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subject: Tearing parameter failure integration with the multilevel solver

Abstract

The tearing parameter criterion and material softening failure method currently used in the multilinear elastic-plastic constitutive model was added as an option to modular failure capabilities. The modular failure implementation was integrated with the multilevel solver for multi-element simulations. Currently, this implementation is only available to the J_2 plasticity model due to the formulation of the material softening approach. The implementation compared well with multilinear elastic-plastic model results for a uniaxial tension test, a simple shear test, and a representative structural problem. Necessary generalizations of the failure method to extend it as a modular option for all plasticity models are highlighted.

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1 Introduction

The tearing parameter criterion and material softening failure method currently used in the multilinear elastic-plastic (MLEP) material model was recently added as an option to modular failure capabilities in the Library of Advanced Materials for Engineering (LAMÉ) [1, 2]. Currently, this implementation is only available to the J_2 plasticity model due to the formulation of the material softening approach. The implementation was verified against analytical solutions for both a uniaxial tension and a pure shear boundary-value problem [1]. The initial J_2 plasticity implementation was for a single element.

The MLEP material model uses the tearing parameter as a failure criterion and a critical crack opening, linear softening model as a failure mechanism. For multi-element simulations, Wellman [3] noted convergence issues with the direct implementation of this failure technique in the MLEP constitutive model and the resolution of these issues with a multilevel solver algorithm. For the initial, single element J_2 plasticity implementation [1], the multilevel solver is not used. This memo details the integration of the failure model implementation in J_2 plasticity with the multilevel solver for multi-element simulations and demonstrates the modular softening failure capabilities on a representative structural problem.

2 Theory

2.1 Tearing parameter

For the modular failure criterion, a tearing parameter damage variable d is implemented [4] as a normalization of the modified tearing parameter t_p in Wellman [3] and defined as

$$d = \frac{t_p}{t_p^{crit}} = \frac{1}{t_p^{crit}} \int_0^{\bar{\varepsilon}^p} \left\langle \frac{2\sigma_{max}}{3(\sigma_{max} - p)} \right\rangle^m d\hat{\bar{\varepsilon}}^p. \tag{1}$$

Here, σ_{max} is the maximum principal stress and pressure $p = \frac{1}{3}\mathrm{tr}(\boldsymbol{\sigma})$ is the trace of the Cauchy stress tensor $\boldsymbol{\sigma}$. Also, $\bar{\varepsilon}^p$ is the equivalent plastic strain and $\langle\cdot\rangle$ are Macaulay brackets. The damage variable has two model parameters; m, a fitting exponent, and t_p^{crit}, the critical tearing parameter. During loading, the damage value increases from $d = 0$ in the initial state to the critical damage value $d = 1$. For uniaxial tension, the integrand in the damage variable is one and the tearing parameter is equal to the equivalent plastic strain, $t_p = \bar{\varepsilon}^p$.

2.2 Failure method

For the failure method in Wellman [3], failure initiates at the critical damage value (Figure 1). Prior to failure initiation, the underlying plasticity model is unaltered; the material response is linear elastic until yield (ε_y) and then elastic-plastic until critical damage (ε_d). Once the damage (tearing parameter) criterion is satisfied ($d \geq 1$), stress decays linearly to zero over a critical crack opening strain of magnitude ε_{ccos}, after which the stress remains zero at failure (ε_f). The Macaulay brackets ensure the damage variable (1) is non-zero, and failure initiates only for tensile stress states. This failure method will be referred to as the critical crack opening strain, linear softening method. Details of the transition between plasticity and failure, the stress decay method, and their algorithmic implementation may be found in SAND2021-13778 R [1].

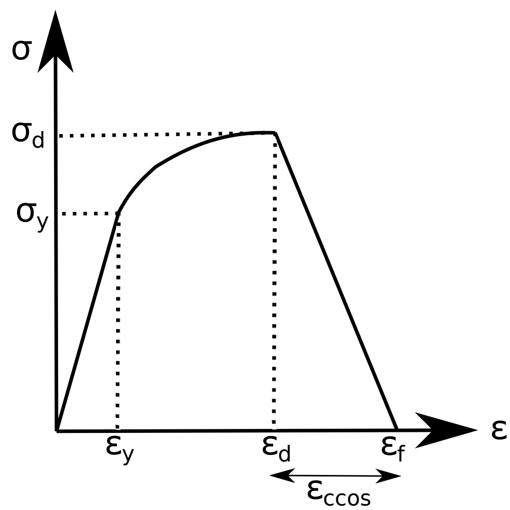


Figure 1: Stress-strain curve for plasticity model with failure.

3 Algorithm

The failure method in Section 2.2 describes the process applicable to any finite element. However, for multi-element simulations, Wellman [3] discusses convergence issues with the direct implementation of this technique in the constitutive model and the resolution of these issues with a multilevel solver algorithm. To assist in the coupling of the constitutive model to the multilevel solver, a set of five crack failure flags is defined for each element as follows:

(1) Crack flag = 0.
The damage criterion, $d_{n+1} \geq 1$, has not been satisfied. Crack initiation has not occurred in the element. Material response is the underlying plasticity model.

(2) Crack flag = 1.
The damage criterion, $d_{n+1} \geq 1$, has been satisfied for the element.

(3) Crack flag = 2.
The element that exceeded the damage criterion the most out of all elements marked with crack flag = 1. Mark as a transition step and use partitioned stress decay.

(4) Crack flag = 3.
The element is in failure. Material response is stress decay.

(5) Crack flag = 4.
The element has failed. Stress is identically zero.

The multilevel solution algorithm is described in Box 1. The plasticity response and stress decay response are local to the constitutive model (multilevel solver Level 0) while the inter-element damage criterion comparison is nonlocal and managed outside the constitutive model by the solver (multilevel solver Level 1).

(1) Start new load step.

(2) Multilevel solver: Level 0

  (a) Converge solution.
    (i) Undamaged elements (crack flag = 0) respond according to the underlying plasticity model.
    (ii) Transition element (crack flag = 2) response is part plasticity and part stress decay.
    (iii) Damaged elements (crack flag = 3) undergo stress decay.
    (iv) Failed elements (crack flag = 4) remain zero stress.
  (b) Mark undamaged elements (crack flag = 0) which exceed the damage criterion by advancing crack flag = 1.
  (c) If an element underwent the transition step (crack flag = 2), advance to the decay regime by setting crack flag = 3.
  (d) For all elements that have decayed to zero stress, advance crack flag = 4.
  (e) Update state variables.

(3) Multilevel solver: Level 1

  (a) Do any elements exceed the damage criterion (crack flag = 1)?
  No – go to (1)
  Yes – continue
  (b) From all possible elements undergoing the transition step from plasticity to failure (crack flag = 1), select the element most exceeding the damage criterion and set crack flag = 2.
  (c) For remaining elements with crack flag = 1, reset crack flag = 0.
  (d) Return to (2)

Box 1 : Multilevel solution algorithm for failure.

At the start of a new load step, the multilevel solver begins in Level 0. All elements are either undamaged (crack flag 0), in the stress decay failure regime (crack flag 3), or failed with the stress identically zero (crack flag 4). For this set of crack flags, the first solution iteration is converged; undamaged elements (crack flag 0) respond according to the underlying plasticity model, stress decay elements (crack flag 3) linearly decay the stress according to the failure technique detailed in [1], and failed elements (crack flag 4) remain unchanged with zero stress. Stress decay elements which decay to zero stress are marked as failed (crack flag 4). After the first solution iteration, the undamaged elements (crack flag 0) are checked against the damage criterion and all that exceed the criterion are marked with crack flag 1.

After the first solution iteration, control is passed to multilevel solver Level 1. Here, the solver checks if any element is marked as exceeding the damage criterion (crack flag 1). If not, the first solution iteration is the converged solution, control is passed back to multilevel solver Level 0, and the next load step is initiated or the simulation completed. However, if elements are marked with crack flag 1, the Level 1 solver identifies the undamaged element which exceeded the damage criterion the most and marks this element (setting crack flag 2) for a transition step from plasticity to failure. All remaining elements that exceeded the damage criterion (crack flag 1) are reset to undamaged elements (crack flag 0). Solution control is returned to the Level 0 solver and a second solution iteration is converged. During this iteration, the transition element (crack flag 2) response is part plasticity and part stress decay according to the failure technique detailed in [1]. After the partitioned stress decay, the transition element (crack flag 2) is advanced to the stress decay regime (crack flag 3). Again, the undamaged elements (crack flag 0) are checked against the damage criterion and all that exceed the criterion are marked with crack flag 1. This set of elements is possibly different than the set in the previous solution iteration as the transitioned element may alter the stress state in the neighboring elements.

Solution control iterates between the Level 0 solver and Level 1 solver as one element completes the transition step per iteration. When no more undamaged elements exceed the damage criterion at the end of a multilevel solution iteration, the next load step is initiated or the simulation completed.

3.1 Implementation

The guide to interactions between LAMÉ material models and Sierra multilevel solver capabilities [5] was followed to implement the failure model for multi-element simulations.

Communication between LAMÉ material models and Sierra/Solid Mechanics for material failure is facilitated by the six element state variables: `crack_flag_old`, `crack_flag_new`, `failure_measure_old`, `failure_measure_new`, `decay_old`, and `decay_new`. These fields are allocated in the `matParams` data structure when the `use_failure` flag is set to true.

The `crack_flag` variables describe the failure state of an element as enumerated above with one exception. For implicit simulations, a crack flag of 2 or 20 are treated as identical; the value of 20 is used to indicate that Sierra/Solid Mechanics set the crack failure flag in its marking routine. The `failure_measure` variables store the damage value d and are used to rank marked elements to select the element which exceeded the damage criterion the most out of all marked elements. The `decay` variables store the stress decay factor used to linearly decay the element stress. The stress decay factor is initially 1 and decreases to 0 when the element fails and the stress is identically zero. These variables are used to proportionally decay other influencing factors, such as hourglass force, in a compatible manner.

4 Verification

The tearing parameter criterion and material softening method was implemented as a modular failure option for the J_2 plasticity model [1]. For a single element, the modular failure implementation was verified analytically for a uniaxial tension test and a pure shear test. Here, the modular failure implementation was integrated with the multilevel solver for multi-element simulations and is compared with MLEP results for a uniaxial tension test, a simple shear test, and a representative structural problem.

4.1 Uniaxial tension

The boundary-value problem is uniaxial tension of a $3 \times 3 \times 1$ block of unit cubes. Two faces of the block, the $(x_1 x_3)$-plane and the $(x_2 x_3)$-plane intersecting at the origin, are fixed in their normal directions. A constant logarithmic strain rate ramp load is applied to the block in the x_1-direction by specifying the applied displacement in this direction as

$$u_1 = (e^{\varepsilon_L \left(\frac{t}{t_L}\right)} - 1) X_1 \tag{2}$$

where ε_L is the loading strain and t_L is the ramp loading time (Figure 2). Here, the loading strain is $\varepsilon_L = 0.05$ and $t_L = 1$ s. The logarithmic strain in the applied displacement direction is

$$\varepsilon_{11} = \varepsilon_L \left(\frac{t}{t_L} \right). \tag{3}$$

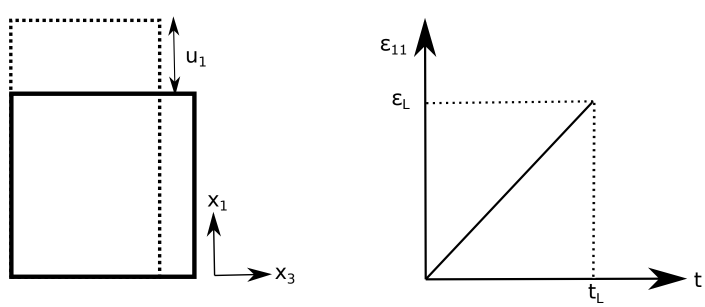


Figure 2: Applied loading and strain history for uniaxial tension.

The material parameters used are shown in Table 1. For the plasticity model, a linear hardening rule is assumed and given by

$$\bar{\sigma}(\bar{\varepsilon}^p) = \sigma_y + H' \bar{\varepsilon}^p, \tag{4}$$

where σ_y is the yield stress and H' the hardening modulus.

For the center element, all material parameters are the same as in Table 1 with the exception of the critical tearing parameter, which was set to $t_p^{crit} = 0.03$. The deformation is homogeneous prior to

E	70 GPa
ν	0.25 MPa

HARDENING MODEL	LINEAR
FAILURE MODEL	TEARING_PARAMETER_SOFTENING

σ_y	200 MPa
H'	500 MPa

t_p^{crit}	0.04
ε_{ccos}	0.005
m	4.0

Table 1: The material properties for the J_2 plasticity model with tearing parameter failure.

failure initiation; the lower critical tearing parameter of the center element will initialize failure in this element.

The results of the analysis, averaged over the nine elements, are shown in Figure 3 and Figure 4. The averaged axial Cauchy stress, damage variable, and yield surface radius match well for the J_2 plasticity model with modular failure and the MLEP model.

The axial stress-strain response in each element of the middle row of the block is shown in Figure 5. Axial stress plots at two axial strains are depicted in Figure 6. Again, the results match well for the J_2 plasticity model with modular failure and the MLEP model.

The averaged axial stress is linear elastic until yield and then elastic-plastic until damage initiates in the center element. The axial stress then softens as the center element stress decays until the center element stress is identically zero at $\epsilon = 0.0336$. As the center element decays, the left and right elements of the middle row bear more of the load. After the center element stress decays to zero, the stress in the left and right elements of the middle row continues to increase until failure initiates simultaneously in these elements and their stress decays to zero at $\epsilon = 0.0361$. At this point, the three elements in the middle row have failed, the stress is zero throughout the block, and the top layer of elements moves as a rigid body under additional displacement loading.

The critical tearing parameter t_p^{crit} of the center element was set lower than the critical tearing parameter of the remaining blocks to initialize failure in this element and demonstrate integration of the multilevel solver. If the critical tearing parameter of the left and right elements of the middle row were reduced to approach the critical tearing parameter of the center element, the stress-strain curves of the left (red) and right (black) elements in Figure 5 shift left toward the center (blue) element stress-strain curve and the magnitude of their peak stress decreases. That is, after failure initiation and stress decay occurs in the center element, the left and right elements of the middle row bear more of the load as before, but also undergo failure initiation at a smaller axial strain due to their reduced critical tearing parameter. As their critical tearing parameter is reduced, the left and right elements of the middle row initiate failure and stress decay prior to full failure of the center element. When the critical tearing parameter of all elements in the middle row are equal, all three elements fail simultaneously.

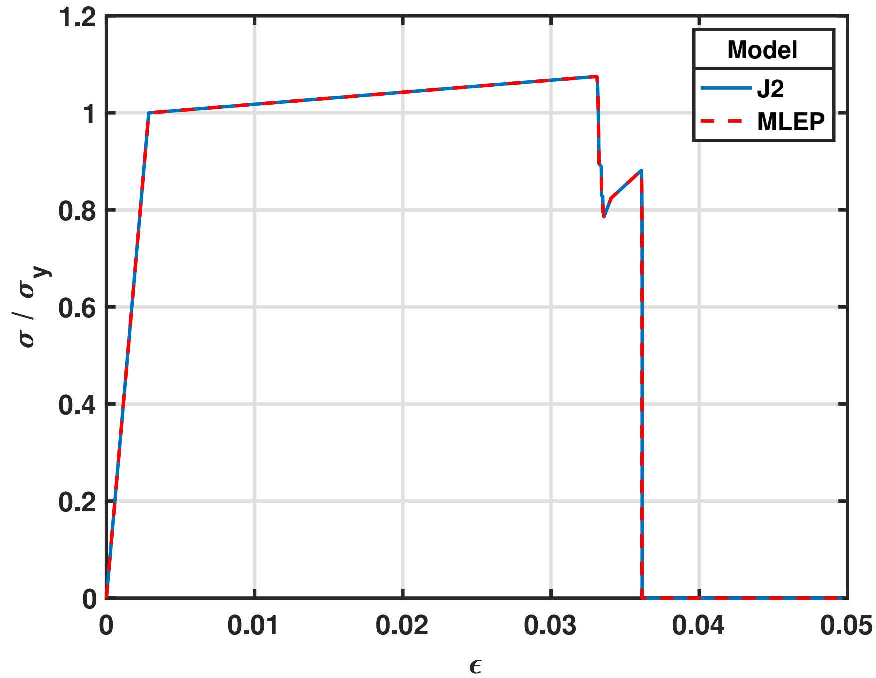
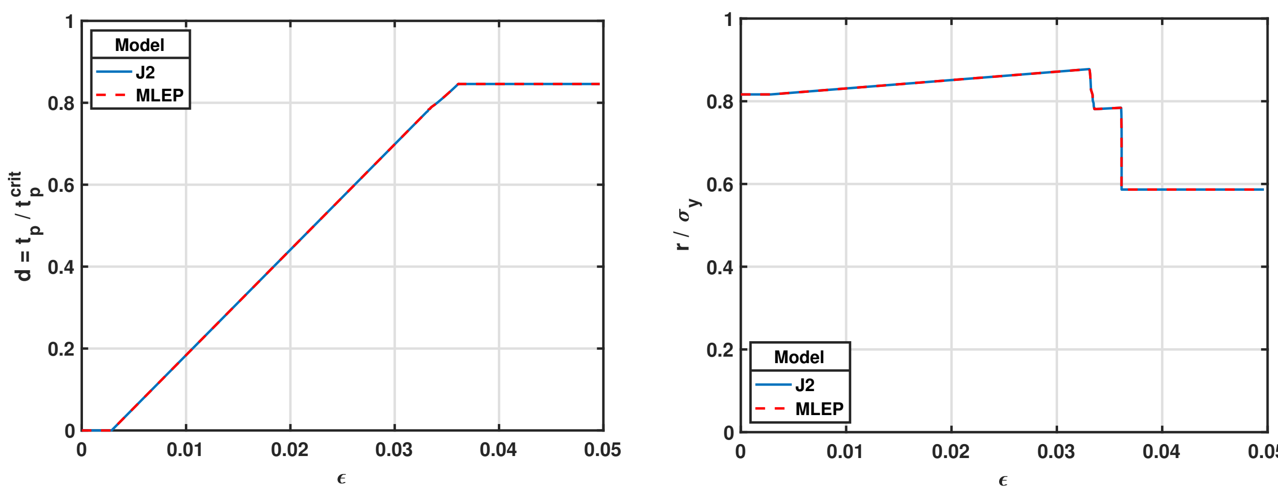


Figure 3: Stress-strain response for uniaxial tension.



(a) Tearing parameter damage variable

(b) Yield surface radius

Figure 4: The tearing parameter damage variable and yield surface radius for uniaxial tension.

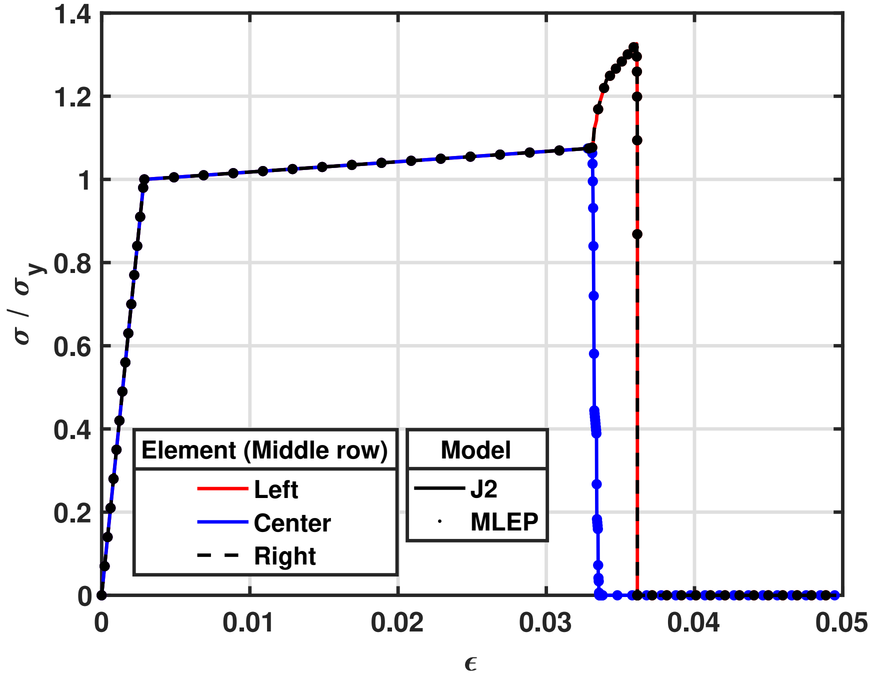
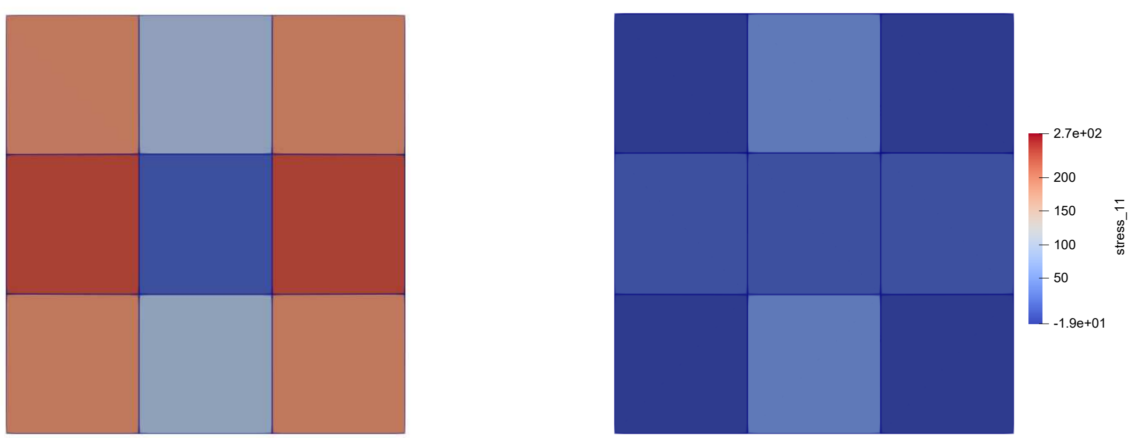


Figure 5: Stress-strain response in individual elements for uniaxial tension.



(a) $\epsilon = 0.0336$ (b) $\epsilon = 0.0361$

Figure 6: Axial stresses at failure strains for the three middle row elements for uniaxial tension.

4.2 Simple shear

A second boundary-value problem is simple shear of a $3 \times 3 \times 1$ block of unit cubes. The $x_2 = 0$ face of the block is fixed. The faces with x_3-normals are fixed in the normal directions. A constant strain rate ramp load (Figure 7) is applied to the top surface of the block in the $(x_1 x_2)$-plane by specifying the applied displacement in the x_1-direction as

$$u_1 = \varepsilon_L \left(\frac{t}{t_L} \right) X_2. \tag{5}$$

Here, the loading strain is $\varepsilon_L = 0.05$ and $t_L = 1$ s. The shear strain is

$$\epsilon_{12} = \frac{1}{2} \varepsilon_L \left(\frac{t}{t_L} \right) \tag{6}$$

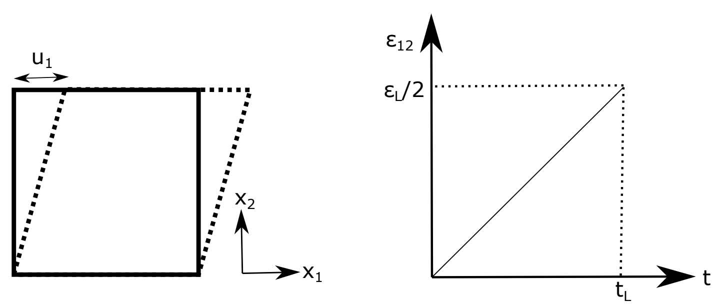


Figure 7: Applied loading and strain history for simple shear.

The material parameters used are the same as in Table 1 with the same exception of the critical tearing parameter for the center element being set to $t_p^{crit} = 0.03$.

The averaged stress-strain results of the analysis are shown in Figure 8. The results match well for the J_2 plasticity model with modular failure and the MLEP model. The comparisons for both the uniaxial tension test and simple shear test show that the multilevel solver was implemented consistently for the two models.

Note, in [1], a pure shear test was used to verify the single element implementation against an analytical solution. Here, the purpose was to compare the results of the J_2 plasticity model implementation to the MLEP implementation for a multi-element simulation involving shear. For ease of boundary condition application, a simple shear test was used instead of a pure shear test.

4.3 Round bar specimen

The final boundary-value problem, for a representative structural problem, is uniaxial tension of a round bar specimen. A quarter model is used due to symmetry. The three symmetry planes are fixed in their normal directions. The same constant logarithmic strain rate ramp load (2) is applied to the top surface of the bar. Here, the loading strain is $\varepsilon_L = 0.005$ and $t_L = 1$ s.

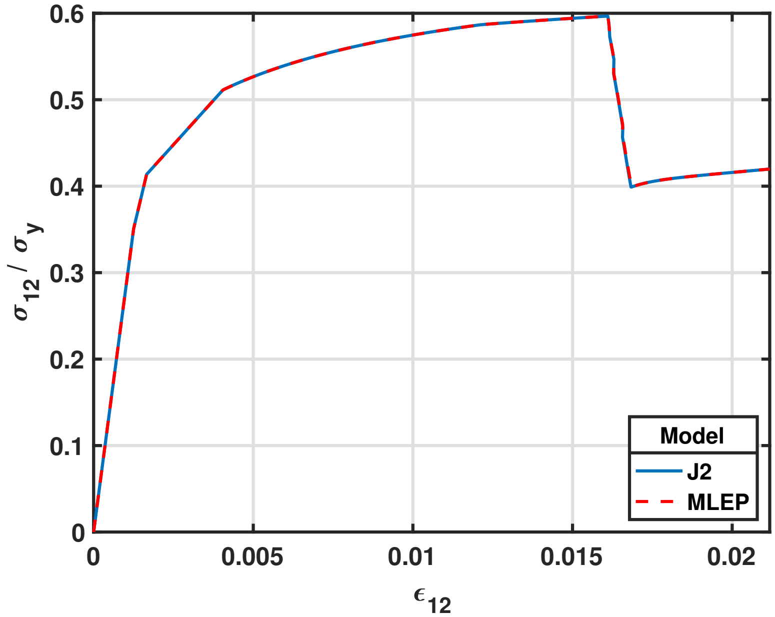


Figure 8: Stress-strain response for simple shear.

Again, the material parameters used are the same as in Table 1 with the exception of the critical crack opening strain being set to $\epsilon_{ccos} = 0.01$. For lower critical crack opening strains, the MLEP model had convergence issues without an adaptive time stepping scheme. The J_2 plasticity model implementation did not have these same convergence issues for lower critical crack opening strains. This may be attributed to the inclusion of a line search algorithm in the J_2 plasticity model return map not available in the MLEP model.

The axial stress-strain results of the analysis, averaged over all elements in the bar, are shown in Figure 9. The results match well for the J_2 plasticity model with modular failure and the MLEP model.

Figure 10 shows the crack failure flags for the round bar specimen at complete failure. Both the J_2 plasticity model implementation and the MLEP model fail in a single cross-sectional band of elements and have similar localized damage patterns surrounding the failure band.

The first element to damage is on the axis of the round bar, roughly at the midpoint of the half-gauge length (Figure 11(a)). Next, elements damage radially outward from the axis of the bar in a single cross-sectional band of elements. The element layers above and below the initiation layer also begin to damage, but with smaller magnitude than the initiation layer (Figure 11(b)). After the damaged elements of the initiation layer reach roughly half the gauge radius, a vertical cylinder of elements, four to five elements in height at half the gauge radius, fails completely (Figure 11(c)). Damage continues to cause elements to fail completely both outward and inward from the gauge half radius until an entire cross-sectional band of elements fails (Figure 11(d)). The stress is then zero throughout this layer and the top section of the bar moves as a rigid body under additional displacement loading.

As the cross-sectional band of elements progressively fails, the load carrying capacity of this layer decreases. The axial stress in any other cross-sectional layer decreases as well and the axial strain in these layers unloads. In the averaged stress-strain response (Figure 9), this resembles unloading although the bar undergoes monotonically increasing displacement loading. The slope of the unloading-type curve is controlled by the failure rate which, in turn, is controlled primarily by the critical crack opening strain, ε_{ccos}.

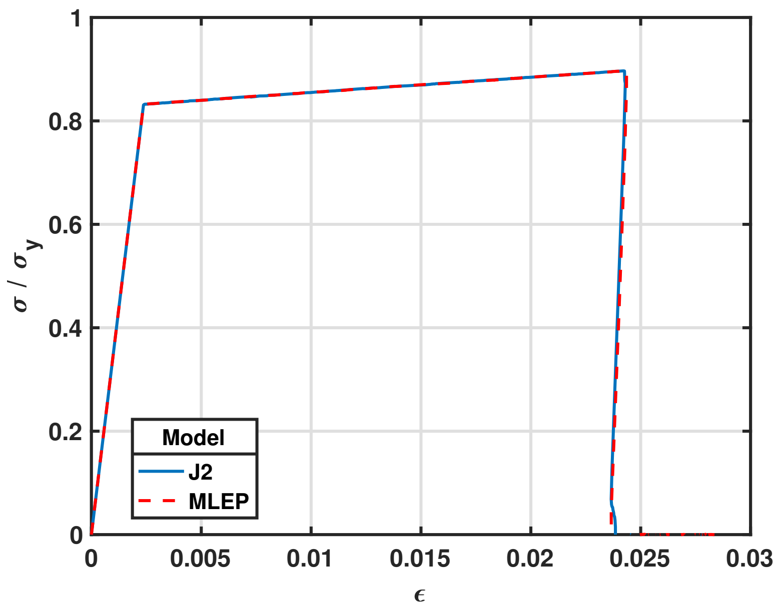


Figure 9: Stress-strain response for the round bar specimen.

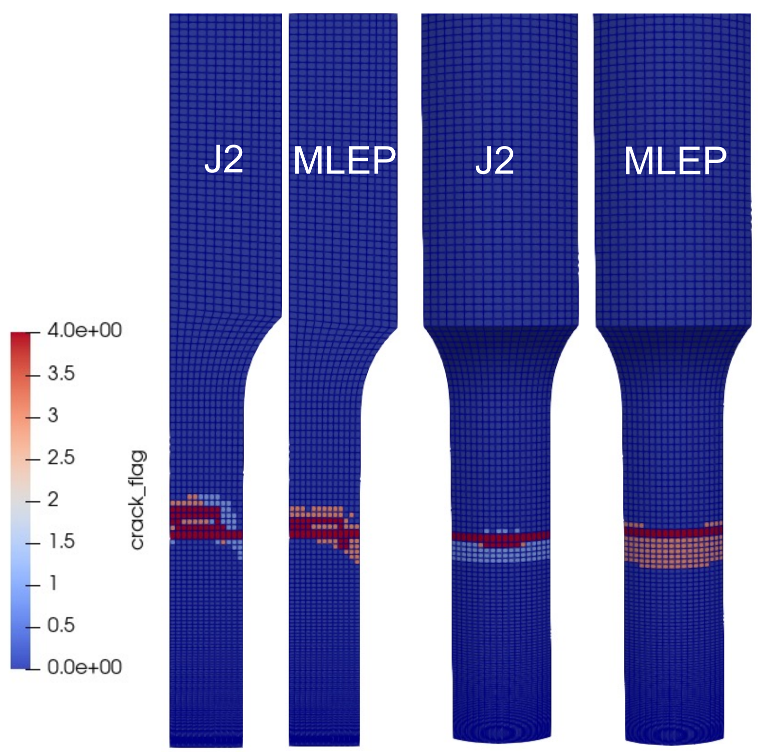
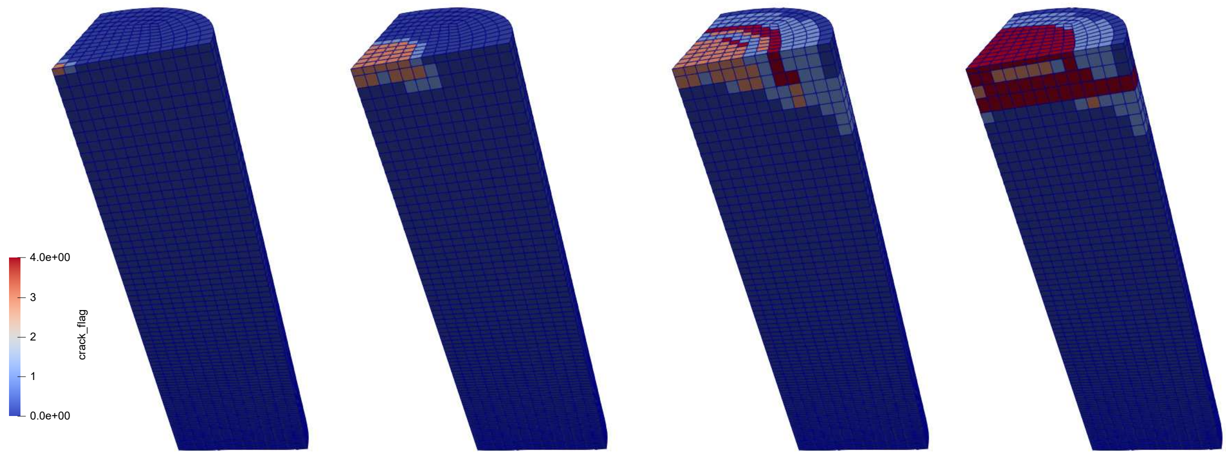


Figure 10: Crack failure flags for the round bar specimen.



(a) (b) (c) (d)

Figure 11: Crack failure flag progression at the failure initiation cross-section for the J_2 results.

5 Future work

The J_2 plasticity model implementation is a first step towards adding the failure model as a modular failure option for any plasticity model. It was selected since both the J_2 plasticity model and the MLEP model use a von Mises yield surface. Many aspects of the failure propagation theory are completely general; however, the stress decay is achieved by scaling of the maximum yield stress. This limits the failure model to isotropic plasticity models with yield surfaces described in terms of a radius. Also, J_2 plasticity is rate-independent. For this model to be modular with any plasticity model, and anisotropic models and rate-dependent models in particular, the theory will need to be generalized.

6 Conclusion

The tearing parameter criterion and material softening method currently used in the multilinear elastic-plastic constitutive model was added as a modular failure option for the J_2 plasticity model. The implementation was integrated with the multilevel solver to add the capability for multi-element simulations. The implementation compared well with MLEP results for a uniaxial tension test, a simple shear test, and a representative structural problem. Necessary generalizations of the failure method to extend it as a modular failure option for all plasticity models were highlighted.

7 Acknowledgements

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