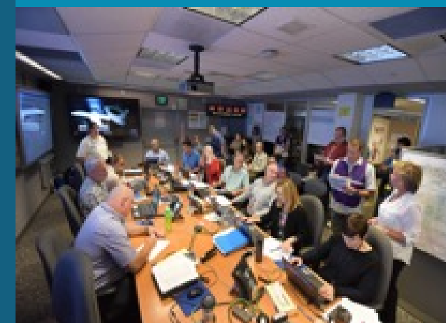
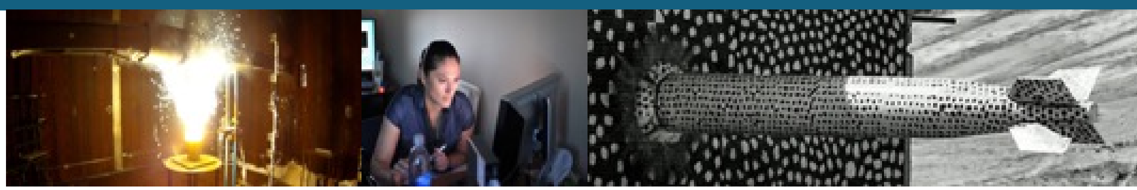
SAND2021-12836C

# Full Kinetic and Drift Kinetic Descriptions of Electrons Within MITLs Near a Load





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Sandia National Laboratories

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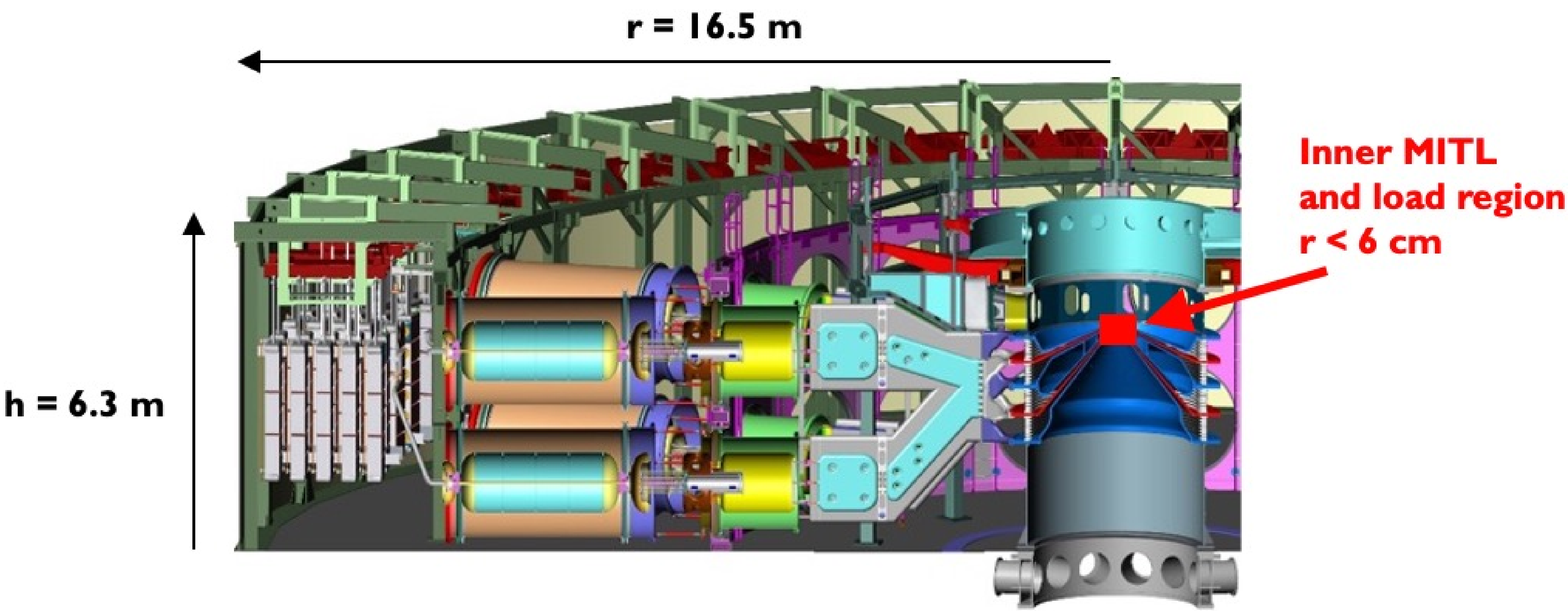
# Why are fields and electron dynamics near a MITL load so important?

- Z machine is the largest pulsed power machine in the world capable of delivering < 30 MA of current.
- The inner MITL conducts power to the load which is located at the center of the machine.
- The following analysis can be used to directly understand fields/electron dynamics near the load. The fields/electron dynamics from this analysis are checked using the fully electromagnetic code EMPIRE developed at Sandia National Laboratories.



# Assumptions*

1. The MITL is cylindrically symmetric.

2. The magnetic field is specified by Ampere's Law in the limit $c \rightarrow \infty$ (no displacement current) for a time-dependent MITL current I(t).

$$\mathbf{B} = -\frac{\mu_0 I(t)\mathbf{e}_\phi}{2\pi r}$$

3. The MITL surfaces are perfect conductors.

4. The load, which defines the "end" of the MITL, is also represented as a perfect conducting surface.

***The following work can be found in M. H. Hess and E. G. Evstatiev, "Electron Dynamics Within a MITL Containing a Load", IEEE Transactions On Plasma Science (accepted for publication 2021). (SAND 2021-11933 J)**

# Electric Field Equations

- The electric fields, which are in the radial and axial directions, can be solved using Maxwell's Equations.

**Gauss's Law:**

$$\nabla \cdot \mathbf{E} = 0$$

**Faraday's Law:**

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

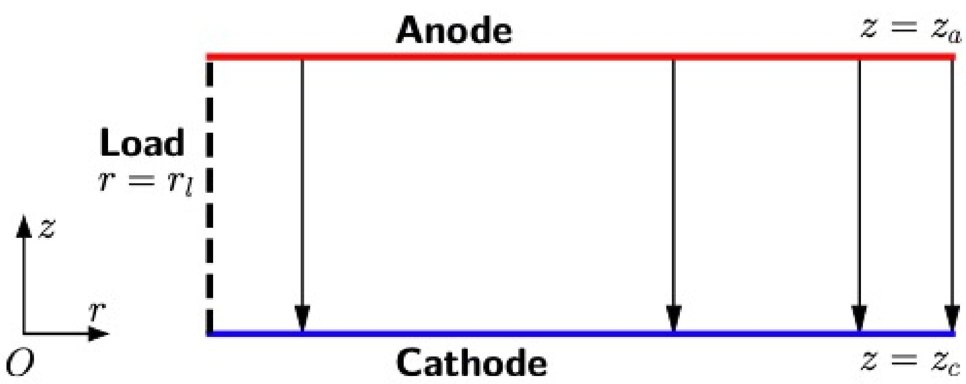
**Boundary Condition at MITL Surface and Load:**

$$\mathbf{n} \times \mathbf{E}|_S = 0$$

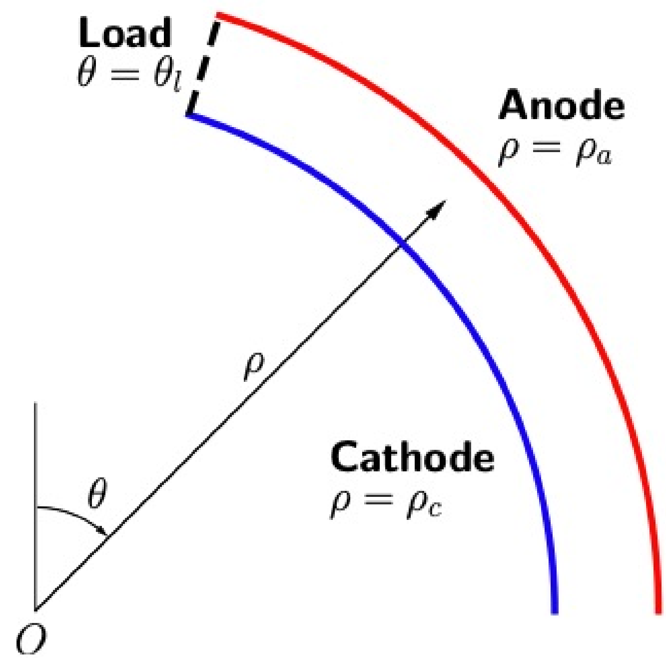
# Types of MITLs Examined

- We examine two different types of MITLs: radial and spherically curved.

## Radial MITL




## Spherically Curved MITL



# Full Kinetic Lagrangian Description of Electron Dynamics

**Lagrangian function of coordinates and velocities:**

$$L(Q_1 \ldots Q_s, \dot{Q}_1 \ldots \dot{Q}_s)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{Q}_i} = \frac{\partial L}{\partial Q_i}$$

## Radial MITL

**Electric Field:**

$$\mathbf{E} = -\frac{\mu_0 \dot{I}}{2\pi}\ln\left(\frac{r}{r_l}\right)\mathbf{e}_z$$

**Vector Potential:**

$$\mathbf{A} = \frac{\mu_0 I}{2\pi}\ln(\frac{r}{R})\mathbf{e}_z$$

**Lagrangian:**

$$L = -mc^2\sqrt{1 - \frac{1}{c^2}\left(\dot{r}^2 + r^2\dot{\phi}^2 + \dot{z}^2\right)} + q\mathbf{v}\cdot\mathbf{A}$$

## Spherical MITL

**Electric Field:**

$$\mathbf{E} = \frac{\mu_0 \dot{I}\rho_a\rho_c}{2\pi\rho^2}\ln\left(\frac{\csc\theta + \cot\theta}{\csc\theta_l + \cot\theta_l}\right)\mathbf{e}_\rho + \frac{\mu_0 \dot{I}(\rho-\rho_a)(\rho-\rho_c)}{2\pi\sin\theta\rho^2}\mathbf{e}_\theta$$

**Vector Potential:**

$$\mathbf{A} = -\frac{\mu_0 I\rho_a\rho_c}{2\pi\rho^2}\ln\left(\frac{\csc\theta + \cot\theta}{\csc\theta_l + \cot\theta_l}\right)\mathbf{e}_\rho - \frac{\mu_0 I(\rho-\rho_a)(\rho-\rho_c)}{2\pi\sin\theta\rho^2}\mathbf{e}_\theta$$

**Lagrangian:**

$$L = -mc^2\sqrt{1 - \frac{1}{c^2}\left(\dot{\rho}^2 + \rho^2\dot{\theta}^2 + \rho^2\sin^2\theta\dot{\phi}^2\right)} + q\mathbf{v}\cdot\mathbf{A}$$

# Full Kinetic Radial MITL Particle Trajectory Equations

In order to simplify our discussion, we assume

$$v_\phi(t) = 0$$

**Momentum Equations:**

$$\frac{dp_r}{dt} = \frac{p_\phi^2}{\gamma m r} + \frac{q\mu_0 I v_z}{2\pi r}$$

$$\frac{d\left(r p_\phi\right)}{dt} = 0$$

$$\frac{dp_z}{dt} = qE_z - \frac{q\mu_0 I v_r}{2\pi r}$$

$$\frac{dp_r}{dt} = \frac{q\mu_0 I v_z}{2\pi r}$$

$$\frac{dp_\phi}{dt} = 0$$

$$\frac{dp_z}{dt} = qE_z - \frac{q\mu_0 I v_r}{2\pi r}$$

**Position Equations:**

$$\frac{dr}{dt} = v_r \qquad \frac{d\phi}{dt} = \frac{v_\phi}{r} \qquad \frac{dz}{dt} = v_z$$

# Full Kinetic Spherically Curved MITL Particle Trajectory Equations

In order to simplify our discussion, we assume

$$v_\phi(t) = 0$$

**Momentum Equations:**

$$\frac{dp_\rho}{dt} = \frac{p_\theta^2 + p_\phi^2}{\gamma m \rho} + qE_\rho - \frac{q\mu_0 I v_\theta}{2\pi\rho\sin\theta}$$

$$\frac{dp_\theta}{dt} = \frac{-p_\rho p_\theta + p_\phi^2 \cot(\theta)}{\gamma m \rho} + qE_\theta + \frac{q\mu_0 I v_\rho}{2\pi\rho\sin\theta}$$

$$\frac{d(\rho\sin\theta p_\phi)}{dt} = 0$$

$$\frac{dp_\rho}{dt} = \frac{p_\theta^2}{\gamma m \rho} + qE_\rho - \frac{q\mu_0 I v_\theta}{2\pi\rho\sin\theta}$$

$$\frac{dp_\theta}{dt} = \frac{-p_\rho p_\theta}{\gamma m \rho} + qE_\theta + \frac{q\mu_0 I v_\rho}{2\pi\rho\sin\theta}$$

$$\frac{dp_\phi}{dt} = 0$$

**Position Equations:**

$$\frac{d\rho}{dt} = v_\rho \qquad \frac{d\theta}{dt} = \frac{v_\theta}{\rho} \qquad \frac{d\phi}{dt} = \frac{v_\phi}{\rho\sin\theta}$$

# Drift Kinetic Approximation

- The guiding center drift motion for a particle in an inner MITL can be described by a combination of ExB and grad B drifts. Since we assume the particle's azimuthal velocity is zero at emission, curvature B drift is also zero.

**Guiding Center Equation*:**

$$\mathbf{v}_{\mathrm{gc}} = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{\mu}{q\gamma}\frac{\mathbf{B}\times\nabla B}{B^2}$$

**Relativistic Adiabatic Invariant (Magnetic Moment)**:**

$$\mu = \frac{p_{\perp}^2}{2mB}$$

*R. J. Goldston and P. H. Rutherford, Introduction to Plasma Physics (1995) p. 51.

**A. J. Brizard and A. A. Chan, Phys. Plasmas 8 4762 (2001).

# Full Kinetic vs. Drift Kinetic (Radial MITL)

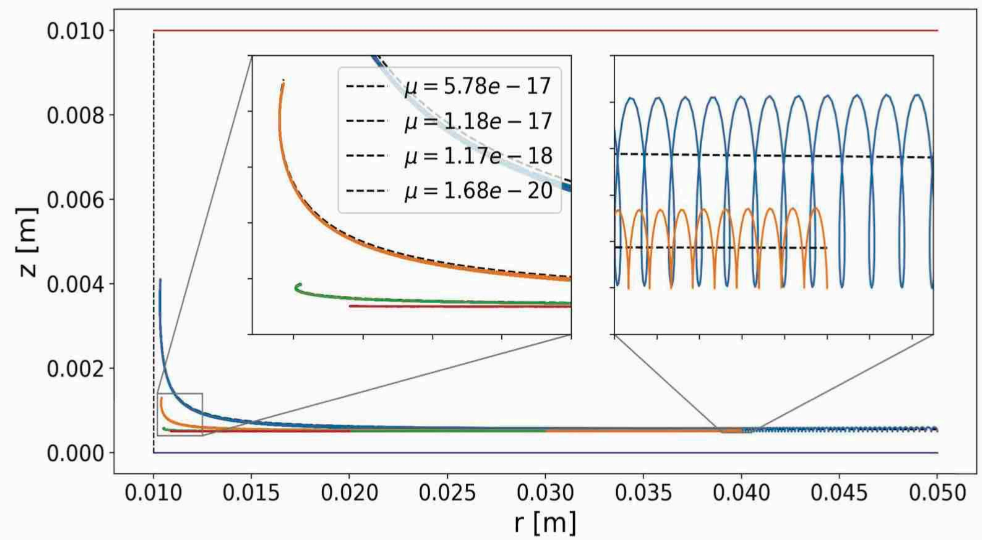
- We use a 20 MA peak current with 120 ns pulse length current drive.

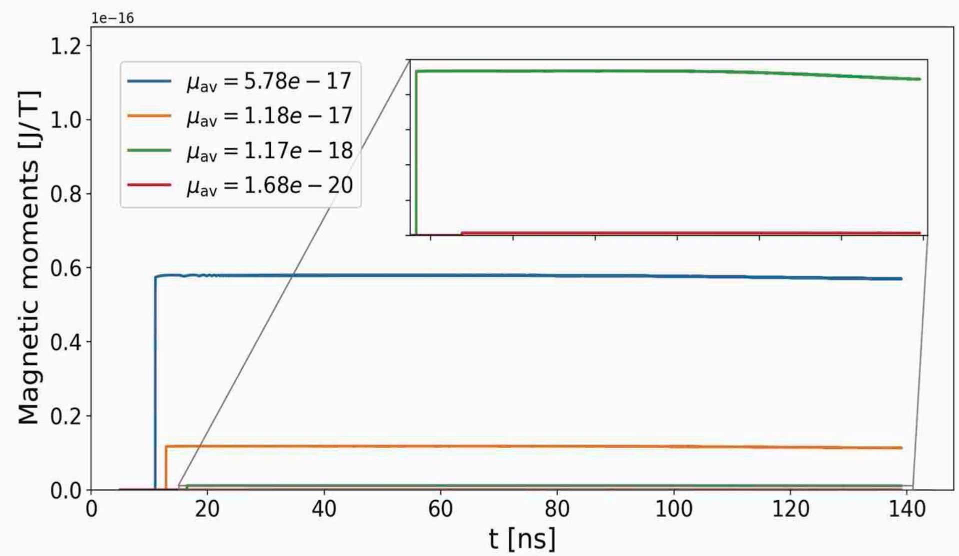
$$I(t) = I_{\text{peak}} \sin^2\left(\frac{\pi t}{2\tau_{\text{peak}}}\right)$$

- We compare a full kinetic 4th order R-K scheme using dt=$10^{-15}$ s and the drift kinetic equations solved with a 2nd order R-K scheme with dt=$10^{-12}$ s.

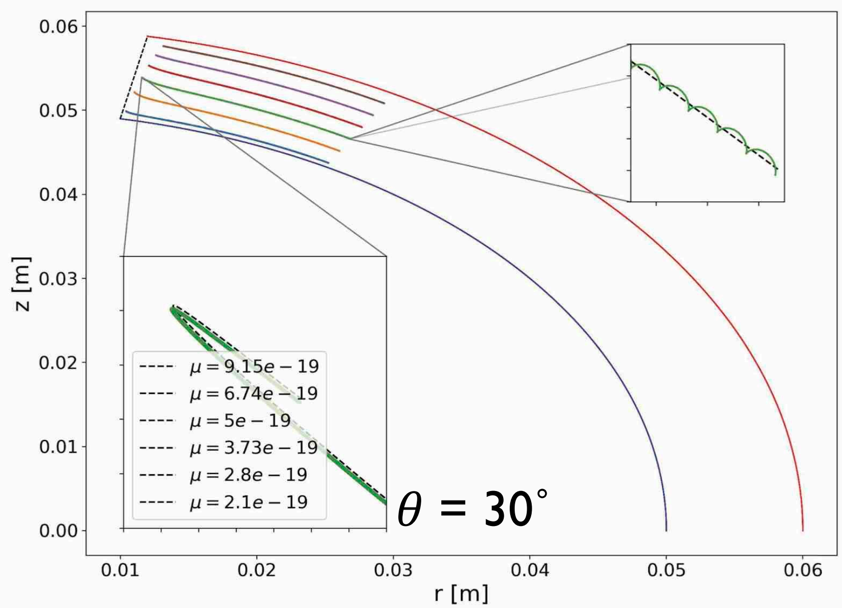
- The initial drift kinetic axial position is set to half the initial cycloidal orbit size of the full kinetic trajectory.

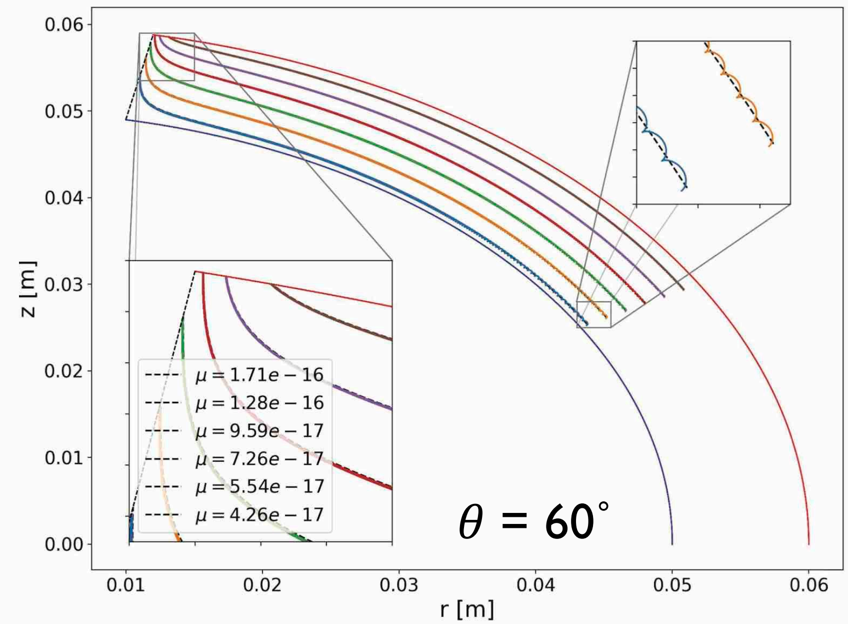
- Electron emission is at 24 MV/m.



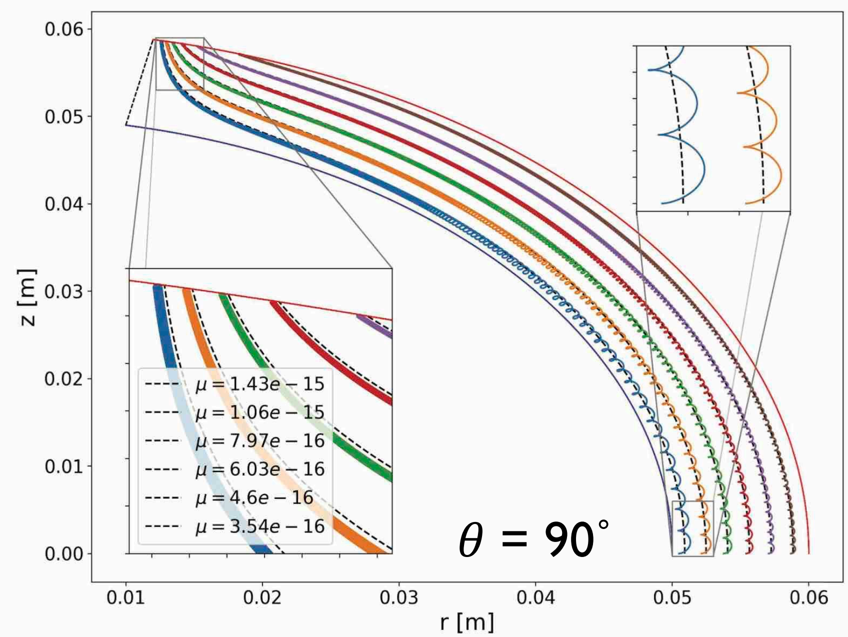



# Full Kinetic vs. Drift Kinetic (Spherically Curved MITL)




- For the spherical MITL, electrons are emitted at different initial MITL angles.
- For smaller initial angles, the initial electric field is smaller → smaller magnetic moment → smaller grad B drift.
- For larger initial angles, the initial electric field is larger → larger magnetic moment → larger grad B drift.



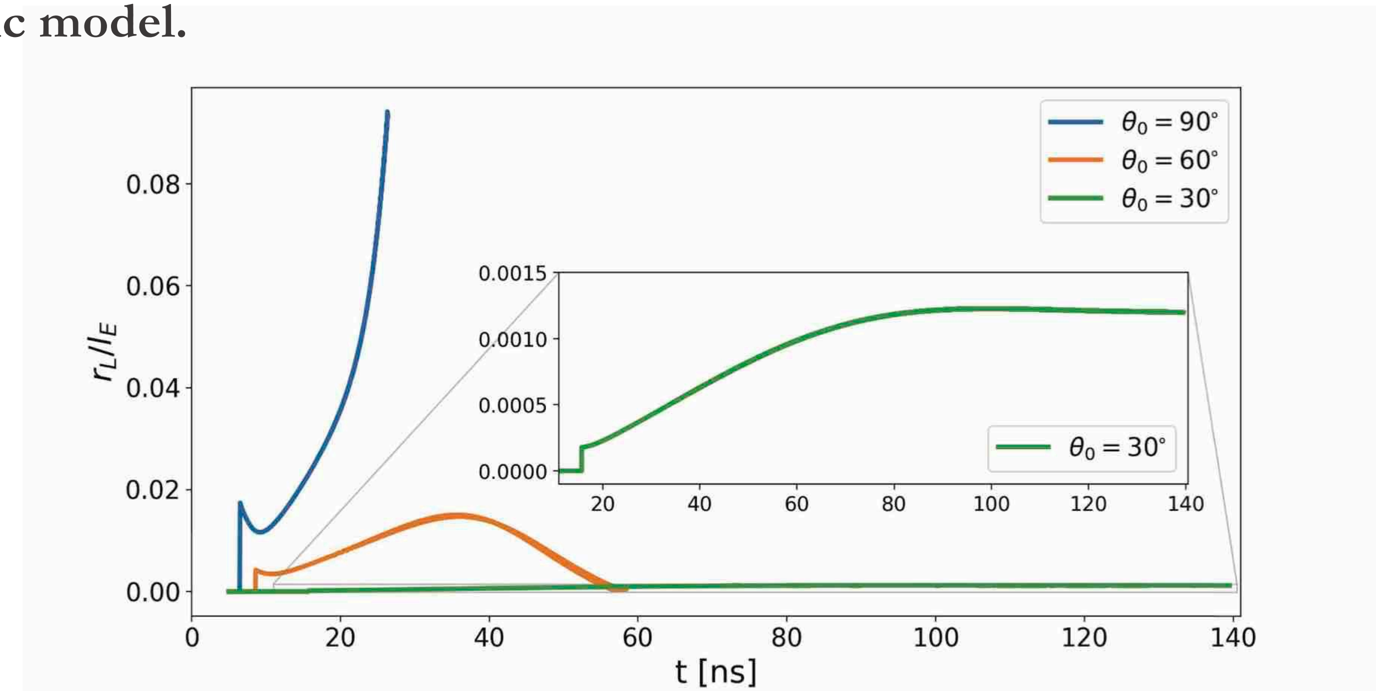
# How good is the comparison of Full Kinetics vs. Drift Kinetics?

- **In general, the smaller the ratio $r_l/l_e$ = Larmor radius/electric field gradient length where**

$$l_E = \frac{E}{|\nabla E|}$$

**the better the drift kinetic model agrees with the full kinetic model.**

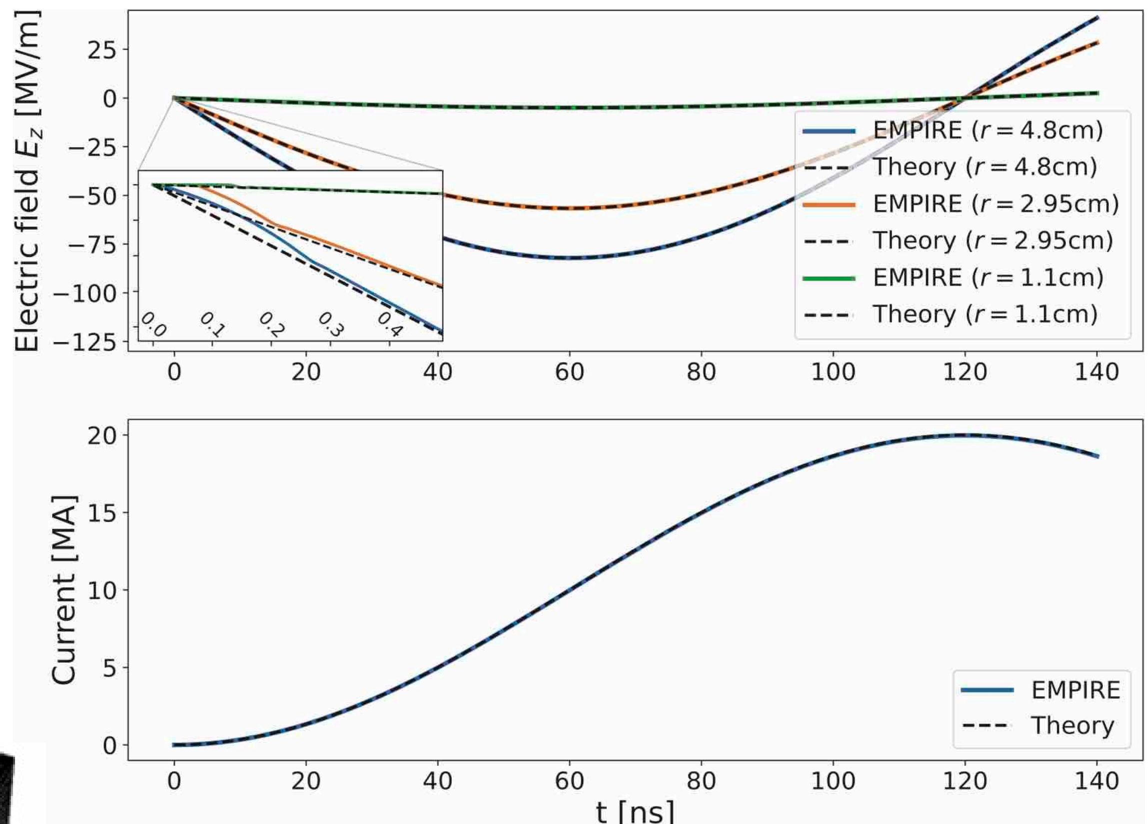
- **Since smaller initial emission angles → smaller Larmor radii, then the drift kinetic model at smaller initial angles agrees better with the full kinetic model.**

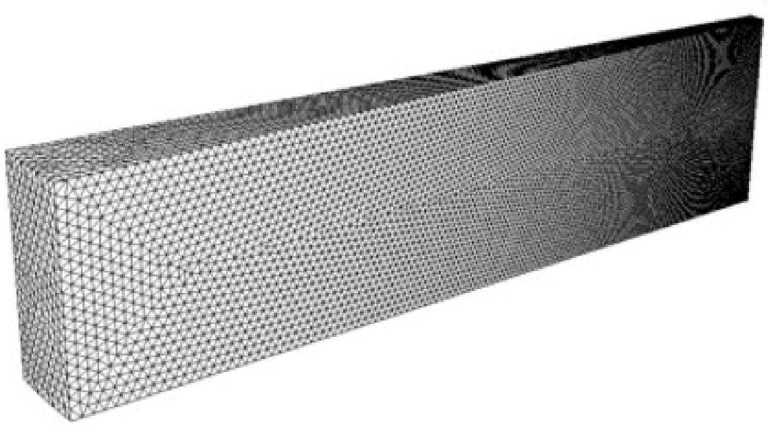


# Theory vs. EMPIRE (Radial MITL Fields)

- We use the radial MITL example with a 20 MA peak current to test the c → ∞ limit model against the fully EM code EMPIRE developed at Sandia.

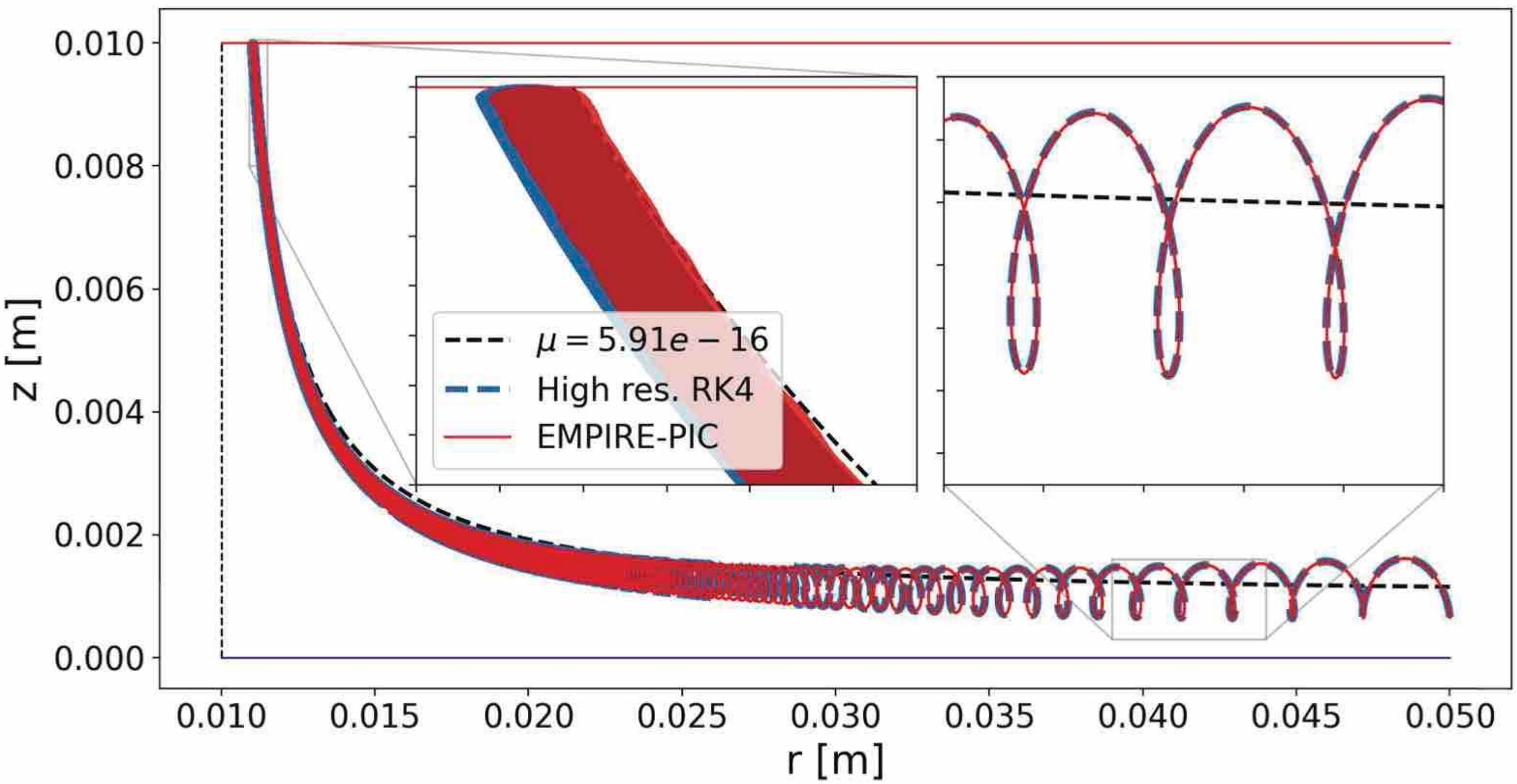
- We get excellent agreement with the spatial dependence of the voltage and electric field.



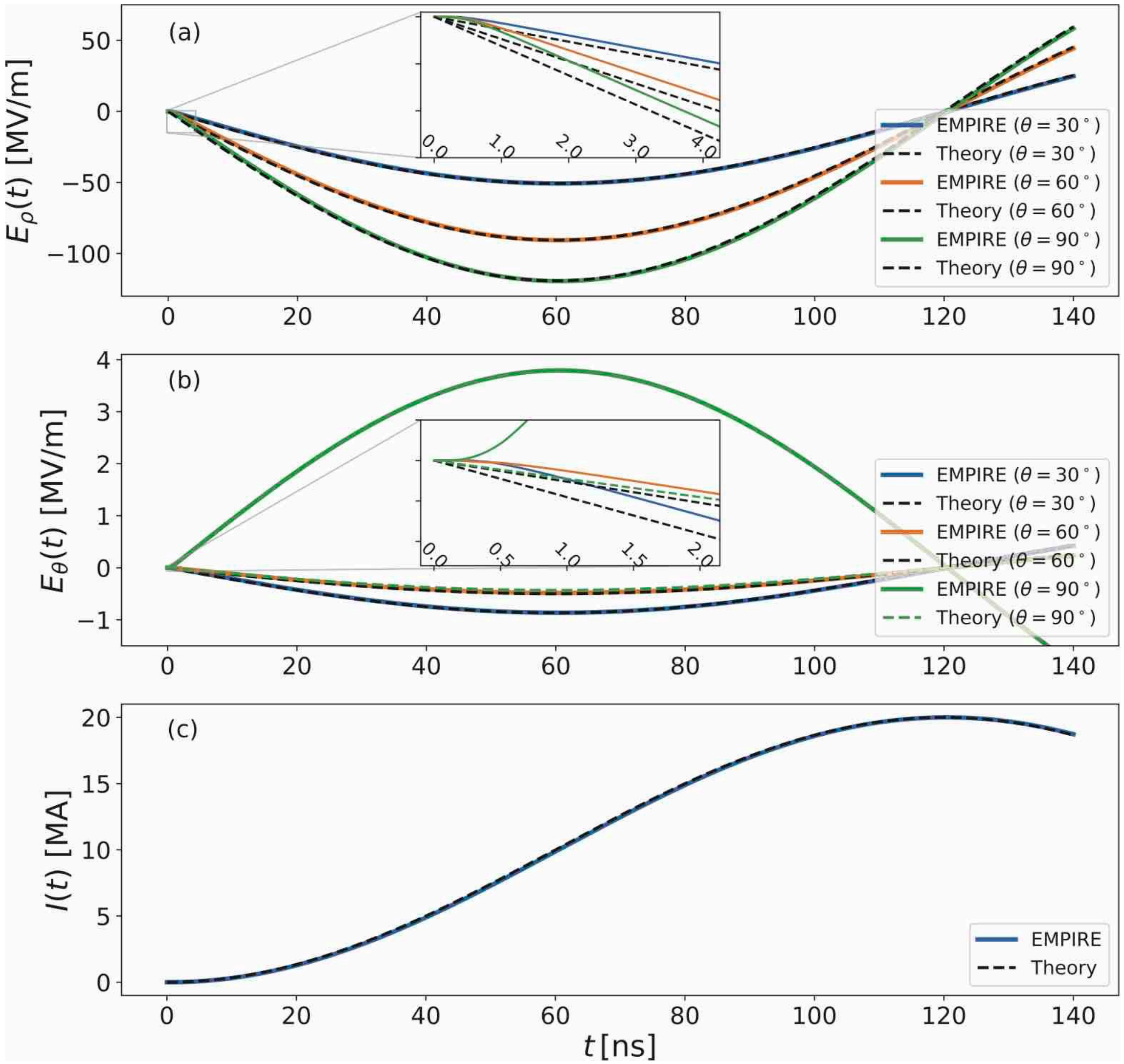
# Full/Drift Kinetic vs. EMPIRE-PIC (Radial MITL Trajectories)

- **In order to better resolve cyclotron orbits near the load, we lower the peak current to 2 MA. We also lower the electric field threshold to 2.4 MV/m.**
- **We get excellent agreement with full kinetic simulation of particle trajectories.**



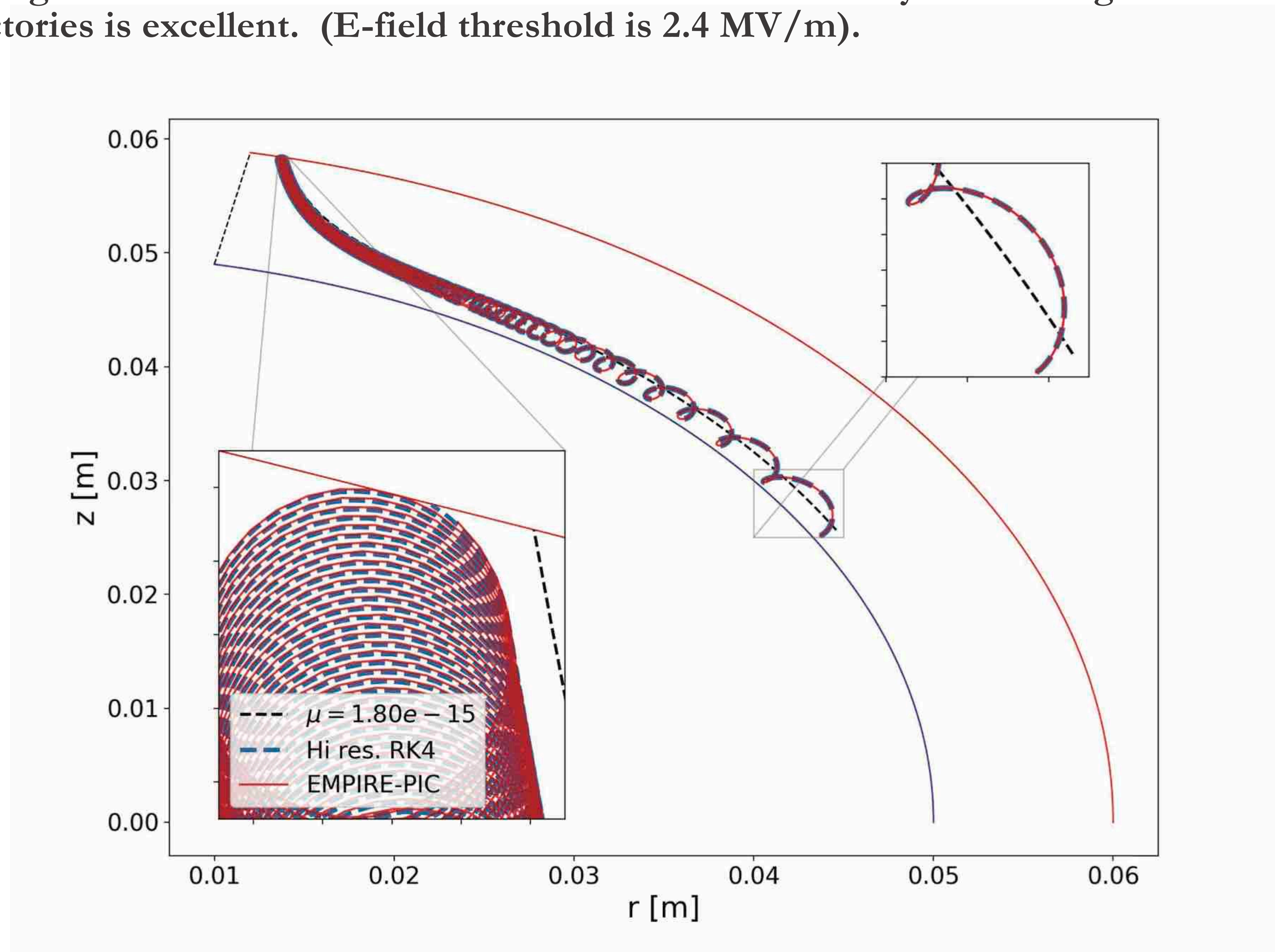
# Theory vs. EMPIRE (Spherically Curved MITL)

- **We include an axial extension into the EMPIRE simulation to provide the correct field BCs into the spherical MITL section.**
- **$E_\rho$ has excellent agreement between theory and EMPIRE.**
- **There is disagreement at larger angles for $E_\theta$ due to either the difference in the mode structure supported by the fully EM model vs. the c → ∞ model or a boundary constraint due to the MITL extension.**



# Full/Drift Kinetic vs. EMPIRE-PIC (Spherically Curved MITL)

- **The agreement between the theoretical model and the fully electromagnetic model trajectories is excellent. (E-field threshold is 2.4 MV/m).**



# Summary

- **We have analyzed the fields and electron trajectories for radial and spherically curved MITLs.**

- **A drift kinetic model that incorporates ExB and grad B drifts provides an overall excellent approximation to the full kinetic electron motion.**
  - **The drift kinetic model shows differences with the full kinetic model when the Larmor radius grows relative to the electric field gradient scale length.**

- **The fields/full kinetic/drift kinetic dynamics for the two MITL problems have been tested against the electromagnetic code EMPIRE.**
  - **We get excellent agreement between theory/EMPIRE for fields/trajectories in the radial MITL case.**
  - **We get excellent agreement for $E_\rho$ in the spherically curved MITL, and some disagreement with $E_\theta$ at large angles between the full EM model and the $c \rightarrow \infty$ model.**
  - **Small differences in trajectories between the full/drift kinetic and EMPIRE are also observed. Overall, both the $c \rightarrow \infty$ and the drift kinetic approximation provide excellent representations of electron trajectories when compared with EMPIRE results.**