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QUANTIFICATION OF CONSERVATISM IN THE MAXI-MAX POWER SPECTRAL DENSITY FUNCTION

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ABSTRACT

The maxi-max Power Spectral Density (PSD) function is frequently used to evaluate non-stationary vibration environments. The resulting PSD are often subsequently used as test specifications, either separably or enveloped with other vibration PSD environments. It is also known that the maxi-max PSD is generally conservative and over-predicts the vibration environment. This paper investigates the range of conservatism inherent in the maxi-max PSD formulation for both stationary and non-stationary vibration environments and its subsequent impact on environmental testing. The paper also presents a recommendation for correcting maxi-max PSD estimates based on event duration and character to better represent the underlying environment.

INTRODUCTION

The Power Spectral Density (PSD) function, also frequently called an Acceleration Spectral Density (ASD) or Auto-Spectral Density (ASD) function, is the most common method of evaluating the random vibration response of a structural system or component. However, one of the fundamental assumptions associated with the PSD is that the input random vibration is a stationary random process. In other words, the statistical parameters associated with the signal are unchanging in time. For many random vibration environments, this requirement is reasonable. Aircraft vibration is relatively constant once a cruising altitude is reached, rail transportation is relatively constant, even cross-country truck transportation is fairly consistent since the truck suspension is set and many roads are nominally paved the same. On the other hand, some environments are substantially non-stationary, but still need to be analyzed. Spacecraft environments are one such environment. Random vibration environments on a typical space vehicle can change substantially as the rocket motors burn and the vehicle is accelerated first through the thicker low-altitude atmosphere and then through the high-altitude thinner atmosphere.

To accommodate the need to develop suitable stationary random vibration test environments for decidedly non-stationary events, the maxi-max PSD algorithm was developed. The maxi-max PSD is defined as the envelope of a series of PSD segments calculated from overlapped one-second segments of the acceleration time history [1]. The standard also states that the frequency resolution bandwidth should be no greater than 1/6 octave. Separating the PSD into segments and enveloping is used to offset the potential lowering of the overall spectrum that naturally results from the averaging inherent in the traditional Welch's method of PSD calculation. However, MIL-STD-810H offers a substantial caution associated with the use of the maxi-max PSD. MIL-STD-810H notes that the normalized error could be very high due to the reduced number of degrees of freedom in the spectrum. The standard goes on to state that the maxi-max spectrum should be used with caution or perhaps not at all [3]. However, in some applications it continues to be extensively used because of its historical application. Thus, a situation arises where the historical use of the maxi-max PSD results in its continued use without the necessary critical interpretation of the resulting spectral results.

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This paper describes objective technical results and analysis. Any subjective views or opinions expressed in the paper do not necessarily reflect the views of the U.S. Department of Energy or the United States Government.

This paper provides a brief review of spectrum theory showing the origins of the ASD / PSD calculation. Examples of maxi-max PSD calculated from both stationary and non-stationary random vibration data are analyzed and compared. While the maxi-max PSD theory is typically not applied to stationary random vibration data, the analysis is relevant to help understand the potential range of differences between the traditional averaging of Welch's method and the enveloping of the maxi-max PSD method. This is especially important given the often historical application of the maxi-max PSD methodology to certain data streams regardless the severity of non-stationary character. Two non-stationary examples are also presented and analyzed using averaging and enveloping PSD methods. Rain-flow analysis of simulated inputs generated from the different PSD methodologies is used to compare the severity of the two methods against the known measured data.

One significant advantage to the work performed here is that maxi-max PSD analysis is almost exclusively performed after the field data is measured. In contrast, a traditional PSD is sometimes calculated in real-time without explicitly saving the acceleration time history for later analysis. As a result, adjustments to the maxi-max PSD methodology are possible by the data analyst or environments engineer with the luxury of time before the resulting test specifications are levied on components or systems.

REVIEW OF SPECTRUM THEORY

The PSD is a very common calculation and with the long ago development of the fast Fourier transform (FFT) the calculation can be performed in essentially real-time. The ubiquitousness of this transformation often leads to a certain carelessness in its use. As such, a very brief review of the PSD origin is appropriate. The PSD is defined as the finite Fourier transform of the of the second-order cumulant of the time history. The second-order cumulant is also know as the covariance when calculated from a random variable with itself. This is the auto-correlation function of the time signal $x(t)$ and can be written as:

$$R_{xx}(t) = \int_{-\infty}^{+\infty} x^*(t)x(t+\tau)dt. \tag{1}$$

The auto-spectral density function is then given as the Fourier transform of the auto-correlation function as:

$$S_{xx}(f) = \mathcal{F}\left[R_{xx}(t)\right] = \int_{-\infty}^{+\infty} R_{xx}(t)e^{-i2\pi ft}dt. \tag{2}$$

One requirement for Eq. 2 is that the integral of the auto-correlation must converge. Since the auto-correlation function tends to the signal mean, it is a requirement that the analyzed random vibration signal be zero mean [4].

$$S_{xx}(f) = \mathcal{F}\left[R_{xx}(t)\right] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^*(t)x(t+\tau)e^{-i2\pi f\tau}dtd\tau \tag{3}$$

As a practical matter, the PSD (ASD) is not actually calculated using the auto-correlation integral as given in Eq. 3. The computational overhead is simply too high. Using the Wiener-Khinchin theorem it can be shown that the PSD for a real-valued signal can be calculated as the product of the signal's Fourier transform with its complex conjugate [5].

$$S_{xx}(f) = X^*(f)X(f) \tag{4}$$

While Eq. 4 represents the theoretical PSD, the actual calculation is more complicated. A PSD calculated using Eq. 4 alone will have significant hash in the output spectrum. and be difficult to use and evaluate. As a result, the common practice is to divide the time signal into overlapping data blocks, apply a standard windowing function, calculate the PSD from this windowed block of data and then average that PSD with many more PSD. This methodology yields a relatively smooth PSD function that can be more easily evaluated. This windowed, averaged PSD is calculated using:

$$S_{xx}(f) = \frac{2}{K}\sum_{j=1}^{K}\frac{X_j^*(f)X_j(f)}{\Delta f W_s}. \tag{5}$$

where K is the number of blocks to be averaged, Δf is the frequency resolution of the resulting PSD, and W_s is the correction factor for the signal window applied. Inherent in this derivation is the fundamental assumption of a stationary random process. When the measured signal is non-stationary, the ASD theory begins to break down. The introduction of ensemble averaging further corrupts the PSD results for time-varying signals. It should also be added here that differences in segment length directly influence the available frequency resolution and the hash in the resulting PSD. This in turn impacts the resulting maxi-max PSD envelope.

EXAMPLE USING STATIONARY RANDOM VIBRATION DATA

PSD theory is predicated on the assumption of a stationary random vibration event. In contrast, the maxi-max PSD fundamentally assumes a non-stationary environment and attempts to apply stationary vibration theory to evaluate the event. To understand the potential errors associated with this discrepancy, it is useful to apply the maxi-max PSD formulation to a known stationary random vibration signal. Figure 1 shows a plot of a sample experimental random vibration time history obtained during some recent research testing at Sandia National Laboratories Vibration Test Laboratory. The measured time history data was truncated to capture approximately 40 seconds of data after the shaker system had stabilized at the desired 0 dB test level. Figure 2 shows a comparison of the PSD calculated using Welch's method with 1/6 octave smoothing compared to the maxi-max PSD from the same time history along with the underlying PSD segments used in the maxi-max calculation. Since MIL-STD-1540C indicates that 1/6 octave frequency spacing is required for a true maxi-max PSD, that frequency smoothing was also applied to Welch's estimate for consistency. Figure 2 shows that the maxi-max PSD is approximately +1.72 dB above the PSD calculated with Welch's method and the maxi-max PSD has a RMS acceleration of 8.08 g compared to 7.01 g for the standard PSD calculation. This test specification should be recognizable as the minimum random vibration unit acceptance test specification from SMC-S-016 [2].

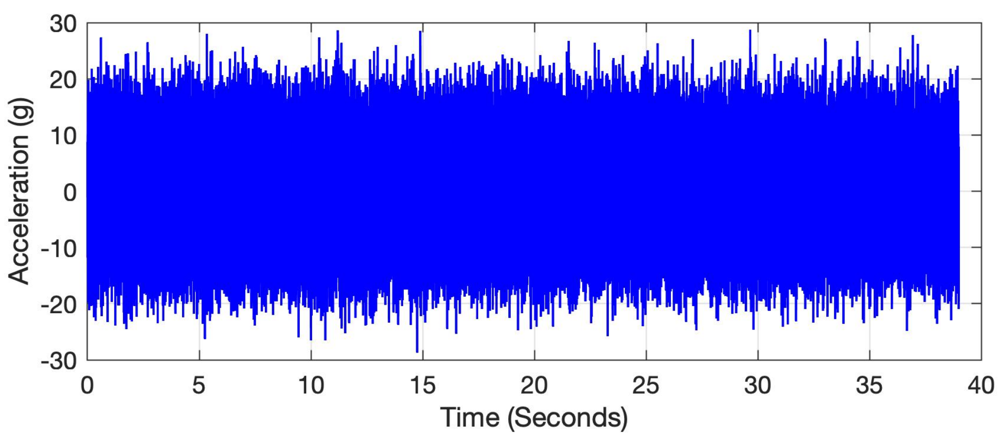

Figure 1: Example experimental stationary random vibration time history

The difference between Welch's method PSD and the maxi-max PSD for a stationary random vibration is somewhat concerning. The reason for the increased spectral levels from the maxi-max calculation is a direct result of taking the maximum of all the underlying one-second PSD segments. Welch's method averages the underlying segments to remove hash in the resulting PSD. In contrast, the maxi-max method averages PSD data within a one-second segment but takes the maximum of the resulting one-second PSD segments. In other words, the resulting segment PSD data is skylined. This is highlighted in Fig. 2, where all the individual one-second PSD segments are shown in grey overlaid with the PSD and the maxi-max PSD results. As can be seen, Welch's method performs an average over all time, and the resulting PSD fares through the cloud of segment data quite nicely. In contrast, the maxi-max method envelops all the underlying segments and is appreciably higher than the accepted result from Welch's method. If a test specification were created to envelop the maxi-max results, that test could easily be 2 dB above the experimental data and result in

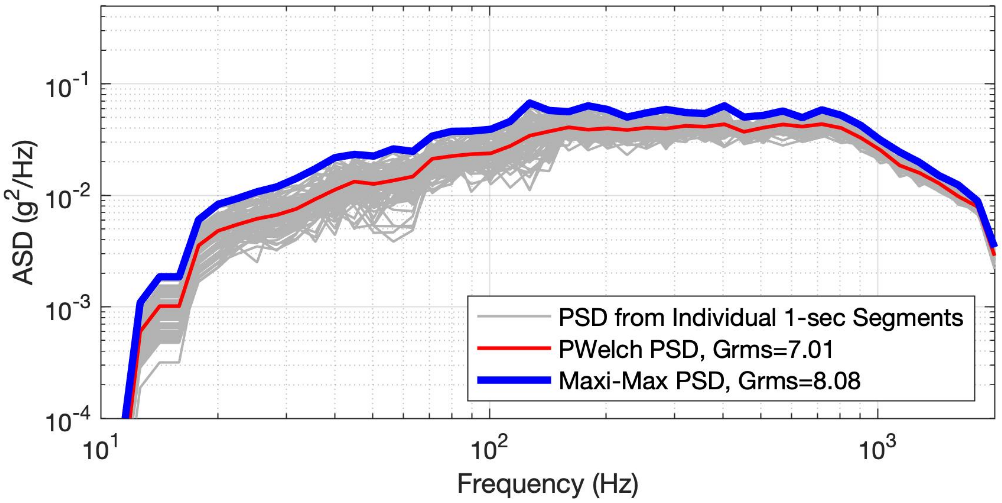

Figure 2: PSD and maxi-max PSD with underlying segment data for the experimental stationary random vibration time history

considerable conservatism. If the new specification were published without reference to the original data, the resulting conservatism would become completely hidden to future applications. Additionally, for this example, the 1/6 octave smoothing was correctly applied to each segment prior to enveloping; however, if the enveloping were performed first, followed by the smoothing operation, the resulting maxi-max PSD would be higher.

While the results shown in Fig. 2 are problematic, they can be worse still. Welch's method by itself yields nominally the same result with variations in blocksize. The results shown in Fig. 2 were calculated with a commonly used 1024-point blocksize. However, the only requirement for Welch's method with regard to blocksize is to select one that yields the desired frequency range. To examine the effects of blocksize on the resulting maxi-max PSD, the calculation was repeated for blocksize values ranging from 512 points up to 4096 points. Figure 3 shows how the resulting maxi-max PSD changes with increasing analysis blocksize. All of the values chosen here are common values for PSD calculation and might be chosen arbitrarily by any analyst. If a 4096-point blocksize is selected, Fig. 3 indicates the resulting maxi-max PSD is approximately +2.86 dB above the Welch's method PSD with an RMS acceleration of 8.76 g. The reason for the increase is due to the fact that longer blocksizes yield fewer averages in a one-second time segment. With fewer averages, the hash in the resulting PSD increases and the skyline of all PSD segments proportionately increases. This increase is somewhat muted with the 1/6 octave smoothing; however, the same comparison without the 1/6 octave smoothing results in the 4096 point maxi-max PSD being +6.46 dB above the baseline Welch's method PSD and having an RMS acceleration of 14.99 g. Those numbers represent a very substantial increase that would be readily apparent without smoothing or to a lesser degree if the octal smoothing were performed after the segment enveloping.

The preceding examples all assume a standard one-second segment length for the maxi-max PSD. While one-second segments are the accepted approach, it is not necessarily required. While SMC-S-016 [2] only refers to one-second time windows, the earlier MIL-STD-1540C [1] suggests one-second segments but also suggests that longer time segments could be used in certain circumstances. While neither document suggests that shorter time windows are acceptable, the authors have seen half-second segments used for analysis in several organizations. The justification provided was that the one-second definition was not codified, therefore it was open to interpretation. Figure 4 shows an example of how the maxi-max PSD can change with time segment length. The maxi-max results shown here were obtained by varying the time window from 0.25 seconds up to 10 seconds. As would be expected, the PSD calculated using the longer segment lengths asymptotically approach the Welch's method PSD. In contrast, the shorter time segments rapidly increase in amplitude when segment times below one-second are used. The resulting maxi-max PSD calculated with quarter-second time segments is incredibly +5.03 dB above Welch's PSD with a RMS of 11.03 g. A design

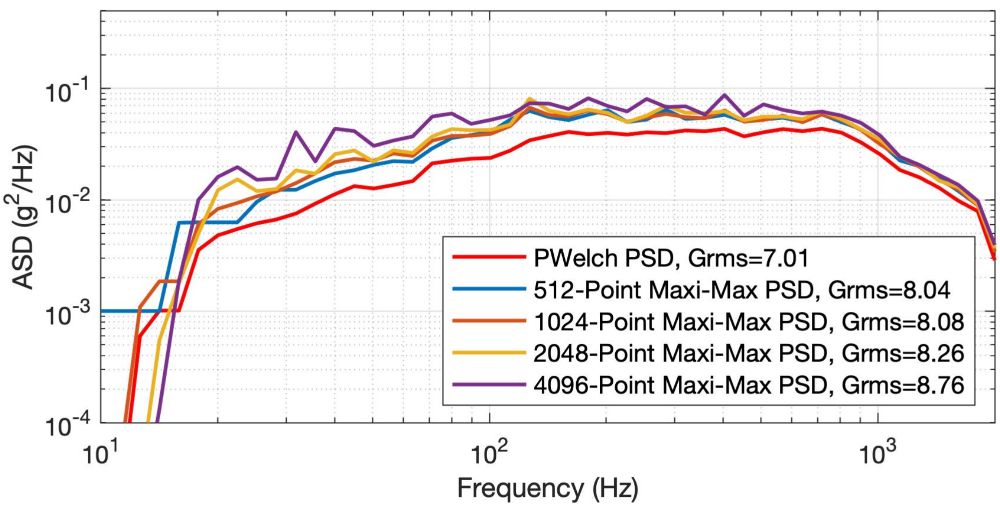

Figure 3: Variations in the maxi-max PSD results with analysis blocksize for the example experimental stationary random vibration time history

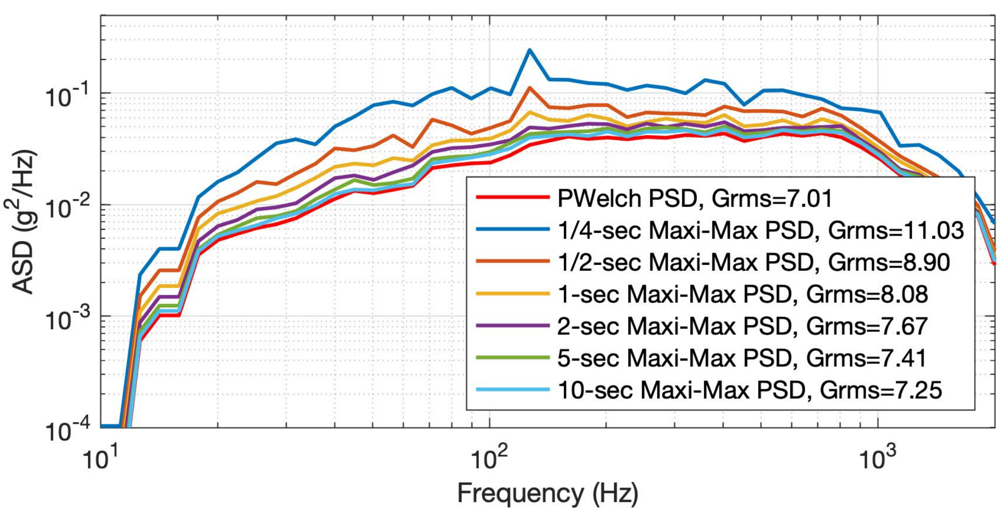

Figure 4: Variations in the maxi-max PSD results with analysis time segment length for the example experimental stationary random vibration time history

specification written from this would result in significant component over-design and likely increased cost and weight. Here again, if the same analysis were performed without the 1/6 octave smoothing, the same quarter-second analysis jumps to +7.12 dB above Welch's PSD with a RMS of 16.14 g. These are very substantial exposure increases.

The above analysis has made mention of the differences between 1/6 octave smoothing and the as-calculated linear results; however, Fig. 5 shows comparisons of maxi-max PSD calculated with 1/6 octave smoothing, 1/12 octave smoothing, and with no smoothing. This comparison shows a steady increase in the resulting maxi-max PSD with reduced smoothing. As stated previous, there is a +1.72 dB difference between Welch's estimate and the 1/6 octave maxi-max PSD estimate. This increases to +2.01 dB when 1/12 octave smoothing is applied. It further increases to +2.74 dB when no smoothing is applied.

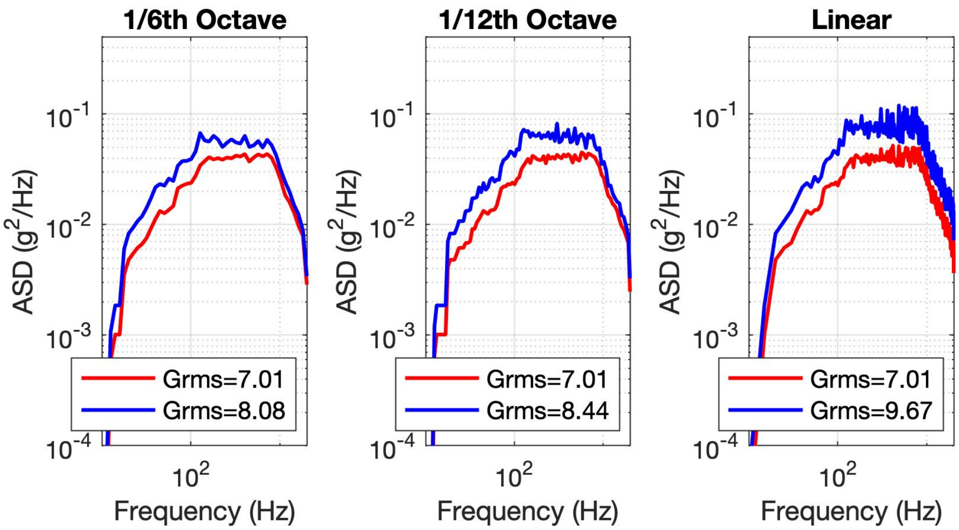

Figure 5: Variations in the maxi-max PSD results with and without octal smoothing for the example experimental stationary random vibration time history

EXAMPLES USING NON-STATIONARY RANDOM VIBRATION DATA

While the stationary random vibration example provides a bounding understanding of the potential conservatism inherent in the maxi-max PSD, the method is decidedly aimed at the non-stationary vibration problem. As such, two experimentally derived non-stationary random vibration examples are analyzed here for comparison with the stationary data results.

Figure 6 shows a plot of an experimentally measured non-stationary random vibration event. The event here is modest and reasonably flat for a few seconds and then tapers down to a significantly lower level and again begins to flatten. Figure 7 shows a plot of the PSD calculated using the traditional Welch's method of averaging and the maxi-max PSD spectrum overlaid with the PSD of the underlying segments. Here again, the maxi-max PSD is substantially higher than the average, approximately +3.96 dB above Welch's average method. Of course it is entirely likely that Welch's method may be under-predicting in this non-stationary case so the dB difference may not be entirely correct. In contrast to the stationary example shown previously in Fig. 2, there is significantly greater variability in the underlying segment spectra shown here. It should be noted that all spectra shown here were calculated with a consistent 1024-point segment length.

While it is easy to assume that Welch's method has dramatically under-estimated the signal in this non-stationary example, it is prudent to investigate that assumption. As a first point of consideration, the spectrum from Welch's method and the maxi-max PSD were used to generate simulated acceleration time history signals of the same length as the original measured data. These simulated signals, along with the original time history were then processed to solve for the input energy spectrum. A plot of the three resulting input energy spectra are provided in Fig. 8. In this interesting example, Welch's method is actually

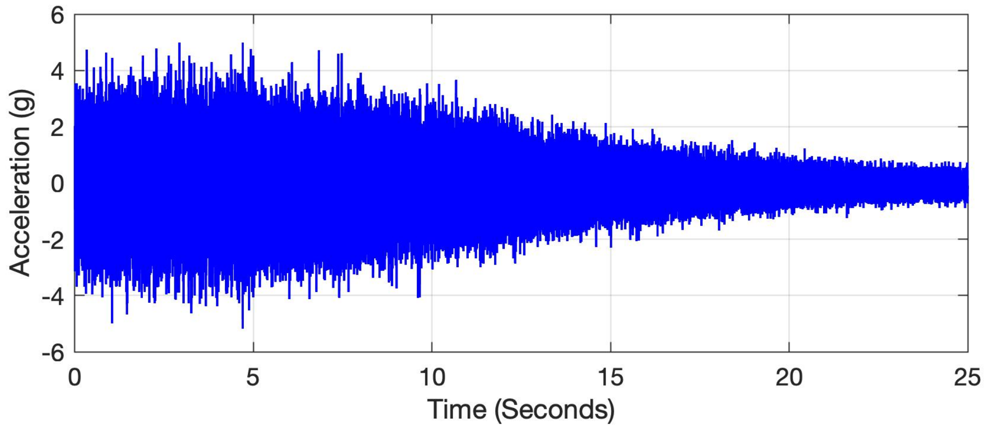

Figure 6: Example experimental non-stationary random vibration time history

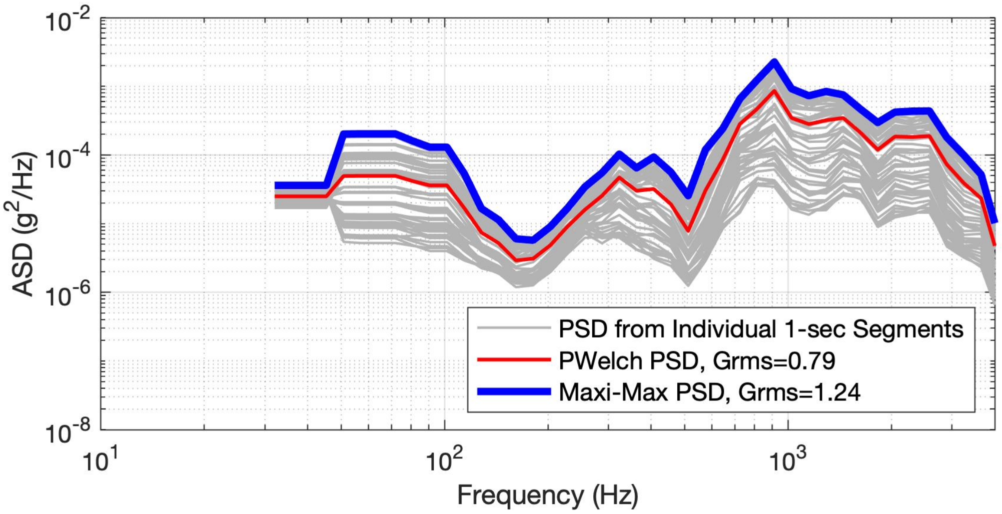

Figure 7: PSD and maxi-max PSD calculated from the first experimental non-stationary random vibration time history

a reasonably good estimate to the energy imparted to the component from the field data and the laboratory test. In contrast. the maxi-max methodology is significantly more severe from an input energy perspective. However, it is also likely that the closeness of the overall input energy spectrum is an artifact of the reasonably split differentiation between the higher level vibrations and the lower level vibrations.

An alternate way of considering these two specifications is with rain-flow cycle counting analysis. Figure 9 shows the result of a rain-flow cycle counting analysis on the original measured data as well as the simulated signals from the Welch's method estimate and the maxi-max PSD estimate. The results shown here are quite interesting in that they show the original signal has significantly more low-amplitude cycles while the maxi-max method has substantially more high-amplitude vibration cycles. The Welch's method estimate is even more unique in that it yields substantially fewer low-amplitude cycles, more cycles in the mid-range, and fewer high-amplitude cycles than the original measured data. Given that the high-amplitude cycles are more stress inducing, the maxi-max PSD is certainly conservative in this example while Welch's estimate is somewhat non-conservative.

Figure 10 shows a plot of a second experimentally measured non-stationary random vibration event. This event is essentially a relatively stationary random vibration for about 13.5 seconds and then has a mild downward trend in the vibration amplitude. This mildly non-stationary example should have characteristics

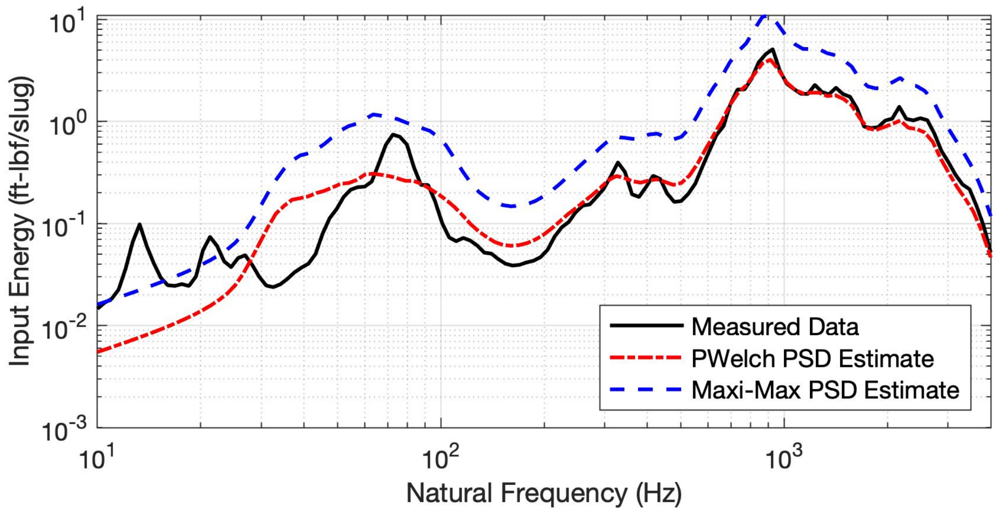

Figure 8: Energy spectrum calculated for the experimental non-stationary random vibration time history as well as simulations from Welch's average and the maxi-max PSD spectrum

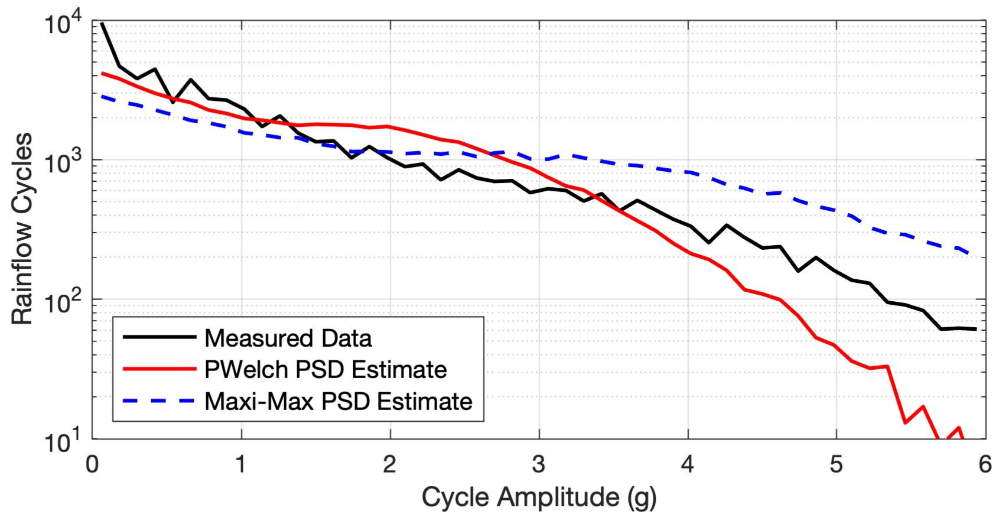

Figure 9: Rain-flow cycle counting for the experimental non-stationary random vibration time history as well as simulations from Welch's average and the maxi-max PSD spectrum

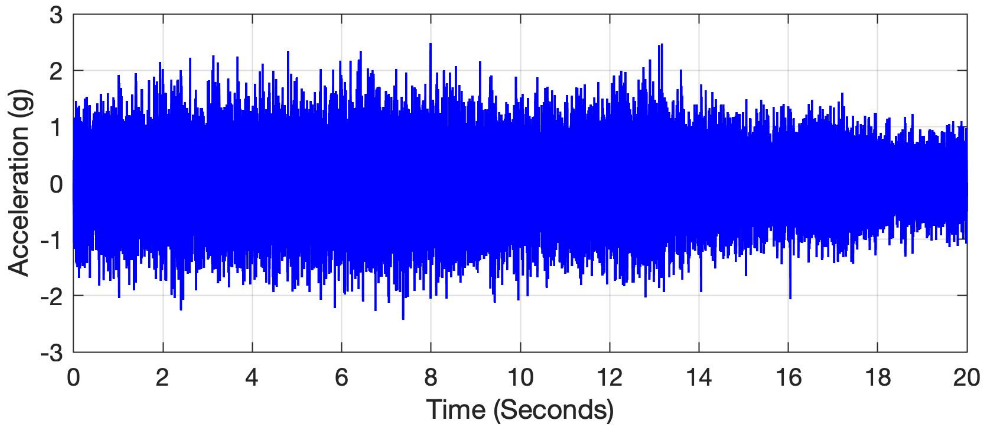

Figure 10: Second example experimental non-stationary random vibration time history

of both stationary and non-stationary data. In fact, the data is likely close enough to stationary that one option for analysis would simply be to analyze the first portion using stationary random vibration techniques and assume that result is applicable to the full data window.

Figure 11 shows a plot of the PSD calculated using the traditional Welch's method of averaging with a 1/6 octave smoothing and the maxi-max PSD spectrum overlaid with the PSD of the underlying segments. Here again, the maxi-max PSD is substantially higher than the average, approximately +4.57 dB. Here again, Welch's method slightly under-predicts in this non-stationary case so the dB difference is slightly overstated here. For reference, if Welch's method is used on the first 13.5 seconds of the record, the portion that is largely stationary, the RMS acceleration is estimated at 0.59 g compared to 0.54 g from the full record. Welch's PSD from the first 13.5 seconds is also approximately +0.98 dB higher than the PSD from the full record. Indicating how sensitive Welch's average method can be for non-stationary signals.

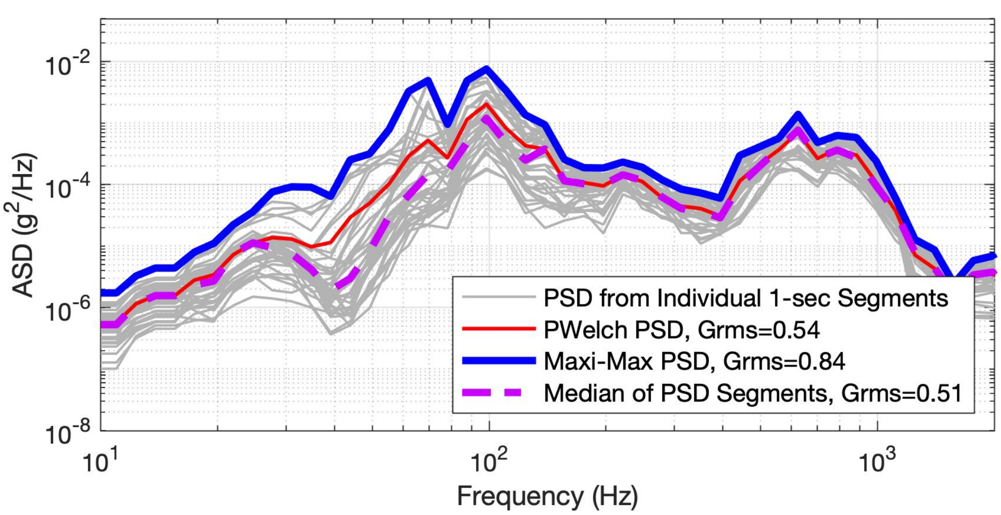

Figure 11: PSD and maxi-max PSD calculated from the second experimental non-stationary random vibration time history

One very interesting result is also apparent in Fig. 11 in the 30 – 80 Hz range. In this range, the bulk of the individual segment PSD are well below both the maxi-max PSD and even Welch's average PSD. A median PSD is overlaid with a dashed line that shows a relatively substantial notch has been filled near 40 Hz. In this case, the notch has been filled because a few high PSD segments bring the average up substantially.

Perhaps this data set offers a cautionary note to blindly accepting the results of even the more traditional analysis methodologies. All spectra shown here were once again calculated with a consistent 1024-point segment length.

Figure 12 shows the results of rain-flow cycle counting on the original time history data from Fig. 10 as well as the simulated signals from the Welch"s method estimate, the maxi-max PSD estimate, and the median PSD estimate. The rain-flow results shown here indicate that both Welch's method and the median PSD estimate produce signals that have very similar rain-flow results compared to the original measured data. In contrast, the maxi-max PSD method again results in a signal with significantly more high-amplitude cycles and significantly reduced low-amplitude cycles.

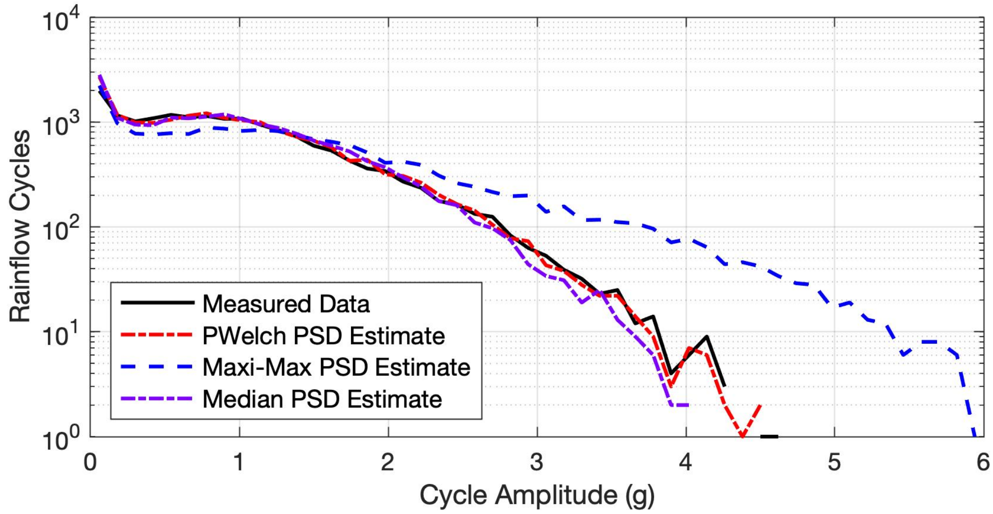

Figure 12: Rain-flow cycle counting for the second experimental non-stationary random vibration time history as well as simulations from Welch's average, the maxi-max PSD spectrum, and the median PSD spectrum

Both of these non-stationary random vibration examples indicate that the maxi-max PSD is significantly more severe than the traditional Welch's average method in both RMS amplitude as well as rain-flow stress cycles. In both of these examples, the maxi-max PSD is approximately $+4\,\text{dB}$ higher than the traditional Welch's method. Additionally, the maxi-max formulation results in substantially more high-amplitude stress reversals compared to the original input signal as indicated by the rain-flow results. This is a double source of conservatism in the resulting environmental or test specification: The overall test level has increased and the number of higher amplitude stress cycles has increased.

CORRECTIONS TO THE MAXI-MAX PSD METHODOLOGY

While the maxi-max PSD methodology is useful, it is also quite severe. The experimental data presented here indicate that the nominal one-second maxi-max PSD can range from $+1.7\,\text{dB}$ up to around $+4.6\,\text{dB}$ above Welch's average method. While the non-stationary cases are difficult to estimate, the $+1.7\,\text{dB}$ stationary random vibration example is more straightforward. Thus, it can be assumed that a 1/6 octave maxi-max PSD is nominally $+1.7\,\text{dB}$ higher than it should be. Certainly there are numerous examples where the results are better or worse as shown here. Segments shorter than the recommended one-second interval result in rapidly increasing PSD envelopes. Likewise, longer and longer segments asymptotically tend toward Welch's average. For this reason, a simple correction factor cannot be blindly applied to a maxi-max PSD. Different correction factors would be required for different PSD parameters such as block size and segment length. Furthermore, severity of the non-stationary data would also factor into the correction. This quickly becomes an untenable situation.

An alternate methodology for calculating a more reasonable PSD estimate from non-stationary data based on a statistical analysis of the underlying 1/6 octave, one-second PSD segments is proposed. Rather than

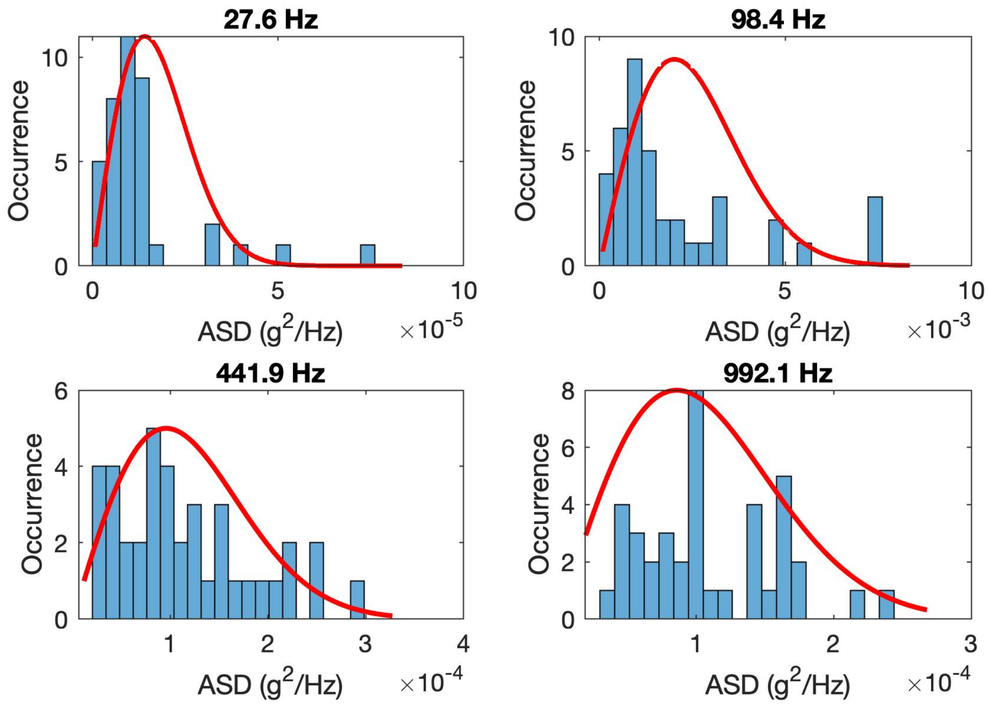

Figure 13: Histograms and Rayleigh distribution fitting of four frequency lines of PSD segment data from the second non-stationary random vibration example

simply envelope all of the underlying segments, it is proposed to examine the PSD magnitude distribution at each frequency and determine a level at which most of the energy is encompassed. While many statistical approaches could be used, a Rayleigh distribution was selected for this analysis. Figure 13 shows sample histograms and Rayleigh distribution fitting at four frequency lines from the PSD segment data shown in Fig. 11. These histograms indicate that the bulk of the energy at each frequency is skewed to the lower end of the range and not evenly distributed nor Gaussian. Given the obvious skewness a Rayleigh distribution seems appropriate here, although a Weibull or other similar distributions may be equally well suited.

To use these Rayleigh distributions to generate a maximal PSD, the 1/6 octave one-second PSD segments from the entire time history were saved and then the amplitudes at each frequency line were fit to a Rayleigh probability distribution function using the function "fitdist" in MATLAB. From this fit, the cumulative distribution function at that frequency was calculated and the PSD amplitude at which 70 percent of the values fell below was selected as the amplitude at that frequency. This calculation was repeated for all frequencies in the experimentally derived PSD. The 70 percent threshold was selected somewhat arbitrarily; however, it should encompass the mean (50 percent value) plus approximately one standard deviation. Obviously selecting a value close to 50 percent should yield a result close to Welch's method and a 100 percent value will result in the original maxi-max PSD.

To understand the impact of this Rayleigh distribution methodology, it was applied to the three experimental random vibration signals analyzed here. Figure 14 shows a plot of the PSD calculated using Welch's method, the traditional maxi-max PSD method, and the proposed 70 percent threshold Rayleigh distribution method for the experimental stationary random vibration signal from Fig. 1. As stated previously, the 1/6 octave maxi-max PSD is +1.72 dB above Welch's method, the 70 percent Rayleigh method is only +0.51 dB higher than Welch's estimate. A considerable improvement.

For the two non-stationary random vibration examples, the corresponding 70 percent Rayleigh distribution examples are shown in Figs. 15 and 16 corresponding to the time history data from Figs. 6 and 10, respectively. In both of these examples, the Rayleigh fit PSD falls between Welch's PSD and the maxi-max PSD, but typically lying closer to Welch's estimate. Figure. 15 shows that the Rayleigh method yields a

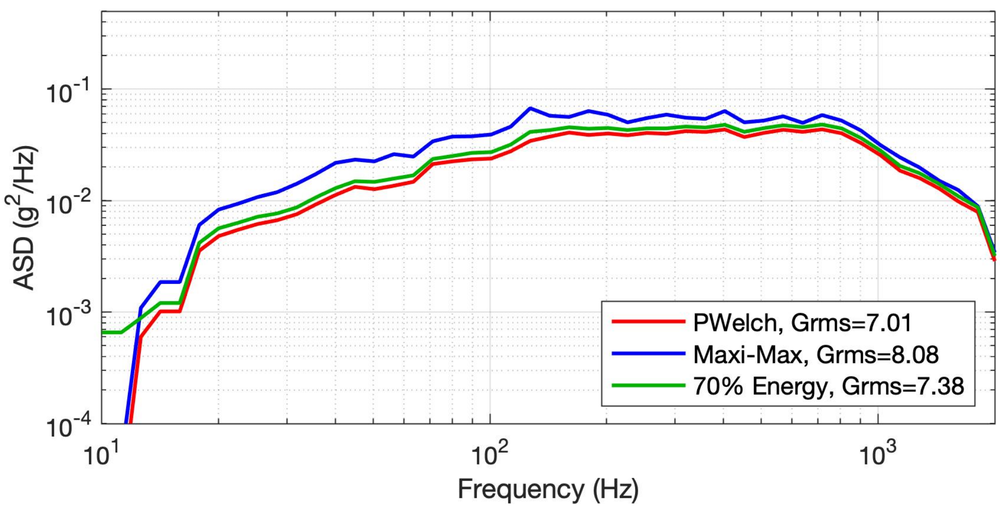

Figure 14: Comparison of Welch's method, the maxi-max PSD method, and the 70 percent Rayleigh distribution method for the stationary random vibration example

PSD estimate +1.32 dB higher than Welch's estimate and far less than the +3.96 dB of the maxi-max PSD method. Similarly, Fig. 16 shows a Rayleigh method PSD estimate that is +1.49 dB higher than Welch's estimate and also far less than the +4.57 dB of the maxi-max PSD method.

While these Rayleigh method predictions seem much nicer from a component testing standpoint, the real question is how well do they represent damage from the original non-stationary environment. To estimate damage, a theoretical time history was developed from the Rayleigh method PSD and rain-flow cycle counting was performed on that time history. Figures 17 and 18 show the rain-flow cycle counting results from the two Rayleigh method PSD estimates for the non-stationary example data. The rain-flow predictions from the first example, shown in Fig. 17, is extremely good with the Rayleigh estimate rain-flow data hitting the high-amplitude cycle counting of the original signal quite well. In contrast, the maxi-max PSD over exposes the number of high-stress cycles while Welch's method under-exposes in the high-stress cycle regime. Figure 18 shows similar rain-flow cycle counting results for the second non-stationary example. In this case, both the Rayleigh method and Welch's estimate give similar estimates with the 70 percent Rayleigh method being slightly more conservative. This is somewhat expected given that this example was only mildly non-stationary. In contrast, the maxi-max PSD methodology applies significantly more high-stress cycles than either of the other methods.

One further comparison is warranted for this new method—making use of a larger PSD blocksize. As was noted previously with the stationary random vibration example, changing the blocksize from 1024-points to 4096-points increased the maxi-max PSD estimate from +1.72 dB above Welch's estimate to +2.86 dB above Welch's estimate. For the two non-stationary examples considered here, the Rayleigh distribution method was repeated with a 4096-point PSD blocksize for comparison. For the first non-stationary example, the 70 percent Rayleigh distribution PSD prediction actually decreased from +1.32 dB above Welch's method to +1.29 dB above. A relatively small change of only 0.03 dB and considerably smaller than the same change from the traditional maxi-max PSD methodology. For the second non-stationary example, the increase to a 4096-point PSD calculation increased the resulting 70 percent Rayleigh distribution PSD to +1.55 dB above Welch's estimate compared to +1.49 dB with 1024-point averages. Here again, a relatively small change of 0.06 dB. These results indicate that fitting the probability distribution reduces the possible variance in the resulting PSD estimates.

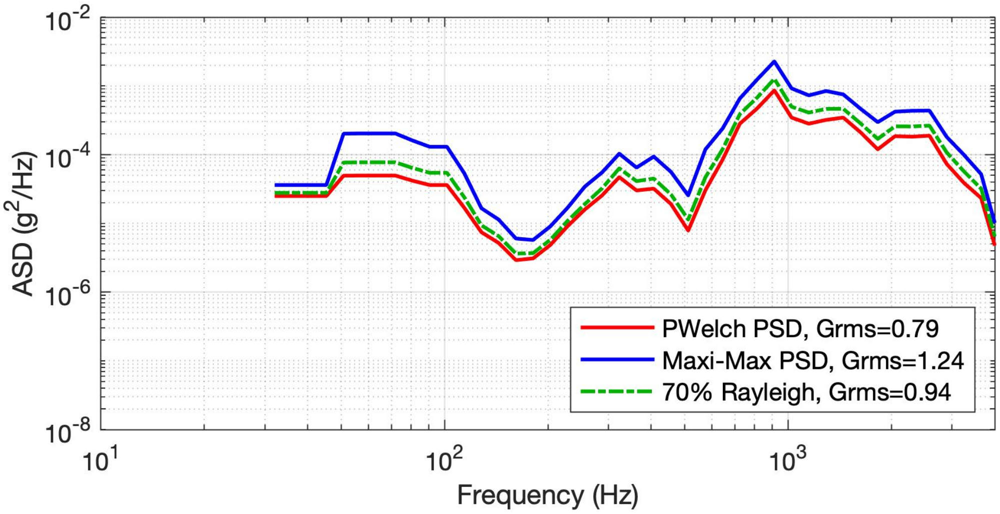

Figure 15: Comparison of Welch's method, the maxi-max PSD method, and the 70 percent Rayleigh distribution method for the first non-stationary random vibration example

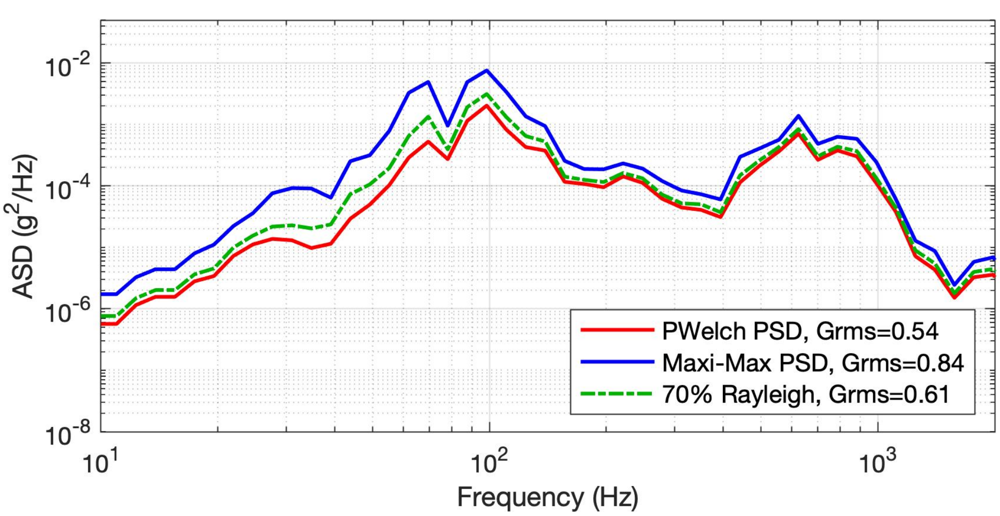

Figure 16: Comparison of Welch's method, the maxi-max PSD method, and the 70 percent Rayleigh distribution method for the second non-stationary random vibration example

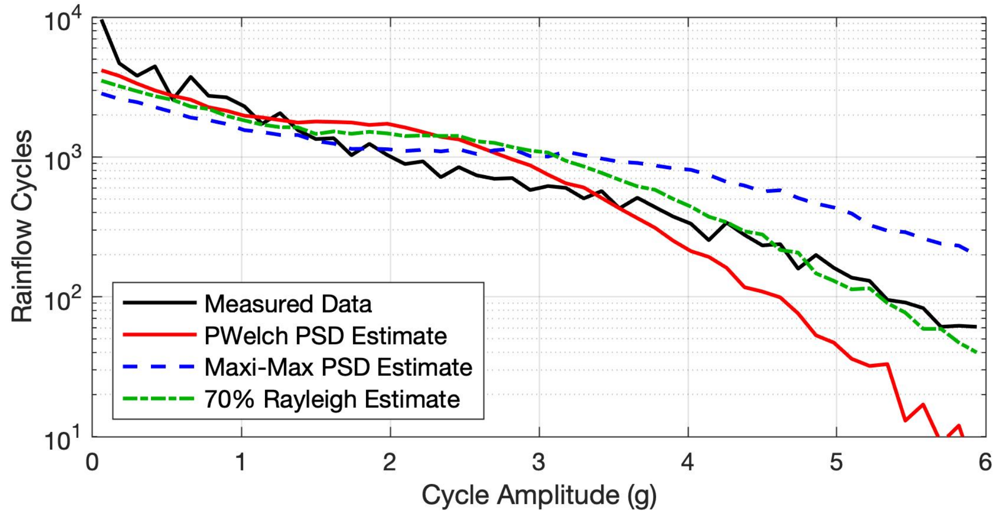

Figure 17: Comparison of rain-flow cycle counting results from Welch's method, the maxi-max PSD method, and the 70 percent Rayleigh distribution method for the first non-stationary random vibration example

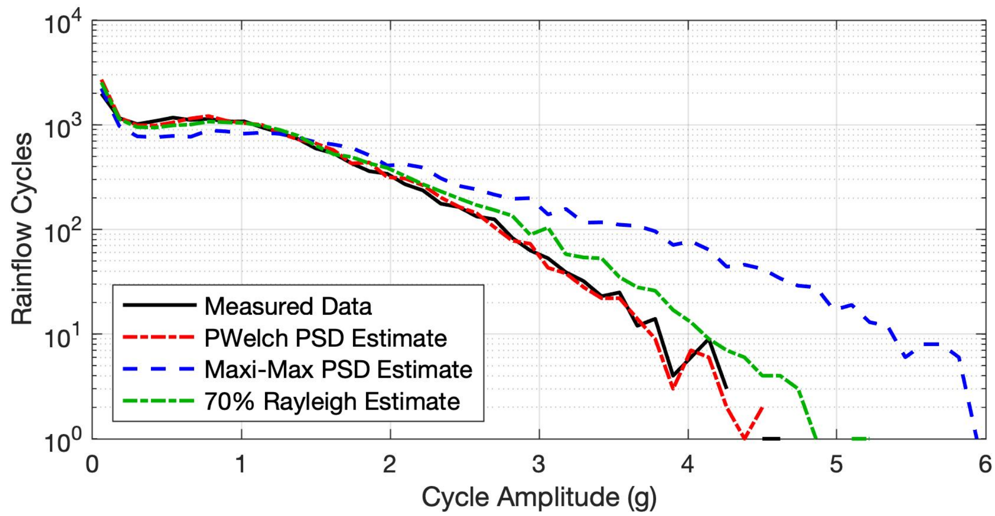

Figure 18: Comparison of rain-flow cycle counting results from Welch's method, the maxi-max PSD method, and the 70 percent Rayleigh distribution method for the second non-stationary random vibration example

CONCLUSION

Welch's method will generally under-predict the severity of a non-stationary random vibration signal due to the segment averaging utilized. In contrast, it is known that the maxi-max PSD methodology over-predicts the severity of a non-stationary random vibration signal due to the enveloping methodology. The maxi-max PSD will also over-predict the severity of a stationary random vibration if the method is applied there. Based on the experimental examples analyzed here, the maxi-max PSD methodology appears to have a nominal minimum over-estimation of +1.7 dB but it may be substantially higher depending on the PSD parameters selected.

A new method for analyzing non-stationary random vibrations using a Rayleigh distribution fit to the underlying PSD segments was developed. This method fits a Rayleigh probability distribution to the PSD segment amplitudes in a frequency bin and then uses the cumulative distribution function to select the PSD amplitude that encompasses 70 percent of the energy in that frequency band. The resulting 70 percent Rayleigh distribution PSD estimates are much closer to Welch's estimates, falling between Welch's estimate and the maxi-max PSD estimate. The resulting rain-flow cycle counting estimates from simulated random vibration time histories fall closer to the rain-flow estimates from the original signals with the number of high-amplitude cycles also matching more closely.

It is understood that the maxi-max PSD methodology is not ideal. It is also understood that it will continue be used for the analysis of non-stationary random vibration data, partly because of its historical use for that same data. What is presented here is an estimate of its conservatism along with a methodology to reduce that known conservatism. Further refinements to the methodology presented here could be made as the breadth of analyzed data increases. A level of standardization should also be pursued if this method gains acceptance.

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