



SECURE

Science and Engineering of Cyber security through
Uncertainty quantification and Rigorous Experimentation

Trilevel Programming for Network Segmentation of Power System Cyber-Physical System

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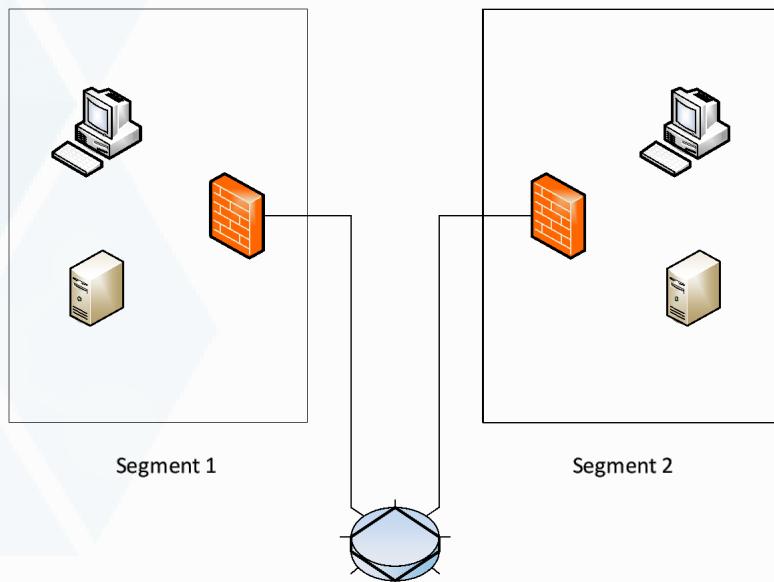
Outline

- Motivation
- Model Description
- Solution Technique
- 9-Bus and 30 Bus Results
- Larger Cyber-Physical Systems



Network Segmentation

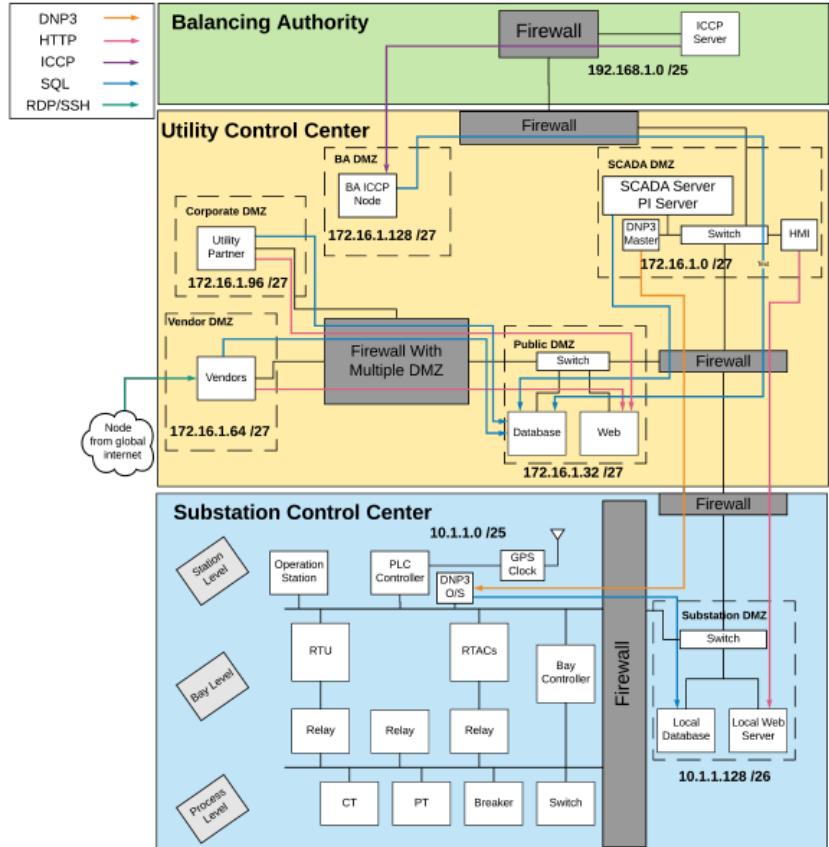
- Network segmentation involves
 - **dividing** a network into a set of sub-networks and
 - **enforcing communication rules** among network devices
- Improves security by limiting an attackers ability to pivot between workstations on network



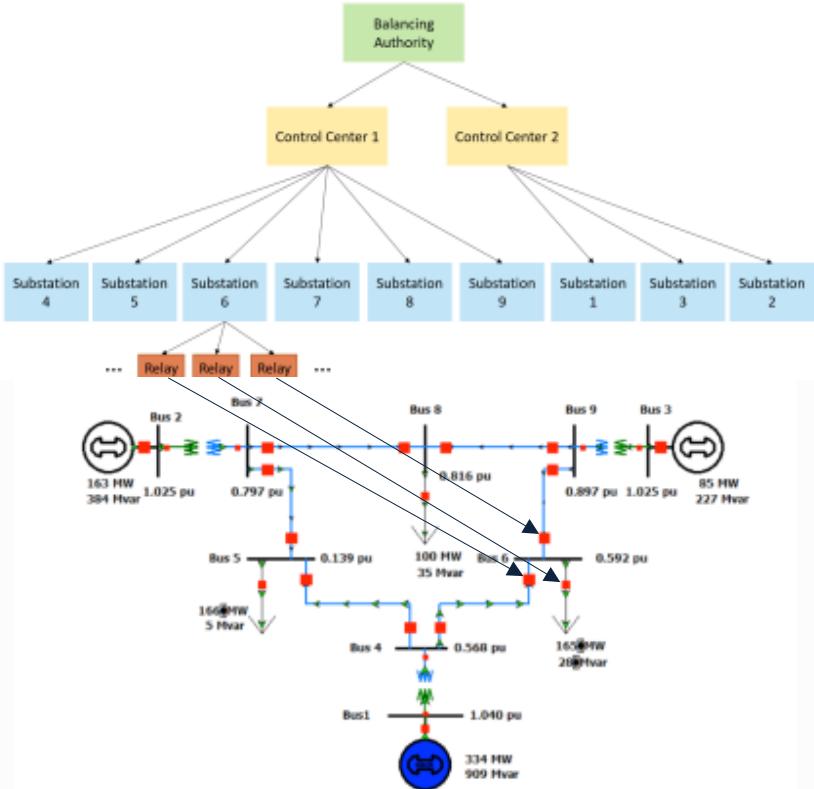


Power System Cyber-Physical Network

SCADA System



Cyber-Physical Model



Gaudet, Nastassja, et al. "Firewall Configuration and Path Analysis for SmartGrid Networks." 2020 IEEE International Workshop Technical Committee on Communications Quality and Reliability (CQR). IEEE, 2020.



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Cyber-Physical Network Segmentation

Defender segments the SCADA system to limit the scope of the attacker

Operator observes the attack and redispatches power to minimize load shed

$$\min_{(x,y,q,t) \in \mathcal{D}} \quad \max_{(\delta,z,u,v,w) \in \mathcal{A}(x,y)} \quad \min_{(\theta,f,p,l) \in \mathcal{O}(u,v,w)} \sum_{d \in \mathcal{L}} l_d$$

Attacker attacks up to N segments on the defender-segmented SCADA and disables relays in compromised substations.

Rules

- Leaders make decisions in anticipation of optimal follower decisions
- Followers must adhere to decisions made by leaders



Network Segmentation Model

\mathcal{D}

$$\sum_{r \in \mathcal{R}} x_{e,r} \geq 1, \quad \forall e \in \mathcal{E}(\mathcal{S}) \quad (2)$$

$$\sum_{e \in \mathcal{E}(\mathcal{S})} x_{e,r} = 1, \quad \forall r \in \mathcal{R} \quad (3)$$

$$q_{s,e} \leq \sum_{r \in \mathcal{R}_s} x_{e,r}, \quad \forall s \in \mathcal{S}, e \in \mathcal{E}_1(\mathcal{S}) \quad (4)$$

$$q_{s,e} \geq x_{e,r}, \quad \forall s \in \mathcal{S}, r \in \mathcal{R}_s, e \in \mathcal{E}_1(\mathcal{S}) \quad (5)$$

$$Q_{s,e} \leq \sum_{r \in \mathcal{R}_s} x_{e,r}, \quad \forall s \in \mathcal{S}, e \in \mathcal{E}_0(\mathcal{S}) \quad (6)$$

$$Q_{s,e} \geq x_{e,r}, \quad \forall s \in \mathcal{S}, r \in \mathcal{R}_s, e \in \mathcal{E}_0(\mathcal{S}) \quad (7)$$

$$\sum_{e \in \mathcal{E}(A)} y_{e,f} = 1, \quad \forall (A, B) \in \mathcal{Z}, f \in \mathcal{E}(B) \quad (8)$$

$$\sum_{n \in T} q_{n,e} = 1, \quad \forall T \in \mathcal{T}, e \in \mathcal{E}_1(T) \quad (9)$$

$$t_{e,n} \leq \sum_{f \in \mathcal{E}_0(B)} y_{e,f} Q_{n,f} + \sum_{f \in \mathcal{E}_1(B)} y_{e,f} q_{n,f}, \quad \forall (A, B) \in \mathcal{Z}, e \in \mathcal{E}(A), n \in B \quad (10)$$

$$t_{e,n} \geq y_{e,f} Q_{n,f}, \quad \forall (A, B) \in \mathcal{Z}, e \in \mathcal{E}(A), n \in B, f \in \mathcal{E}_0(B) \quad (11)$$

$$t_{e,n} \geq y_{e,f} q_{n,f}, \quad \forall (A, B) \in \mathcal{Z}, e \in \mathcal{E}(A), n \in B, f \in \mathcal{E}_1(B) \quad (12)$$

$$t_{e,n} \leq Q_{m,e}, \quad \forall (A, B) \in \mathcal{Z}, e \in \mathcal{E}_0(A), m \in A, n \in B_m \quad (13)$$

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$$y_{e,f} \in \{0, 1\}, \forall (e, f) \in (\mathcal{E}(\mathcal{C}) \times \mathcal{E}(\mathcal{S})) \cup (\mathcal{E}(\mathcal{B}) \times \mathcal{E}(\mathcal{C})) \quad (15)$$

$$x_{e,r} \in \{0, 1\}, \quad e \in \mathcal{E}(\mathcal{S}), \forall r \in \mathcal{R} \quad (16)$$

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$\mathcal{A}(x, y)$

$$\sum_{e \in \mathcal{E}} z_e \leq U \quad (22)$$

$$z_f \leq \sum_{e \in \mathcal{E}(A)} y_{e,f} z_e, \quad \forall (A, B) \in \mathcal{Z}, \forall f \in \mathcal{E}(B) \quad (23)$$

$$\delta_r = \sum_{e \in \mathcal{E}(\mathcal{S})} x_{e,r} z_e, \quad \forall r \in \mathcal{R} \quad (24)$$

$$v_k \leq (1 - \delta_r), \quad \forall k \in \mathcal{K}, r \in \mathcal{R}_k \quad (25)$$

$$v_k \geq \sum_{r \in \mathcal{R}_k} (1 - \delta_r) - |\mathcal{R}_k| + 1, \quad \forall k \in \mathcal{K} \quad (26)$$

$$w_g \leq (1 - \delta_r), \quad \forall g \in \mathcal{G}, r \in \mathcal{R}_g \quad (27)$$

$$w_g \geq \sum_{r \in \mathcal{R}_g} (1 - \delta_r) - |\mathcal{R}_g| + 1, \quad \forall g \in \mathcal{G} \quad (28)$$

$$u_d \leq (1 - \delta_r), \quad \forall d \in \mathcal{L}, r \in \mathcal{R}_d \quad (29)$$

$$u_d \geq \sum_{r \in \mathcal{R}_d} (1 - \delta_r) - |\mathcal{R}_d| + 1, \quad \forall d \in \mathcal{L} \quad (30)$$

$\mathcal{O}(u, v, w)$

$$\sum_{k \in \{k' | d(k') = s\}} f_k - \sum_{k \in \{k' | o(k') = s\}} f_k + \sum_{g \in \mathcal{G}_s} p_g = \sum_{d \in \mathcal{L}_s} (D_d - l_d) \quad \forall s \in \mathcal{S} \quad (31)$$

$$f_k = B_k v_k (\theta_{o(k)} - \theta_{d(k)} - \Theta_k) \quad \forall k \in \mathcal{K} \quad (32)$$

$$-\bar{F}_k \leq f_k \leq \bar{F}_k \quad \forall k \in \mathcal{K} \quad (33)$$

$$0 \leq p_g \leq w_g \bar{P}_g \quad \forall g \in \mathcal{G} \quad (34)$$

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$$-\pi \leq \theta_s \leq \pi \quad \forall s \in \mathcal{S} \quad (36)$$

DCOPF operator model

Attacker precedence-based selection model

Designer assignment model



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Attacker precedence-based selection model

Designer assignment model

DCOPF operator model

Results in the coming slides will clarify the model



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A Naïve Solution Technique for Small Systems

- Step 1: Linearize bilinear terms using **McCormick Relaxation**

$$z_f \leq \sum_{e \in \mathcal{E}(A)} y_{e,f} z_e \quad f_k = B_k v_k (\theta_{o(k)} - \theta_{d(k)} - \Theta_k)$$

- Step 2: **Dualize** third (**operator**) model and **reduce** **attacker** and dualized **operator** into a single max model, transforming the trilevel model into a bilevel min-max model.
 - Note that the combined max model has binaries, so it cannot be dualized.
- Step 3: Use **bilevel branch-and-bound** to solve mixed bilevel model
 - Apply branch-and-bound to high-point relaxation (constraints from both levels with leader objective) and obtain cuts to remove follow-suboptimal solutions through callbacks.
 - **Fischetti et. al.** has made their CPLEX-based academic solver available for research purposes.
 - MibS is open-source and uses COIN-OR framework.

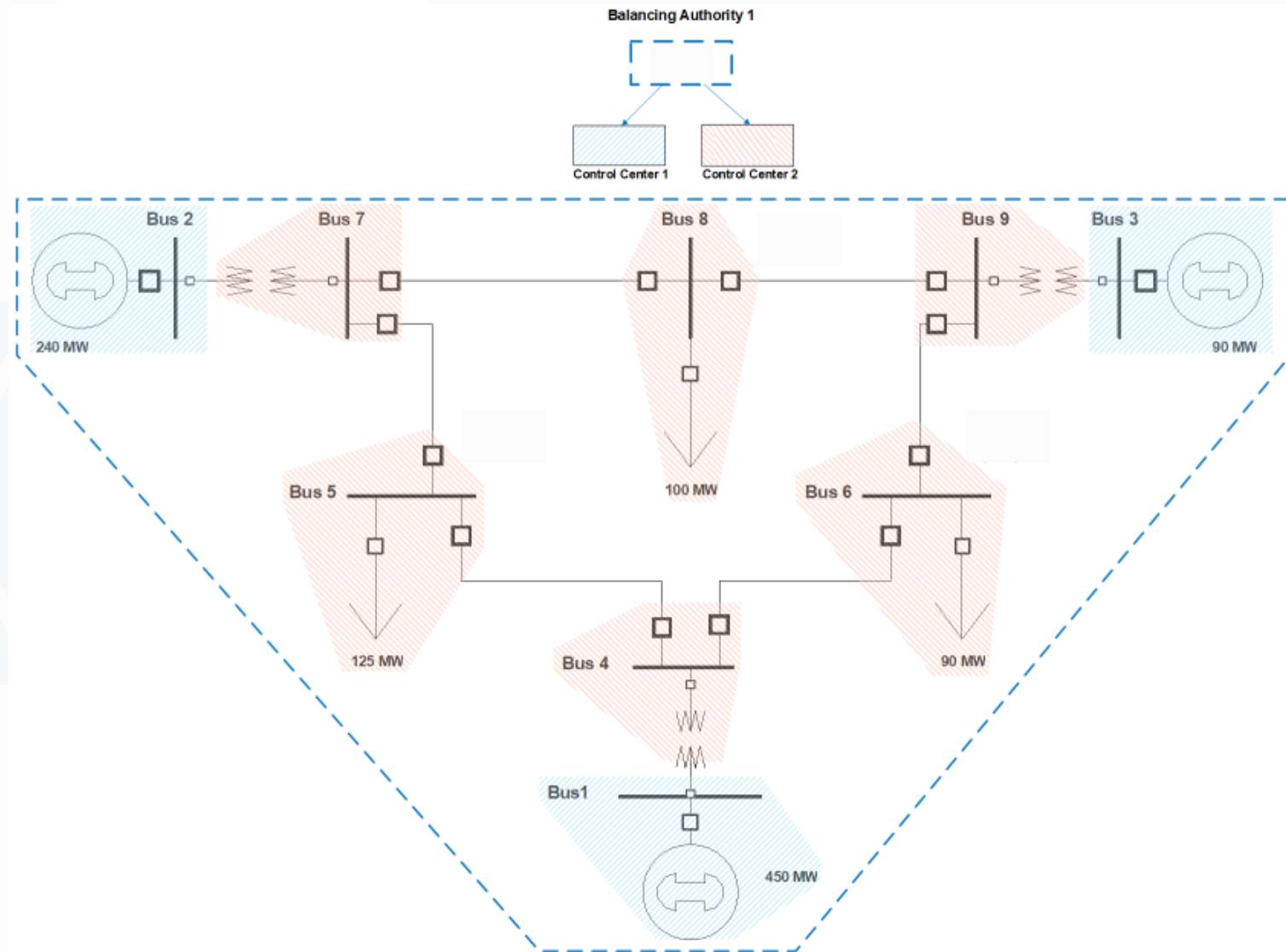


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9-bus System Before Segmentation

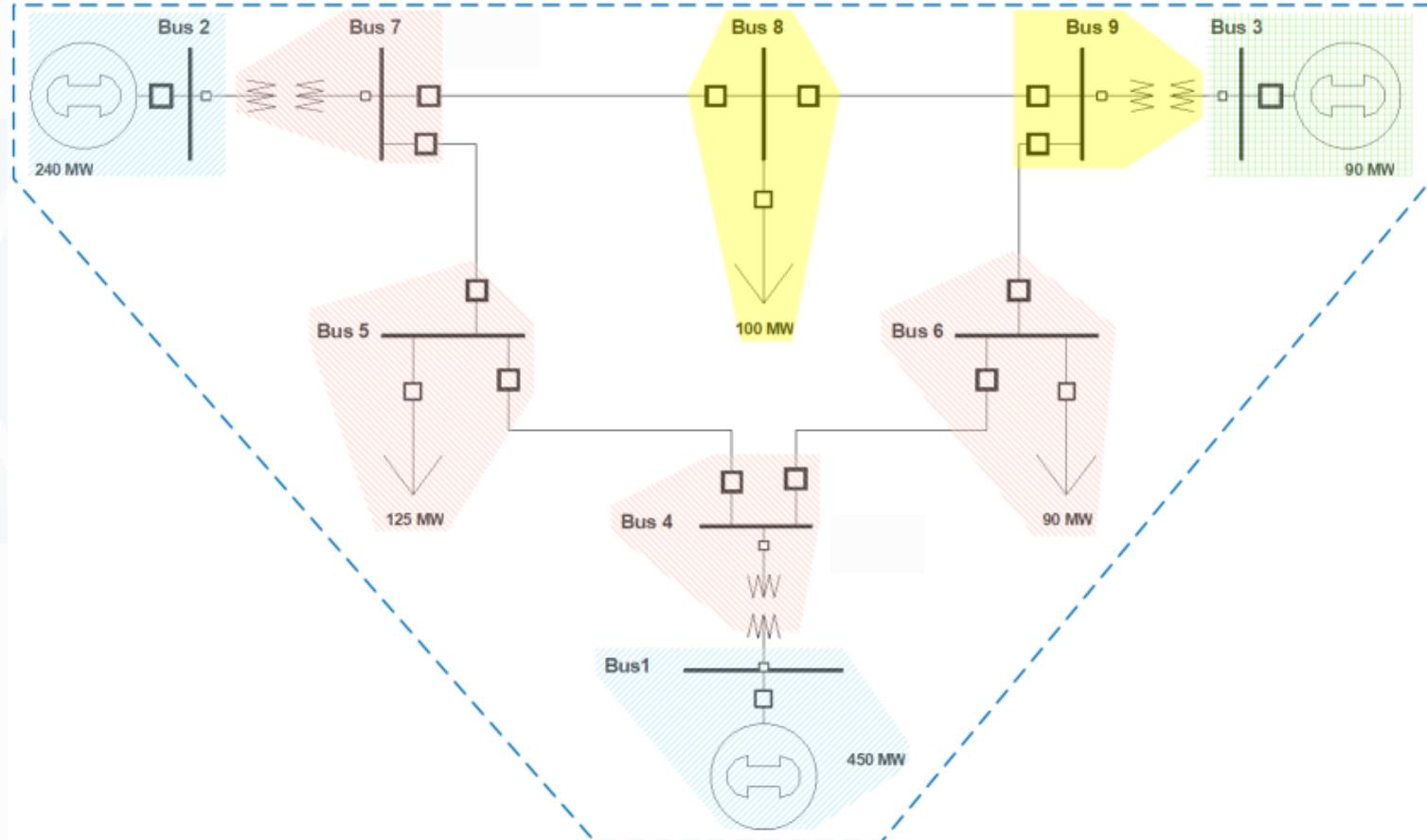
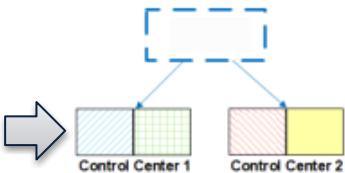




9-bus After Segmentation

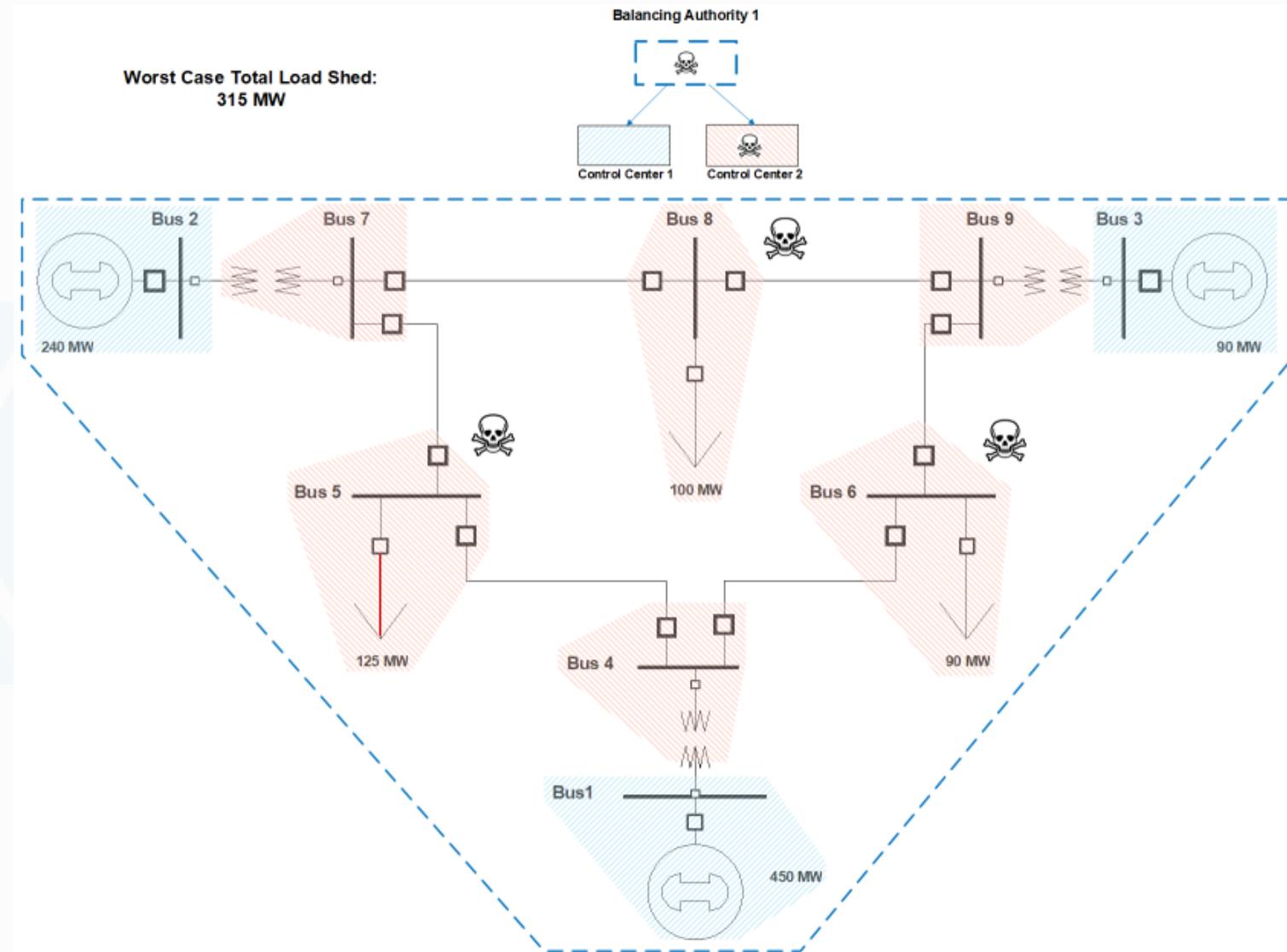
Each Control Center
Split in Two

Balancing Authority 1



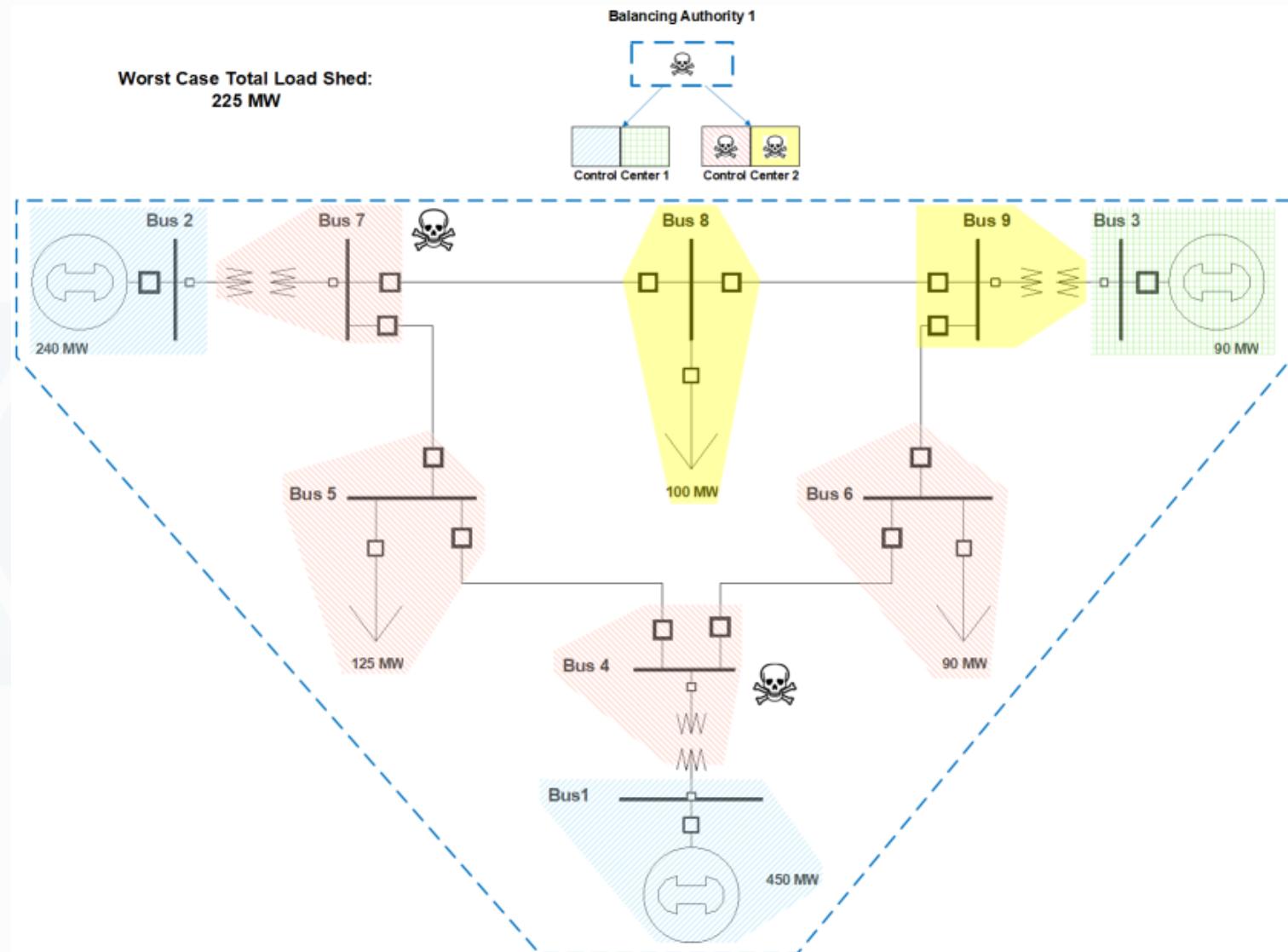


9-bus (Attacker Budget = 5) Before Segmentation



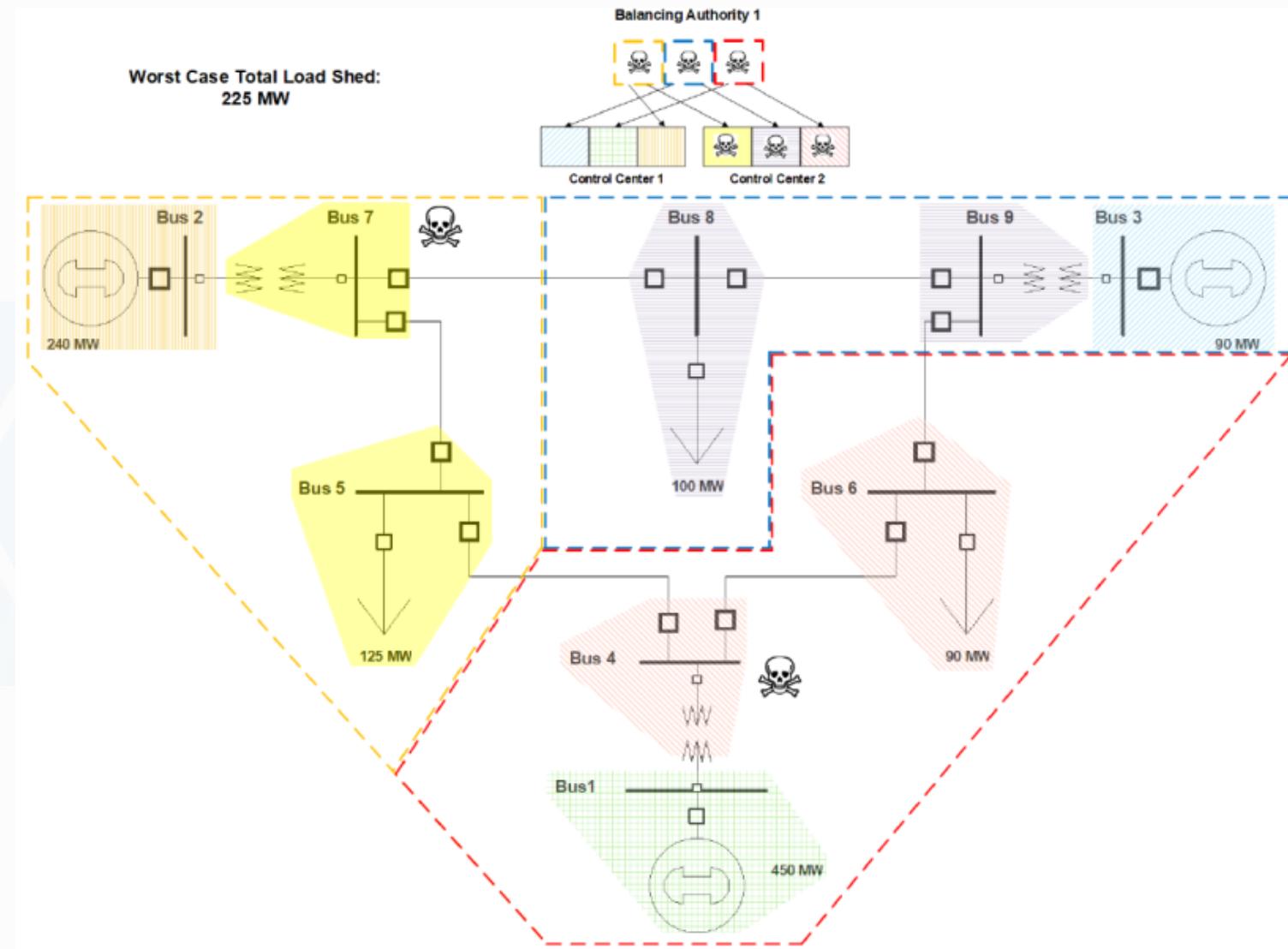


9-bus (Attacker Budget=5) After Segmentation



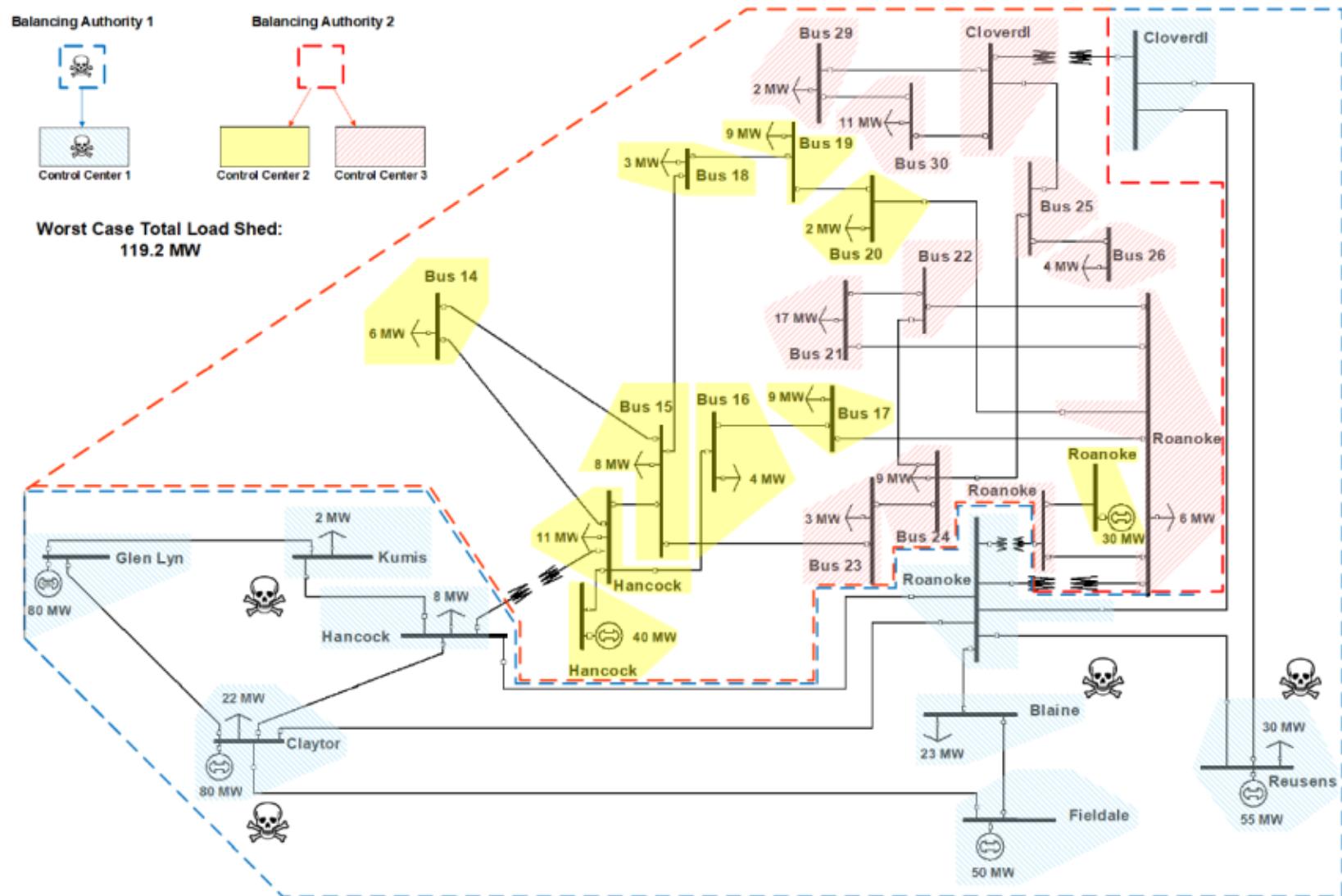


9-bus (Attacker Budget = 8) After Segmentation

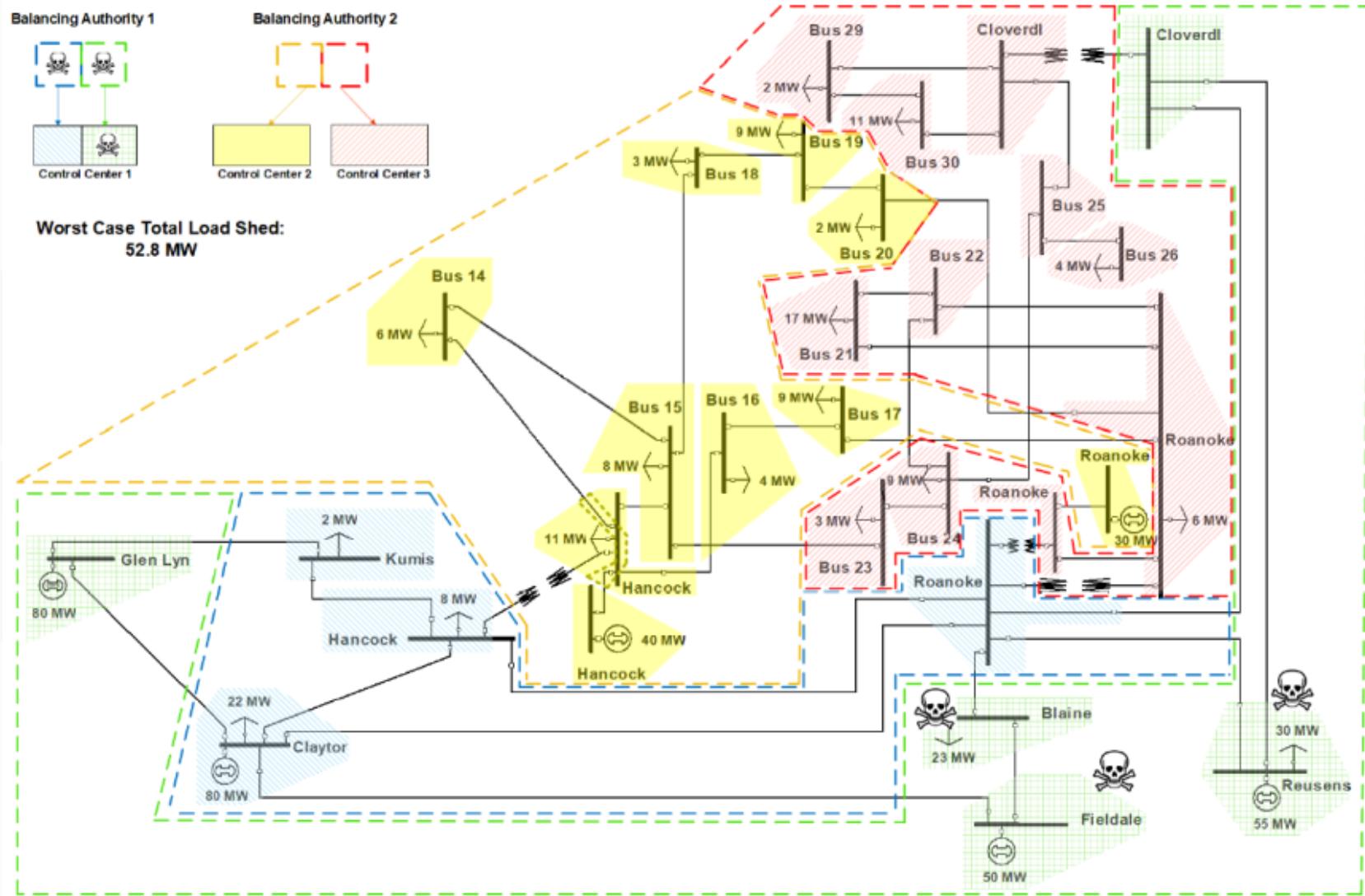




30-bus (Attacker Budget = 6) Before Segmentation



30-bus (Attacker Budget = 6) After Segmentation





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Scaling to Larger Systems

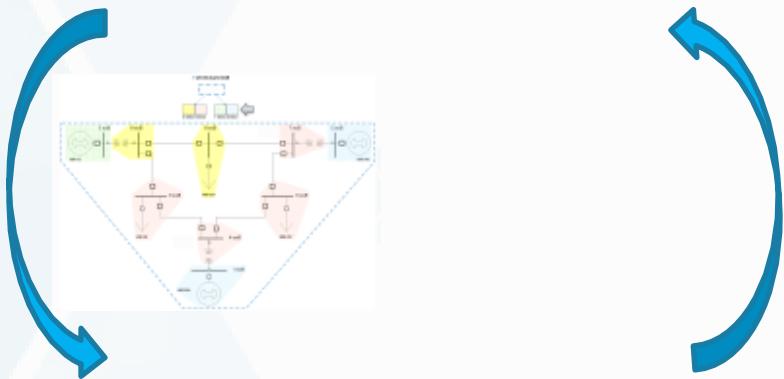
- Bilevel branch-and-bound is ineffective for **interdiction** models.
 - Follower and leader objectives are the same.
 - This leads to **weak dual bounds**.
 - Does not scale well for larger power systems than the 30-bus test case.
- Partnered with Emma Johnson and Santanu Dey (Georgia Tech), Jonathan Eckstein (Rutgers), and Cynthia Phillips (Sandia) to develop **decomposition algorithm for trilevel interdiction**.
 - Based on the **Covering Decomposition Algorithm** for bilevel programming by Israeli and Wood (2002)



A Trilevel Decomposition Algorithm

$$\min_{\substack{(x,y,q,t) \in \mathcal{D} \\ \text{***}}} \quad \min_{(\theta,f,p,l) \in \mathcal{O}(u,v,w)} \sum_{d \in \mathcal{L}} l_d$$

Network design that can block all discovered worst-case attacks is passed to attacker subproblem



Worst-case attack is returned to master

$$\max_{(\delta,z,u,v,w) \in \mathcal{A}(x,y)}$$



Large-scale Results

- Can solve small instances far more quickly with trilevel decomposition than with the bilevel branch-and-bound approach.
- To further help with scaling, the DC optimal power flow was simplified to **capacitated network flow**
 - Relaxing B-theta constraint has empirically been shown to yield high-quality lower bounds for the inner two problems (Johnson and Dey 2021)
 - Can solve:
 - **500-bus** system with a SCADA system that communicates with the whole grid.
 - **2000-bus** synthetic system with a SCADA system that communicates with 30 buses.



References

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Thank you!