placing the expression (3) back into (1) allows for the *inf* and *min* to be combined and yields the following investment optimization problem:

$$\min_{x \in \mathcal{X}, \beta \in \mathbb{R}} \beta + \frac{1}{\epsilon} \sum_{e=1}^E \phi(e) \max\{\tilde{\ell}(x, e) - \beta, 0\}. \quad (4)$$

We now arrive at the following practical reformulation by introducing variables $t \in \mathbb{R}_+^E$ that intuitively represent the values $\max\{\tilde{\ell}(x, e) - \beta, 0\}$, by replacing the first stage loss function $\tilde{\ell}(x, e)$ with the expression in (2), and by denoting the restoration decision variables for each $e \in [1, E]$ as y_e :

$$\min_{\substack{x \in \mathcal{X}, \beta \in \mathbb{R}, t \in \mathbb{R}_+^E \\ y_e \in \mathcal{Y}(x, e) \quad \forall e \in [1, E]}} \beta + \frac{1}{\epsilon} \sum_{e=1}^E \phi(e) t_e \quad (5a)$$

$$\text{st: } \ell(y_e, e) - \beta \leq t_e \quad \forall e \in [1, E]. \quad (5b)$$

This is a practical implementation of the CVaR objective because the objective function is now linear in the decision variables and the constraint (5b) is convex in decision variables under the assumption that $\ell(y_e, e)$ is convex in y_e ,

III. SPECIFIC FORMULATION

This section provides a specific example of the general formulation. In this example, the investment decisions represent the winterization of generators, making them immune to any winter storm. The following subsections define specific sets \mathcal{X} and $\mathcal{Y}(x, e)$, the second stage loss function $\ell(y_e, e)$, and the random variable \mathbf{e} . These definitions result in a risk-averse investment optimization problem (5) that represents a Mixed-Integer Linear Program.

A. Probabilistic Model of a Winter Storm

Part I of this paper [2] constructed a probability distribution that represents a random winter storm. This probability distribution is used in the investment stage and represents a distribution of generator outages for the next winter storm. As a result, the investment optimization problem aims to improve resilience to the next winter storm that will occur.

Part I of this paper also provided a method of sampling the proposed probability distribution to determine scenarios of relative capacity loss for each generator during a winter

storm. In this section we empirically construct the distribution of generator outages by evenly weighting the samples provided by Part I of this paper. In this context the random variable e represents the random index of a winter storm scenario, where the scenarios are indexed $[1, E]$ and each scenario will occur with probability $\phi(e) := \frac{1}{E}$. This type of distribution is often referred to as an empirical distribution.

We will denote the total number of generators by g and the generators will be indexed by $[1, g]$. Each scenario $e \in [1, E]$ will correspond to a set of relative capacity loss fractions $\alpha_{e,j}$ for each generator $j \in [1, g]$, where $0 \leq \alpha_{e,j} \leq 1$ and g is the total number of generators in the network. Under scenario e each generator j is limited to produce at most $1 - \alpha_{e,j}$ of its typical generation capacity. (Note: the relative capacity loss $\alpha_{e,j}$ is denoted $x_{g,\omega}$ in reference [2]).

B. Investment Set \mathcal{X}

The vector $x \in \{0, 1\}^g$ will represent integer variables where x_j is one if generator j is chosen for winterization investment, protecting it against any possible winter storm. The investment cost for each generator is denoted $c \in \mathbb{R}^g$, where the cost of winterizing generator j is c_j . We will be constrained by a budget for investments. The budget is denoted b and the set of feasible investments is written as follows:

$$\mathcal{X} = \{x \in \{0, 1\}^g \mid c^\top x \leq b\}. \quad (6)$$

C. Restoration Set \mathcal{Y}

The restoration decisions for each scenario $e \in [1, E]$ are denoted $y_e = L_e$ where $L_e \in \mathbb{R}_+^n$ represents the load loss at each bus. The restoration stage is modeled using DC Optimal Power Flow (OPF). We introduce intermediate variables $G_e \in \mathbb{R}_+^g$, which represent the generation for each generator. We denote the total number of buses by n and the buses are indexed by $[1, n]$. The matrix M is a sparse matrix used to map generators to buses. Only one element in each row of $M \in \mathbb{R}^{n \times g}$ is non-zero and is equal to 1. Generator j is located on bus i if $M_{ji} = 1$. Furthermore, $D_e \in \mathbb{R}^n$ represents the fixed demand at each bus. The matrix of shift factors is denoted S and maps the net power injections to the power flow along each transmission line. The transmission line limits are denoted \bar{F} and \underline{F} . The generator capacities are denoted \bar{G} . The constraints defining the set $\mathcal{Y}(x, e)$ are as follows:

$$\mathbf{1}^\top (G_e + L_e - D_e) = 0 \quad (7a)$$

$$\underline{F} \leq S(MG_e + L_e - D_e) \leq \bar{F} \quad (7b)$$

$$0 \leq L_e \leq D_e \quad (7c)$$

$$0 \leq G_{e,j} \leq x_j \bar{G}_j + (1 - x_j)(1 - \alpha_{e,j})\bar{G}_j \quad \forall j \in [1, g]. \quad (7d)$$

Constraint (7a) represents the overall power balance constraint. Constraint (7b) represents the transmission line limits. Constraint (7c) places bounds on the load shed variables L . Constraint (7d) represents the generator limits. If a generator is invested in, then its capacity is \bar{G}_j . If a generator is not invested in, then its capacity is $(1 - \alpha_{e,j})\bar{G}_j \leq \bar{G}_j$.

The set $\mathcal{Y}(x, e)$ can now be formally defined as follows:

$$\mathcal{Y}(x, e) := \{y_e = L_e \in \mathbb{R}_+^n \mid \exists G_e \in \mathbb{R}_+^g \text{ st: (7a)-(7d)}\}. \quad (8)$$

Future work may consider alternative definitions of $\mathcal{Y}(x, e)$ and y_e that account for the AC transmission network.

D. Second Stage Loss Function

The loss function in the restoration stage will simply represent load shed. That is, the loss function in the second stage is defined as $\ell(y_e, e) = \mathbf{1}^\top L_e$. Future work may consider alternative loss functions that prioritize the shedding of certain loads through a weighted sum.

IV. NUMERICAL RESULTS

This section provides results from the risk-averse investment optimization problem explained in Section III using winter storm scenarios explained in the partner paper [2] and using the ACTIVSg2000 test case that intends to represent the Texas system and is created from publicly available data [12]. This test case has 432 active generators with a total generation capacity of 81,201 MW and a total demand of 53,964 MW.

The winter storm scenarios are sampled from the winter storm probability distribution explained in Part I of this paper [2]. Reference [2] explains how the relative capacity loss $\alpha_{e,j}$ was sampled and how the load was scaled proportionally to the severity of the storm. The costs of each generator investment is assumed equal to the generators' capacities $c_j = \bar{G}_j$. As a result, the budget b limits the MW capacity of total investment. With a budget b in units of MW we can increase the winter resilience level of b MW of capacity and we can not partially harden a generator.

We solve the risk-averse investment optimization problem using 5,000 randomly sampled winter storm scenarios. Of the 5,000 winter storm scenarios considered in the optimization problem 1,446 scenarios result in zero load loss when no investments are made. These scenarios are represented as a single scenario in the optimization problem that results in zero load loss, e.g. $\ell(x, e) = 0$, and occurs with probability $\frac{1,446}{5,000}$. As a result, we only need to optimize over 3,554 scenarios, easing the computational burden. This scenario reduction approach is accurate given that winterization investments do not increase load loss for any of the scenarios.

The risk-averse investment optimization problem is solved using Pyomo and Gurobi. Gurobi's MIP solver was used with a MIP gap stopping criterion of 0.1%. The DC OPF constraints that define the restoration set (8) were imported from the Electrical Grid Research and Engineering Tools (EGRET) software package [13]. This DC OPF model efficiently handles Power Transfer Distribution Factors, which represent matrix S , allowing us to efficiently scale up to a large number of scenarios. Each problem required approximately 15 minutes to build in Pyomo and less than 15 minutes to solve.

It is important to analyze the optimal investment decisions on out-of-sample scenarios to ensure the chosen investments are not tailored specifically to the scenarios used for optimization. We analyze the load loss of 5,000 validation scenarios

that were randomly sampled separately from the scenarios used in the optimization problem. To determine the load loss of each of these validation scenarios, we solved the load loss minimization problem for each individual scenario.

Fig. 1 illustrates the histogram of load loss for each of the 5,000 validation scenarios. Three histograms are shown, each representing an investment optimization problem with a different budget and with a risk metric of $\text{CVaR}_\epsilon^\epsilon$ with $\epsilon = 0.025$, which will be denoted $\text{CVaR}_\epsilon^{0.025}$. The histograms are transparent and overlaid on top of each other. The budgets of 0, 10,000 MW, and 25,000 MW are shown. When the budget is 0, there are no investments and the distribution exhibits a peak around 10,000 MW of load shed. An increased budget significantly decreases the load shed and shifts the distribution to the left. All budgets result in a significant amount of scenarios with zero load shed, represented as a peak at the origin. A budget of 25,000 MW results in 84% of the validation scenarios having zero load loss. In comparison, making no investments results in 33.48% of the validation scenarios having zero load loss.

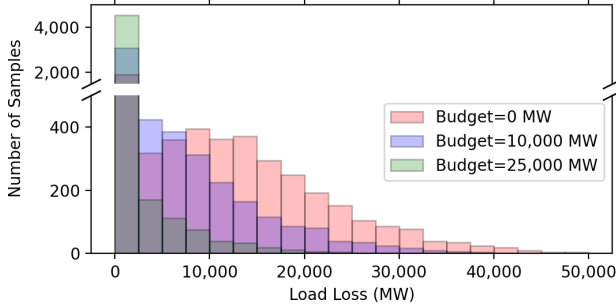


Fig. 1. Three overlaid histograms of load loss with varying budget. The investment optimization problem minimizes the risk metric $\text{CVaR}_\epsilon^{0.025}$. Purple represents overlap between the red and blue histograms, cyan represents overlap between the blue and green histograms, and gray represents overlap between all histograms.

Fig. 2 illustrates two overlaid histograms of load loss for each of 5,000 validation scenarios. One histogram is constructed by optimizing the risk metric $\text{CVaR}_\epsilon^{0.025}$ and the other is constructed by optimizing CVaR_ϵ^1 , which represents the expected value. Both histograms fix the budget to $b = 25,000$ MW. The CVaR minimization chooses investments that reduce load loss for the most severe scenarios. Indeed each of the bins that represent load loss greater than 12,500 MW are either reduced in size or remain the same size when minimizing $\text{CVaR}_\epsilon^{0.025}$ as compared to minimizing expected value CVaR_ϵ^1 . This demonstrates risk-aversity. In contrast, the expected value minimization chooses investments that increase the number of scenarios located in the first bin, which contains a load loss of 0.

Fig. 2 also shows the measured values of $\text{CVaR}_\epsilon^{0.025}$ and CVaR_ϵ^1 on the validation scenarios, which are represented as solid and dotted lines. The red lines correspond to investment optimization over CVaR_ϵ^1 and the blue lines correspond to investment optimization over $\text{CVaR}_\epsilon^{0.025}$. The measured $\text{CVaR}_\epsilon^\epsilon$ value is lower when optimizing $\text{CVaR}_\epsilon^\epsilon$ directly in the

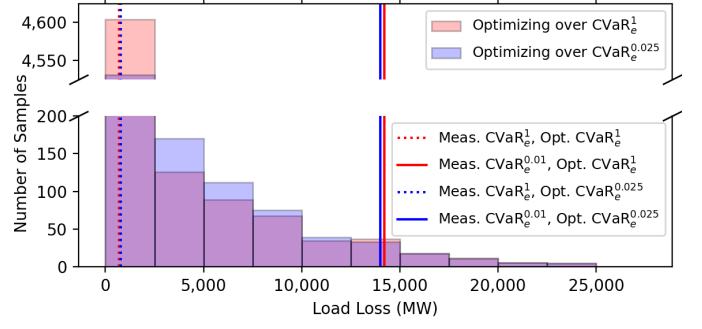


Fig. 2. Two overlapping transparent histograms of load loss with different values of ϵ . The budget is $b = 25,000$ MW. Histogram overlap is purple.

investment optimization problem for $\epsilon = 1$ and $\epsilon = 0.025$. This validates that our investments are having the intended impact on out-of-sample scenarios.

Fig. 3 illustrates the $\text{CVaR}_\epsilon^\epsilon$ of load loss as measured on the validation scenarios for different values of ϵ when optimizing four different objective functions. The $\text{CVaR}_\epsilon^\epsilon$ measurements are illustrated as colored tick marks where different colors represent different values of ϵ . The vertical axis represents load loss in MW and the horizontal axis represents each of the four different objective functions in the optimization problem. The objective functions considered are $\text{CVaR}_\epsilon^\epsilon$ where ϵ is 1, 0.05, 0.025, and 0.001. The scenario with the largest load loss lies between 24,500 MW and 25,000 MW for each objective function and this value is illustrated by a black tick mark. Furthermore, the vertical axis is cut in three locations and scaled to visualize the differences between each objective function.

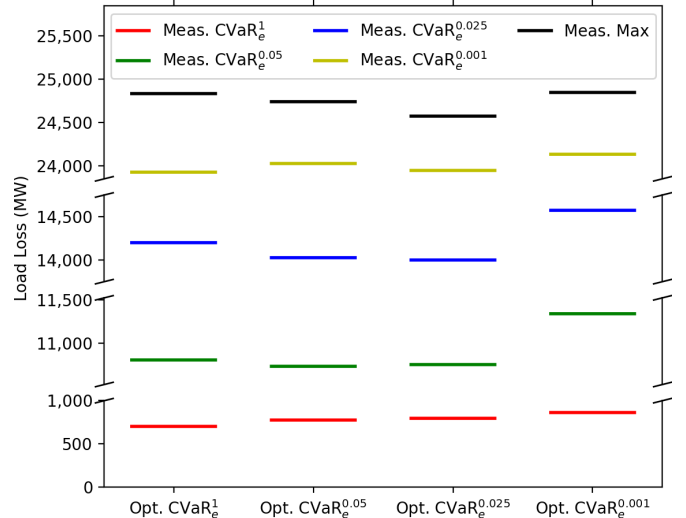


Fig. 3. $\text{CVaR}_\epsilon^\epsilon$ of load loss as measured on the validation scenarios for different values of ϵ when optimizing four different objective functions.

As illustrated in Fig. 3, optimizing over the expected value CVaR_ϵ^1 results in the lowest measured value of CVaR_ϵ^1 on the validation scenarios, which is illustrated by the red tick mark. Similarly, optimizing over $\text{CVaR}_\epsilon^{0.05}$ and $\text{CVaR}_\epsilon^{0.025}$ result in the lowest measured value of $\text{CVaR}_\epsilon^{0.05}$ and $\text{CVaR}_\epsilon^{0.025}$ respectively, which are illustrated by the green and blue

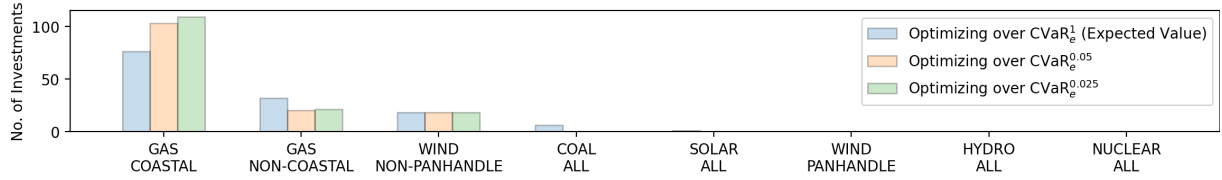


Fig. 4. Number of generators chosen for investment by generator type. The budget is $b = 25,000$ MW and the risk metric is $CVaR_{\epsilon}^{\epsilon}$ for varying values of ϵ .

tick marks. This indicates that the investment optimization problem is working as intended on out-of-sample scenarios by minimizing the $CVaR_{\epsilon}^{\epsilon}$ value that appears in the objective. Furthermore, the tails of the distribution are improved when optimizing over $CVaR_{\epsilon}^{0.05}$ and $CVaR_{\epsilon}^{0.025}$ as compared to $CVaR_{\epsilon}^1$. This is because they focus more on the tail of the distribution as compared to $CVaR_{\epsilon}^1$. Indeed they both reduce the measured $CVaR_{\epsilon}^{0.05}$ and $CVaR_{\epsilon}^{0.025}$ and reduce the worst case load loss illustrated by the black tick mark.

As shown in Fig. 3, optimizing over $CVaR_{\epsilon}^{0.001}$ does not result in the lowest measured value of $CVaR_{\epsilon}^{0.001}$ on the validation scenarios. This is because such a low value of ϵ causes the investment optimization problem to heavily focus on the worst few scenarios and thus the corresponding investments are heavily dependent on the specific random samples used in the investment optimization problem. As a result the investments chosen when optimizing over $CVaR_{\epsilon}^{0.001}$ do not perform well on out-of-sample scenarios.

Fig. 4 illustrates the generator classes that were chosen for investment when optimizing over $CVaR_{\epsilon}^{\epsilon}$ for epsilon values of $\epsilon = 1$, $\epsilon = 0.05$, and $\epsilon = 0.025$ with a budget fixed to $b = 25,000$ MW. For all values of ϵ we see significant investment in the generator classes that experienced the most failure during Winter Storm Uri, namely, Gas-Coastal, Gas-Non-Coastal, and Wind-Non-Panhandle. We do not see significant investment in generator classes Hydro-All, Solar-All, Nuclear-All, or Wind-Panhandle for any values of ϵ . Furthermore, we see a trend where smaller values of ϵ shift away from Gas-Non-Coastal and Coal-All investments and shift toward Gas-Coastal investments. Indeed the total number of generators chosen for winterization investment also tends to increase as ϵ becomes smaller as illustrated in Table I. In this context, the CVaR metric invests in more generators with smaller capacities, which effectively diversifies investments. This observation may be an artifact of our assumption that investment costs are proportional to generator capacities.

TABLE I
OPTIMAL INVESTMENT WITH BUDGET $b = 25,000$ MW.

Optimized Risk Metric	$CVaR_{\epsilon}^1$	$CVaR_{\epsilon}^{0.05}$	$CVaR_{\epsilon}^{0.025}$
Number of Investments	133	141	148
Average Gen. Capacity (MW)	187.96	177.26	168.90

V. CONCLUSIONS

This paper presented a scenario-based stochastic optimization formulation to determine investments that target power system resilience. We proposed minimizing the ϵ -CVaR metric to focus on low-probability high-impact events and illustrated the effect of varying the degree of risk-aversity by adjusting

the parameter ϵ . With application to winter storms in Texas, we illustrated that our risk-averse formulation determines investments that reduce the tail of the distribution of load loss as compared to the risk-neutral formulation. This result was validated using out-of-sample scenarios. Assuming that generator winterization investment costs are proportional to the capacity of the generator, we found that risk averse investments tend to winterize more generators with smaller capacities as opposed to fewer generators with larger capacities.

REFERENCES

- [1] J. W. Busby, K. Baker, M. D. Bazilian, A. Q. Gilbert, E. Grubert, V. Rai, J. D. Rhodes, S. Shidore, C. A. Smith, and M. E. Webber, "Cascading risks: Understanding the 2021 winter blackout in Texas," *Energy Research & Social Science*, vol. 77, p. 102106, 2021.
- [2] B. Austgen, M. Garcia, B. Pierre, J. Hasenbein, and E. Kutanoglu, "Winter storm scenario generation for power grids based on historical generator outages," in *2022 IEEE Power Engineering Society Transmission and Distribution Conference (Submitted as Part I of this paper)*. IEEE, 2022.
- [3] J.-P. Watson, R. Guttromson, C. Silva-Monroy, R. Jeffers, K. Jones, J. Ellison, C. Rath, J. Gearhart, D. Jones, T. Corbet *et al.*, "Conceptual framework for developing resilience metrics for the electricity oil and gas sectors in the united states," *Sandia National Laboratories, Albuquerque, NM (United States), Tech. Rep.*, 2014.
- [4] M. Mahzarnia, M. P. Moghaddam, P. T. Baboli, and P. Siano, "A review of the measures to enhance power systems resilience," *IEEE Systems Journal*, vol. 14, no. 3, pp. 4059–4070, 2020.
- [5] E. D. Vugrin, A. R. Castillo, and C. A. Silva-Monroy, "Resilience metrics for the electric power system: A performance-based approach," *Sandia National Lab.(SNL-NM), Albuquerque, NM (United States), Tech. Rep.*, 2017.
- [6] R. Tyrrell Rockafellar and J. O. Royset, "Engineering decisions under risk averseness," *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering*, vol. 1, no. 2, p. 04015003, 2015.
- [7] M. Bynum, A. Staid, B. Arguello, A. Castillo, B. Kneueven, C. D. Laird, and J.-P. Watson, "Proactive operations and investment planning via stochastic optimization to enhance power systems' extreme weather resilience," *Journal of Infrastructure Systems*, vol. 27, no. 2, p. 04021004, 2021.
- [8] K. Garifi, E. Johnson, B. Arguello, and B. J. Pierre, "Transmission grid resiliency investment optimization model with SOCP recovery planning," *IEEE Transactions on Power Systems*, 2021.
- [9] B. J. Pierre, B. Arguello, and M. J. Garcia, "Optimal investments to improve grid resilience considering initial transient response and long-term restoration," in *2020 International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*. IEEE, 2020, pp. 1–6.
- [10] R. T. Rockafellar, S. Uryasev *et al.*, "Optimization of conditional value-at-risk," *Journal of Risk*, vol. 2, pp. 21–42, 2000.
- [11] R. T. Rockafellar and S. Uryasev, "Conditional value-at-risk for general loss distributions," *Journal of Banking & Finance*, vol. 26, no. 7, pp. 1443–1471, 2002.
- [12] A. B. Birchfield, T. Xu, K. M. Gegner, K. S. Shetye, and T. J. Overbye, "Grid structural characteristics as validation criteria for synthetic networks," *IEEE Transactions on Power Systems*, vol. 32, no. 4, pp. 3258–3265, 2017.
- [13] B. Kneueven, C. Laird, J.-P. Watson, M. Bynum, A. Castillo *et al.*, "Egret v. 0.1 (beta)," *Sandia National Lab.(SNL-NM), Albuquerque, NM (United States), Tech. Rep.*, 2019.