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Method for Determining the Estimated Timing Uncertainty for Digital Sampling Instruments

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The overall uncertainty in digital captured data points is often misunderstood in our organization and is typically accepted as only the manufacturer uncertainty specification of the time base clock typically on the order of 10-100 parts per million. The time base clock of digital sampling technologies is critically important to maintain timing control of the internal electronics and to achieve the specified sampling rate of the instrument. The time base clock must remain within the manufacturer specification tolerance throughout the calibration interval to assure accurate performance. However, the time base uncertainty does not adequately account for the additional measurement errors accompanying the capture and evaluation of the time values for any cardinal points of interest when periodically sampling analog waveforms generated by other instruments or Units Under Test (UUTs). The proposed methodology described here details a general approach used to estimate the magnitude of the digital instrument sampling error when capturing analog waveforms based upon the instrument sampling rate, the frequency of a nominally equivalent sinusoidal waveform, as well as, whether the time value of any cardinal points is selected by a 'Next Point After' or Interpolation method for our purposes. Finally, the overall estimated timing uncertainty is quantified by arithmetically combining the error contributions for the sampling rate, the cardinal point selection method, and the instrument time base specification. The results of this method aid in selecting the appropriate digital sampling technology based upon waveform rise time requirements and provide general engineering guidance. Since the estimated error is a portion of the sampling timestep interval, the percentage error could be significant based upon the measured rise time. Sandia National Laboratories is a multi-mission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy National Nuclear Security Administration under contract DE-NA0003525.

1. Introduction

A review in FY2021 of our current measurement systems MAPs showed that the timing uncertainty value for any digital sampling instrument was listed as only the manufacturer uncertainty specification of the time base clock. Typically, for the digital sampling instruments implemented in our measurement systems, the time base uncertainty specification ranged from ±30 ppm (±0.003%) to ±100 ppm (±0.01%). The time base clock is critically important to maintain timing control of the internal electronics and the specified sampling rate(s) of the instrument. The time base clock must remain within the manufacturer specification tolerance throughout the calibration interval to assure accurate performance.

However, the time base uncertainty does not adequately account for the additional measurement errors accompanying the capture and evaluation of the time values for any cardinal points of

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interest when periodically sampling analog waveforms generated by other instruments or Units Under Test (UUTs).

This manuscript documents a methodology that details a general approach used to estimate the magnitude of the digital instrument sampling error when capturing analog waveforms based upon the instrument sampling rate, the frequency of a normally equivalent ideal sinusoidal waveform, as well as, whether the time value of any cardinal points is established by a 'Next Point After' or Interpolation method.

Finally, the overall estimated timing uncertainty is quantified by arithmetically combining the error contributions for the sampling rate, the cardinal point selection method, and the instrument time base specification.

2. Standard Method for Determining the Estimated Timing Uncertainty for Waveform Rise Time Measurements

Waveform rise time, t_r, is defined as the observed time for a measured signal to transition from 10% to 90% relative to the peak of the waveform.[1.] In this definition, the 10% and 90% locations are cardinal points for the time-voltage pairs on the initial transition portion of the measured waveform as demonstrated in the following figure:

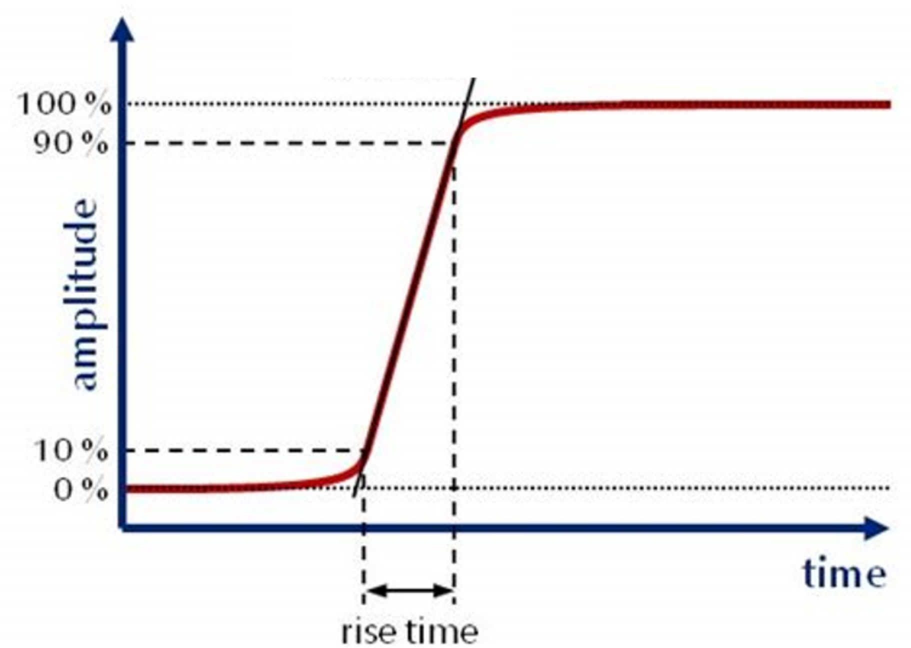

FIG 1. Amplitude vs Time graph to represent signal t_r.

The estimated uncertainty of the t_r has two main error contributors; the sampling rate of the digitizing instrument and the overall voltage measurement error for the specific measurement channel that combines the instrument voltage accuracy with the errors of any additional components.

The first step in achieving a low degree of sample rate uncertainty is to determine the number of samples available within the waveform rise time interval. Using the instrument sampling rate, f_s , the total number of samples, $Sa_{\#}$, within the interval is:

$$Sa_{\#} = t_r[sec] * f_s. \qquad (1)$$

$Sa_{\#}$ should be equal to or greater than 25 so to obtain enough data resolution for the rise time interval calculation using either the 'Next-Point-After' or Interpolation method. If $Sa_{\#}$is less than 25, then a higher sampling rate instrument should be selected.[2.]

Next, assign a sampling rate error for any individual data point, $f_{serr\ single\ point}$, within the defined interval. This is the estimated timing error associated with each single point along the interval.

The sampling step error is defined as:

$$f_{serr\ single\ point}[sec] \ = \frac{0.5}{f_s}. \qquad (2)$$

This assumes a 'Next-Point-After' Method.[3.]

For any waveform attribute metric taking the difference between two data points, $f_{serr\ t_r}$, the timing error results in one sampling step shown as:

$$f_{serr\ t_r}[sec] = \frac{1}{f_s} \qquad (3)$$

This is the overall timing error associated with the total interval, such as a rise time.

For the Measurement Assurance Plan (MAP), the sampling rate contribution to the overall uncertainty is defined in units of time and is stated whether the evaluation was done for a single point or a time interval. However, if so desired to convert the error into a percentage, use the following expression(s):

$$f_{serr\ single\ point}[\%] = \frac{Measured(single\ point\ value)sec}{f_{serr\ single\ point}[sec]} * 100 \qquad (4)$$

$$f_{serr\ t_r}[\%] = \frac{t_r[sec]}{f_{serr\ single\ point}[sec]} * 100 \qquad (5)$$

The contribution to the uncertainty in t_r due to voltage is derived from the manufacturer's specification for the instrument. This uncertainty is commonly expressed as "vertical uncertainty" and is a function of "% of reading". Additionally, this uncertainty is calculated to include additional conservativism that will make up a considerable portion of the uncertainty budget and is thus a sufficient representation of the error due to voltage.

However, if the PRT (Product Realization Team) defines a critical necessity to do so, additional uncertainty to the voltage contribution can be estimated according to the following expression:

$$V_{err}[sec] \cong 2U * t_r[sec], \qquad (6)$$

where U is the overall voltage error for the measurement channel, converted from its percentage error value (k=2) to decimal form, $t_r[sec]$ is the measured waveform rise time in seconds and, $V_{err}[sec]$ is the estimated t_r error, in seconds, based upon the channel voltage error that is found in the MAP.

The resulting $V_{err}[sec]$ value will have a coverage factor of k=2 (~95% confidence level) because it utilizes the specific channel voltage percentage error k=2 value.

3. Supplemental Theory

The rise time, t_r, the fall time, t_f, and the Full Width Half Max time, *FWHM*, cardinal points of a measured waveform can be approximated by a portion, typically the first quarter or second quarter cycle, of an ideal sine wave of some appropriately matched frequency of the following general form:

$$y(t) = Asin(\omega t), \quad (3A)$$

where $y(t)$ is the periodic signal magnitude as a function of time, A is a constant that establishes the peak values (both positive and negative values), ω is the angular frequency, in units of radians and also expressed as $2\pi f$, f is the appropriate signal-matched-sine-wave frequency in units of hertz (Hz: cycles per sec), and t is the time in units of seconds.
For any location of interest on the first full cycle of an ideal sine wave, such as: individual cardinal points, or sets of cardinal points, a trigonometric unit circle is used to determine the t(s) value for any point when the frequency value is known. For a trigonometric unit circle the value of A is equal to one for the peak values.

Figure 1A illustrates the algebraic results, as a function of frequency, when applying the trigonometric unit circle general equations to determine the t(s) values for the 90% and 10% locations. For these specific locations, the constant A in Eqn. (1) is replaced by the variable A_cp which is equal to: 0.90 for the 90% location and 0.10 for the 10% location. For the FWHM locations the value of A_cp equals 0.50 on both the positive and negative portions of the first half-cycle.
(Note: Ideal t_r, t_f, and FWHM time positions are independent of digitizer sample rate. They are only dependent upon frequency of ideal sine wave).

Trigonometric unit circle general equations:

$$x^2 + y^2 = 1, \quad (2A)$$

$$y = A\_cp\,, \quad (3A)$$

(amplitude of cardinal point of interest normalized for unit circle)

$$x = \left(1 - A\_cp^2\right)^{\frac{1}{2}}, \quad (4A)$$

$$\theta = arctan\,\left(\frac{x}{A\_cp}\right), \quad (5A)$$

$$\omega = \frac{\pi}{2} - \theta, \quad (6A)$$

$$\frac{\omega}{s} = 2\pi f, \quad (7A)$$

$$t = \omega/\left(\frac{\omega}{s}\right) \quad (8A)$$

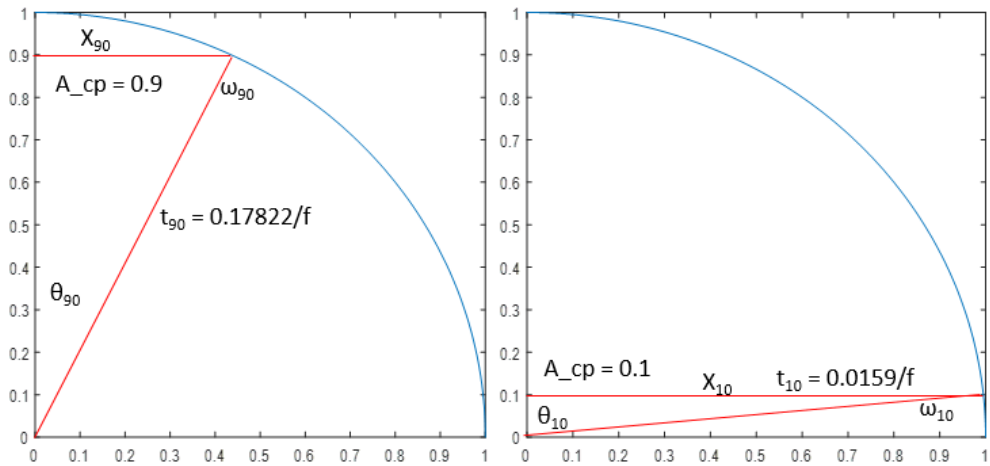

FIG 1A. Trigonometric Unit Circle locations for the 90% and 10% positions as a function of frequency, f, in the first quarter cycle of an ideal sinusodial waveform. The A_cp value for the 90% cardinal position is 0.90 on the positive y-axis and the A_cp value for the 10% cardinal position is 0.10 on the positive y-axis. The values for t(s), as a function of frequency for the 90% and 10% locations are the theoretical time values where the cardinal points occur on an ideal sine waveform.

The values for t(s) as a function of frequency expressed in Figure 1A are the ideal times at which the cardinal points in a given interval *should* occur in an idealized sine waveform.
Listed in Table IA. And Table IIA. are the theoretical time equations, as a function of frequency, for the 10%, 50%, 90%, and peak cardinal points for an ideal sine wave.

Cardinal Point / Location	Time value equation as a function of frequency, f in Hz
t_10%	0.01594 / f
t_50%	0.83333 / f
t_90%	0.17822 / f
t_peak	0.25000 / f

Table IA. Theoretical time equations, as a function of frequency, for the 10%, 50%, 90%, and peak cardinal points for an ideal sine wave.

Rise Time of Ideal Sine Wave	Frequency, f (Hz)	Number of points in first quarter cycle at 250MSa/s
50 ns	3.25M	19
75 ns	2.165M	29
150 ns	1.082M	58
250 ns	649k	96
500 ns	324.5k	193
1000 ns	162.25k	385
2000 ns	81k	772
4000 ns	40.575k	1540

Table IIA. Theoretical time values, with the associated frequencies and number of points in the first quarter cycle using a 250MSa/s example digitizer.

4. Methods of defining Cardinal Point(s) Time Occurrence

There are two specified methods of determining the time for cardinal points of interest within the test equipment measured waveform. These two methods are called the 'Next-Point-After' method, and the Interpolation method.

The 'Next-Point-After' method determines the time value for any cardinal point of interest in a digitially captured analog waveform data set at the data point immediately after the occurence of the desired amplitude level if that level is not available in the data set.

As an example, Table IIIA. lists a digitally captured ideal sine waveform (first quarter cycle) data set normalized by the a peak value of one. If the ideal unit circle approximation determined the time at 90% *RT* to be 55 ns at A_cp = 0.9, the user with the data set in Table IIIA. would select the time at which the closest, larger value of A_cp=0.9 occures. In this example, A_cp=0.910. The time associated with A_cp= 0.934 is equal to 56 ns.

Example: 250M Sa/s; RT ≈ 50 ns
Ideal Sine Wave
f = 3.25 MHz; t_10% = 4.9 ns; t_90% = 55 ns
t_r (actual) = 49.94 ns

t(ns)	A(t)
0	0
4	0.082
8	**0.163**
12	0.243
16	0.321
20	0.397
24	0.471
28	0.541
32	0.608
36	0.671
40	0.729

44	0.782
48	0.831
52	0.873
56	**0.910**
60	0.941
64	0.965
68	0.983
72	0.995
76	1.000

Table IIIA. Example laboratory data set to demonstrate the 'Next-Point-After' method of defining the time at which the 90% t_r and 10% t_r occur.

The timestep error in this example equals 0.1 steps, which is rounded to 0.5 steps.

The interpolation method uses the linear interpolation equation to define the time at which cardinal point(s) occur.
Using the same example as in Table IIIA., if the ideal unit circle approximation determined the time at 90% RT to be 55 ns. The user with the data set in Table IVA. would interpolate between the two closest, larger and smaller values that surround A_cp = 0.9.

Example: 250M Sa/s; RT ≈ 50 ns
Ideal Sine Wave
f = 3.25 MHz; t_10% = 4.9 ns; t_90% = 55 ns
t_r (actual) = 49.94 ns

t(ns)	A(t)
0	0
4	0.082
8	0.163
12	0.243
16	0.321
20	0.397
24	0.471
28	0.541
32	0.608
36	0.671
40	0.729
44	0.782
48	0.831
52	**0.873**
56	**0.910**
6.00E-08	0.941
6.40E-08	0.965
6.80E-08	0.983
7.20E-08	0.995

7.60E-08	1.000

Table IVA. Example laboratory data set to demonstrate the Interpolation method of defining the time at which the 90% RT occurs.

Via the interpolation equation, the time at which 90% RT occurs in this example is:

$$t_{0.9} = \frac{[A_{0.9} * (t_2 - t_1) - (A_1 * t_2) + (A_2 * t_1)]}{A_2 - A_1} = 54.9\ ns, \quad (9A)$$

where $A_{0.9}$ = 0.9, $(t_1(ns),A_1) = (52, 0.873)$, and $(t_2\ (ns),A_2) = (56, 0.910)$.

The timestep error in this example equals 0.01 steps, which is rounded to 0.125.

5. References

[1] IEEE, "IEEE Instrumentation and Measurement Society," Waveform Generation, Measurement, and Analysis Committee, New York, New York, IEEE Std 181, 2011.

[2] These observations can be attributed in part to the Nyquist Sampling Theorem, which states that in order to avoid aliasing the sampling frequency should be at least two times the highest frequency contained in the signal (NI 2015a), and to NI's recommendation that sampling be done at a rate of 10 times the highest frequency of the signal (NI 2016b).
NI. (2015a). "Acquiring an Analog Signal: Bandwidth, Nyquist Sampling Theorem, and Aliasing." http://www.ni.com/white-paper/2709/en/.
NI. (2016b). "How to Choose the Right DAQ Hardware for Your Measurement System." http://www.ni.com/white-paper/13655/en/.

[3] Interpolation methods for the calculated uncertainty in RT will yield a lower uncertainty given by the following:

$$f_{serr\ single\ point}[sec] = \frac{0.125}{f_s},$$
$$f_{serr\ t_r}[sec] = \frac{.25}{f_s},$$

and thus, reduces the $f_{serr\ t_r}[\%]$ by a factor of 4. It is recommended to use the interpolation method if there are less than 100 data points in the interval of interest i.e., if $Sa_{\#}$ is less than 100.