



Progress in Line-Shape Theory: the BALROG code



Thomas Gomez



Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

Common Approximations in Line-Shape Calculations



- Line shape theory is a multi-disciplinary field, requiring
 - Atomic Physics
 - Collision Physics
 - Plasma Physics
 - Statistical Mechanics
- Drawing from so many sources usually means that approximations have to be employed to keep calculations tractable
- Approximations include
 - Static Ion Approximation
 - No-Quenching Approximation
 - Dipole Approximation
 - 2nd-Order Approximation
 - Classical Approximation
 - Factorized Density Matrix Approximation
 - Binary-Collision Approximation
 - Straight-Path & Screening Approximations

The Xenomorph Code



- The Xenomorph code was developed during my master's thesis and has been improved upon by P. Cho
 - P. Cho will discuss more after this
- Xenomorph easily removes some of these approximations
 - As a simulation, it easily tackles several approximations all at once

- Approximations include
 - ~~Static Ion Approximation~~
 - ~~No Quenching Approximation~~
 - Dipole Approximation
 - ~~2nd-Order Approximation~~
 - Classical Approximation
 - Factorized Density Matrix Approximation
 - ~~Binary Collision Approximation~~
 - Straight-Path & Screening Approximations

The Xenomorph Code



- The Xenomorph code was developed during my master's thesis and has been improved upon by P. Cho
 - P. Cho will discuss more after this
- Xenomorph easily removes some of these approximations
 - As a simulation, it easily tackles several approximations all at once
- But, it is missing some important physics
 - Dipole approximation and Classical Approximation cannot be removed within the framework of Xenomorph
 - Removing these approximations requires a new approach

- Approximations include
 - ~~Static Ion Approximation~~
 - ~~No Quenching Approximation~~
 - **Dipole Approximation***
 - ~~2nd-Order Approximation~~
 - **Classical Approximation**
 - Factorized Density Matrix Approximation
 - ~~Binary Collision Approximation~~
 - ~~Straight-Path & Screening Approximations~~

We Can Include Other Physics By using Different Approach



- By changing to an analytic approach, we can include different physics than simulations
- Analytic calculations are able to incorporate
 - Finite size of plasma electrons [1]
 - Penetration of plasma electrons into the radiator wavefunctions [2]
 - Include more general density matrix [3,4]
 - Include plasma electrons becoming bound to atom [5]

Xenomorph cannot capture any of these effects

- Approximations include
 - Static Ion Approximation
 - ~~No Quenching Approximation~~
 - ~~Dipole Approximation~~
 - ~~2nd-Order Approximation~~
 - ~~Classical Approximation~~
 - ~~Factorized Density Matrix Approximation~~
 - ~~Binary-Collision Approximation~~
 - ~~Straight-Path & Screening Approximations~~

Analytic Calculations have a Severe Problem. Until Now



- However, analytic codes so far have had difficulty removing the 2nd-Order Approximation

$$T(E) = \frac{1}{1 - V[E - H_0]^{-1}} V$$
$$\approx V + V[E - H_0]^{-1} V$$

The 2nd-order approximation poses a problem because it does not preserve unitarity

$$U^\dagger(t)U(t) = 1$$

$$|\langle \psi(t) | \psi(t) \rangle|^2 = 1$$

- Approximations include
 - Static Ion Approximation
 - ~~No Quenching Approximation~~
 - ~~Dipole Approximation~~
 - 2nd-Order Approximation
 - Classical Approximation
 - ~~Factorized Density Matrix Approximation~~
 - Binary-Collision Approximation
 - ~~Straight-Path & Screening Approximations~~

We Learned how to Remove the 2nd-order Approximation in Analytic Methods (Balrog Code)



- Consultation with collision physicists gave us the tools to remove the 2nd-order approximation
- The technique involves re-arranging the problem to be $Ax=b$ and use linear solver

$$T(E) = \frac{1}{1 - V[E - H_0]^{-1}} V$$

$$\{1 - V[E - H_0]^{-1}\} T(E) = V$$

- Balrog presents a significant update to the current state of line-shape models

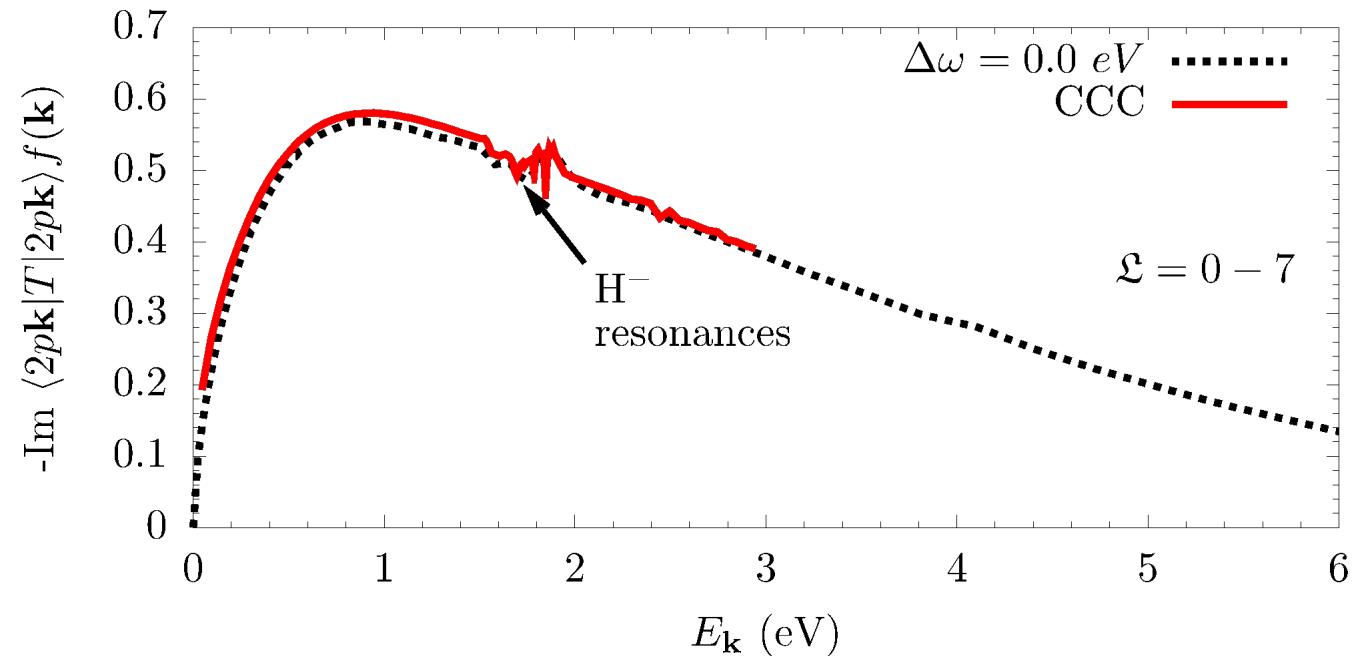
- Approximations include
 - Static Ion Approximation
 - ~~No Quenching Approximation~~
 - ~~Dipole Approximation~~
 - ~~2nd-Order Approximation~~
 - ~~Classical Approximation~~
 - ~~Factorized Density Matrix Approximation~~
 - ~~Binary-Collision Approximation~~
 - ~~Straight-Path & Screening Approximations~~

We Need to Confirm the Accuracy of Our Calculations by Comparing against CCC



- Before we start applying our T-matrices to line-shapes, we need to confirm that our code is accurate
- The CCC [1] code is a collision code that calculates T-matrices that treats the plasma particles quantum mechanically and uses the full-Coulomb treatment for the interaction (with Pauli exclusion/exchange)
- For these reasons, we choose to compare our T-matrices against those

We are now free to remove certain physics to test approximations



9 Balrog Preserves Unitarity to Machine Precision



- The S-matrix defined in terms of the T-matrix

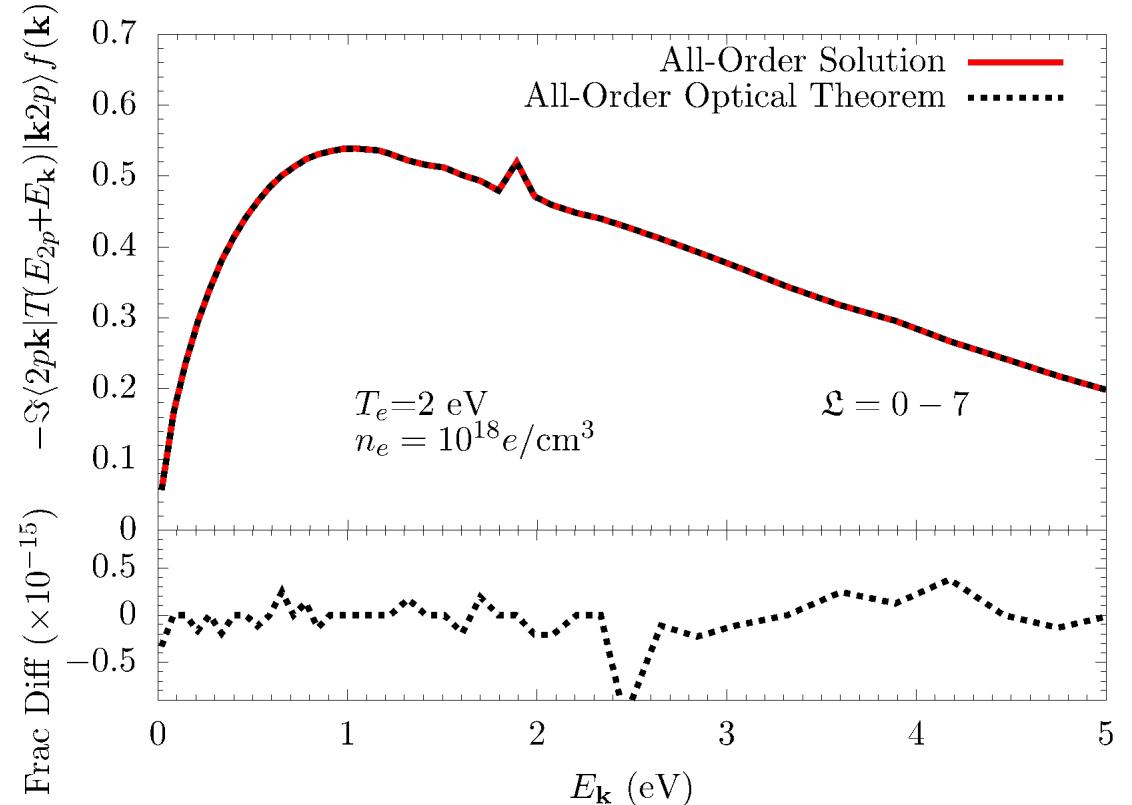
$$S = 1 - 2i\pi\delta(E - H_0)T(E)$$

- From the unitarity condition of the S-matrix, one can derive the optical theorem [1]

$$SS^\dagger = 1$$

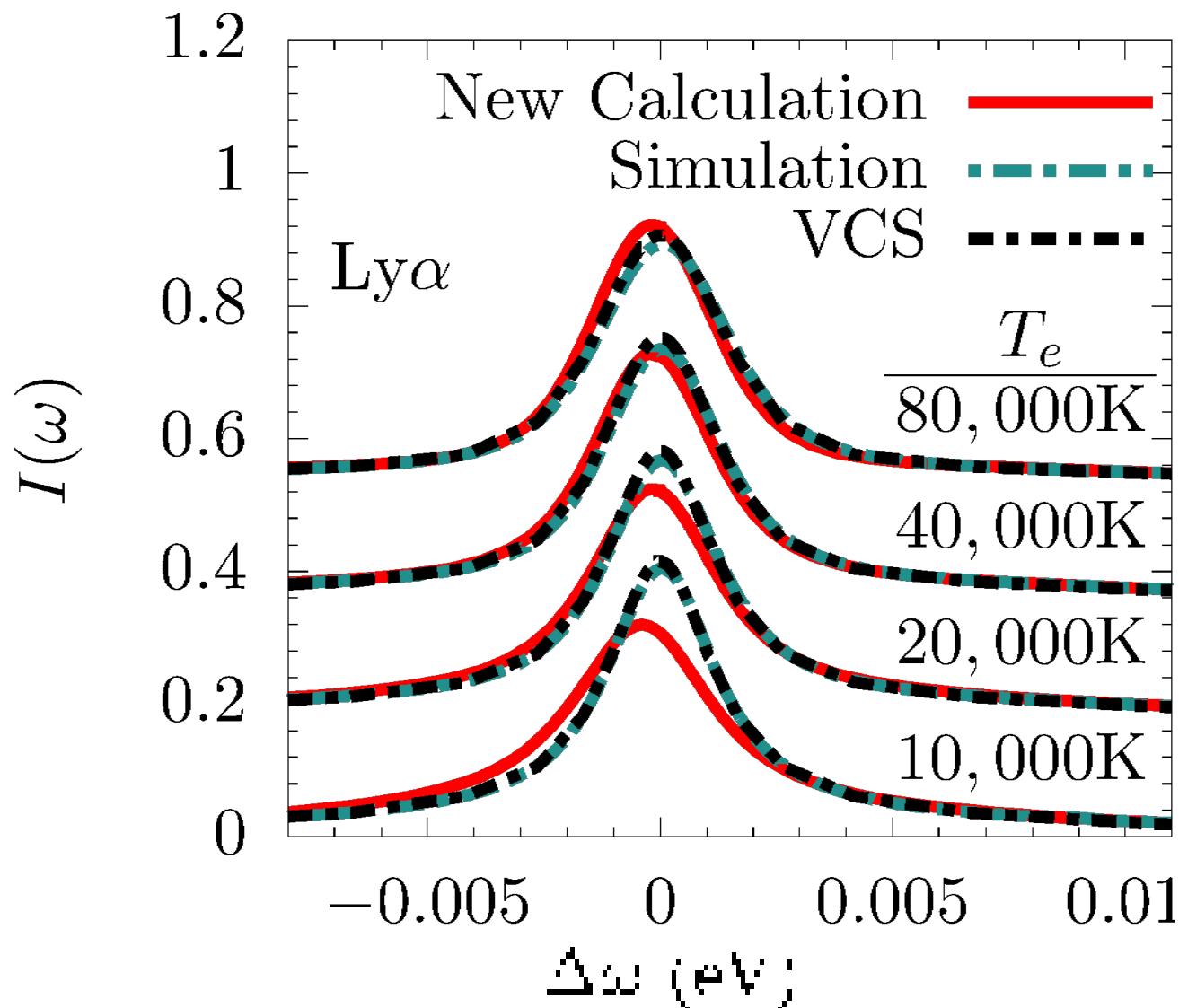
$$\Im T(E) = -\pi T(E)\delta(E - H_0)T^\dagger(E)$$

- Therefore, we know that if our T-matrix solution preserves the optical theorem that it preserves unitarity



Optical Theorem is Satisfied to one part in 10^{15} —which is the double precision limit

Quantum Calculations Are Important For Some Transitions



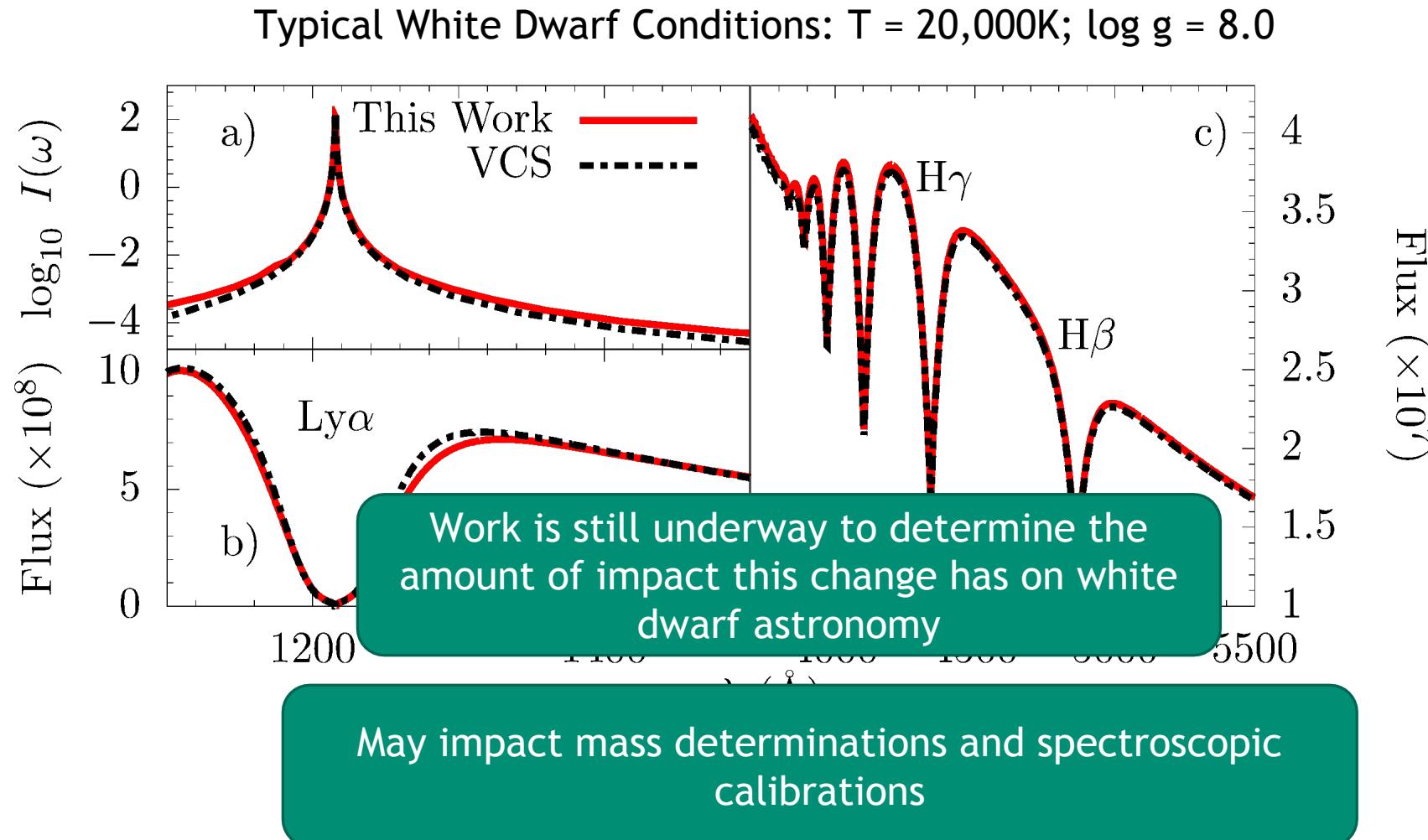
- Ly α is the strongest line in stellar spectra
- Dominant source of opacity
- Changes in Ly α spectra can have wide ranging effects in stellar spectra
- Current line-broadening models are based on the Vidal-Cooper-Smith (VCS) model [1]
 - Classical Approximation
 - Dipole Approximation
 - All-Order
- New model is broader than VCS at low temperatures but is roughly equivalent at high temperatures

These Differences in Ly α Result in Increases in the White Dwarf Visible Spectrum



- The calculation with additional physics resulted in broader line shapes
- This raised the opacity in the area where most of the flux comes out of the star
 - The radiation has to get out elsewhere, so it comes out in the UV and visible
 - Thus raising the continuum flux by a few percent

(Larger than the <1% error estimates of WD fluxes [1,2])



The Fe opacity experiment at Sandia National Laboratories



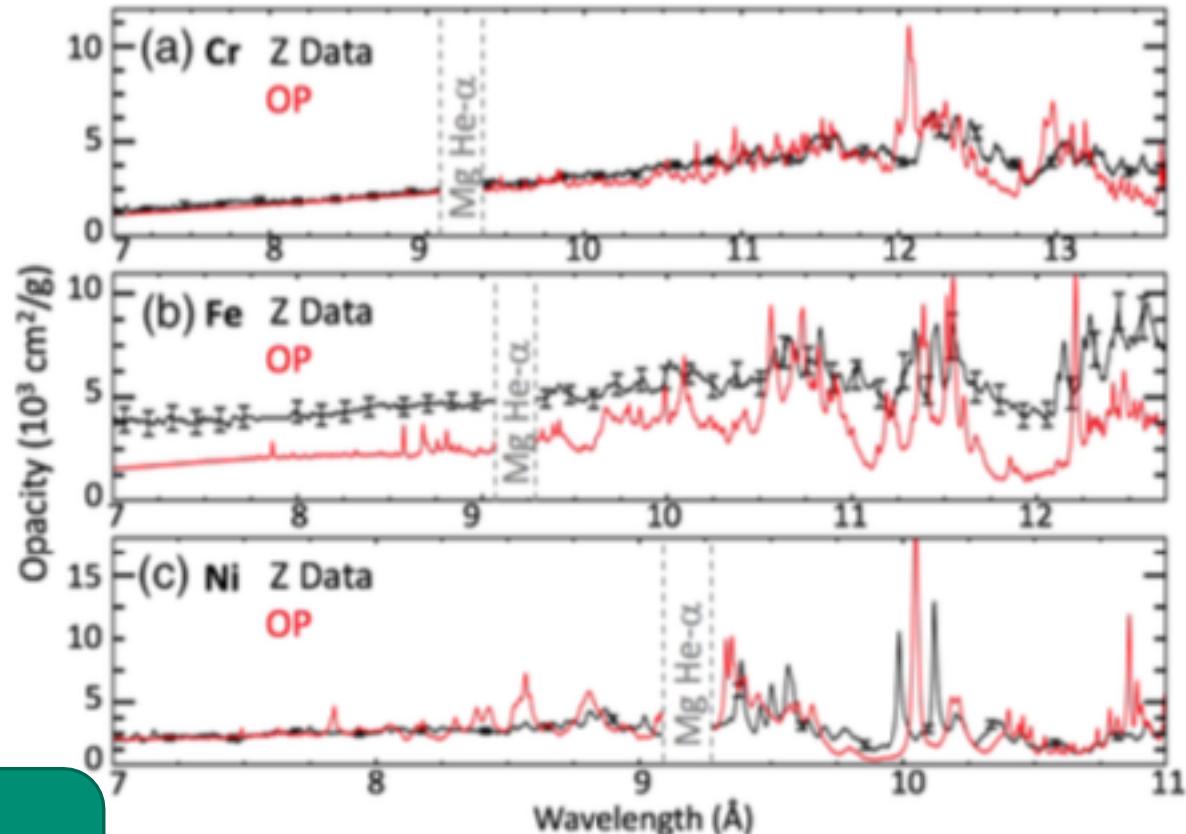
➤ At the Z-machine at Sandia, we are trying to measure the opacity of Fe at solar interior conditions

➤ Opacity of Fe is substantially greater than predicted by models [1,2]

➤ One of the tools used to determine the plasma conditions is fitting line shapes of K-shell transitions of Mg

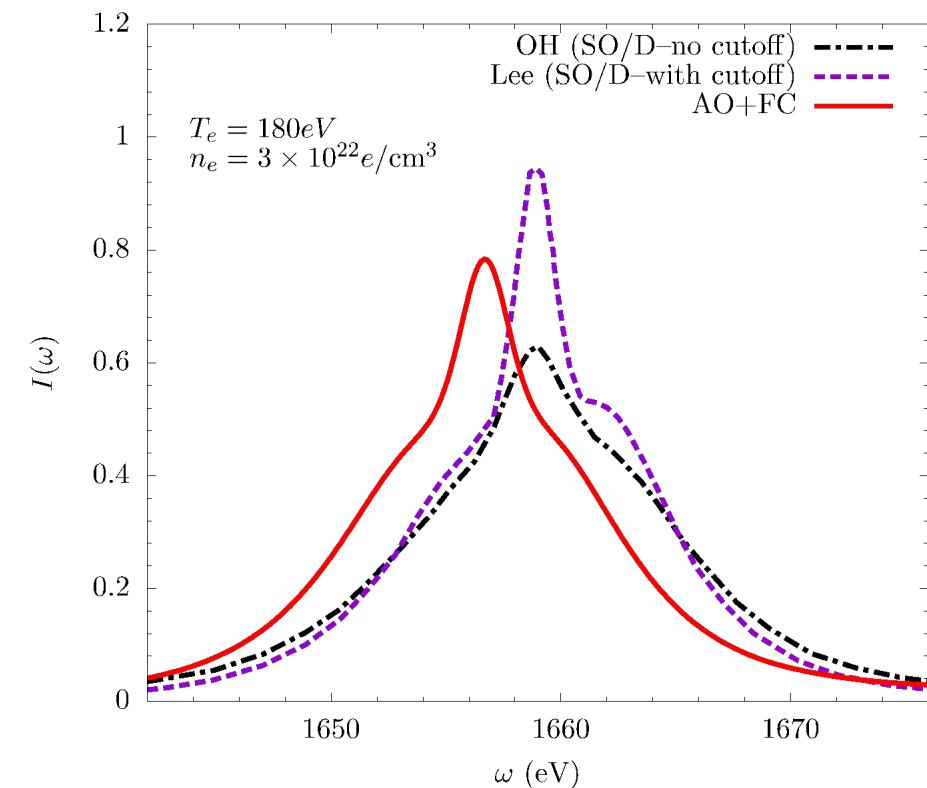
➤ Mg Ly β
➤ Mg He γ

Improving line-shape model will help determine true discrepancies between data and model



- Nagayama [2] and Iglesias [3] showed how two commonly-used models (O'Brien & Hooper [4] and Lee [5]) differ in inferred electron density by more than 50%
 - Both use second-order and dipole approximations
 - Lee uses a strong collision treatment, OH does not
- New calculation is between OH and Lee and has additional shift

Preliminary fits to data indicate 30% higher density than estimated by OH



Using Balrog to Calculate Fe opacity



- Balrog can also be used to calculate opacities at solar interior conditions
- The full Balrog treatment of the O spectra
 - Modifies line intensities
 - Raises opacity between Ly α and Ly β
 - Gives structure to the wings of Ly α
- It's always been assumed that line shapes are area normalized
- However, this doesn't seem to be the case in balrog, where the oscillator strength is increased with plasma effects
 - i.e. may violate the TRK f-sum rule for H-like oxygen
- This raises questions about the line shapes
- May have

