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BayesMT: A Probabilistic Bayesian Framework for the Seismic Moment Tensor

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Summary

Moment tensors (MTs) have long been used in earthquake and explosion source analysis, and there has been a renewed interest in how they can inform us about the seismic source, particularly in the geophysical monitoring community due to its application in event identification and yield analysis. However, parameter uncertainties in seismic MT inversion are rarely available. The inverse procedure often does not quantify MT model errors such as event location, data noise and Earth model that are essential for estimating solution robustness. To address this need, we propose to adopt the Bayesian probabilistic framework to incorporate uncertainties in MT inversions. In this study, we present the theoretical background of a probabilistic Bayesian framework for MT inversion accounting for model and measurements errors and illustrate the implementation of the method using a synthetic example.

1 Introduction

Moment tensors (MTs) have long been used in earthquake and explosion source analysis, and there has been a renewed interest in how they can inform us about the seismic source particularly in the geophysical monitoring community due to its application in event identification and yield analysis (Ford et al., 2020; Pasyanos & Chiang, 2021). Some of this interest derives from our recent ability to routinely determine six-component MT that takes advantage of the full description of the MT to characterize isotropic and non-isotropic radiation of seismic sources. The elements of the tensor are used to derive the source-type and subsequently tested against theoretical mechanisms such as explosion, earthquake and collapse. MTs and their source-types have been shown to be valuable geophysical monitoring tools in identifying explosions (Alvizuri & Tape, 2018; Chiang et al., 2018; Mustać et al., 2020) and other nuisance signals (Boyd et al., 2018; Mustać et al., 2018; Shuler et al., 2013), as well as discriminating explosions from earthquakes when varied-data type inversion is applied to the analysis (Ford et al., 2012).The MTs can be used to augment traditional semi-empirical based methods that utilize surface-to-body-wave magnitude ratios (Fisk et al., 2002; Selby et al., 2012) and regional phase amplitude ratios (Bottone et al., 2002; Walter et al., 2018). The effort to develop and improve the MT discriminant for monitoring and enforcement of nuclear test-ban treaties continue to be an area of active research.

However, parameter uncertainties in seismic MT inversion are rarely available. The inverse procedure often does not quantify MT model errors such as event location, data noise and Earth model that are essential for estimating solution robustness. Here we propose to adopt the Bayesian probabilistic framework to incorporate uncertainties in MT inversions. The probabilistic formulation described by Tarantola and Valette (1981) casts the inverse problem in a Bayesian framework where information on the model parameters is represented in probabilistic term. With this approach the solution is given as the complete posterior probability density function of the data and model parameters, instead of a single best-fit

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solution. Subsequent works such as those by Duputel et al. (2012); Mustać and Tkalčić (2015); Phạm and Tkalčić (2021); Stähler and Sigloch (2014) presented the methods for incorporating model uncertainties in seismic source and MT inversions.

In this report we will cover the theoretical background for MT inversion using a probabilistic Bayesian model and demonstrate the incorporation of data noise and Earth model uncertainty in MT inversion using a synthetic experiment. The analysis is done using open source Python packages NumPy, SciPy and ObsPy.

2.1 Bayesian Source Inversions

Based on the formulation of Gesret et al. (2015) we can describe the general forward problem that predicts the observations $\mathbf{d_{obs}}$ for a set of MTs **m** at a spatiotemporal location **l** in a velocity field **v** as,

$$d_{obs} = g(m, l, v) + \epsilon = d + \epsilon \tag{1}$$

where **m** contains the six independent MT elements and scalar moment m_0, and **g**, the deterministic part of the forward problem, is the function that computes the observations $\mathbf{d_{obs}}$ with error $\boldsymbol{\epsilon}$.

For a simple case of a point source inversion at a given location the predicted observations **d** can be expressed as the linear combination of weighted basis Green's functions (Jost & Herrmann, 1989),

$$d = Gm \tag{2}$$

where **G** is the impulse response of the Earth at the seismic station for a point force applied at the source, including near-, intermediate-, and far-field terms for body and surface waves. The classical approach assumes there is a true MT **m** that explains the data and finds the best solution by minimizing the misfit between observations and predictions. But because our knowledge of the Earth's structure is imperfect, we cannot predict exactly the observations for a given **m**.

In a probabilistic Bayesian framework, all the information of the inverse problem is formulated in terms of probability densitiy functions (PDFs). The solution is given by the posterior distribution (or *a posteriori* distribution) p(**x**|**d**), which is the probability density of the model parameters **x**, given the observed data **d**. The posterior is given by Bayes' theorem, such that data and modeling uncertainties enter as prior (or *a priori*) information into the inversion,

$$p(x|d) = \frac{p(d|x)p(x)}{p(d)} \tag{3}$$

where p(**x**) quantifies the *a posteriori* information we have for **x** and p(**d**|**x**) is the likelihood function quantifying the probability of the measured data **d** for different values of **x**. The prior represents our previous knowledge about the model and enable us to reject physically implausible solutions. The marginal likelihood p(**d**) is not a function of **x** but a constant factor ensuring that the integral of the posterior distribution equals to one. Therefore, the Bayes' theorem can also be written as

$$p(x|d) = cp(d|x)p(x) \tag{4}$$

$$c^{-1} = p(d) = \int p(d|x)p(x)dx \tag{5}$$

Suppose that the error $\boldsymbol{\epsilon}$ follows a Gaussian probability distribution, we can rewrite Eq. 1 as

$$d_{obs} \sim N(g(G, m), \Sigma) \tag{6}$$

where **N** is the multivariate Gaussian distribution with mean vector and variance matrix Σ. Both measurement and modelling errors are accounted for by Σ. Here we can separate two sources of error, the modeling uncertainty for a given **G** and **m**, and the background noises recorded at the station (Duputel et al., 2014). Given the forward problem from Eq. 2, Duputel et al. (2014), Phạm and Tkalčić (2021) and others have shown that the likelihood function for the measurement process **m** and a reference model **Ĝ** is

$$p(d_{obs}|d) = \frac{1}{\sqrt{(2\pi)^N |C|}} \exp\left(-\frac{1}{2}\left[d_{obs} - g\left(\hat{G}, m\right)\right]^T C^{-1}\left[d_{obs} - g\left(\hat{G}, m\right)\right]\right) \tag{7}$$

where the covariance matrix C (Fig. 1) is defined as the sum of the data noise covariance matrix C_d (measurement error) and model prediction covariance matrix C_t(**m**), which is the result of physical and mathematical approximations in the forward model (theory error).

$$C(m) = C_d + C_t(m) \tag{8}$$

$$C_t(m) = \frac{1}{N-1}\sum_{i=1}^{N}([g(G_i, m) - \bar{g}(m)]^T[g(G_i, m) - \bar{g}(m)]) \tag{9}$$

where $\bar{g}(m)$ is the sample mean over the predicted data vectors (mean Green's function). In the simplest form, we can represent **G** as a collection of velocity models deviating around a reference model **Ĝ**. This works well when the true Earth model lies within the population of models described by the reference model and its uncertainty. For the data noise covariance matrix, its simplest form is proportional to the identity matrix (ignoring correlated noise) $C_d = \sigma^2 \mathrm{I}$, where $\boldsymbol{\sigma^2}$ is the noise variance. Works by Mustać and Tkalčić (2015) and Vasyura-Bathke et al. (2021) explored and presented different parameterizations of the measurement error and theory error in MT estimation.

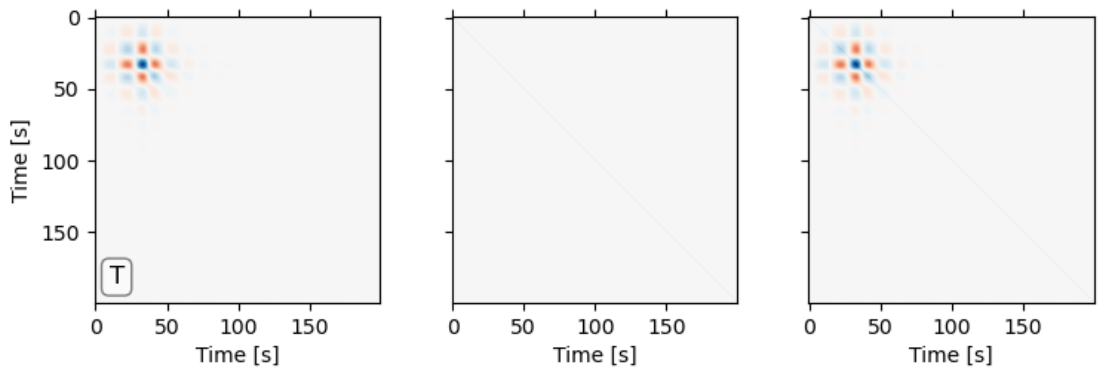

Figure 1. An example of the complete data covariance matrix of transverse component synthetic Green's functions. From left to right is the model covariance matrix Ct(m), data noise covariance matrix Cd, and the combined covariance C(m)

2.2 Random Moment Tensor Sampling

The seismic MT is a 3x3 matrix consists of nine force couples that represent the equivalent body forces for seismic sources of different geometries (Jost & Herrmann, 1989), which due to conservation of angular momentum reduce to six independent couples and dipoles. In the Bayesian framework, an appropriate choice of a priori moment tensor probability is important, but what constitutes as a random moment tensor is not straightforward and depends on the coordinate domain of the parameterized MT. Tape and Tape (2015) have shown that uniformly distributed MTs have uniformly distributed orientations (eigenvectors) but not uniformly distributed source-types (eigenvalues). In fact uniformly distributed MTs favors double-couples in the source-type space.

Tape and Tape (2015) have provided two approaches to generate uniformly distributed moment tensors based on the 5-D space of all MTs of unit norm, in which one of the approaches uses a parameterization of the MT that is closely related to the MT orientations and source types. The Tape parameterizations has five parameters and have finite upper and lower bounds: **κ** (strike), **h** (dip cosine), **σ** (slip), **u** (similar to lune colatitude) and **v** (similar to lune longitude), where the pair (**u**,**v**) determines the eigenvalues of MT (source type). We can now construct the uniform priors using the five Tape parameterization and the moment magnitude (or seismic moment).

A method of sampling to explore the parameter space is needed to evaluate the posterior PDF. In practice, an exhaustive search of all possible parameters can be impractical or require large amounts of computational resources. Here we will sample the posterior distribution with the Markov Chain Monte Carlo (MCMC) method, which is a random walk through the parameter space guided by the likelihood values, and the commonly used Metropolis-Hastings algorithm (Hastings, 1970; Metropolis et al., 1953). The chain starts at an initial model and is accepted unconditionally. In the second iteration, a new model is proposed as a perturbation of the current model, where the deviation is randomly drawn from a proposal distribution. The proposal is accepted and added to the chain if it is within the pre-set bounds and with the probability

$$\alpha = \min\left(\frac{p(x')p(d|x')p(x'|x)}{p(x)p(d|x)p(x|x')}, 1\right) \tag{10}$$

If the proposal distribution is symmetric the ratio can be dropped from the acceptance calculation. In the next section for the synthetic test case, the Gaussian distribution is used as the proposal distribution.

3 Synthetic Test Case: Earthquake

We designed a simple synthetic setup to demonstrate the incorporation of Earth model in regional MT inversion. In this setup, we simulated the observed data of a pure double-couple source and added uncorrelated noise to the data. We pre-computed the ensemble Green's functions of 100 velocity models, where the velocity models were generated from random perturbations to a reference model. The Green's functions are calculated using a 1D wave propagation solver from the software package Computer Programs in Seismology (Herrmann, 2013), a frequency–wavenumber integration method. Both the simulated data and synthetics are bandpass filtered from 20 to 50 seconds. The station coverage is set to mimic a realistic configuration of stations in regional MT inversion. For this simple test case we did not solve for the noise hyperparameters and only sample the MT parameters while accounting for Earth model error in the inversion. Each Markov chain consists of 25,000 steps where the first half (called the burn-in period) is discarded and models from the post-burn-in thinned chain are collected for the ensemble. Here we show results from one Markov chain but in practice it would be essential to run multiple Markov chain to ensure the target posterior distribution is fully explored. It can be done iteratively or mixing several independent chains to get the mean solution.

First, we show the result from a single Markov chain where the true velocity model is used in the inversion. The initial MT is randomly chosen from a uniform prior, as described in the previous section. Not surprisingly, the source models converge relatively quickly to the true solution and with very narrow posterior distributions (Fig. 2). The ensemble source models (shown in the focal mechanism plot) have a mean solution very close to the input. However, in practice we do not know the exact velocity model, only an approximation of it to the real Earth. Thus, Figure 3 shows the result of an inversion from a randomly selected initial MT and randomly selected Green's functions. The Green's functions are randomly selected from the ensemble Green's functions used to calculate the model covariance matrix. In this case we observe some source parameters converged quickly with very narrow posterior distributions, like $\boldsymbol{\kappa}$ (strike) and magnitude, while others have broader distributions compared to the previous example. But the variation with each step resembles a white noise process with no obvious trends, producing a mean solution that is still closely located to the input source model.

The two examples illustrate the advantages of a Bayesian probabilistic MT inversion. The ability to quantify uncertainties are especially important when examining the non-double-couple components of the MT. As shown here the velocity model error has a clear impact on the inverted solutions.

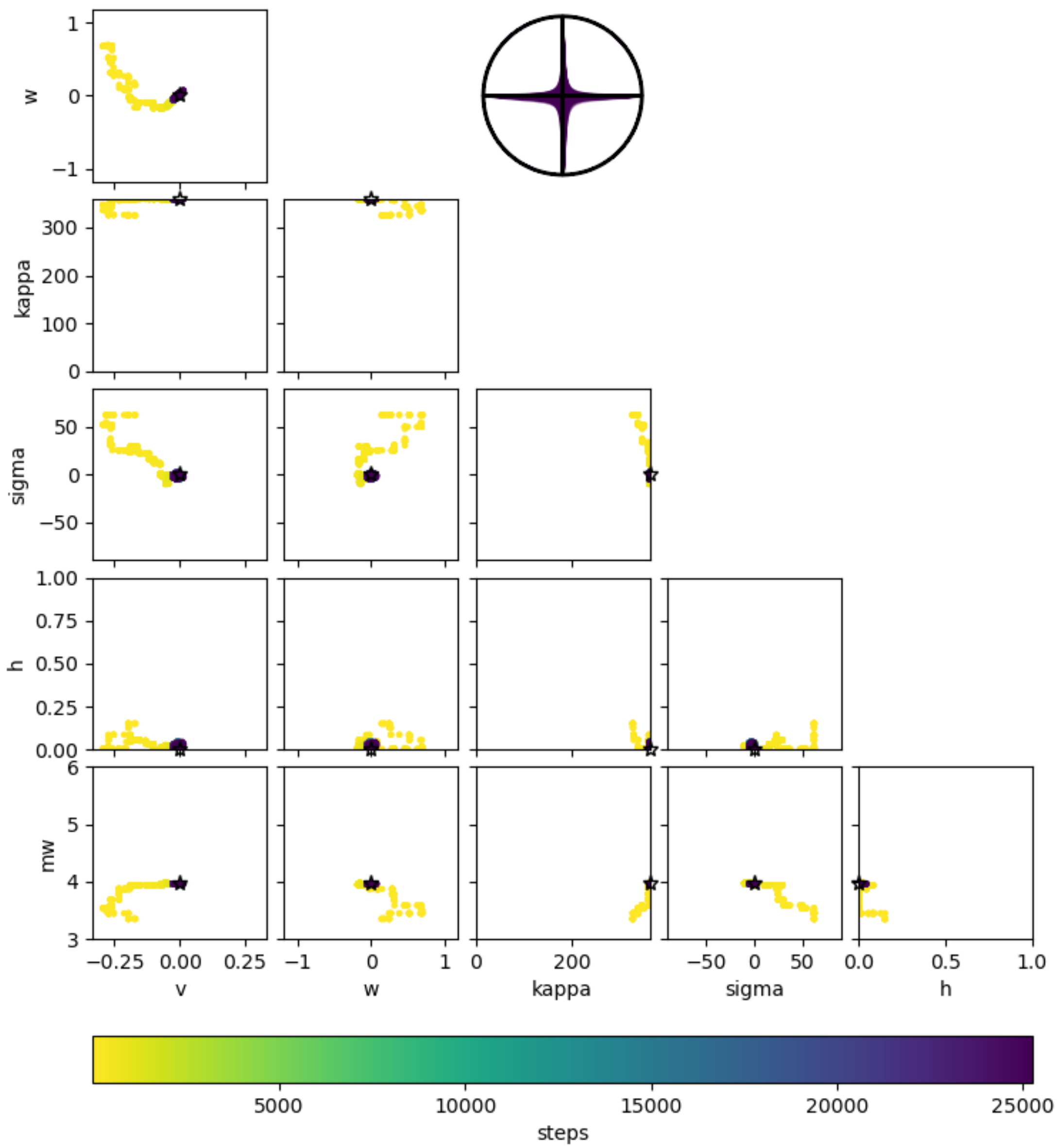

Figure 2. Inversion results when the true Earth model is known. The solution converges relatively quickly to the input source parameter (denoted as a black star). The focal mechanisms of the input earthquake source and the post-burn-in solutions are also plotted.

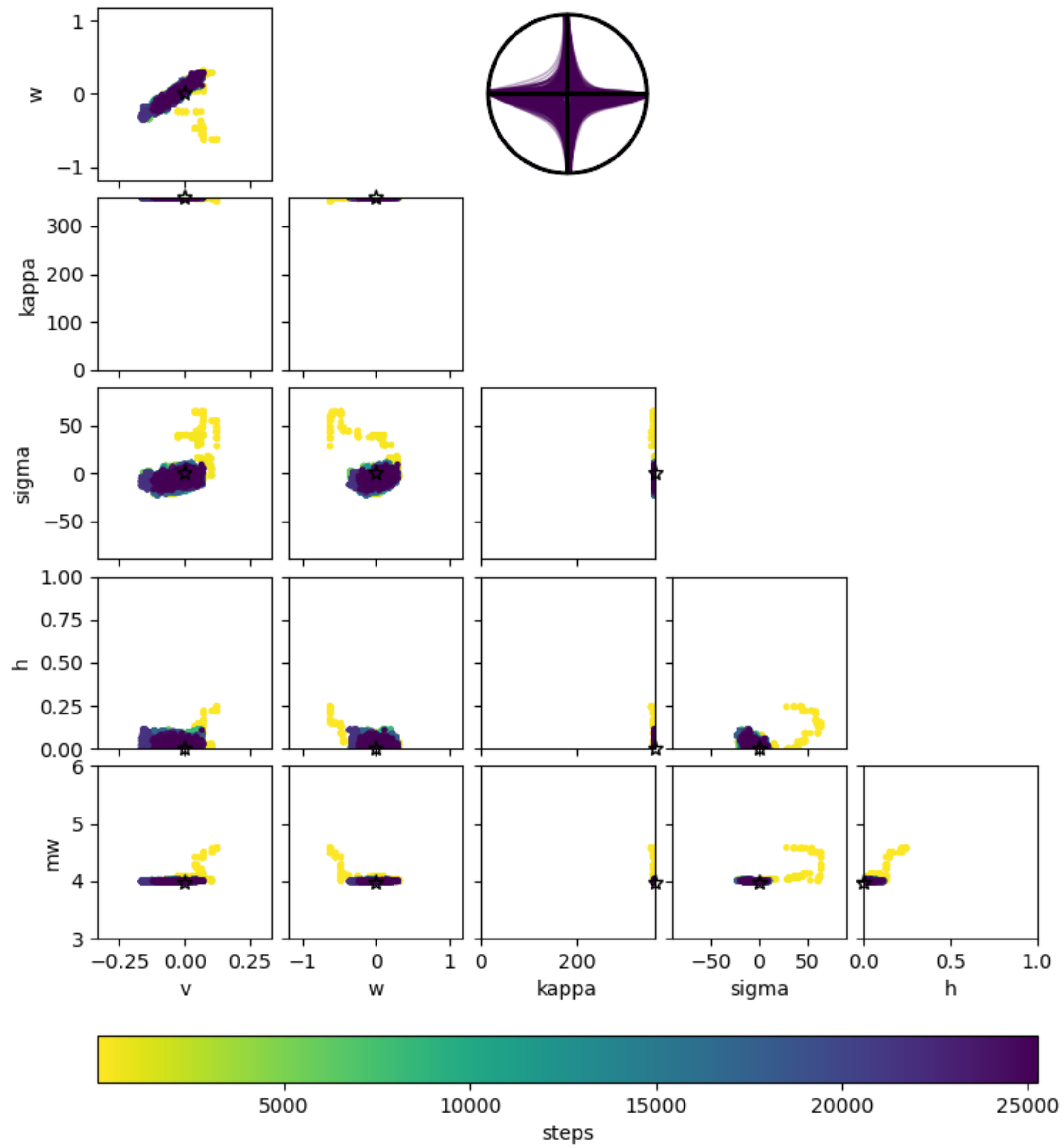

Figure 3. Inversion result when the Green's function is randomly selected. In this case we observe some source parameters converged relatively quickly and the mean solution matches closely to the input source.

4 Conclusions

MTs are valuable tools for event identification and yield estimation but the uncertainties, if provided, are often significantly underestimated. As we move towards monitoring smaller events, an estimation of the velocity model becomes increasingly more important. To address this need, we presented the theoretical background of a probabilistic Bayesian framework for MT inversion accounting for model and measurements errors, and we illustrated the method using a simple synthetic example. Future work should focus on the complete sampling of the source model space where we draw location parameters (latitude, longitude and depth) from a prior distribution defined by a given hypocenter or a previous calculation, as well as the velocity model from a prior distribution. Then for a location with a model, calculate **G**, the set of covariates for the linear problem, and find the likelihood for a uniform distribution of source parameters. This is done for several draws of the location parameters and velocity models, and the likelihoods for the given source parameters are integrated to find the posterior distribution. It is also possible to extend the approach to include other datasets such as polarity information (Jia et al., 2022; Pugh & White, 2018) and incorporate correlated noise models.

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