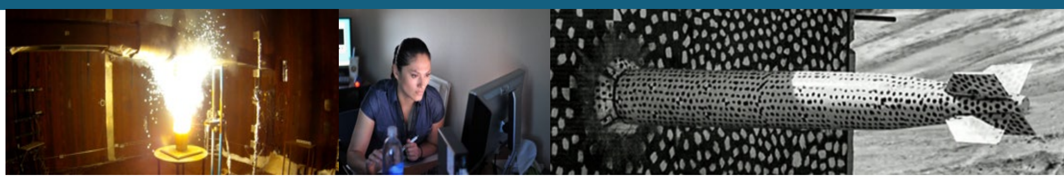
SAND2021-12791C

Sandia National Laboratories

Application of Multi-harmonic Balance for Nonlinear Frequency Response Analysis in MSC Nastran



Presented at IMAC XL in February 2022

Presented by

David A. Najera-Flores, ATA Engineering, Inc.

Robert J. Kuether, Sandia National Laboratories

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Motivation: Nonlinear Forced Vibration

Nonlinear Forced Response: determines the steady-state periodic response of a **nonlinear system** to a harmonically driven input excitation at various forcing frequencies

Two common approaches to calculate the nonlinear steady-state response:

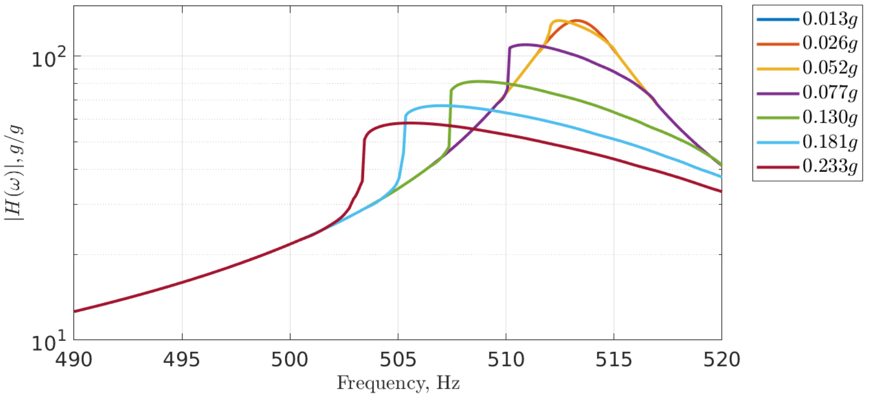
1.) Direct time integration to reach steady-state conditions – too costly for complex nonlinear structures

2.) Path-following continuation – requires specialized software

2a.) Shooting method

2b.) Multi-harmonic balance – available in MSC Nastran as SOL128!

2c.) Collocation methods



The goal is to investigate the capabilities and limitations of Nastran's MHB solver to understand the range of problems that can be solved with this approach.

Nonlinear Forced Response

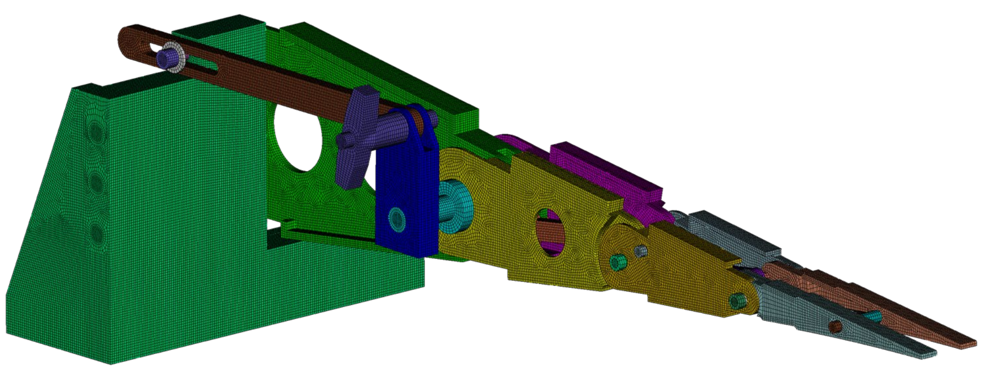
Harmonic balance methods well suited to compute periodic responses of forced, nonlinear vibrations of mechanical systems

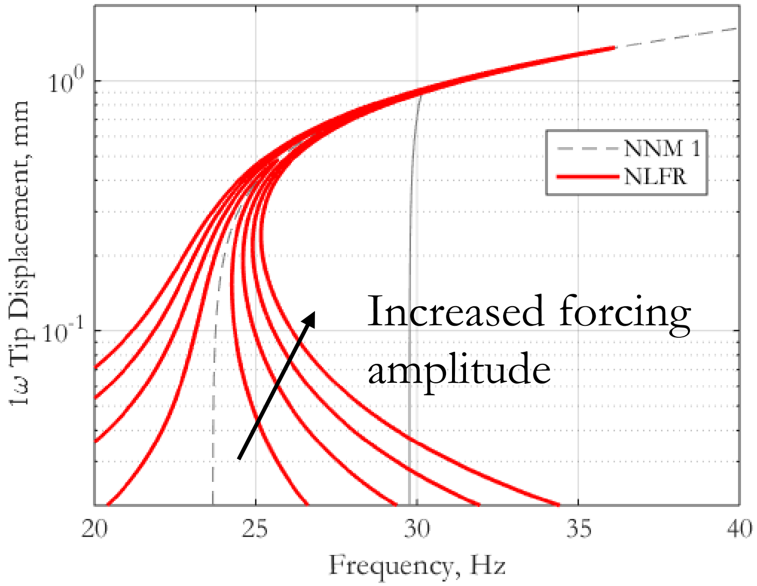
- Applicable to large-scale models
- Numerical schemes to address various forms of nonlinearity
- Filtering property of truncated Fourier series

Methods well-documented in literature and open-source research codes available (e.g. NLVib, Mousai)

Limited options available for nonlinear finite element models within commercially available software

- MSC Nastran SOL128, Abaqus Direct Cyclic Analysis
- Production use requires more general access within FEA codes with multiple nonlinear physics





Brief Review of Multi-Harmonic Balance Theory [1,2]

Goal: calculate the steady-state, periodic response of the harmonically forced system

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) + \mathbf{f}_{nl}\big(\mathbf{x}(t), \dot{\mathbf{x}}(t)\big) = \mathbf{f}_{ext}(t)$$

Assume truncated Fourier series for the periodic response and nonlinear restoring force

$$\mathbf{x}(t) = \frac{\mathbf{c}_0^x}{\sqrt{2}} + \sum_{k=1}^{N_h} [\mathbf{s}_k^x \sin(k\omega t) + \mathbf{c}_k^x \cos(k\omega t)]$$

$$\mathbf{f}_{nl}(\mathbf{x}) = \frac{\mathbf{c}_0^f}{\sqrt{2}} + \sum_{k=1}^{N_h} \left[\mathbf{s}_k^f \sin(k\omega t) + \mathbf{c}_k^f \cos(k\omega t)\right]$$

Fourier-Galerkin projection results in the nonlinear algebraic system of equations

$$\mathbf{r}(\mathbf{z}, \omega) = \mathbf{A}(\omega)\mathbf{z} + \mathbf{b}(\mathbf{z}) - \mathbf{b}_{ext} = \mathbf{0}$$

MSC Nastran SOL128 computes the HB residual at discrete frequencies defined by the user (i.e. frequency stepping)

[1] T. Detroux, L. Renson, L. Masset, and G. Kerschen, "The harmonic balance method for bifurcation analysis of large-scale nonlinear mechanical systems," *Computer Methods in Applied Mechanics and Engineering,* vol. 296, pp. 18-38, 2015.

[2] M. Krack and J. Gross, *Harmonic Balance for Nonlinear Vibration Problems*, 1st ed. Springer International Publishing, 2019.

Introduction to MSC NASTRAN SOL128

```
SOL 128
PARAM,POST,-1
TITLE = mhb
ECHO = UNSORTED
SUBCASE 1
SUBTITLE = LANCZOS
SPC = 1
DLOAD = 10
MPC = 100
NLHARM = 2000
NONLINEAR = 1000
VECTOR=ALL
DISPLACEMENT(PLOT)=ALL
SET 99 = 2
DISPLACEMENT(SORT2,PRINT,PUNCH,PHASE) = 99
BEGIN BULK
PARAM,AUTOSPC,YES
PARAM,NLHTOL,1.0e-4
GRID,1,,0.0,0.0,0.0
GRID,2,,0.0,0.0,0.0
GRID,3,,0.0,0.0,0.0
GRID,4,,0.0,0.0,0.0
SPOINT,5
MPC,100,3,1,1.0,2,1,-1.0,,,5,,-1.0
CONM2,21,1,,1.0,,,,
CONM2,22,2,,1.0,,,,
CONM2,23,3,,1.0,,,,
CONM2,24,4,,1.0,,,,
NOLIN3,1000,5,,-0.5,5,,3.0
NOLIN4,1000,5,,-0.5,5,,3.0
CELAS2,1,1.0,1,1,2,1
CELAS2,2,1.0,2,1,3,1
CELAS2,3,1.0,3,1,4,1
CDAMP2,11,0.01,1,1,2,1
CDAMP2,12,0.01,2,1,3,1
CDAMP2,13,0.01,3,1,4,1
SPC,1,4,123456,0.0,1,123456,0.0,2,23456,0.0,3,23456,0.0
RLOAD2,10,11,,,12,,0
DAREA,11,2,1,0.005
TABLED1,12,,,,,,,,0.001,1.0,100.0,1.0,ENDT
$NLFREQ1,88,1.3e-1,5.0e-6,20000
NLFREQ1,88,3.20e-1,-3.0e-6,30000
NLHARM,2000,2,2,88
ENDDATA
```

The key parameter is NLHARM, which indicates the use of the nonlinear harmonic response solution. NLHARM can only be used with NOLIN entries, so only for nonlinearities that can be modeled with a general force-displacement relationship.

Relative Tolerance for MHB solver (PARAM,NLHTOL)

Number of harmonics (NLHARM entry, 4th field: 88)

Factor for capturing sub-harmonic response (NLHARM entry, 2nd field: 2)

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CDAMP2,13,0.01,3,1,4,1
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The key parameter is NLHARM, which indicates the use of the nonlinear harmonic response solution. NLHARM can only be used with NOLIN entries, so only for nonlinearities that can be modeled with a general force-displacement relationship.

- PARAM,NLHTOL: Tolerance for MHB solver
- RLOAD2 / DAREA / TABLED1: Periodic loading, location (DAREA), and frequency dependent amplitude (TABLED1)
- NLFREQ1: Forcing frequencies for nonlinear harmonic response
- NLHARM 2,2,88: Number of harmonics; Factor for capturing sub-harmonic response

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Tolerance for MHB solver

Nonlinear transient forcing functions for positive (NOLIN3) and negative (NOLIN4) displacements. Defined at the relative DOF (u_5).

Periodic loading, location (DAREA), and frequency dependent amplitude (TABLED1)

Forcing frequencies for nonlinear harmonic response

Number of harmonics

Factor for capturing sub-harmonic response

Introduction to MSC NASTRAN SOL128

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GRID,4,,0.0,0.0,0.0
SPOINT,5
MPC,100,3,1,1.0,2,1,-1.0,,,5,,-1.0
CONM2,21,1,,1.0,,,,
CONM2,22,2,,1.0,,,,
CONM2,23,3,,1.0,,,,
CONM2,24,4,,1.0,,,,
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CDAMP2,12,0.01,2,1,3,1
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RLOAD2,10,11,,,12,,0
DAREA,11,2,1,0.005
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Tolerance for MHB solver (PARAM,NLHTOL)

u_5 defined as a scalar point (SPOINT)

MPC used to define relative DOF:

$$1.0u_3 - 1.0u_2 - 1.0u_5 = 0$$
$$u_3 - u_2 = u_5$$

Nonlinear transient forcing functions for positive (NOLIN3) and negative (NOLIN4) displacements. Defined at the relative DOF (u_5).

Periodic loading, location (DAREA), and frequency dependent amplitude (TABLED1)

Forcing frequencies for nonlinear harmonic response

Number of harmonics

Factor for capturing sub-harmonic response

Demonstration of algorithm

Demonstration of the algorithm

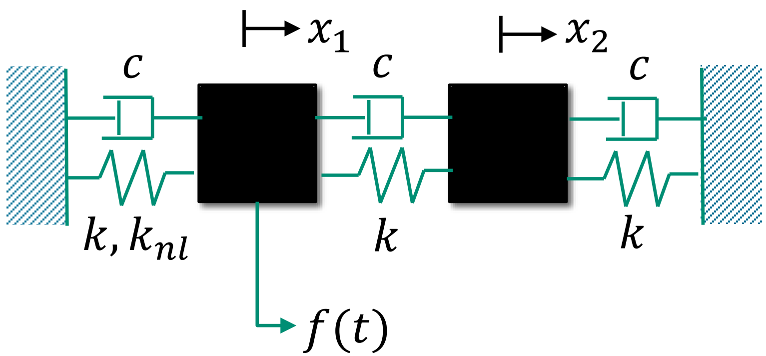
The following slides present a few examples showing Nastran's capabilities.

To obtain the nonlinear forced response curves, the solver was run twice to perform an **upward and downward frequency sweep**.

The results for the example problems are compared to reference curves obtained from an MHB implementation based on [1] within MATLAB. **The reference curves are plotted with a dashed line.**

[1] T. Detroux, L. Renson, L. Masset, and G. Kerschen, "The harmonic balance method for bifurcation analysis of large-scale nonlinear mechanical systems," *Computer Methods in Applied Mechanics and Engineering*, vol. 296, pp. 18-38, 2015.

Examples: 2DOF mass-spring-damper oscillator with grounded cubic spring

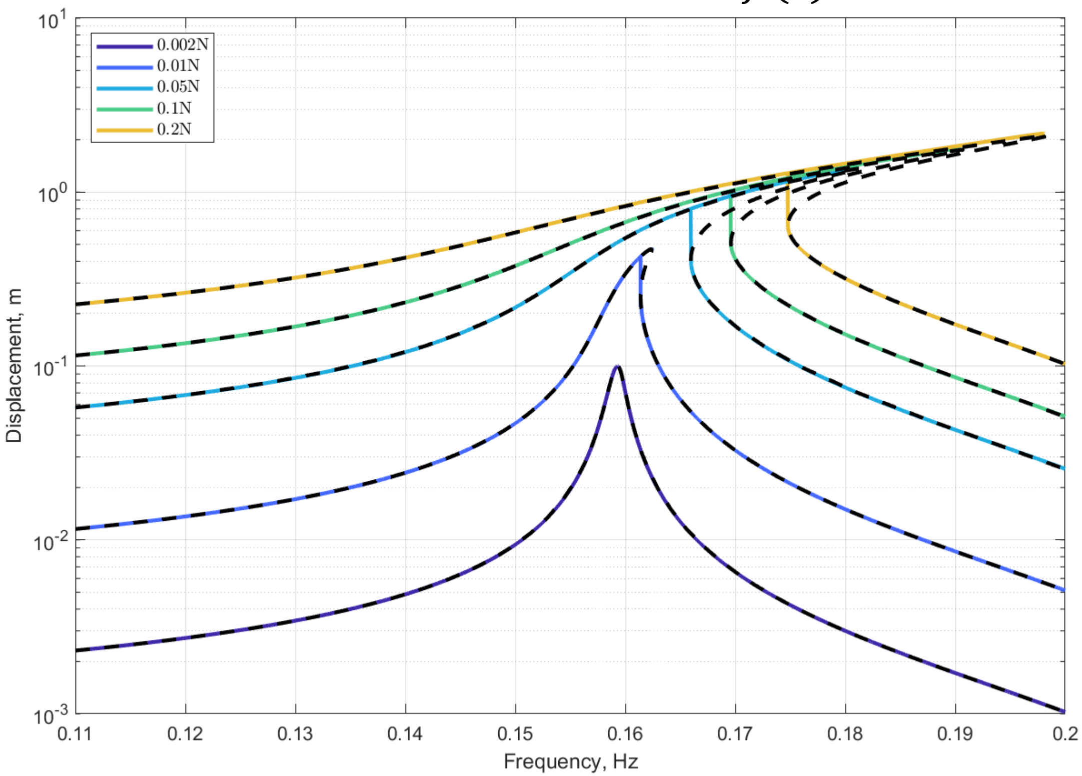


$k = 1.0$

$m = 1.0$

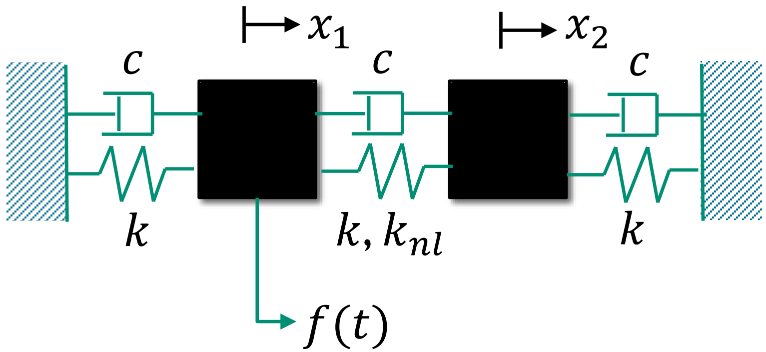
$c = 0.01$

$k_{nl} = 0.5$



30,000 frequency lines used
8 minute runtime per curve
on 1 CPU

Examples: 2DOF mass-spring-damper oscillator with centered cubic spring

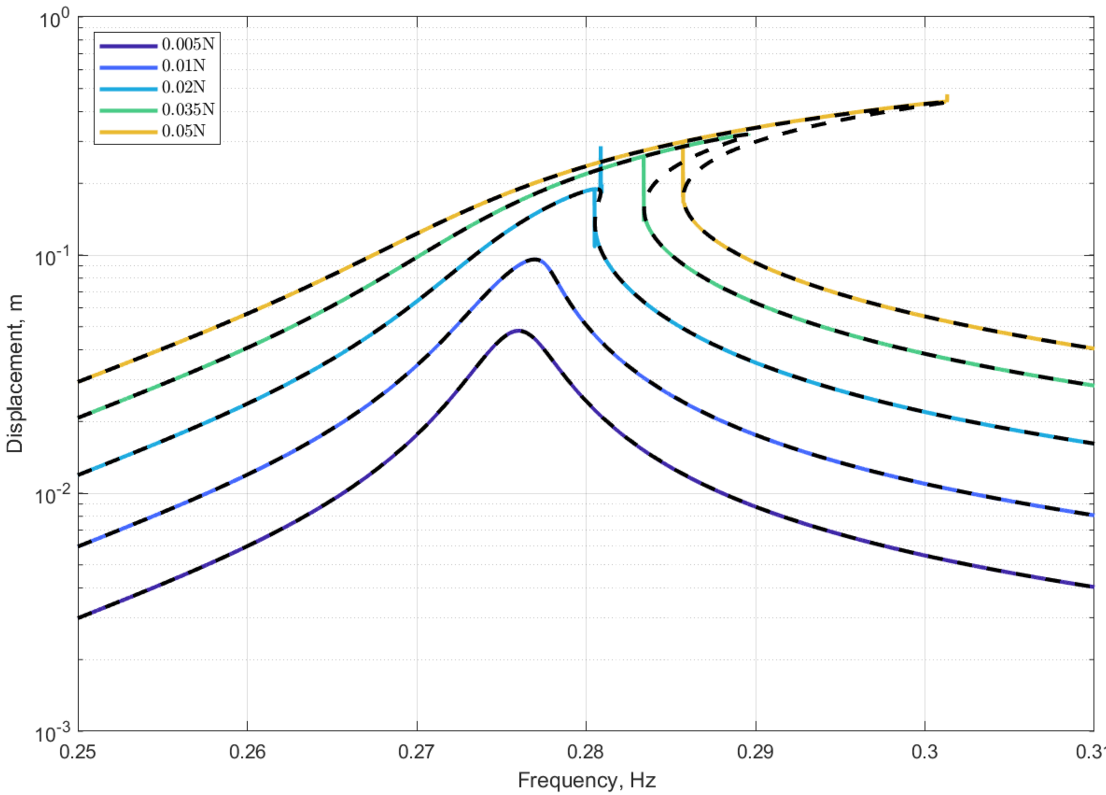


$k = 1.0$

$m = 1.0$

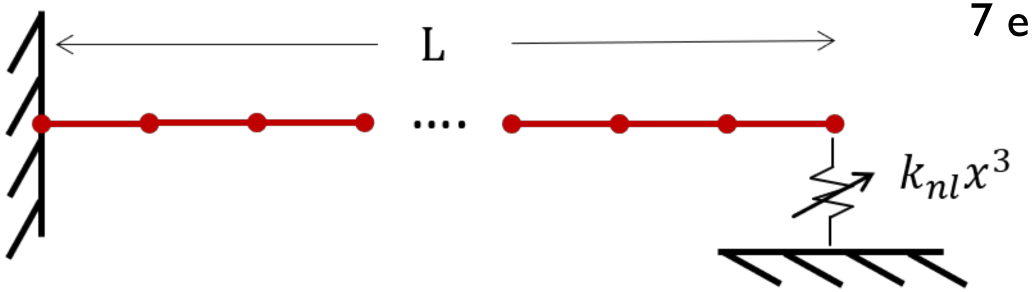
$c = 0.01$

$k_{nl} = 0.5$



30,000 frequency lines used
8 minute runtime per curve on 1 CPU

Examples: Cantilever Euler-Bernoulli beam with grounded cubic spring at tip



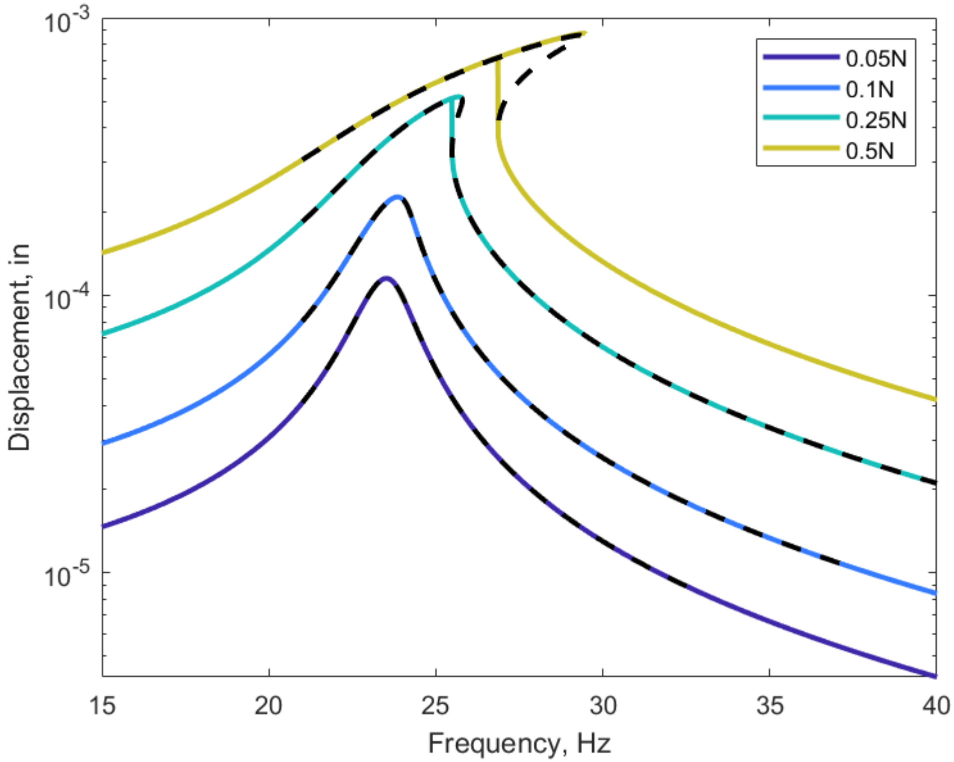
7 elements, generic steel material

$$L = 0.7$$

$$k_{nl} = 6 \times 10^9$$

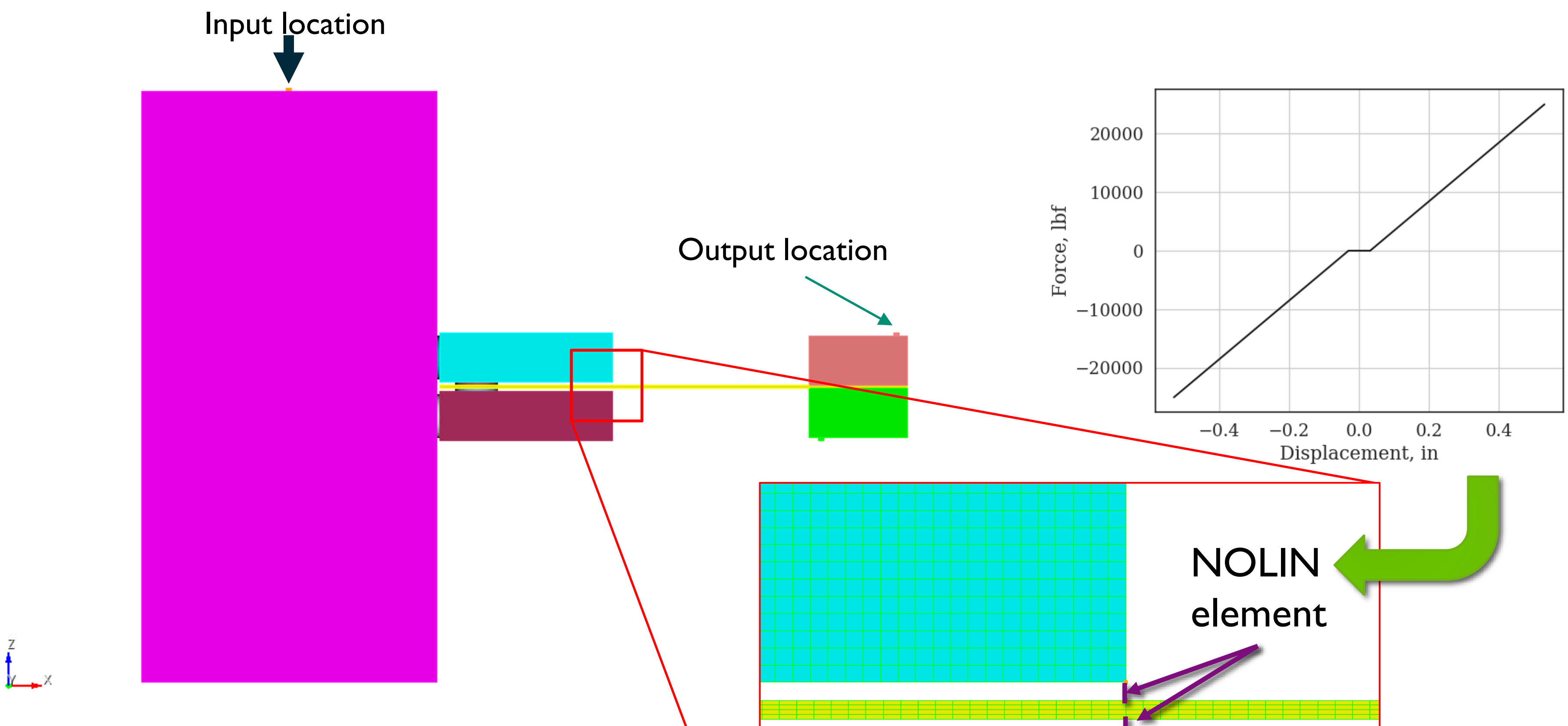
$$C = \alpha M + \beta K$$

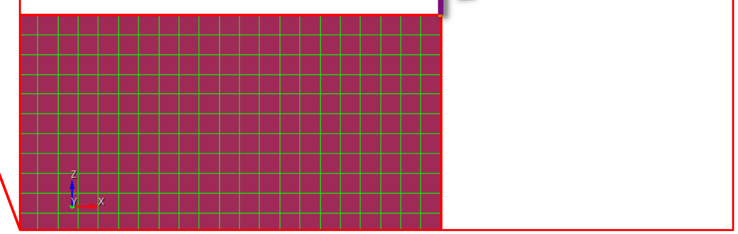
$$\alpha = 0.001, \beta = 0.0005$$

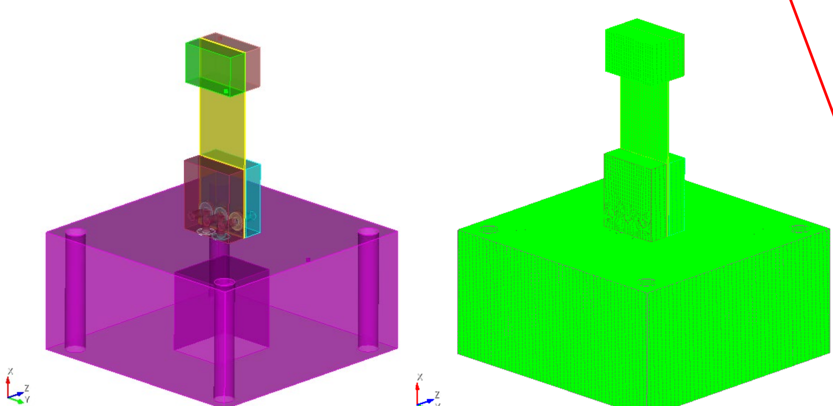


30,000 frequency lines used
7 minute runtime per curve on 1 CPU

Examples: 3.9M Element Pylon FEM







Work in progress. Results may be available by conference presentation date.

Conclusion and Summary of Capabilities and Limitations

MSC Nastran SOL128 can be used to obtain nonlinear forced response curves for cases where the nonlinearity can be modeled with NOLINi elements.

NOLINi elements provide the **capability to model arbitrary nonlinear force-displacement/velocity functions** (e.g., in tabular form or through polynomials). One limitation of NOLINi elements is that they are meant to be used as nonlinear force functions, not necessarily as connector elements. For this reason, they are typically a function of a single DOF. One workaround demonstrated in this presentation is to define relative DOFs through MPCs and SPOINTs.

Unfortunately, other types of nonlinearities, such as **hysteretic behavior cannot be currently used in combination with SOL128**. This means that the SOL128 **cannot handle plasticity or frictional contact**. In addition, **geometric nonlinearities cannot be used either**.

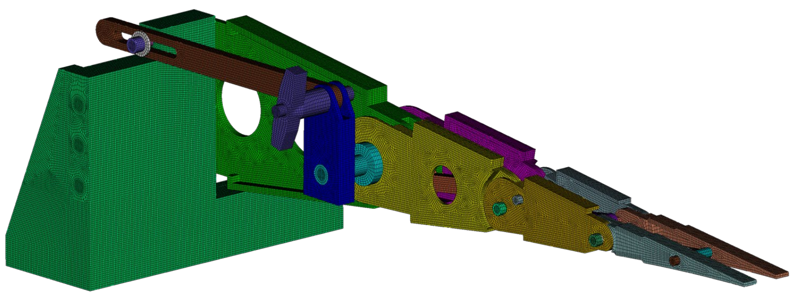
Moreover, **SOL128 cannot be used in combination with a static preload** which heavily restricts the type of problems that can be solved. Intrusive techniques can be used to work around this limitation (e.g., importing linearized stiffness matrix) but these were not explored in this work.

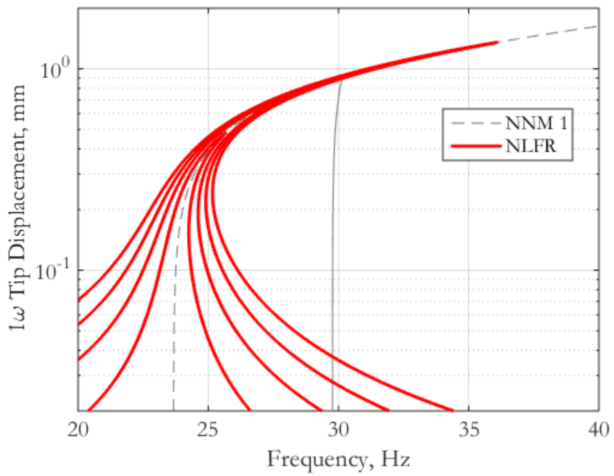
For more general cases, tools such as NLvib can be used to perform MHB, especially when a large part of the model can be assumed to be linear and reduced with substructuring techniques. [https://www.ila.uni-stuttgart.de/nlvib/].

Any Questions?

Contact information

- David Najera, david.najera@ata-e.com
- Robert Kuether, rjkueth@sandia.gov





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