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Computation of Sobol' indices using Embedded Variance Deconvolution

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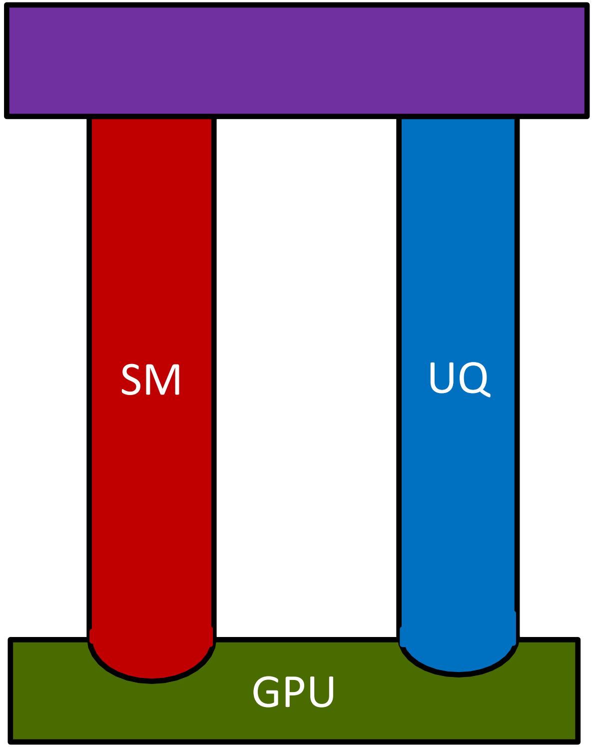
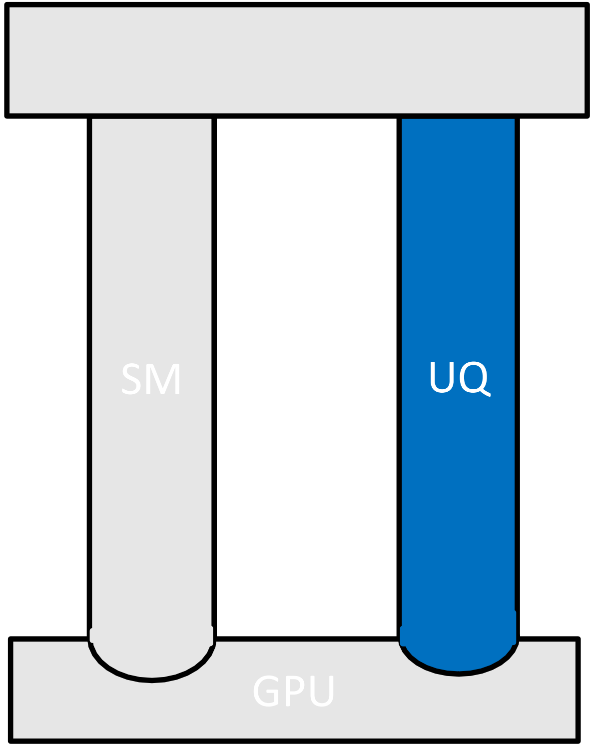
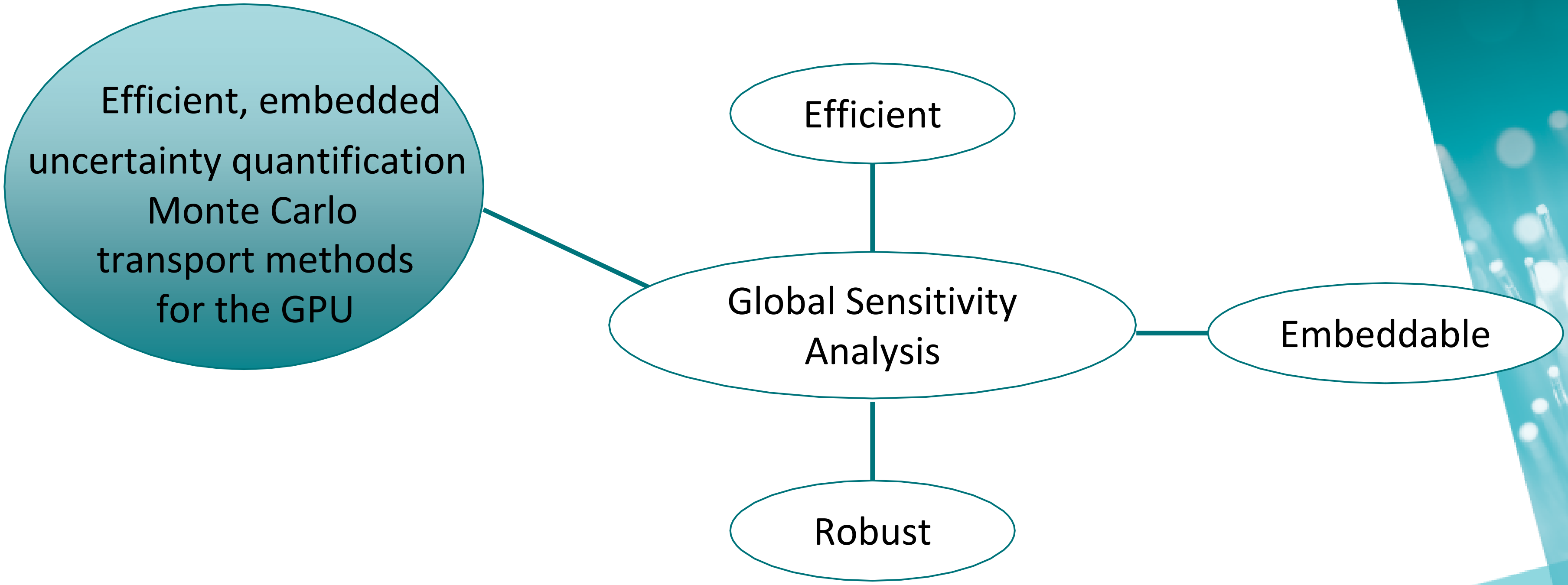
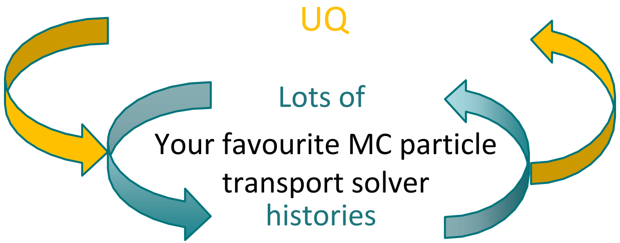
Develop efficient, embedded stochastic media (SM) and uncertainty quantification (UQ) Monte Carlo transport methods for the GPU.



Develop efficient, embedded stochastic media (SM) and uncertainty quantification (UQ) Monte Carlo transport methods for the GPU.




Efficient, embedded uncertainty quantification Monte Carlo transport methods for the GPU
Efficient
Global Sensitivity Analysis
Embeddable
Robust

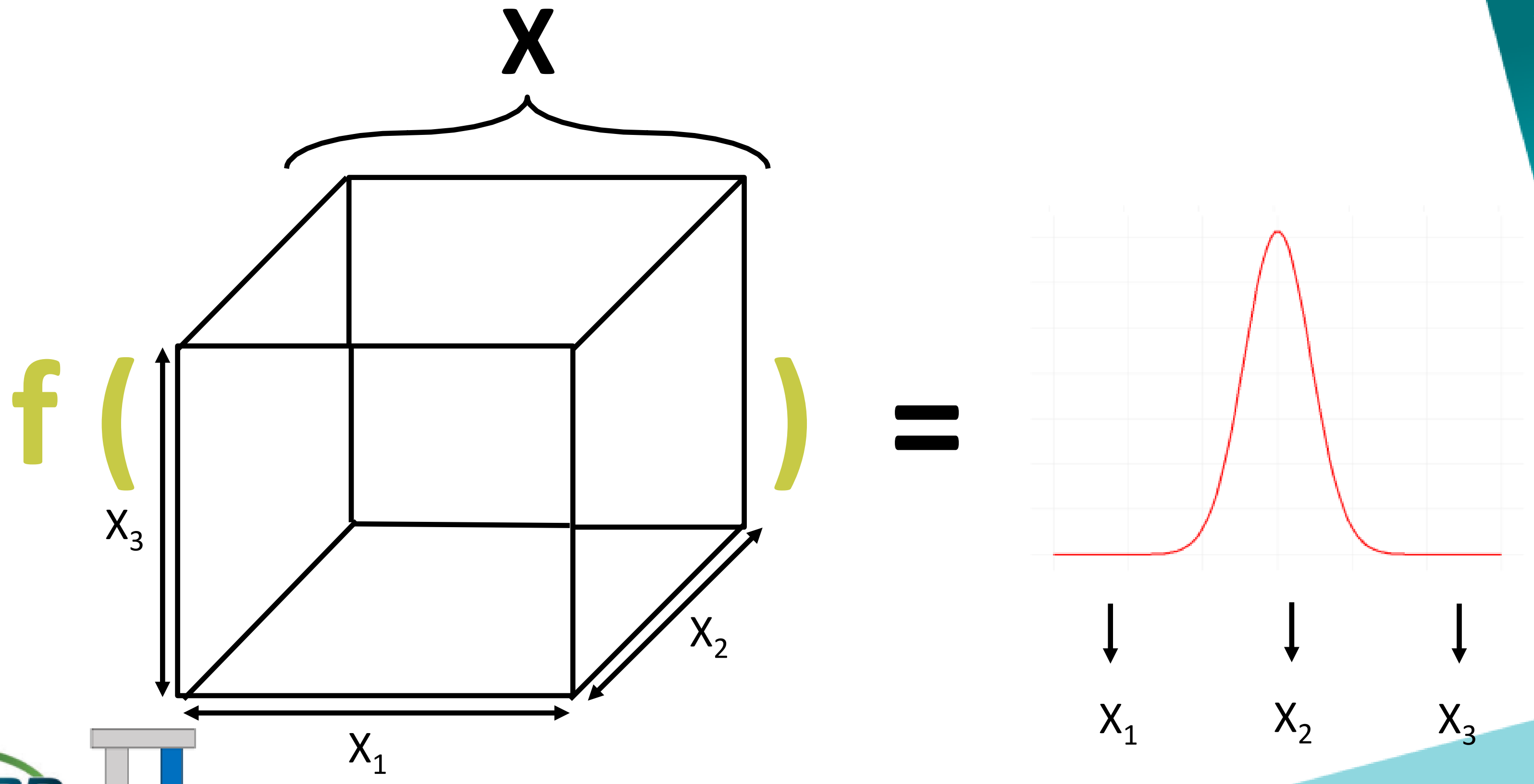


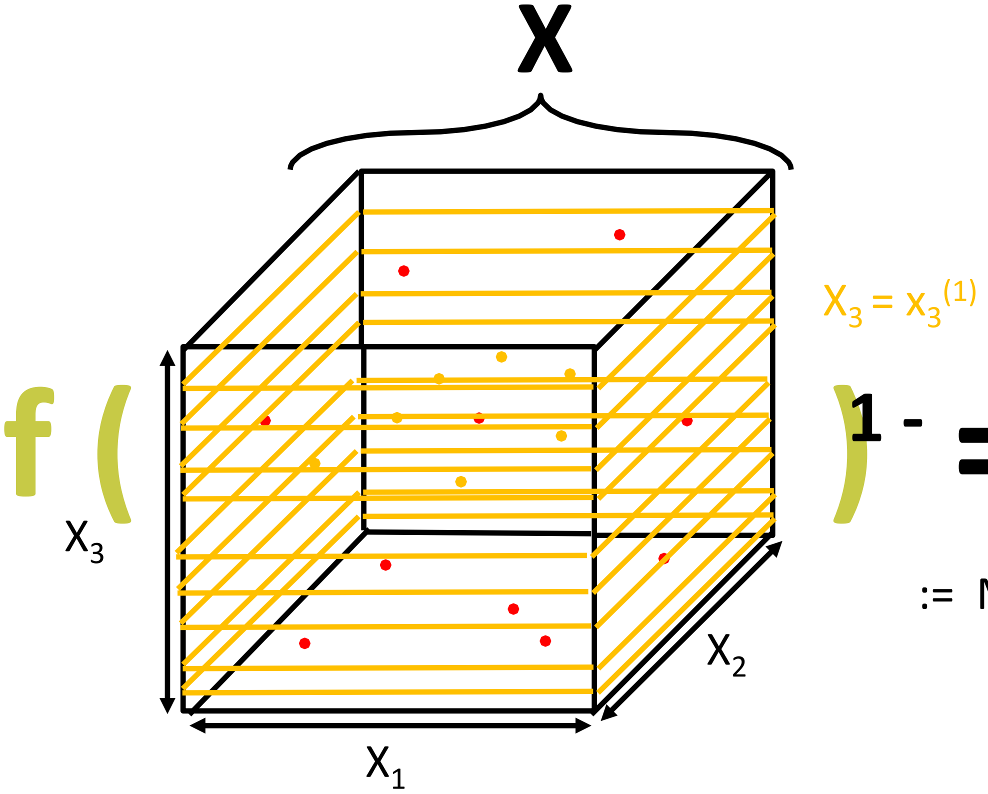
$$= \boldsymbol{O}\,(\text{ expensive })^{2}$$

	Screening, emulators, reliability methods	Embedded variance-based sensitivity analysis
Efficient	✓	✓
Embeddable	✗	✓
Robust	✗	✓

1. Sobol' indices

2. EVADE


X
f
X3
X2
X1
X1
X2
X3



$$1 - \frac{E(Var[Y|X_3])}{Var(Y)} =$$

$Var[Y|X_3 = x_3^{(1)}]$

Y

Var(Y)

:= Main effect Sobol' index for X_3

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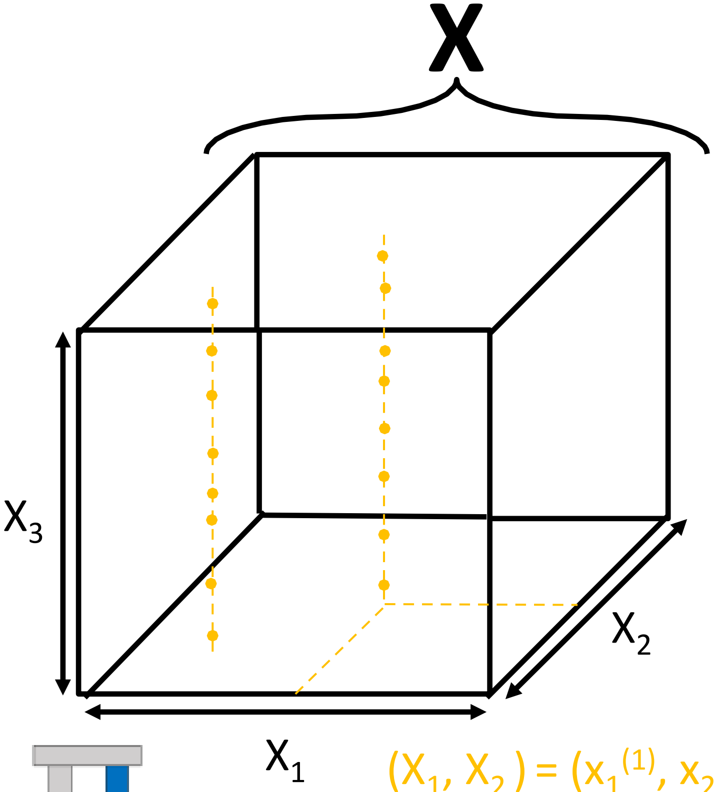

$$\frac{E\left(Var\left[Y \mid (X_1, X_2) = (x_1^{(1)}, x_2^{(1)})\right]\right)}{Var(Y)}$$

:= Total effect Sobol’ index for X_3

$$1 - \frac{E\left(Var[Y \mid X_3] \right)}{Var(Y)}$$

:= Main effect Sobol’ index for X_3

$(X_1, X_2) = (x_1^{(1)}, x_2^{(1)})$

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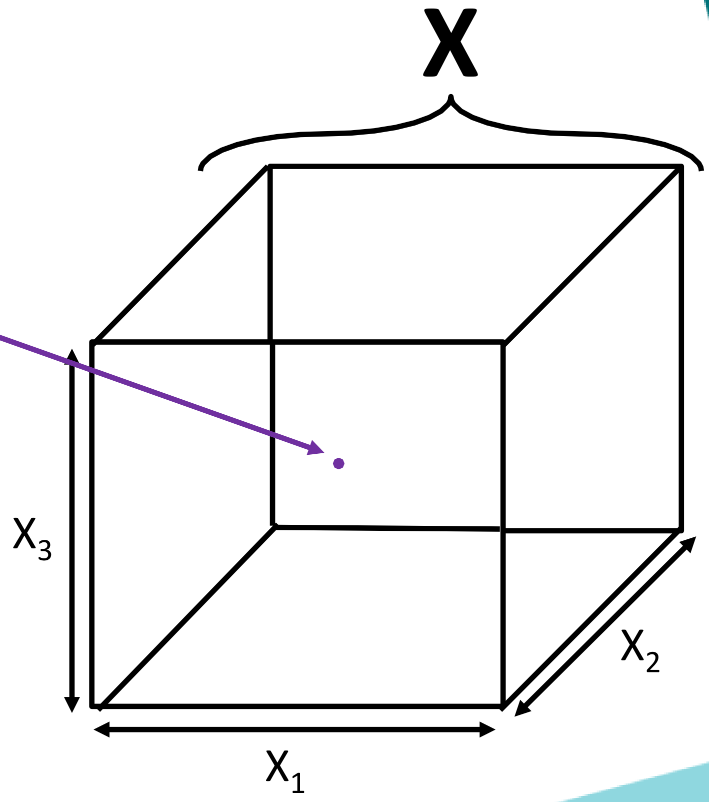
$$\mathbf{x}^{(1)} := (x_1^{(1)}, x_2^{(1)}, x_3^{(1)})$$

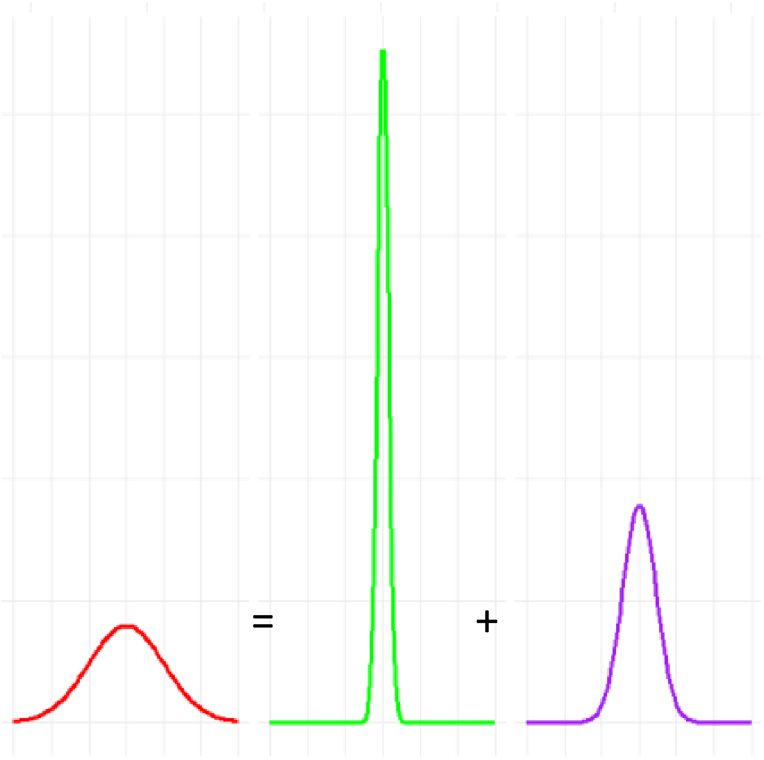
$$y^{(1)} := f(\mathbf{x}^{(1)})^{(1)}$$

$$y^{(2)} := f(\mathbf{x}^{(1)})^{(2)}$$

$$y^{(1)} \neq y^{(2)}$$

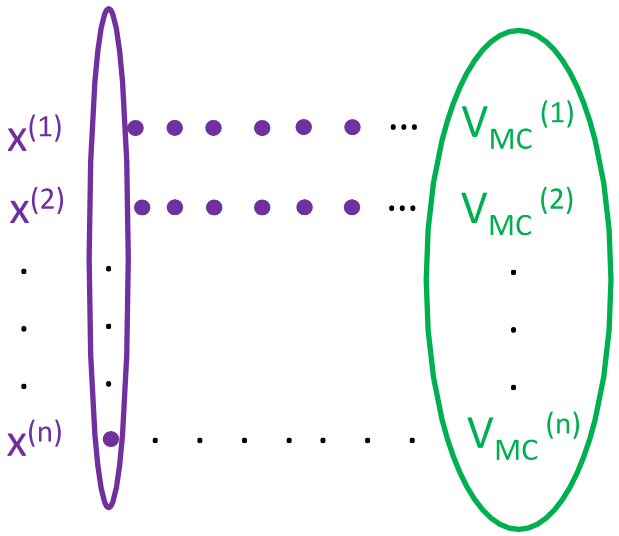
$$V_{MC}(Y | X = \mathbf{x}^{(1)}) = Var(f(\mathbf{x}^{(1)}))$$



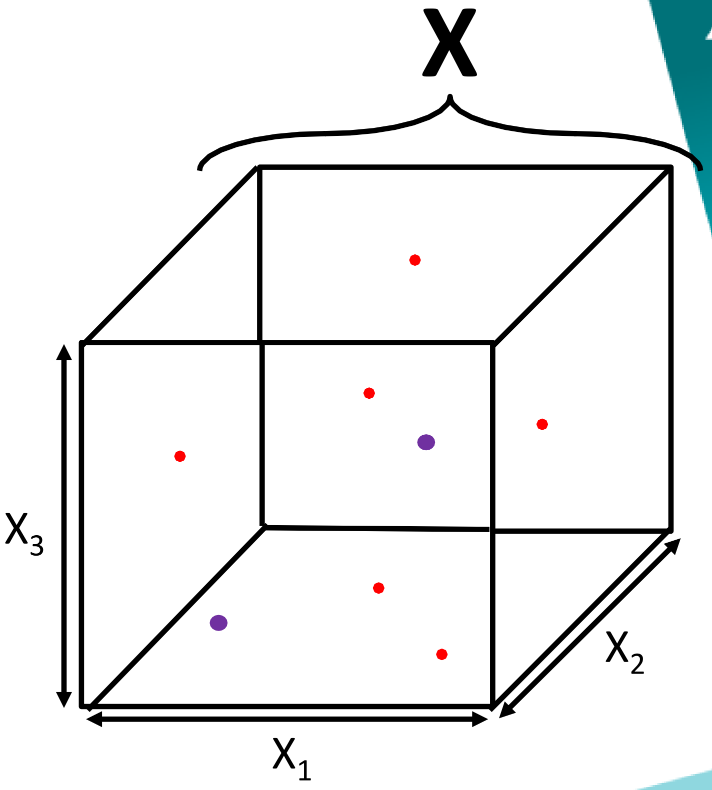


$V(Y) = E(V[Y|X]) + V(E[Y|X])$

V_p



$$V_{TOT} - E(V_{MC}) = V_p$$



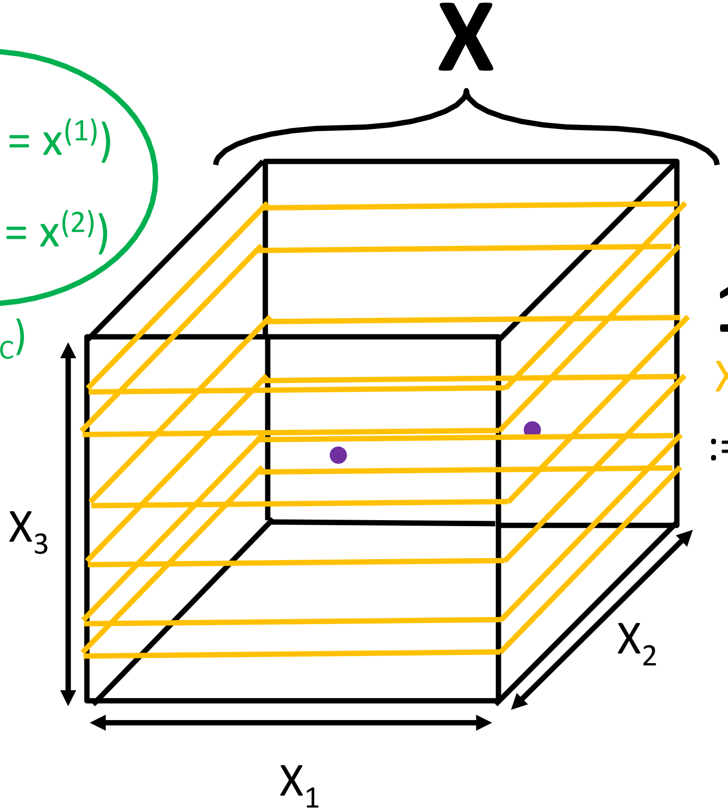
Embeddable

$x^{(1)}$ • $V_{MC}{}^{(1)}(X = x^{(1)})$

$x^{(2)}$ • $V_{MC}{}^{(2)}(X = x^{(2)})$

$V_{TOT}\,[X_3 = x_3{}^{(1)}]$ - $E(V_{MC})$

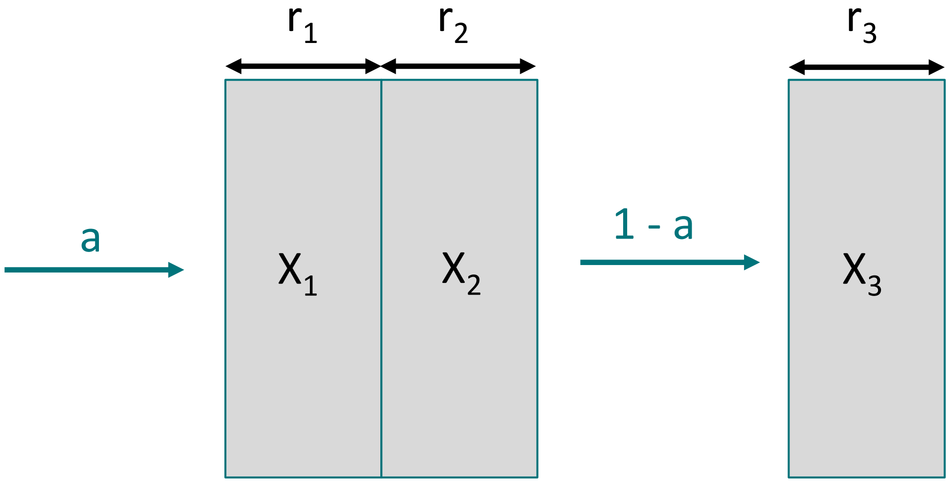
$= V_p[Y \mid X_3 = x_3{}^{(1)}]$



$$1 - \frac{E(\,Var[Y \mid X_3]\,)}{Var(Y)}$$

$X_3 = x_3{}^{(1)}$

:= Main effect Sobol' index for X_3



$$f(X_1, X_2, X_3) = a\exp(-r_1X_1 - r_2X_2 - X_3) + (1-a)\exp(-r_3X_3)$$

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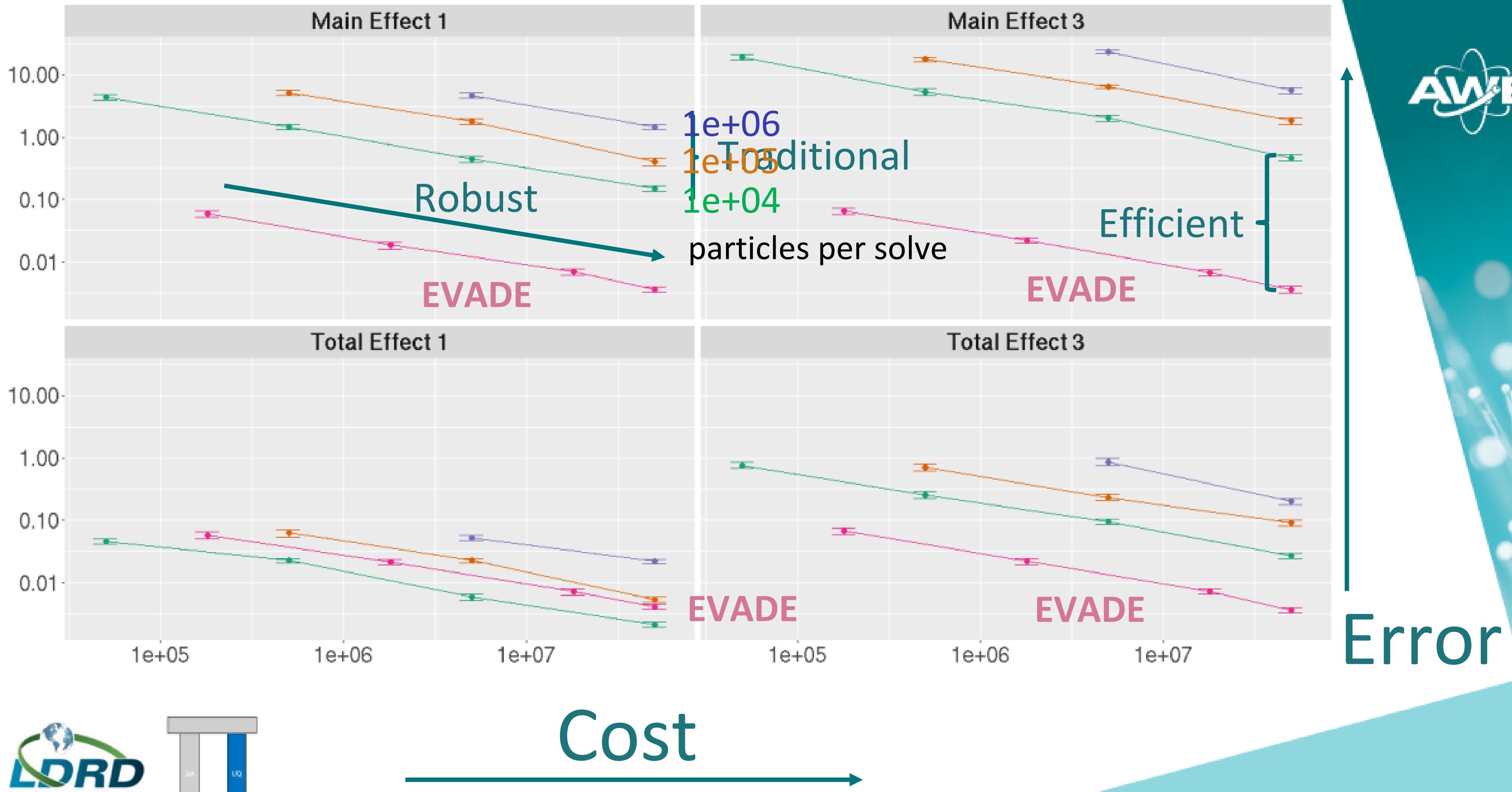
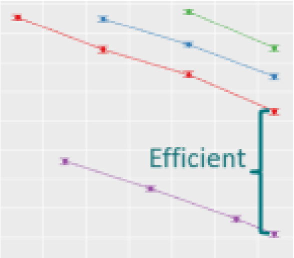

Main Effect 1
Main Effect 3
Total Effect 1
Total Effect 3
10.00
1.00
0.10
0.01
1e+05
1e+06
1e+07
1e+06
1e+05
1e+04
Traditional
Robust
particles per solve
Efficient
EVADE
EVADE
EVADE
EVADE
Error
Cost

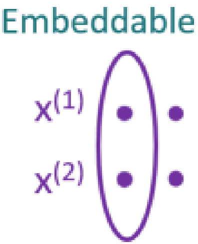
Efficient, embedded uncertainty quantification Monte Carlo transport methods for the GPU

Global Sensitivity Analysis

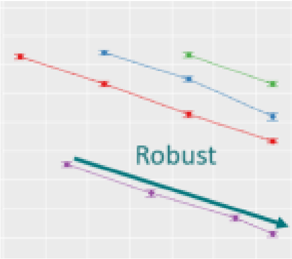
Efficient



Embeddable



Robust



Develop efficient, embedded stochastic media (SM) and uncertainty quantification (UQ) Monte Carlo transport methods for the GPU.

CLS/LRP for three+ materials

Vu, Paper #33712
Transport in Stochastic Media I

Markovian three+ materials

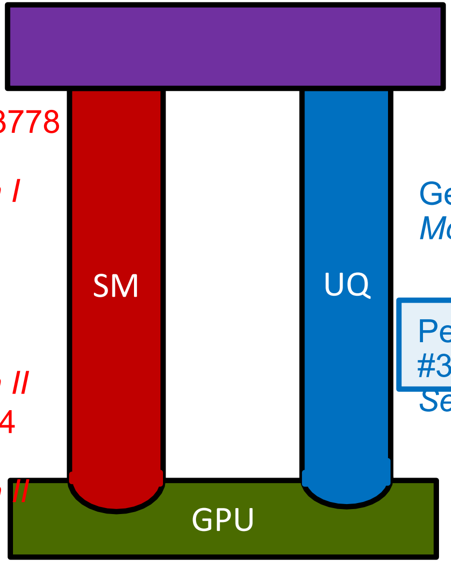
Olson, Paper #33778
Transport in Stochastic Media I

CoPS for generalized mixing

Davis, Paper #33784
Transport in Stochastic Media II

CoPS memory/runtime efficiency

Vu, Paper #33614
Transport in Stochastic Media II



Geraci, Paper #33671
Monte Carlo Methods

PCE surrogate models

Petticrew, Paper #33657
Sensitivity Analysis

Global sensitivity analysis

CoPS on the GPU

Kersting, Paper #33673
Monte Carlo Algorithms

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Backup

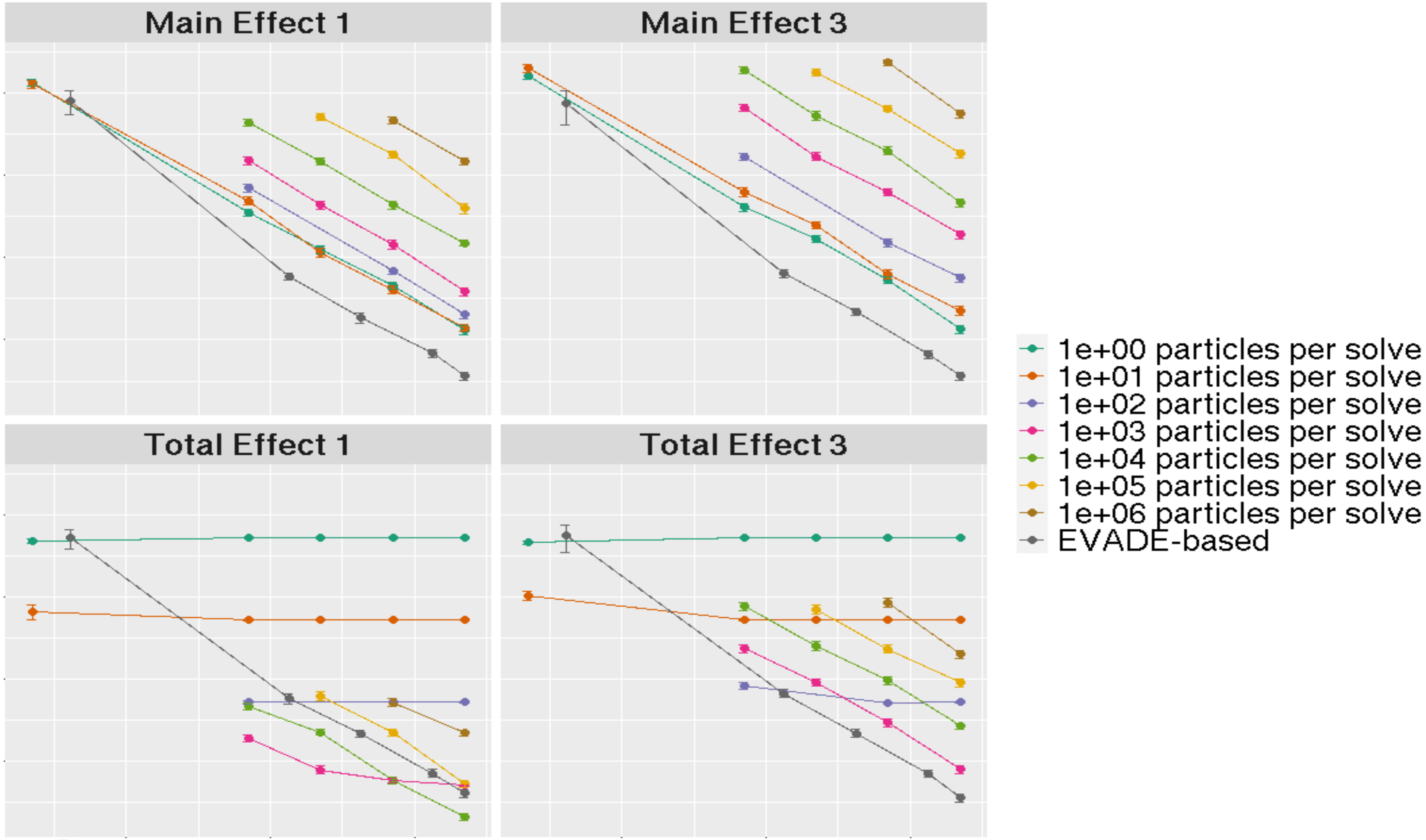

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Table 1: Benchmark function numerical results after 3e+07 particle histories

Method	$\mathbf{s_1}$	$\mathbf{s_2}$	$\mathbf{s_3}$	$\mathbf{s_{T_1}}$	$\mathbf{s_{T_2}}$	$\mathbf{s_{T_3}}$
Analytic	0.0635	0.0635	0.8725	0.0638	0.0638	0.8730
1e4 hist. solve	0.0462 ± 0.0250	0.0870 ± 0.0268	0.749 ± 0.0844	0.0644 ± 0.0004	0.0644 ± 0.0003	0.876 ± 0.0045
1e5 hist. solve	0.0920 ± 0.0810	0.0366 ± 0.0812	0.761 ± 0.339	0.0641 ± 0.0009	0.0646 ± 0.0008	0.892 ± 0.0156
1e6 hist. solve	-0.1024 ± 0.2578	0.5619 ± 0.2742	0.9059 ± 1.0310	0.0653 ± 0.0037	0.0623 ± 0.0035	0.9215 ± 0.0371
EVADE 4 hist. plane	0.0627 ± 0.0006	0.0623 ± 0.0009	0.8730 ± 0.0007	0.0638 ± 0.0007	0.0627 ± 0.0009	0.8737 ± 0.0006
EVADE 30 hist. plane	0.0634 ± 0.0007	0.0645 ± 0.0007	0.8729 ± 0.0004	0.0642 ± 0.0005	0.0638 ± 0.0005	0.8722 ± 0.0007