Numerical Prediction of Boundary Layer Flow Around An Immersed Plate in Carbopol Yield Stress Fluid

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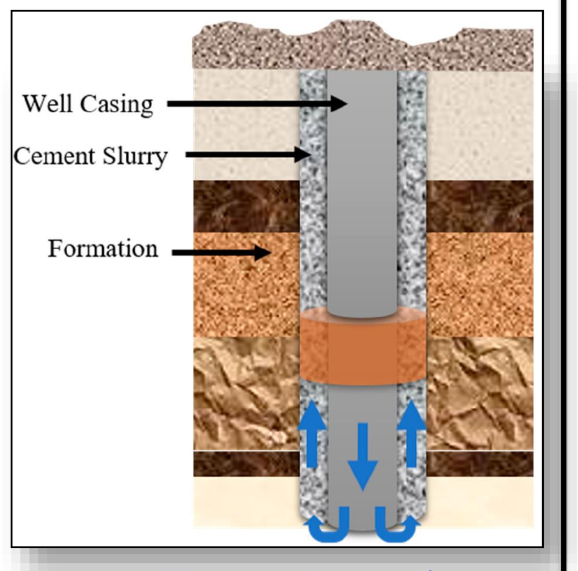
Background/Motivation

- **Understand Cement Slurry Rheological Properties**
- **Wellbore Cement Gas Migration Problem[1,2]**
  - Cement annulus leakage, producing and abandoned wells
    - Microstructures, fractures
  - Causes of 25-30% gas migration issues



Wellbore Schematic

Why is boundary layer prediction relevant[3]?

- Narrow viscoplastic regions develop in non-Newtonian fluids
  - Potential failure surfaces between plugs
- Lubrication between plug flows or with walls
  - Rigid blocks sliding over one another
- Buffers for predominantly plastic deformation

Wellbore cement slurry is a yield stress, thixotropic, shear-thinning,… non-Newtonian fluid

➔ Boundary layers can provide channeling pathways for gas migration and potentially increase likelihood of operation failures

Mathematical Model

Conservation Equations

- Mass conservation: $$\frac{\partial \rho}{\partial t} + div(\rho \mathbf{v}) = 0$$
- Linear momentum conservation: $$\rho \frac{d\mathbf{v}}{dt} = div(\boldsymbol{T}) + \rho \boldsymbol{b}$$
- No-slip wall

$\boldsymbol{b}$ is the body force, ρ is the density and $\mathbf{v}$ the velocity; d/dt is the total time derivative

Constitutive Relations

- Cauchy stress tensor: $\boldsymbol{T} = -p\boldsymbol{I} + \boldsymbol{\tau}$
- 3D Herschel-Bulkley[4] viscous stress $\boldsymbol{\tau}$:

$$\boldsymbol{\tau} = \left[k\left|II_{\boldsymbol{A_1}}\right|^{(n-1)/2} + \frac{\tau_0}{\left|II_{\boldsymbol{A_1}}\right|^{1/2}}\right]\boldsymbol{A_1} \text{ for } |II_{\boldsymbol{\tau}}|^{1/2} > \tau_0$$

$$\boldsymbol{A_1} = \boldsymbol{0} \text{ for } |II_{\boldsymbol{\tau}}|^{1/2} \le \tau_0$$

$$\text{with } \boldsymbol{A_1} = \boldsymbol{L} + \boldsymbol{L}^T, \boldsymbol{L} = grad(\mathbf{v})$$

p is the pressure and $\boldsymbol{I}$ is the identity tensor. $II_{\boldsymbol{\tau}}$ and $II_{\boldsymbol{A_1}}$ are the second invariants of the deviatoric tensor $\boldsymbol{\tau}$ and the strain-of-rate tensor $\boldsymbol{A_1}$

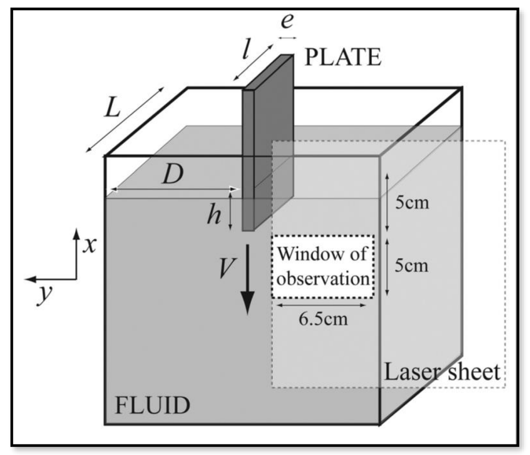
τ_0 is the yield stress; k and are n the consistency index and power-law exponent, resp.

Case Study

Plate Immersion Experiments in Carbopol

(Boujlel et al.[5], Journal of Rheology, 2012)

- ***Problem Statement***
  - Quantify liquid region thickness surrounding moving object in yield stress fluid
  - Identify flow properties/parameters affecting liquid region thickness



Experimental Flow Conditions

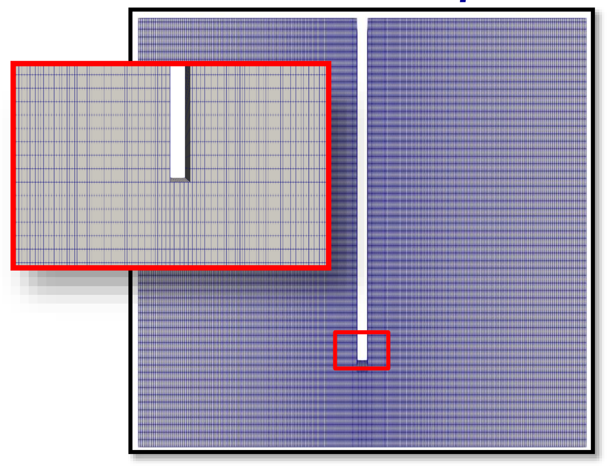
- ***Fluid***: Carbopol gel solution (0.5%) with τ_0 = 59.5 Pa, k=23.6 Pa.s^n and n=0.38 (flow curve fitting using Herschel-Bulkley)
- ***Box (fluid)***: H=25cm, W=10cm and L=16cm
- ***Plate***: e=1.5mm, H=25cm and l=7cm
- Plate immersion velocity: 1mm/s, 3mm/s and 5mm/s

Numerical Method Overview[6]*

- **Viscosity regularization using Papanastasiou method**
- **Numerical Schemes**
  - **Spatial**: *Gradient*: linear and *Convection*: linear
  - **Temporal**: Semi-implicit Euler
- **Mesh (Overset technique)**
  - Background mesh (uniform in each direction)
    - Δx = 2mm, Δy = 2mm
  - Overset (with 89,940 cells)
    - $\Delta x_{min} = 0.15$mm, $\Delta y_{max} = 0.3$mm

CFL<1 ($\delta t_{max} \approx 2\text{x}10^{-3} s$)

2D Flow Assumption

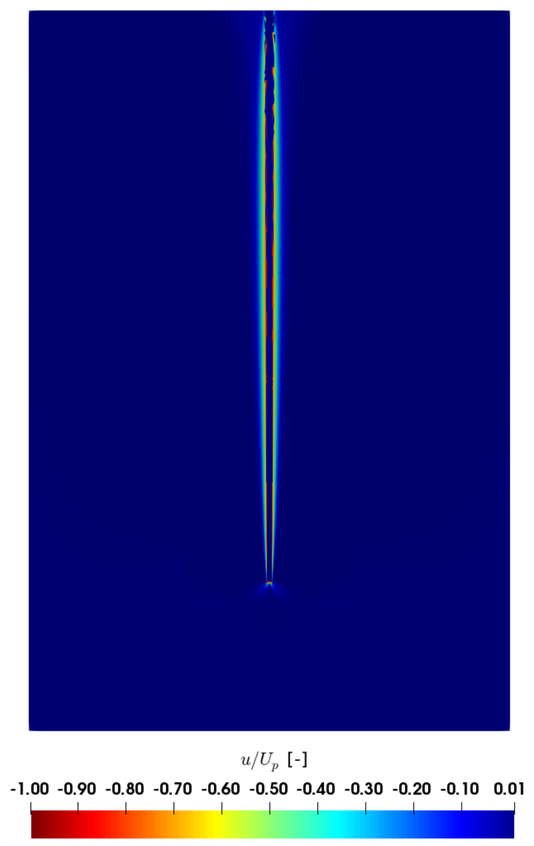


(*) OpenFOAM v2012, Dec. 2020 (with customized non-Newtonian viscosity libraries)

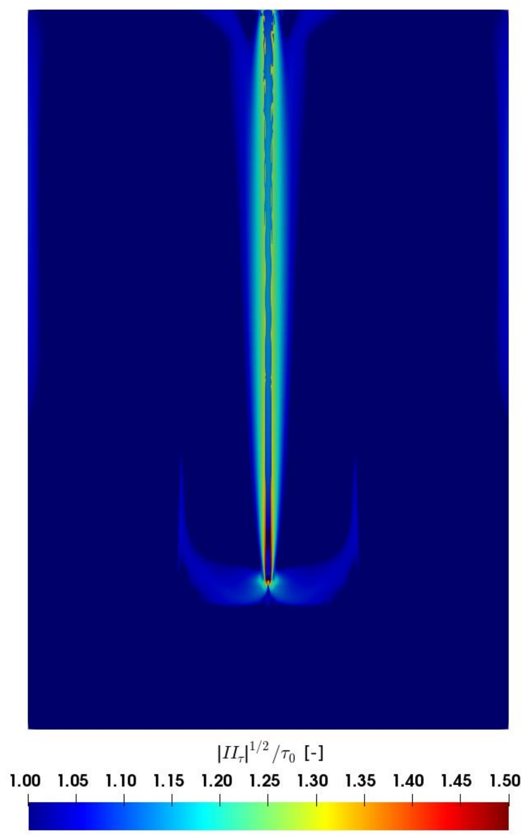
Results

Flow Field Visualizations

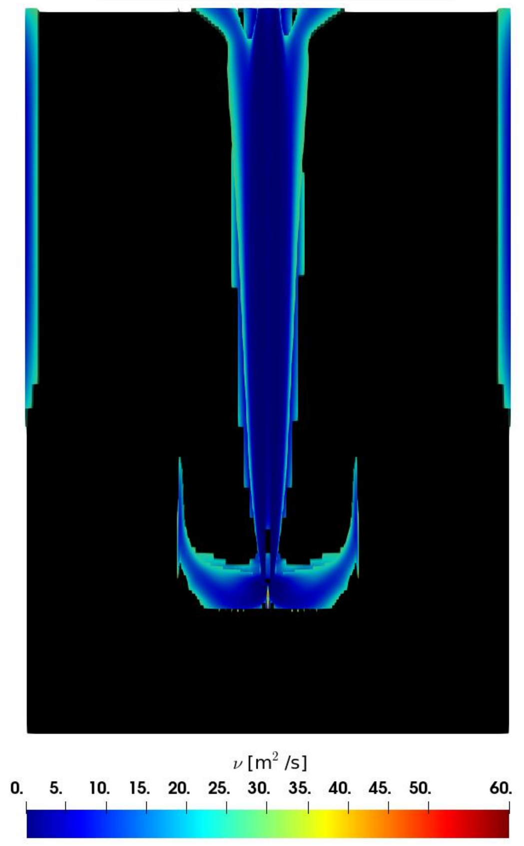
Carbopol Velocity Distribution



Viscous Stress 2nd Invariant Distribution



Liquid Region* (or Boundary Layer)



Instantaneous flow field distributions for plate immersion velocity of 1mm/s

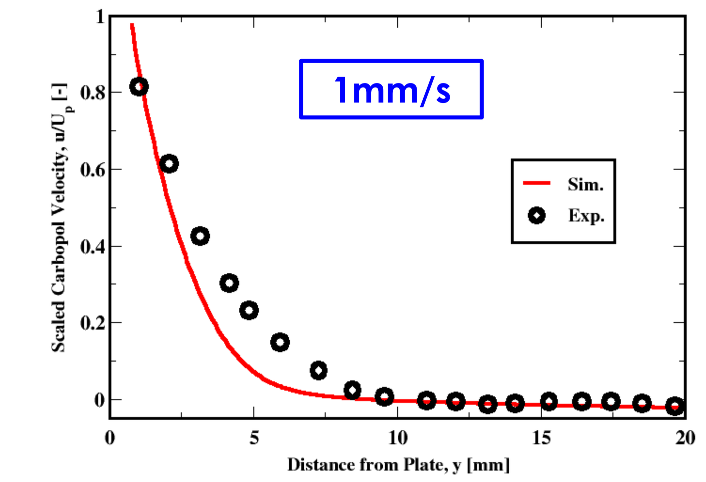
Snapshots above show:

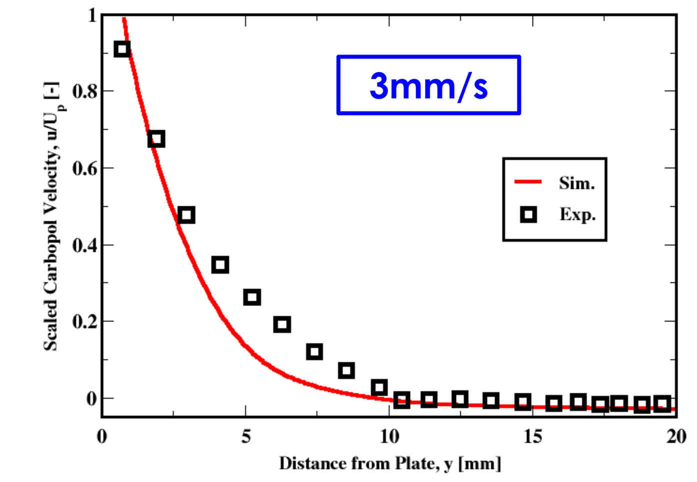
- Carbopol material mainly moved within a slim envelope around the plate
- Second invariant of viscous stress only exceeded the yield stress in the vicinity of the plate
- Anchor-like shape liquid region: Carbopol material yielded around the plate, below plate leading edge and at the boundary of the container

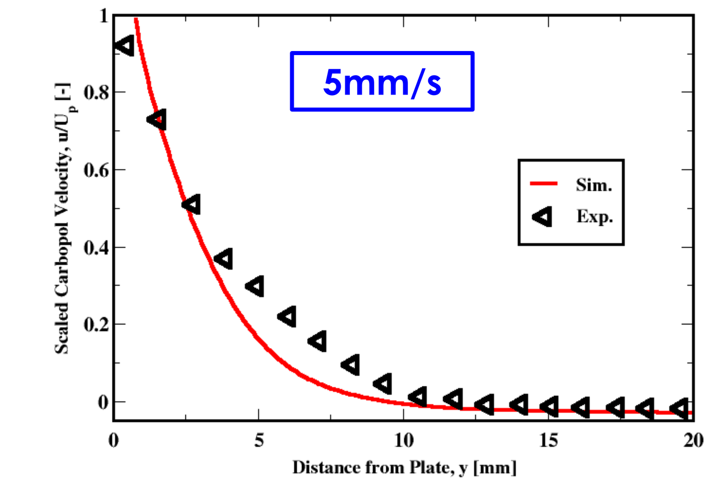
(*) Black region shows the unyielded region, while yielded region is colored by the kinematic viscosity.

Flow Statistical Properties

Carbopol Velocity Distribution

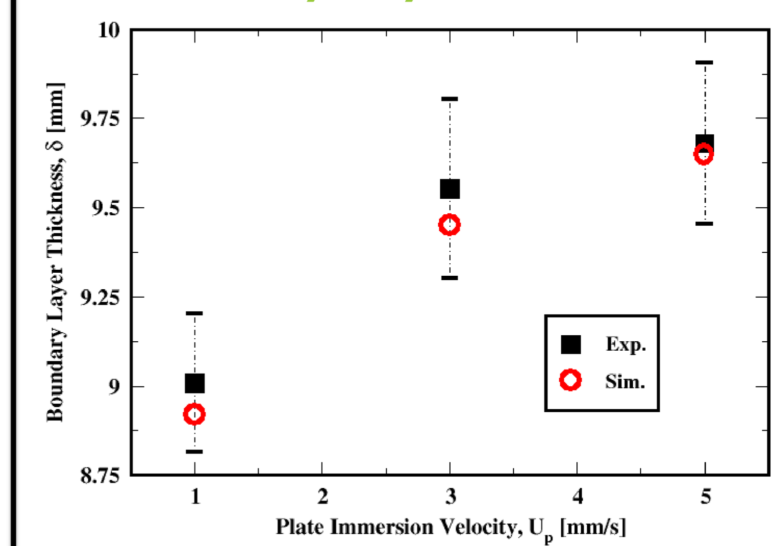


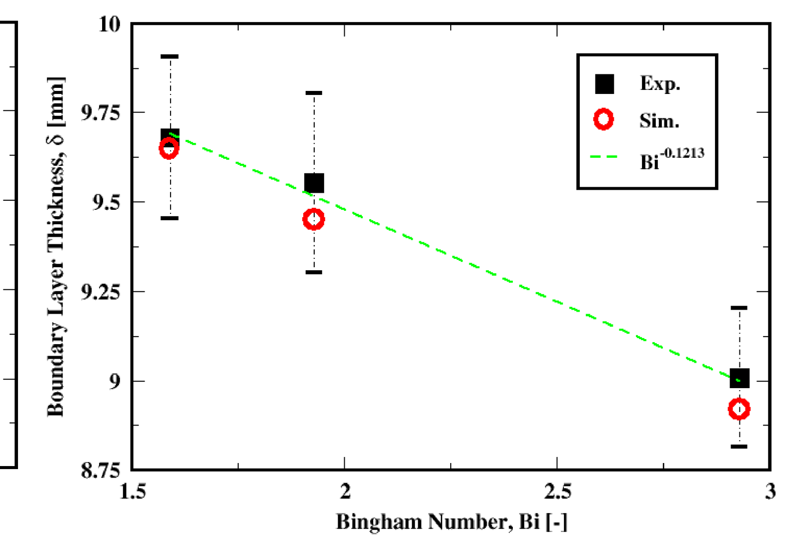




- Accurate predictions of solid-like region velocities slow upward displacements
- Satisfactory predictions of velocity distribution in the yielded regions; slight mismatch probably due to uncertainties in rheological properties.

Boundary Layer Thickness





- Boundary layer thickness predicted within measurement uncertainties for the three plate immersion velocities
- Yielded region thickness decreases with Bingham number[7] and both experiments and simulations scale as Bi$^{-0.1213}$

Conclusions/Outlook

❖ Simulations of non-inertial flow boundary layers of Herschel-Bulkley fluids using regularization method showed:

- Flow physics are adequately captured
- Velocity distribution around the plate compared favorably against measurements
- Excellent agreement of the boundary layer thickness against experiments

❖ Future work will include effects of particle local concentration in the fluid

References

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7. **Piau J.-M., Debiane K.**, "The adhesive or slippery flat plate viscoplastic boundary layer for a shear-thinning power-law viscosity". (**2004**) Journal of Non-Newtonian Fluid Mech., vol, 117. pp. 97-107

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