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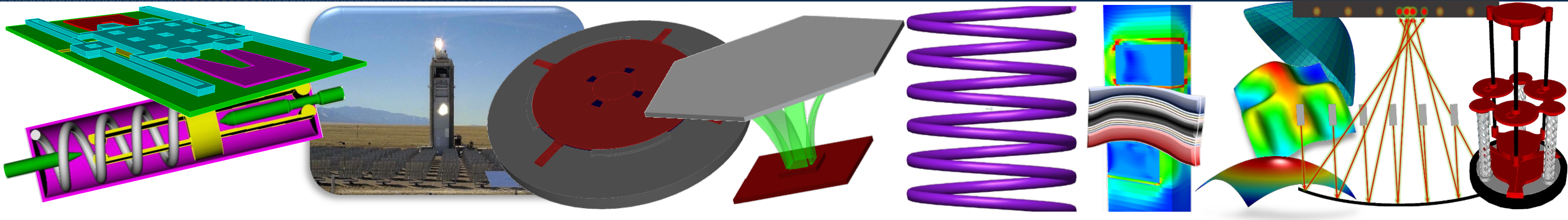

A Case Study of Applied Mathematics at Sandia National Laboratories: Design of Electromagnetic Reflectors with Integrated Shape Control

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Albuquerque, NM

SAMSI/NCSU (Telepresence)
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U.S. DEPARTMENT OF ENERGY
NNSA National Nuclear Security Administration

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Sandia National Laboratories

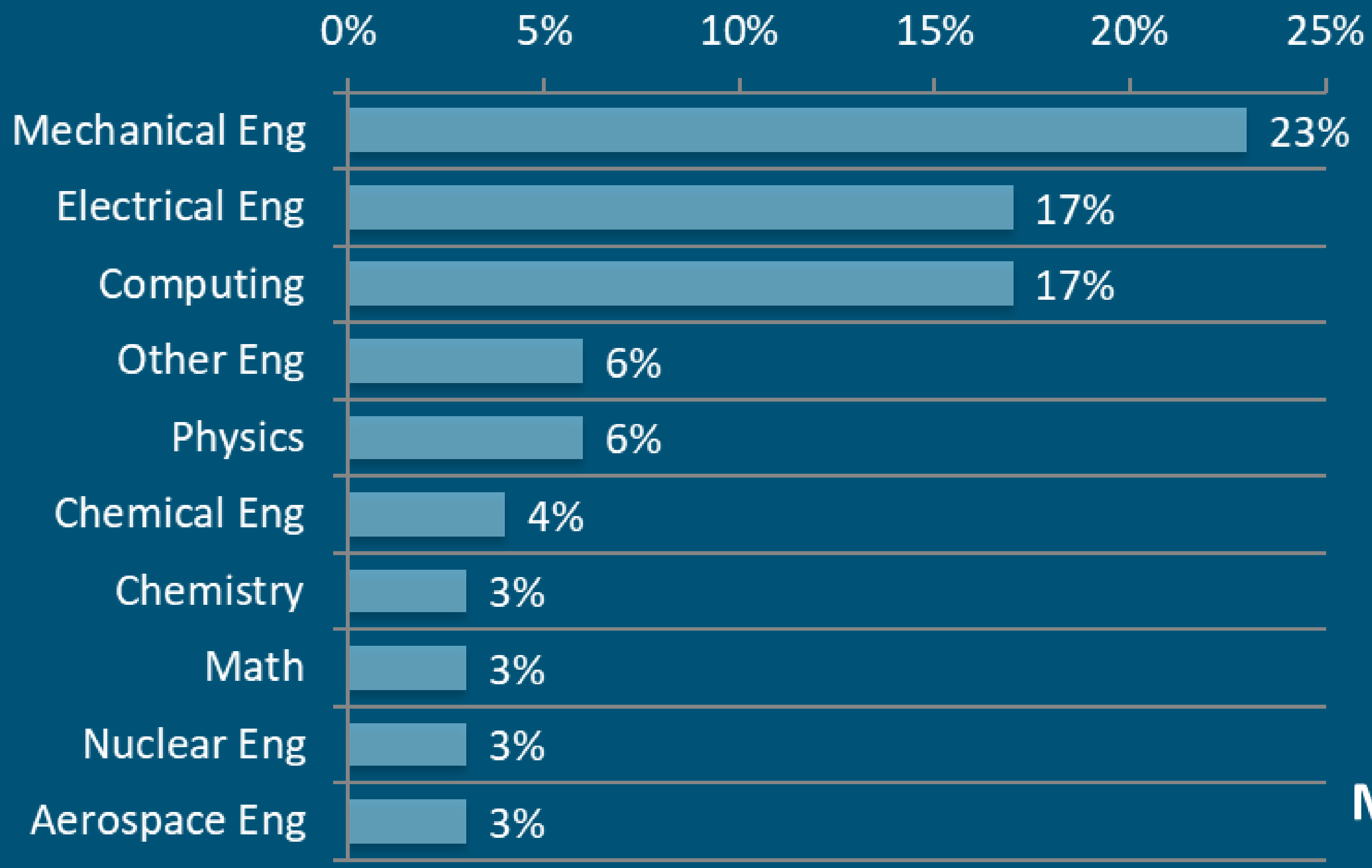
- A multi-faceted national security laboratory.
- **Core Purpose**: help our nation secure a peaceful and free world through technology.
- Provide objective, multidisciplinary technical assessments for complex problems.
- Focus on solutions with large science and technology content.
- Create prototypes for production and operation by industry.

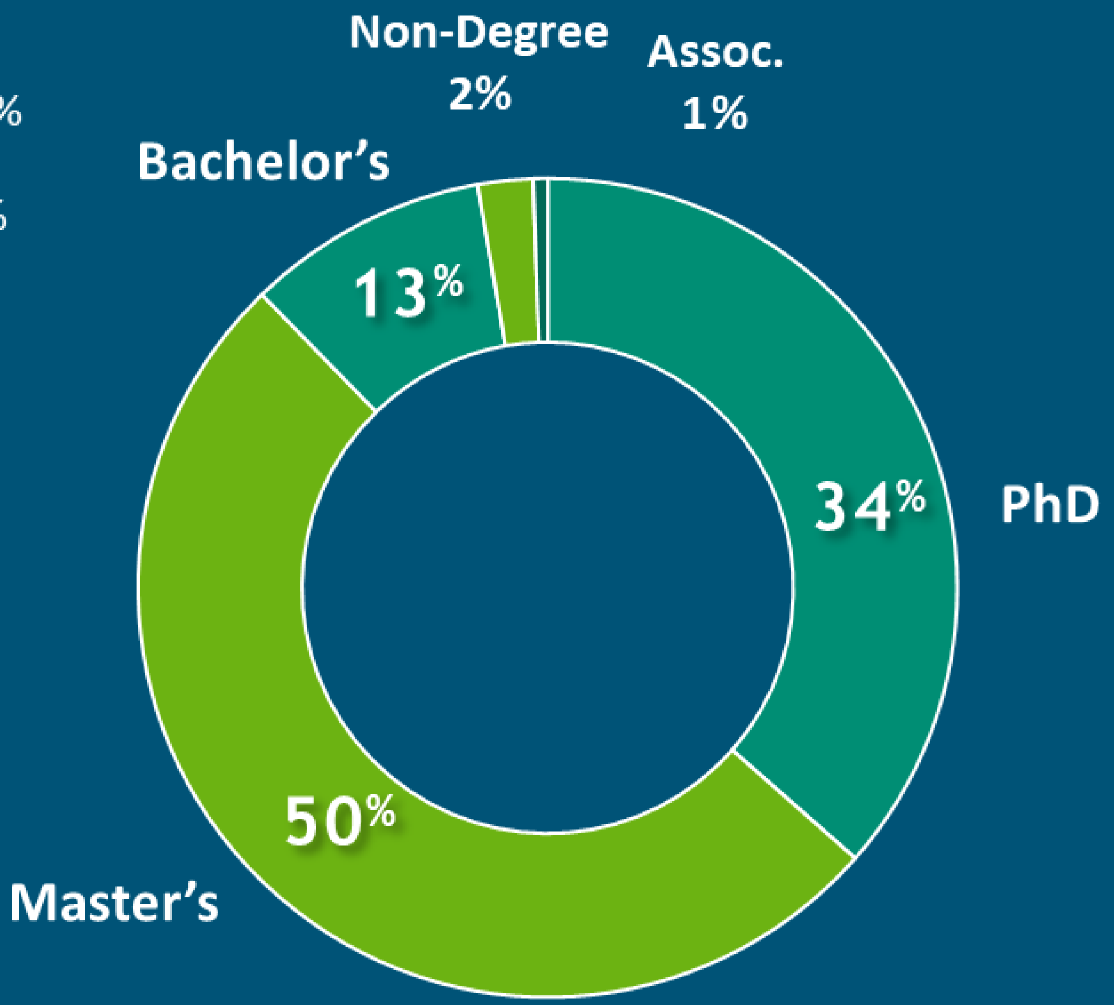
U.S. DEPARTMENT OF ENERGY

Research Disciplines Drive Capabilities

Research Disciplines

R&D by Discipline & Degree





Data as of July 2020

Sandia Mathema/Statis-ticians

Mathematicians and Statisticians work in almost every area across SNL in 80+ organizations.

Center for Computing Research

Discrete Mathematics, Optimization, & Uncertainty Quantification
Scalable System Algorithms, Software, Analysis, & Visualization
Multiscale/Cognitive Science, Data-driven & Neural Computing

Mission Engineering & Information Systems Analysis

Sensor, Data, Imaging Analysis
Data Science, Cyber Security, Cryptography, Analytics
Digital & Quantum Information Sciences & Systems

Engineering Sciences

Diagnostic, Shock, Structural, Climatic, Fluid & Reactive Processes, Fire S&T
Computational Solid/Structural/Thermal/Fluid Mechanics & Dynamics
Verification & Validation, Uncertainty Quantification, Credibility Processes

- Statisticians work mostly in areas of Risk/Reliability Analysis, Quality Engineering, Quantification of Margins and Uncertainty (QMU).

My Route to SNL Engineering Sciences

Theoretical particle physics (SUNY Stony Brook)

Undergraduate Education Influence

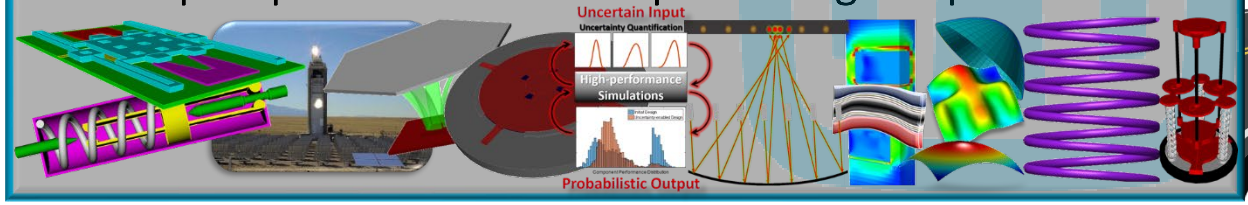
- **Goal:** nuclear *engineering*, but decided it was too empirical, insufficiently *theoretical/fundamental*.
- **Degrees**: Physics (quantum mechanics), Mathematics (Steklov eigenvalues)
- COMAP Mathematical Contest in Modeling (MRI image analysis): exposed to Applied Mathematics.

Graduate Education Influence

- Industrial Mathematical Modeling Workshop (exposed to smart materials, quick thinking).
- **Degree**: Computational/Industrial Applied Math
- **Dissertation**: Shape Memory Alloy (SMA) modeling.
- Graduate **Internships**: The Boeing Company (sparse optimization), SNL (SMAs).

Career (Phase 1)

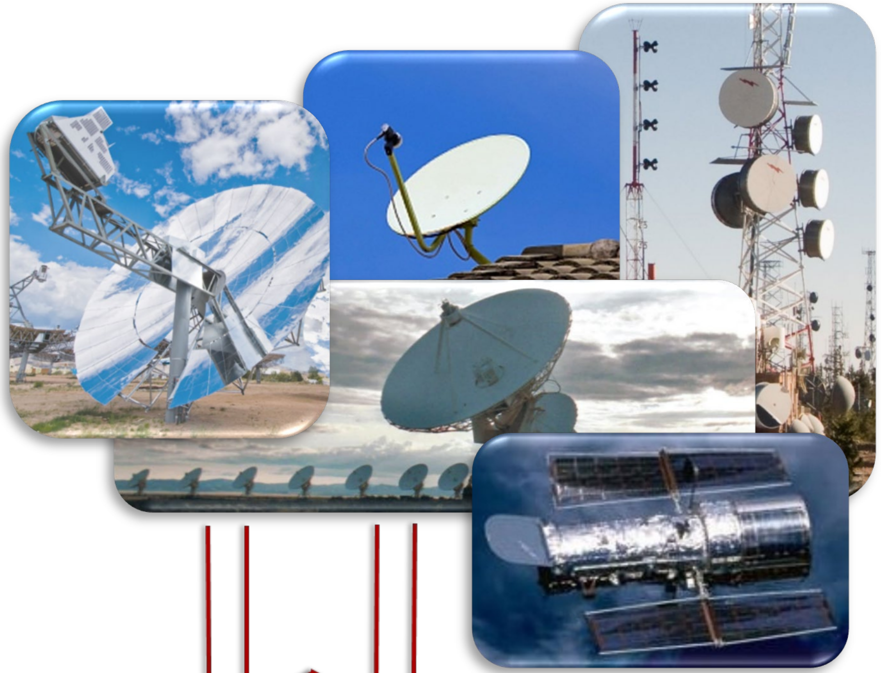
- Many-disciplinary analysis & *engineering*.
- Mechanical design with uncertainty quantification and optimization.
- Complex process from concept to design to production.

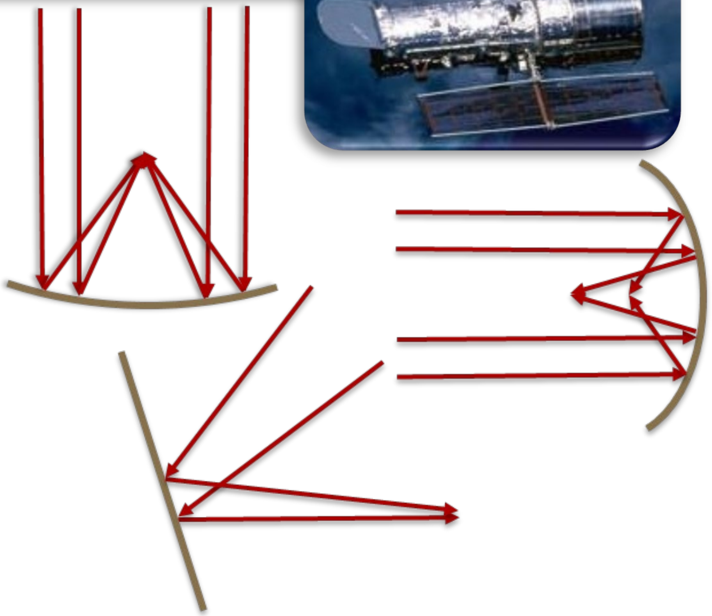


Electromagnetic Reflectors

- Surfaces that reflect electromagnetic radiation (often radio and visible light).
- Typically in antennas, receivers, and telescopes:

  satellite TV receivers, communications systems,
  solar concentrators, radio observatories, reflecting telescopes…
- Reflected signal pattern is directly related to *reflector shape*.
- Paraboloidal reflectors are common:
  shape allows sharp focus.
- Some applications demand highly precise shapes, especially when looking far away.
- Many reflectors are rigid, particularly to satisfy small shape tolerances.





Shape Matters: An Infamous Example

The Hubble Space Telescope

2400 mm Mirror

Degraded Images

Culprit: *0.0022 mm* shape error.

Solution: correct for shape error in orbit…3 years later!

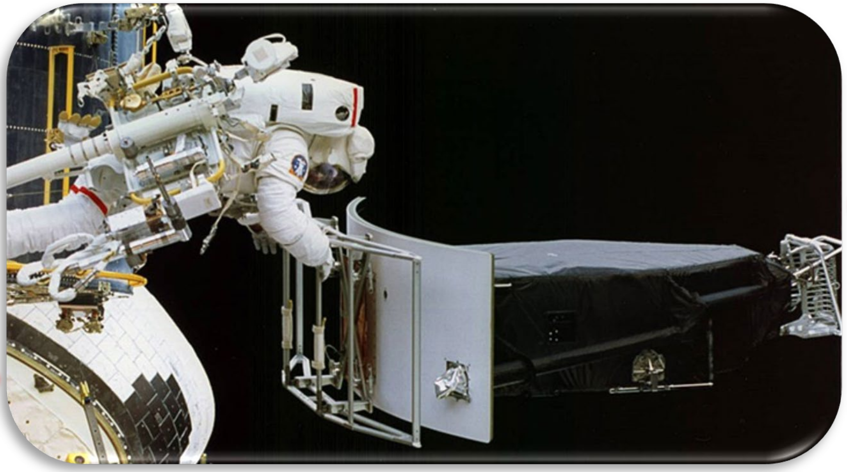
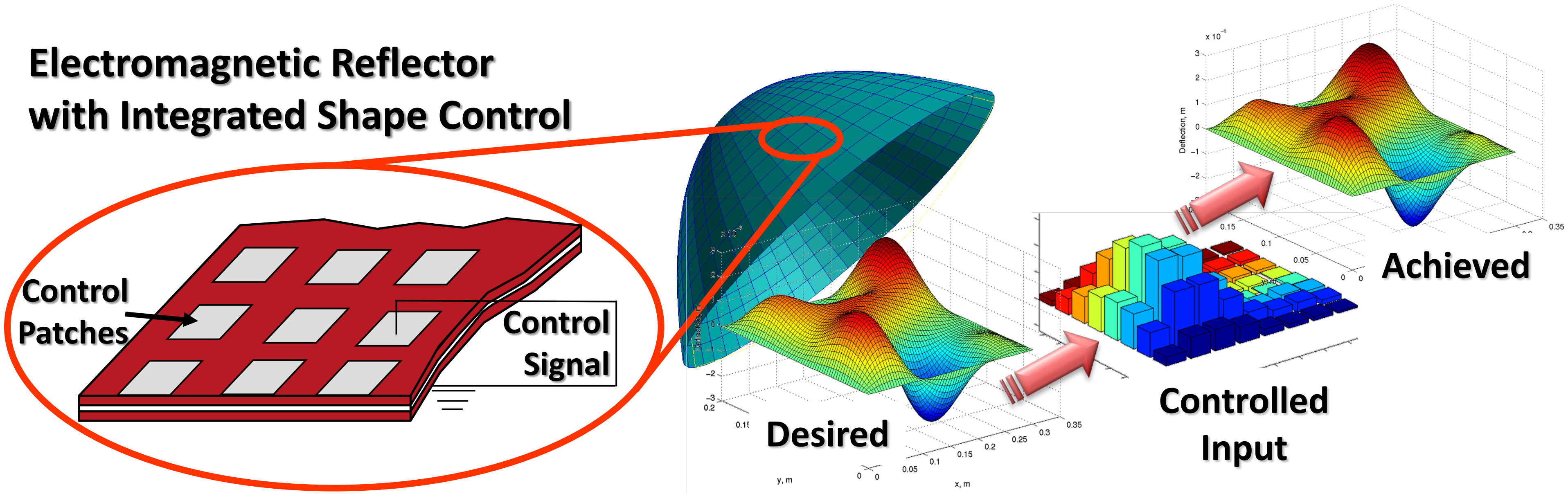


Image Quality Restored

Shape-controlled Reflectors?

- Reflector shape control technology is available.
- For typical rigid reflectors, options and amount of control are limited, and controlling mechanism can be bulky.
- Shape errors also can be mitigated using additional hardware.

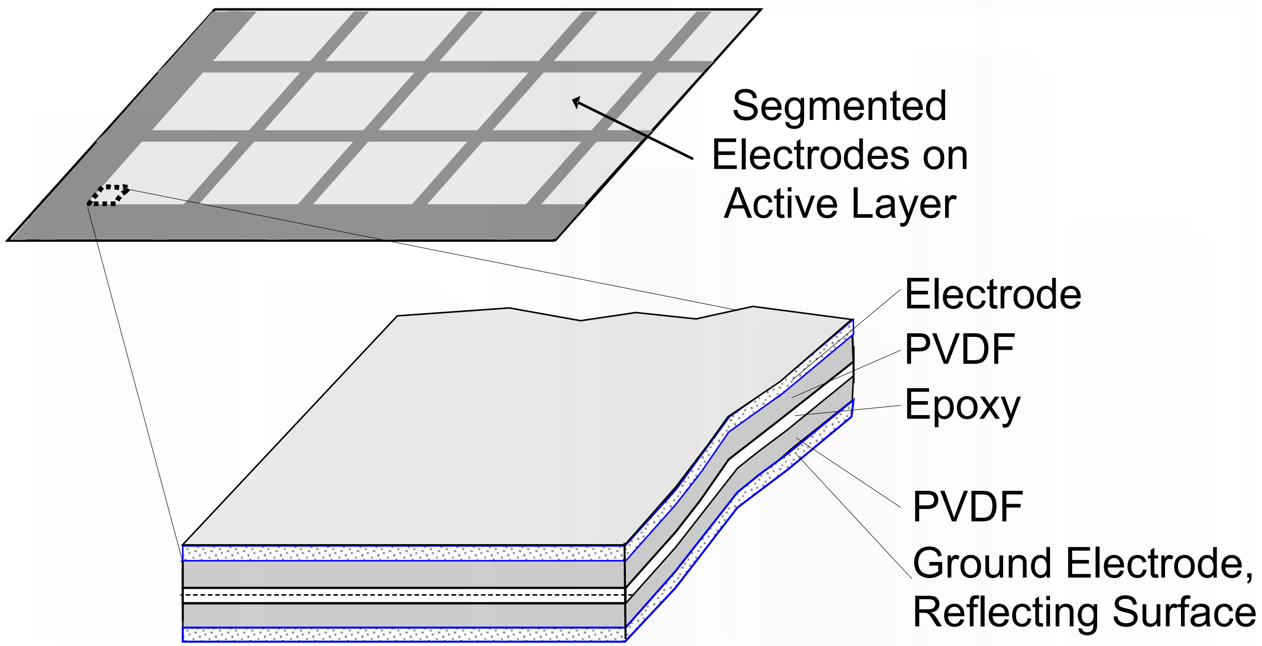
More control, larger deflections, smaller footprint, less overhead?



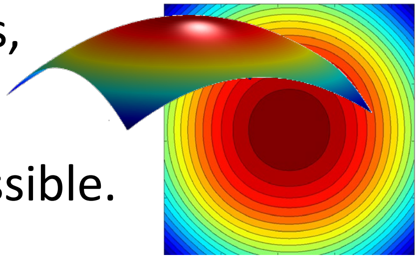
(Sumali, Martin, Chaplya, Massad, et alia)

Sandia Smart Laminate Concept

Thin, Square, Active Membrane

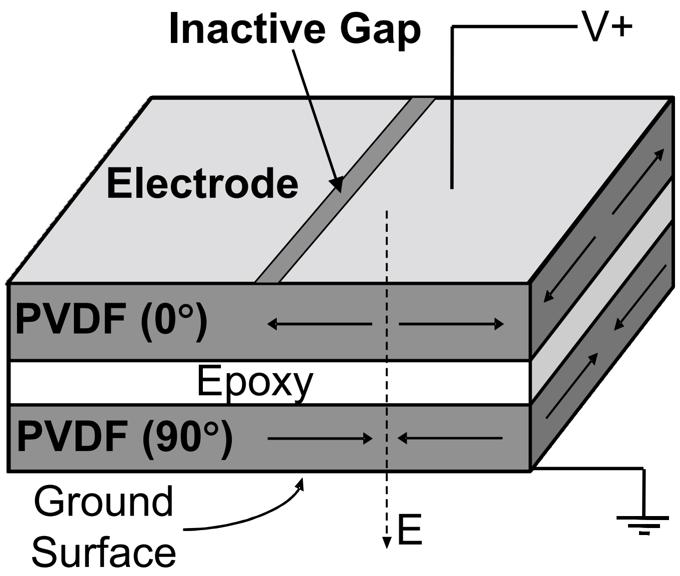


- Natural actuation into paraboloid with ideal corner supports,
- Improved flexibility,
- Large deflections possible.



How it Deforms: Bimorph Action

- PVDF layers have opposing poling directions.
- Positive field induces simultaneous expansion (top) and contraction (bottom).



Initial Linear Model

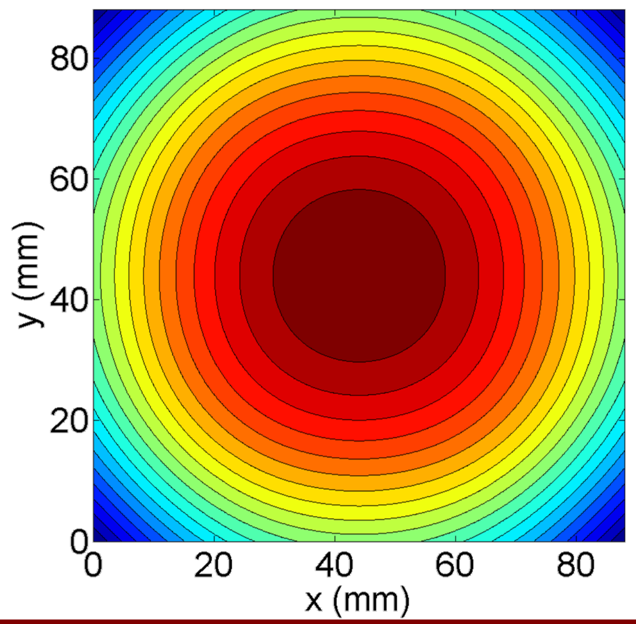
Electromechanical Actuation | Mechanical Response

$$\mathbf{R}V = \mathbf{H}u$$

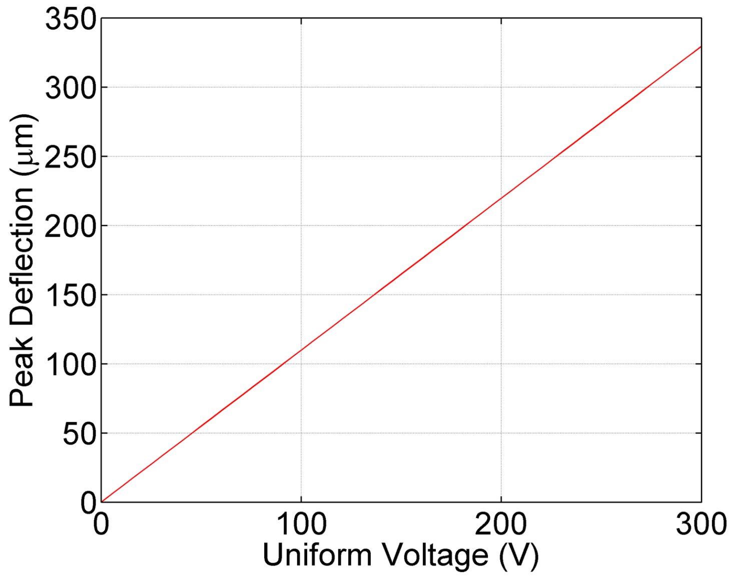
Voltage Array | Deformation

- Based on Kirchhoff-Love plate theory and Ritz Method:
  - Describes bending-dominated deflection;
  - Yields linear mapping between input voltage to output deflection.
- Corner supports: sliding corners (constrained out-of-plane only).
- Formulation facilitates shape control, quick to execute.
- **Observations**: simulates *uniformly circular contours* and *linear* rise in peak deflection with increasing uniform actuation voltage.

Deflection Contours

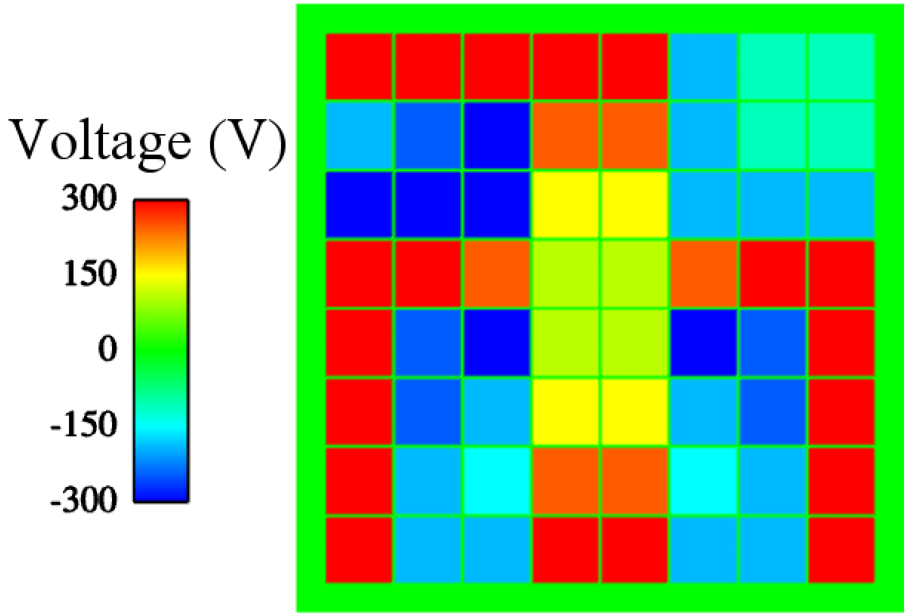


Peak Deflection vs. Uniform Voltage

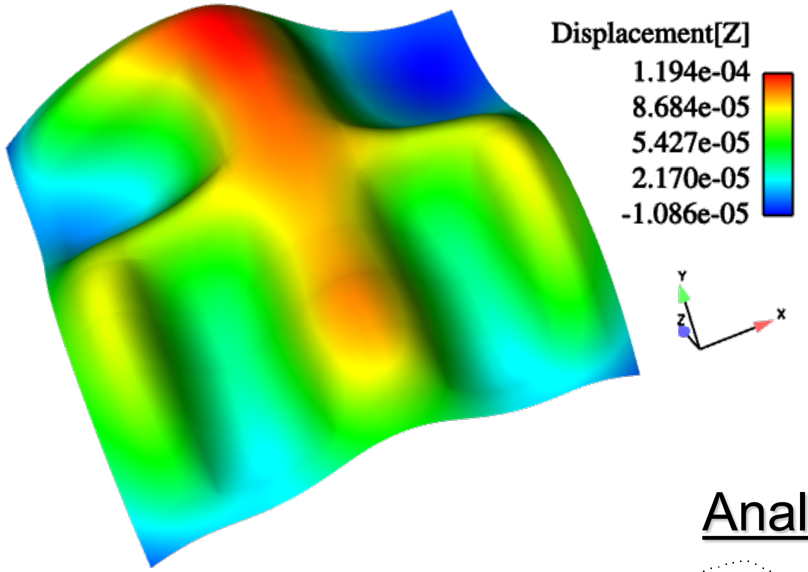


A Model-Model Comparison

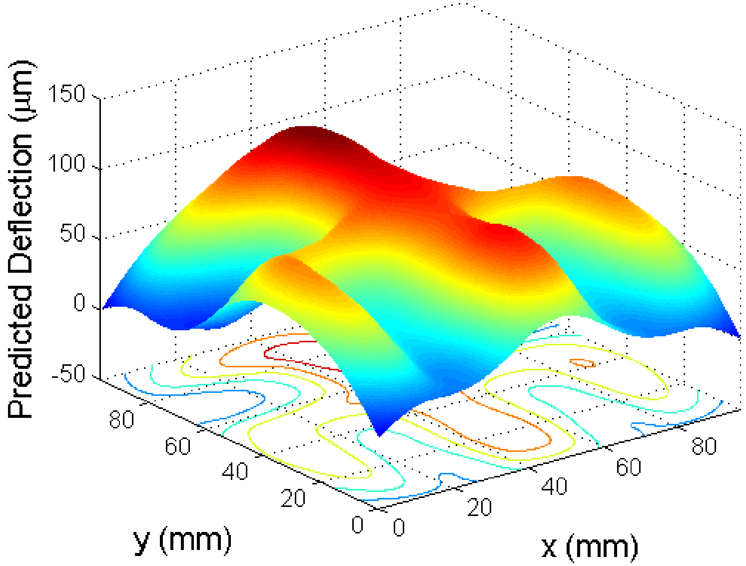
Voltage Distribution



Finite Element Model



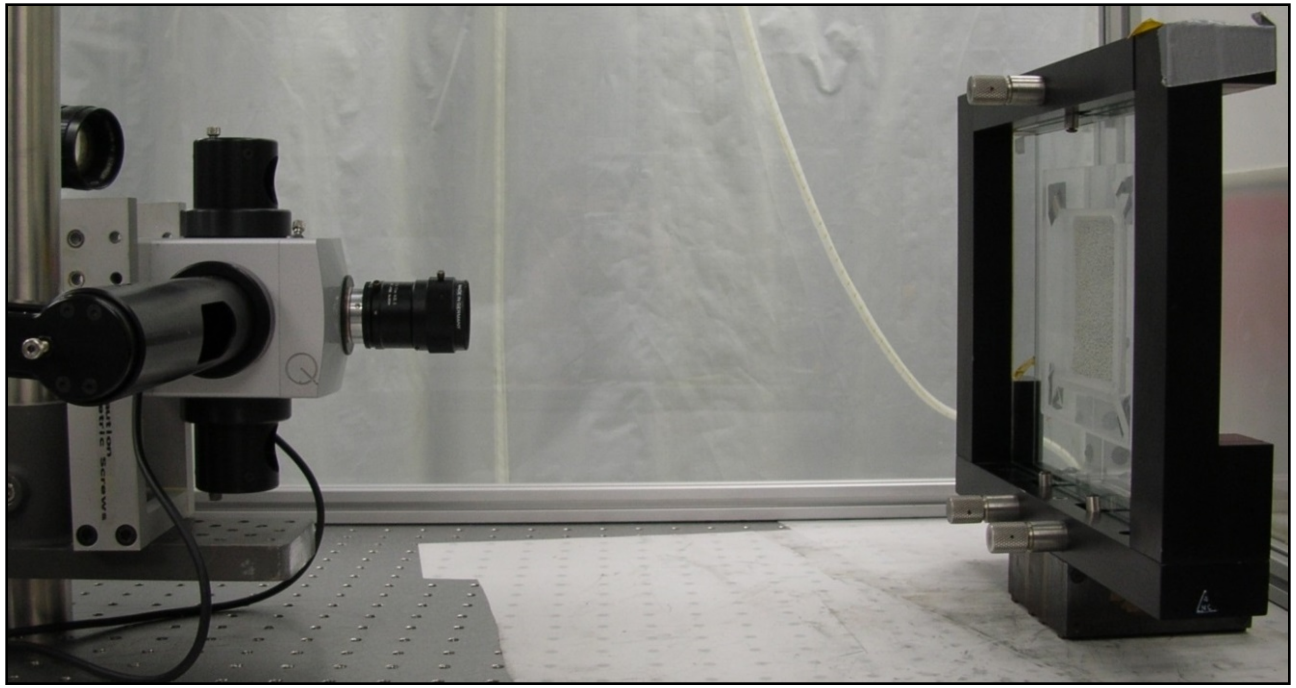
Analytical Model



- FEM: layered-shells, 150k quad elements, corner-supported boundary conditions.
- Total relative difference between analytical and FEM is 2.8%.

Smart Laminate Experiments

- Fabricated corner-supported laminate with single electrode (test case).
- Corner-support boundary condition approximated with corner tabs.

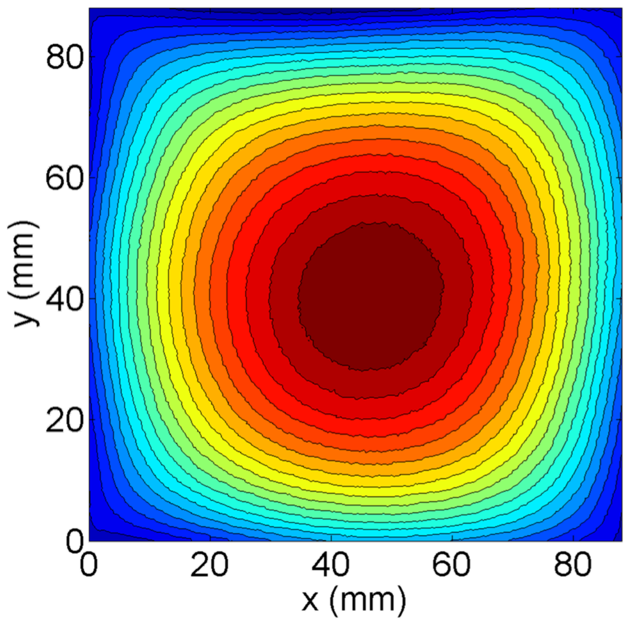


- Electronic Speckle Pattern Interferometry (ESPI): full-field displacement measurements with out-of-plane measurement resolution ≤45 nm.
- Optical fringe measurement is sensitive to vibrations (HVAC, etc.).
  - fixture designed to suppress vibrations;
  - tightened corner supports to facilitate repeatable measurements.

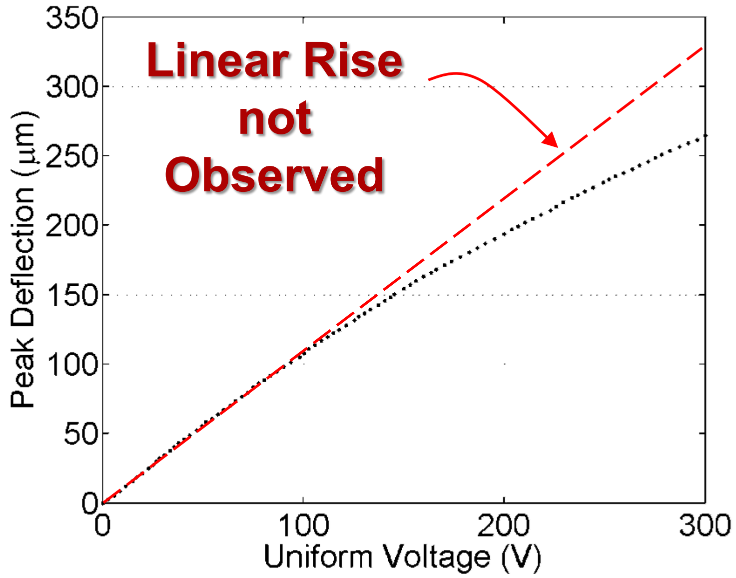
Experiment Results

- Observations:
  - *squared contours* become circular only away from boundary;
  - *nonlinear* rise in peak deflection with increasing uniform actuation voltage.

Deflection Contours



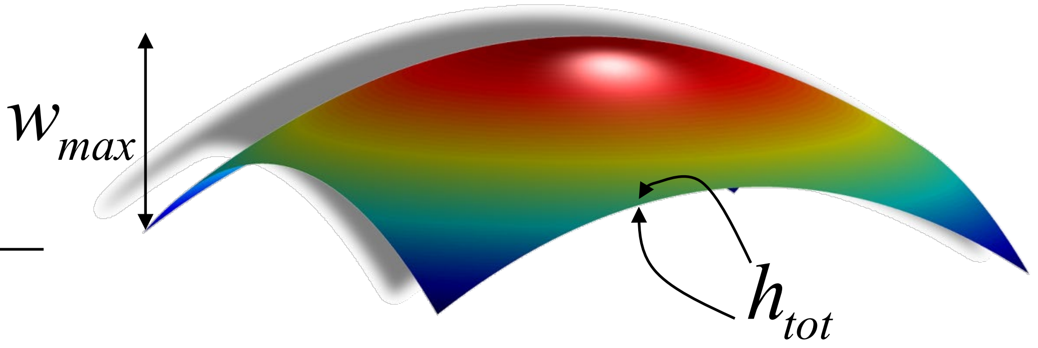
Peak Deflection vs. Uniform Voltage



Why the difference?

- Size of membrane deflections is quantified by the ratio

$$\frac{\textbf{Peak Deflection}\ \ w_{max}}{\textbf{Total Membrane Thickness}\ \ h_{tot}}$$



Small Deflections

$$\frac{w_{max}}{h_{tot}} \leq 0.2$$

- Negligible stretching of middle surface.
- Bending is dominant.
- Kirchhoff linear theory adequate.

Large Deflections

$$\frac{w_{max}}{h_{tot}} \geq 0.3$$

- Significant stretching of middle surface.
- Membrane deformation ≥ bending.
- Nonlinear geometry changes and significant in-plane deformation.

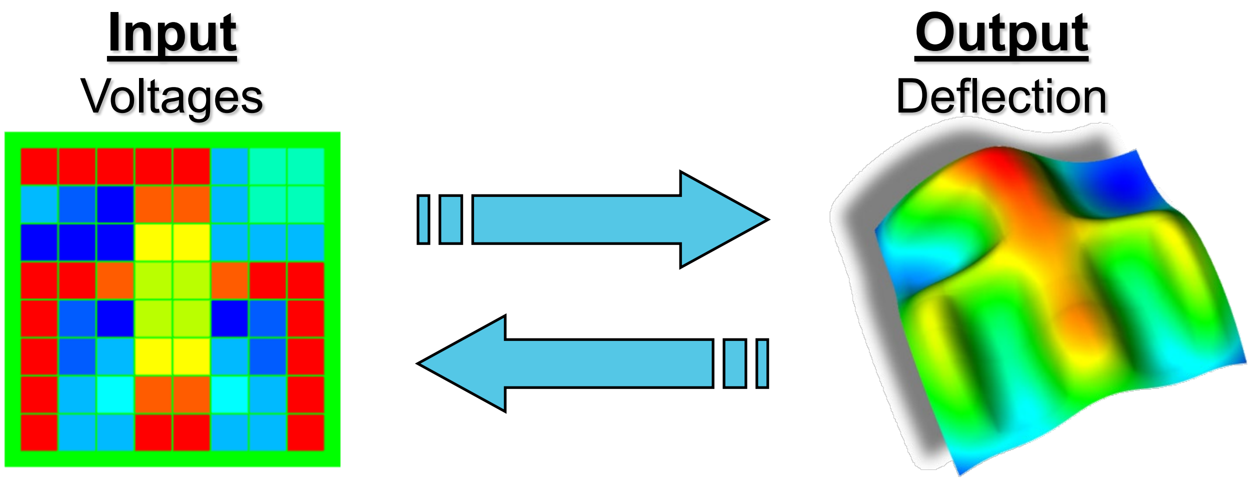
- Desired and measured deflections ≥ 250 µm.
- Typical membrane thicknesses 100 - 250 µm.

$$\Rightarrow \frac{w_{max}}{h_{tot}} \geq 1.0$$

Large deflection theory of membranes must be used to adequately model laminate deflections.

Nonlinear (Large) Deflection Model

- Develop nonlinear model using framework of the initial linear, sliding-corner model.
- Predict large membrane deflections.
- Treat fixed corners.
- Preserve current model formulation as mapping:



Critical: formulate model to be suitable for deflection control.

Energy-based (Ritz) Framework

① Construct membrane deformation energy in terms of deformations and input voltage.

② Express deformations in terms of known functions with undetermined constants.

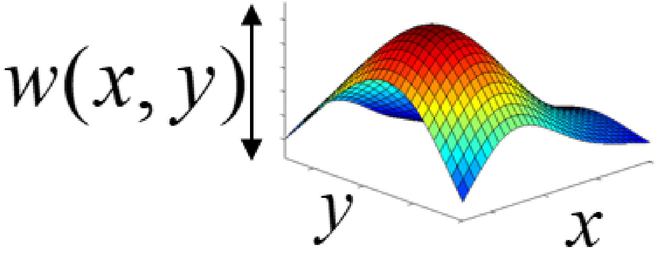
③ Find constants that minimize energy.

Step 1: Deformation Energy

Total Strain Energy	=	Deflection Energy	+	Actuation Energy

$$U = U_\varepsilon(u, v, w) + U_{act}(u, v, w; V)$$

Deflection



In-plane Deformations

$u(x,y)$
$v(x,y)$

Goal: find energy-minimizing deformation given voltage array V.

Deflection Energy

$$U_\varepsilon = \frac{1}{2}\int_0^a\int_0^b\int_{-h_g-h/2}^{h_{el}+h/2} \boldsymbol{\varepsilon}(x,y,z)^{\mathsf{T}}\boldsymbol{T}(z)\,dzdydx$$

Plane Stress

$$\boldsymbol{T}(z) = \boldsymbol{S}(z)\boldsymbol{\varepsilon}(x,y,z)$$

layer-dependent

von Karman Strain Relations

$$\varepsilon = \varepsilon_b + \varepsilon_m + \varepsilon_{nl}$$

Linear Model

Bending Strain

$$\boldsymbol{\varepsilon}_b(z) = -z\boldsymbol{\kappa}$$

Membrane Curvature

$$\boldsymbol{\kappa} = \begin{bmatrix} w_{xx} & w_{yy} & 2w_{xy} \end{bmatrix}^{\mathsf{T}}$$

Membrane Strain

$$\varepsilon_m = \begin{bmatrix} u_x & v_y & u_y + v_x \end{bmatrix}^{\mathsf{T}}$$

Nonlinear Strain

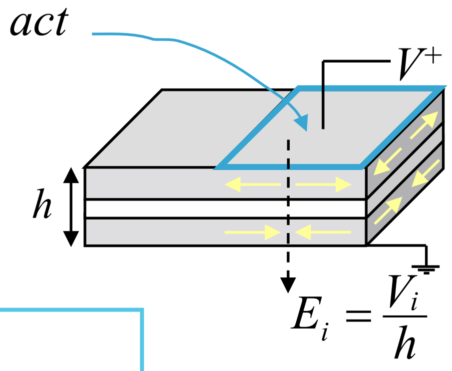
$$\varepsilon_{nl} = \frac{1}{2}\begin{bmatrix} w_x^2 & w_y^2 & 2w_x w_y \end{bmatrix}^{\mathsf{T}}$$

$$U_\varepsilon = U_b + U_m + U_{lc} + U_{nlc} + U_{nl}$$

Bending, Membrane, Linear-coupled, Nonlinear-coupled, and Nonlinear Energy Components.

Actuation Energy

$$U_{act} = \sum_{i=1}^{i_{max}} \iint_{act_i} \boldsymbol{\kappa}^{\mathsf{T}} \boldsymbol{M}_{act_i} \, dA$$



Membrane Curvature

$$\boldsymbol{\kappa} = -\begin{bmatrix} w_{xx} & w_{yy} & 2w_{xy} \end{bmatrix}^{\mathsf{T}}$$

Moment

$$\boldsymbol{M}_{act_i} = \int_{act_i} \boldsymbol{S}(z) \boldsymbol{\varepsilon}_{act_i}(z) z dz$$

Actuation Strain

$$\boldsymbol{\varepsilon}_{act_i}(z) = \begin{cases} \begin{bmatrix} d_{31} & d_{32} & 0 \end{bmatrix}^{\mathsf{T}} E_i & \frac{h_{ep}}{2} \le z \le \frac{h}{2} \\ 0 & -\frac{h_{ep}}{2} < z < \frac{h_{ep}}{2} \\ \begin{bmatrix} -d_{32} & -d_{31} & 0 \end{bmatrix}^{\mathsf{T}} E_i & -\frac{h}{2} \le z \le -\frac{h_{ep}}{2} \end{cases}$$

- Integrate energy expression thru laminate thickness:

$$U_{act} = \frac{D_{act}}{h} \sum_{i=1}^{i_{max}} V_i \iint_{act_i} \left(w_{xx} + w_{yy} \right) dA$$

Voltages
V_i

Actuation Stiffness Constant
D_{act}

Step 2: Energy Expansion

- Assume expansions for tri-axial deformations:

$u(x,y)=\sum_{j=1}^{\infty}\mu_j(x,y)$	$\mu_j(x,y) = a_{u_j}\sin\left(n_j\pi\frac{x}{a}\right)\cos\left(m_j\pi\frac{y}{b}\right)$	Vanishing strain at edges.
$v(x,y)=\sum_{j=1}^{\infty}\psi_j(x,y)$	$\psi_j(x,y) = a_{v_j}\cos\left(m_j\pi\frac{x}{a}\right)\sin\left(n_j\pi\frac{y}{b}\right)$	
$w(x,y)=\sum_{j=1}^{\infty}\varphi_j(x,y)$	$\varphi_j(x,y) = a_{w_j}\cos\left(m_j\pi\frac{x}{a}\right)\sin\left(n_j\pi\frac{y}{b}\right) + b_{w_j}\cos\left(m_j\pi\frac{y}{b}\right)\sin\left(n_j\pi\frac{x}{a}\right)$	Zero displacement at corners.

- Truncate sums, simplify energy in terms of expansions:

$$U\left(a_u, a_v, c_w, V\right) = U_\varepsilon\left(a_u, a_v, c_w\right) + \left(\mathbf{R}V\right)^T c_w$$

Voltage Array V

Actuation Block Matrix $\mathbf{R}$

In-plane Expansion Coefficient Vectors a_u, a_v

Out-of-plane Expansion Coefficient Vector $c_w = \left[a_w \quad b_w\right]$

Step 3: Energy Minimization

Find energy-minimizing deformation. ⟹ Find energy-minimizing expansion coefficients.

- Minimum conditions:

$$\nabla_{a_u} U = 0 \qquad \nabla_{a_v} U = 0 \qquad \nabla_{c_w} U = 0$$

- Solve nonlinear system for expansion coefficients:

$$G_\varepsilon(a_u, a_v, c_w) + \mathbf{R}V = 0$$

Gradient Function

G_ε

couples expansion coefficients nonlinearly

- Resulting Map:

Input: V ⟹ **Output:** ~~$u(x,y)$, $v(x,y)$~~, $w(x,y)$

- Inverse map requires knowledge of in-plane deformation.
- Typically out-of-plane information is known (e.g., ESPI, error surface), in-plane is *unknown*.

De-couple In-plane Strain

- Minimum conditions allow in-plane coefficients (a_u, a_v) to be cast explicitly in terms of out-of-plane coefficients.

$$\nabla_{a_u} U = 0 \qquad \Longrightarrow \qquad a_u = F_u(c_w)$$

$$\nabla_{a_v} U = 0 \qquad \qquad \qquad a_v = F_v(c_w)$$

- Recast nonlinear system:

$$\mathbf{H} c_w + G_{nl}(c_w) + \mathbf{R} V = 0$$

Decoupled Energy Hessian
$\mathbf{H}$

Nonlinear Gradient Function
G_{nl}
nonlinear component of decoupled gradient

- Resulting Map:

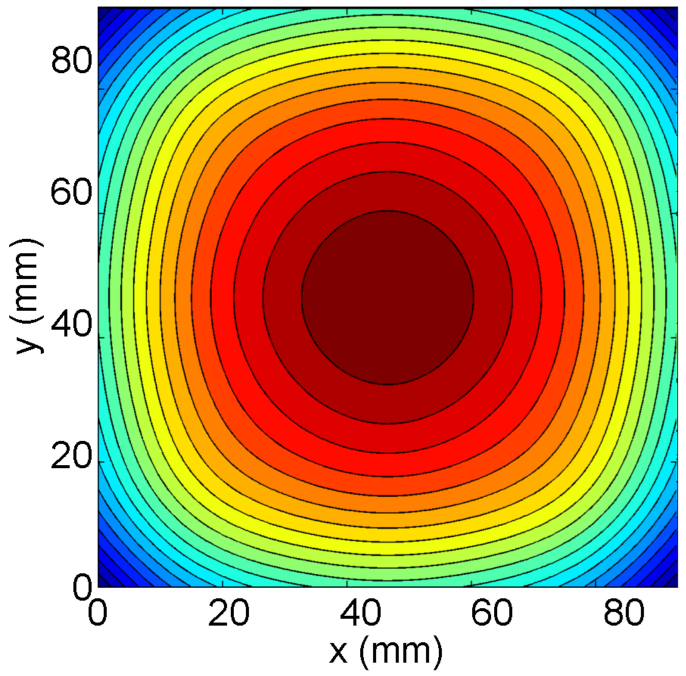
Input: V ⟹ **Output:** $w(x,y)$

- Inverse map requires only *out-of-plane* deformation.
- Deflection control now feasible.

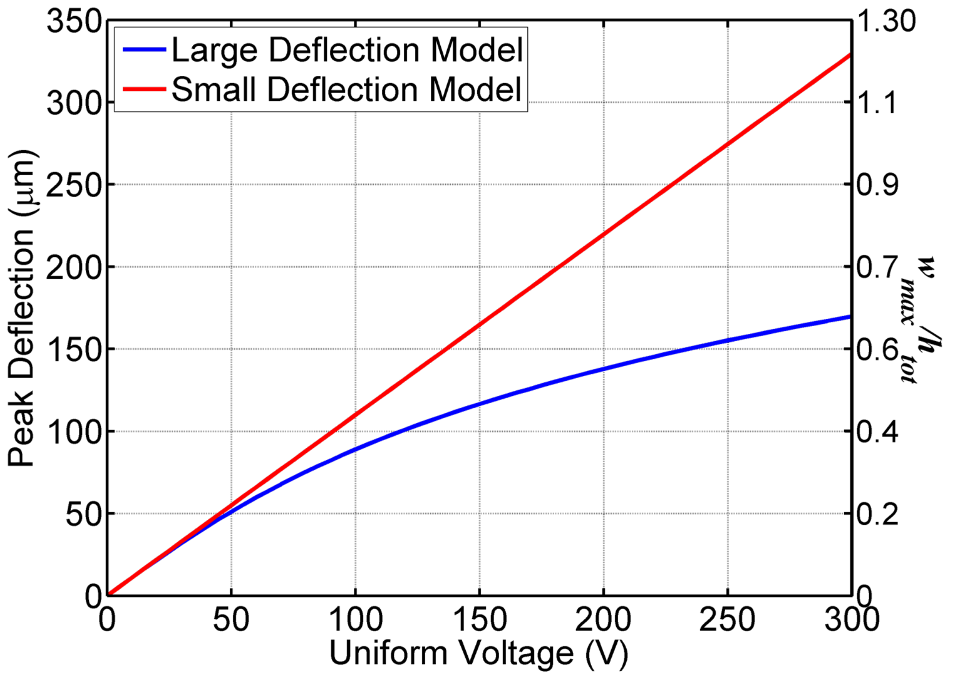
Nonlinear Model Results

- Deflection contours show squaring effects.
- Nonlinear rise in peak deflection predicted.
- Source: nonlinear geometric changes; membrane forces due to large deflections and pinned corners.

Model Deflection Contours



Peak Deflection vs. Uniform Voltage



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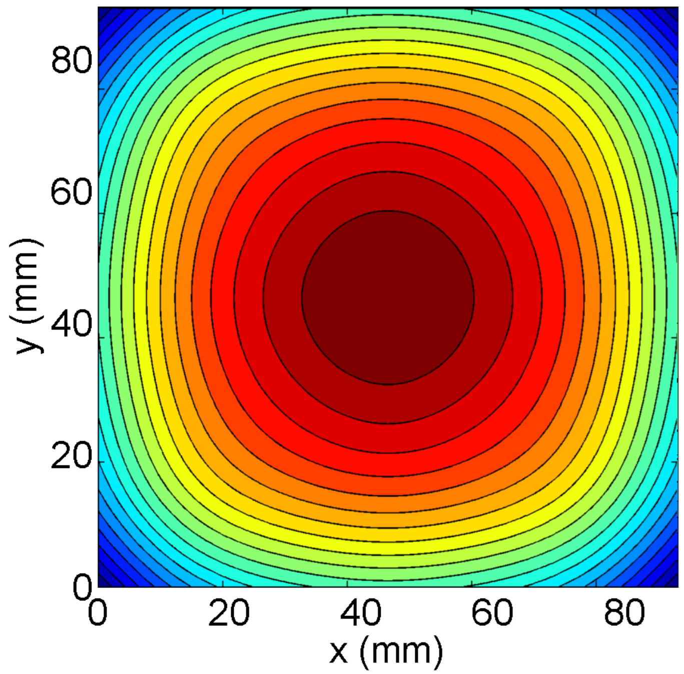
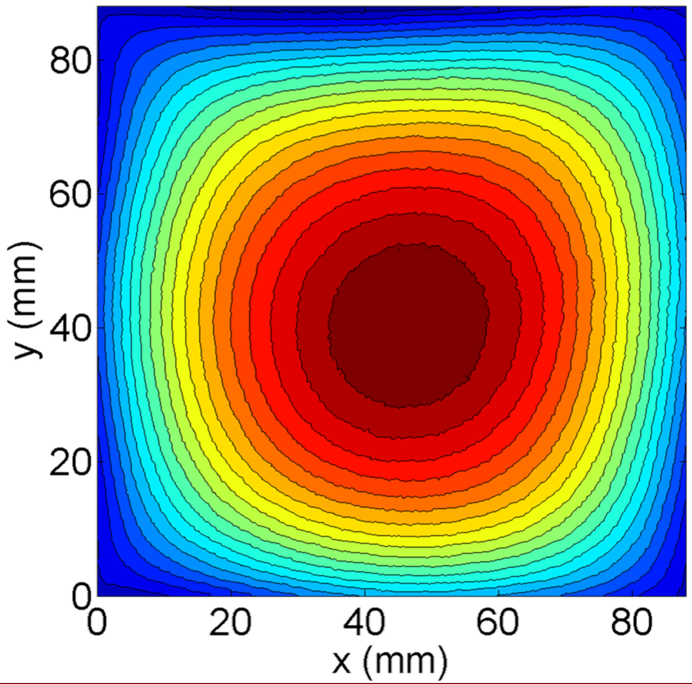
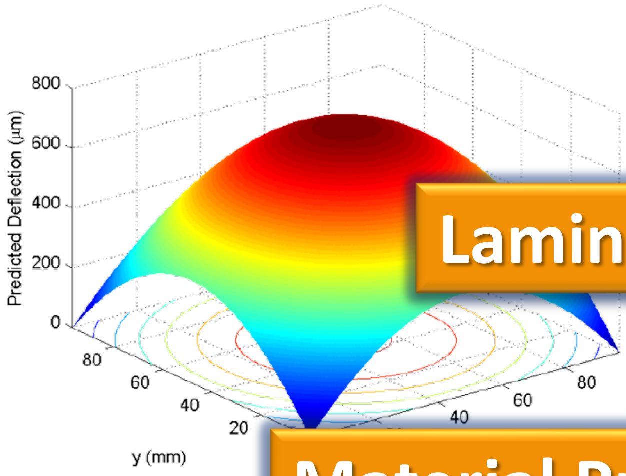


Measured Deflection Contours

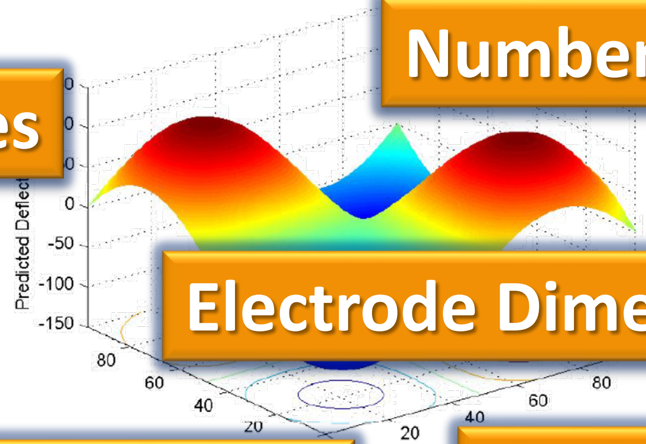


So What Determines Reflector Shape?



Laminate Dimensions

Boundary Conditions



Number of Electrodes

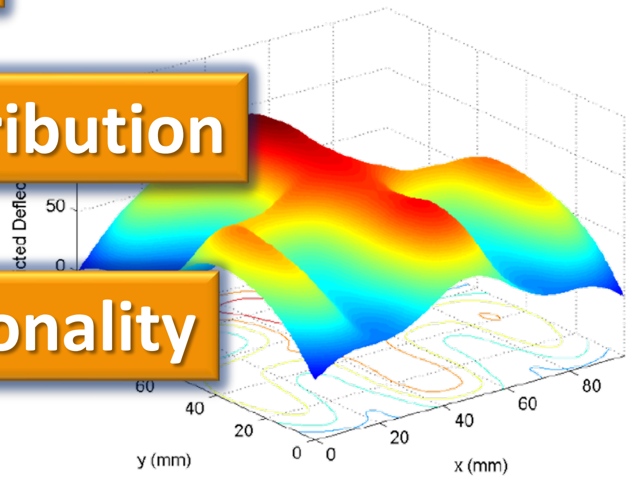
Material Properties

Layer Thicknesses

Electrode Dimensions

Electrode Pattern

Voltage Distribution

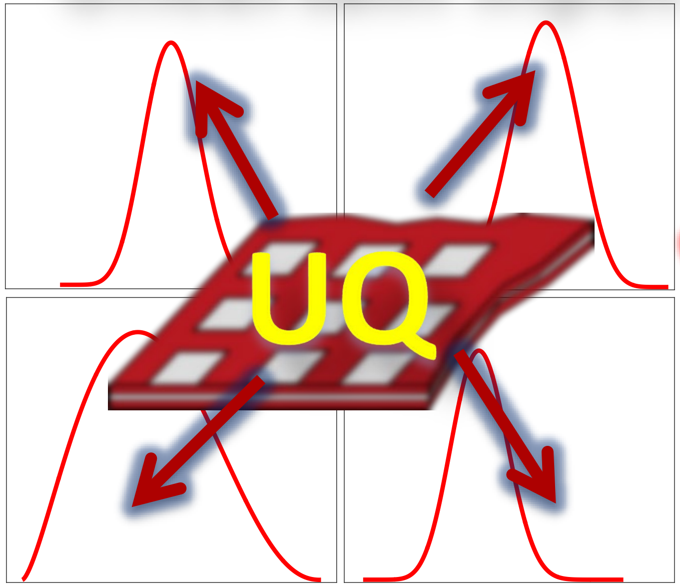


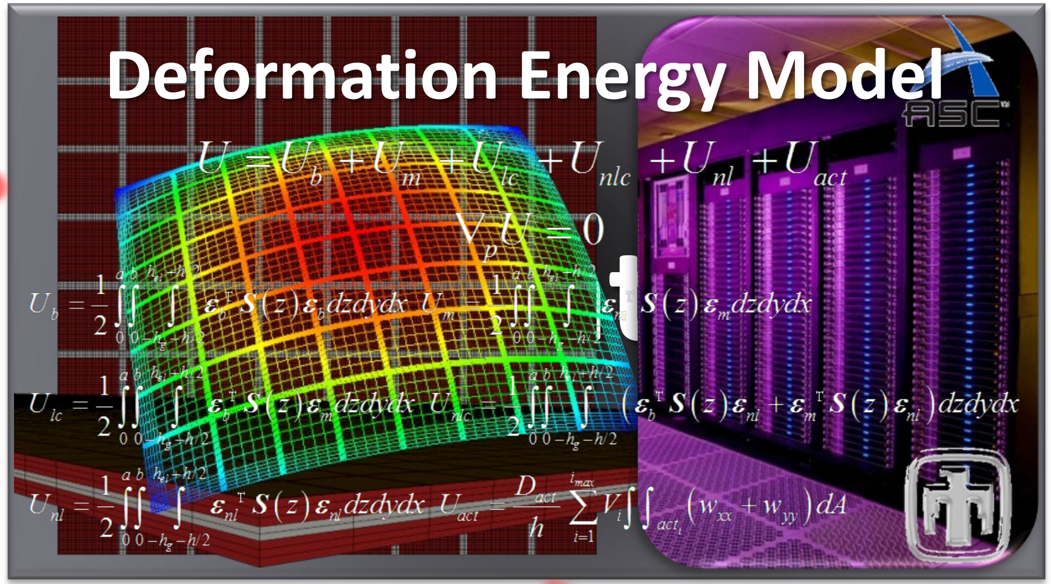
Electrode Funtionality

…what if there is variation & uncertainty?

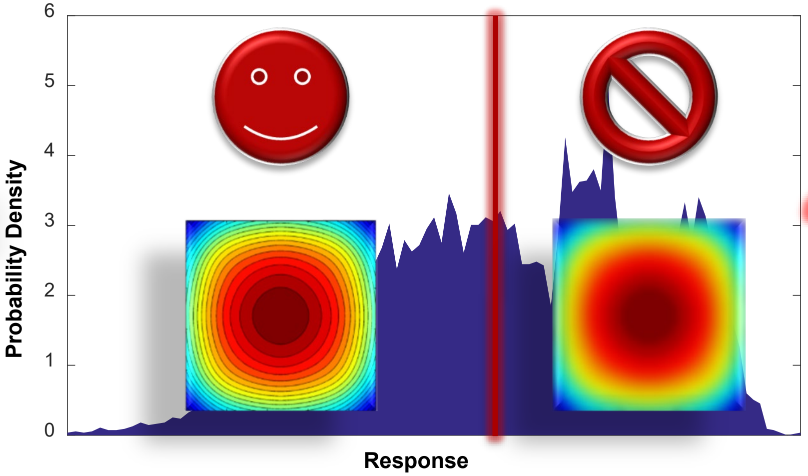
Impact of Uncertainty

Uncertain Input

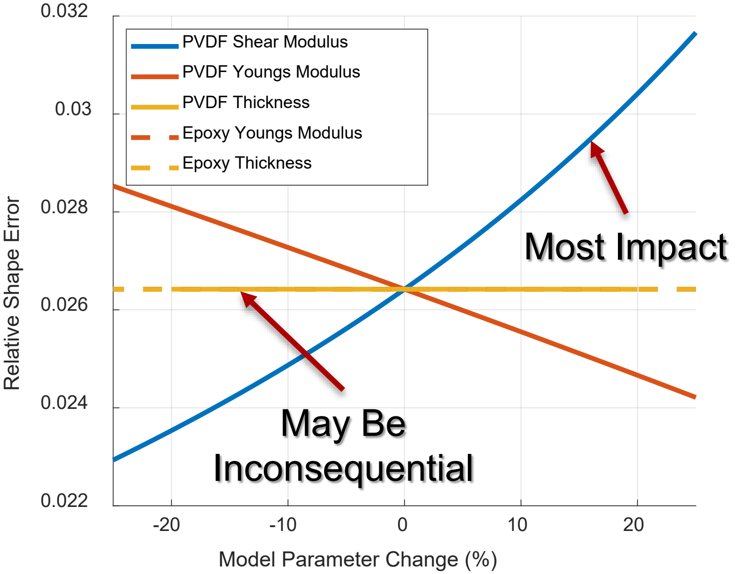


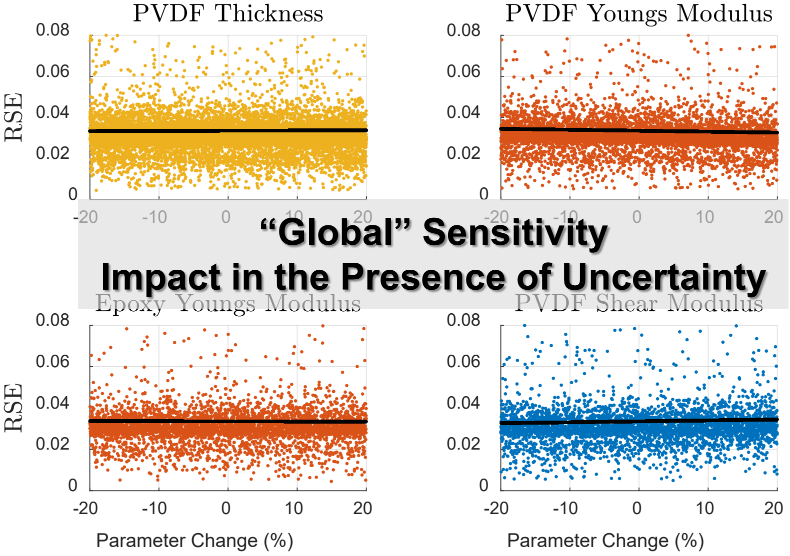


Probabilistic Output



What Matters Most?

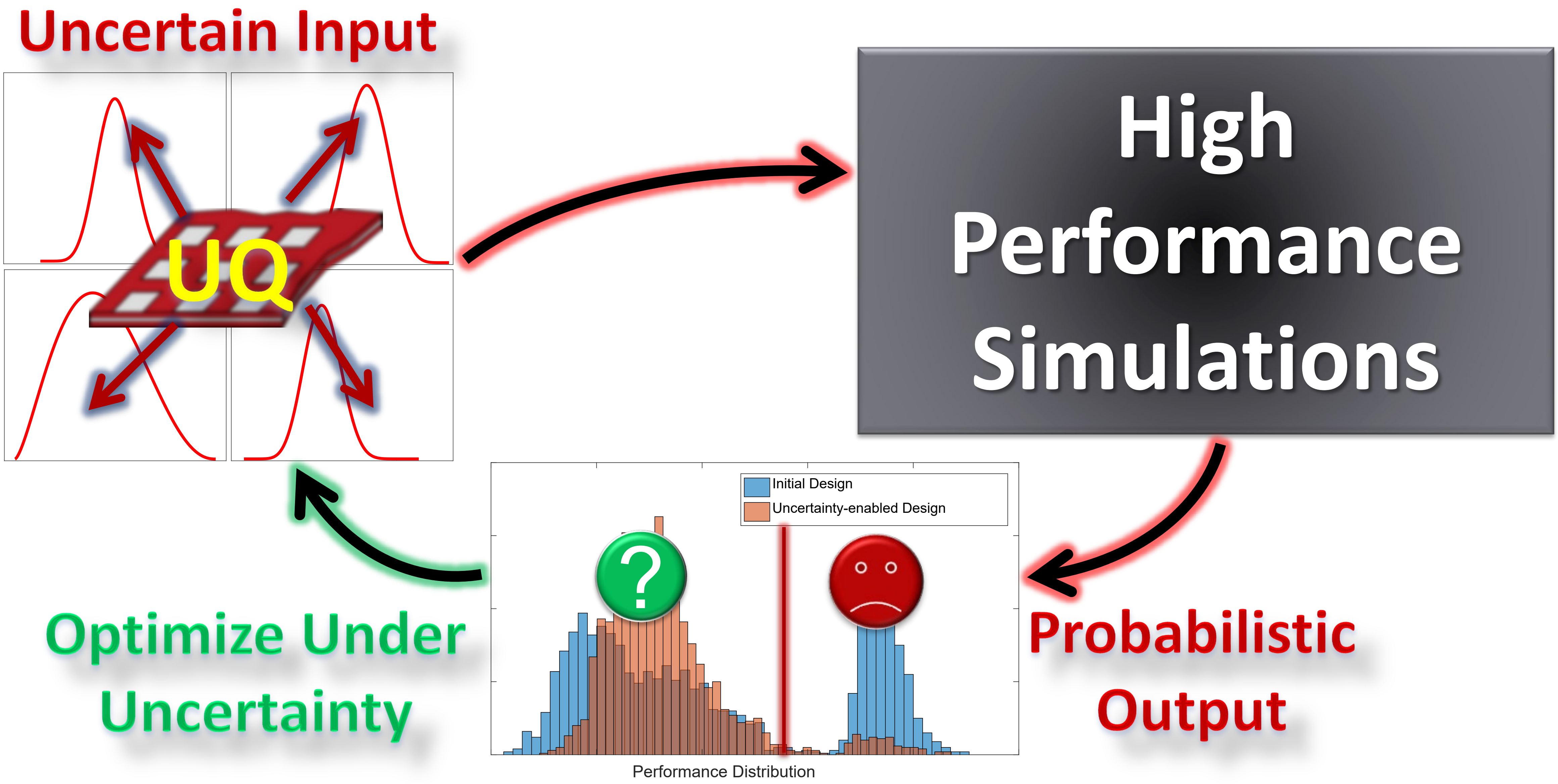




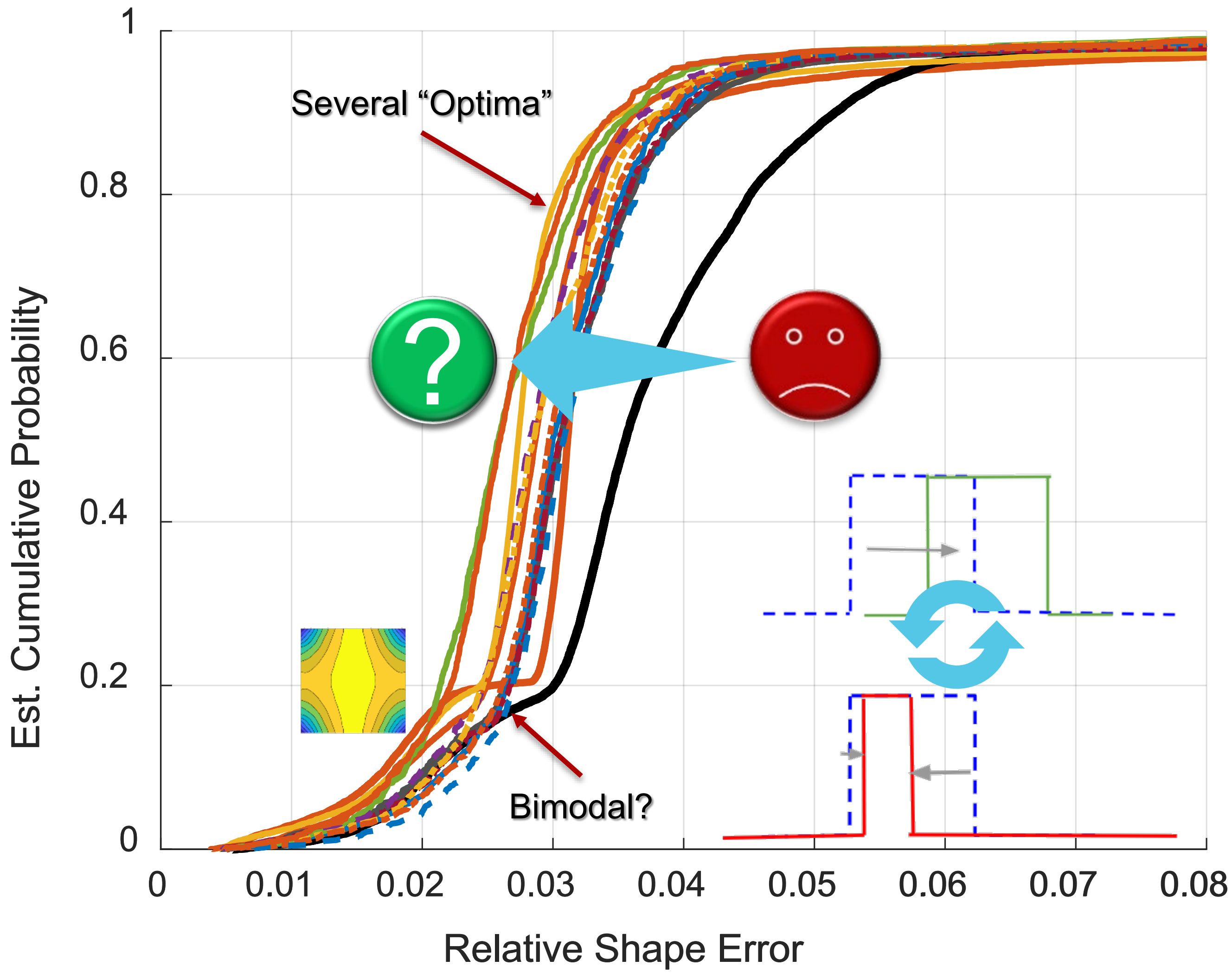
- Define "what": quantify metric(s), like a relative shape error.
- Probe model and gain understanding of simulated correlations.
- Investigate metric sensitivities w.r.t. model input.
- Consider deterministic and statistical (e.g., Latin Hypercube) sensitivities, afforded by computational efficiency.
- Sensitivity analysis and practicality help select tuning parameters.

Parameter	Sensitivity rank (OAT)	LHS sensitivity rank (Latin Hypercube)	Significance rank	PRCC	PRCC rank
h_p	9	11	7	0.0121	11
h_e	10	13	7	0.0110	12
a	2	1	5	0.3866	2
b	2	1	5	-0.4387	1
Y_{11}	5	10	3	-0.0307	6
Y_{22}	4	5	2	0.2411	4
Y_e	11	12	7	0.0063	13
G_{12}	3	6	1	-0.2712	3
ν_{12}	7	2	7	0.0277	7
ν_e	8	7	7	-0.0153	10
D_{31}	13	8	7	-0.0168	9
D_{32}	12	9	7	0.0234	8
Bor	1	3	4	0.1042	5
Sep	6	4	6	0.0017	14

Uncertainty-enabled Design



Optimization Under Uncertainty



- Assumed uniform distributions of chosen design parameters based on guidance and known design tolerances.
- Estimate distribution of shape error via uncertainty propagation (tolerance stackup).
- Adjust distribution of chosen design parameters.
- Iterate to find improved distribution; Simplex, Constrained Differential Evolution, etc.
- Consider practicality of optimized scenarios.

Smart Laminate Recommendations

- Getting just a solution is often insufficient: what do we do with the results?
- Our 2017 Industrial Math and Statistics Modeling (IMSM) Workshop for Graduate Students team provided recommendations to SNL based on their sensitivity and uncertainty analyses.

- **Design Change**: to make the shape error less sensitive to uncertainties:
  - minimize inactive border;
  - use stiffer active material.
- **Tolerance Change**: characterize and reduce the uncertainty of PVDF shear modulus (its uncertainty is a large contributor to shape error variation).
- **Resource Allocation**: impact of variations in bonding layer properties are relatively insignificant; likely they need less expense/attention/precision.
- **Refinement**: build/conduct experiments for continued validation and UQ.

Case Study Remarks

- Integrated shape control of electromagnetic reflectors offers significant advantages.
- To expedite design and experimentation of a smart laminate, model-based support was involved early.
- A useful small-deformation, bending-dominated (linear) model was shown to be insufficient when compared to non-idealized laboratory experiments.
- Better agreement achieved when accounting for large and in-plane deformation.
- Computationally-efficient implementation facilitates sensitivity and uncertainty analyses.
- “Simulated experiments” reveal potential improvement to overall fabrication and performance.

Real-world Problem Solving

- As an applied mathematician and engineering scientist at SNL, my goal is to provide science-based solutions and capabilities.
- Often approaches are constrained by response time.
- Sometimes novel methods are developed/used, other times, the problem demands creative use of existing methods.
  Just get it done!
- Defining and quantifying "solved" or "optimal" typically is non-trivial; problem ID is crucial.
- **Uncertainty-enabled** designs/solutions attempt to maximize the *probability* of performance with uncertain conditions/properties.