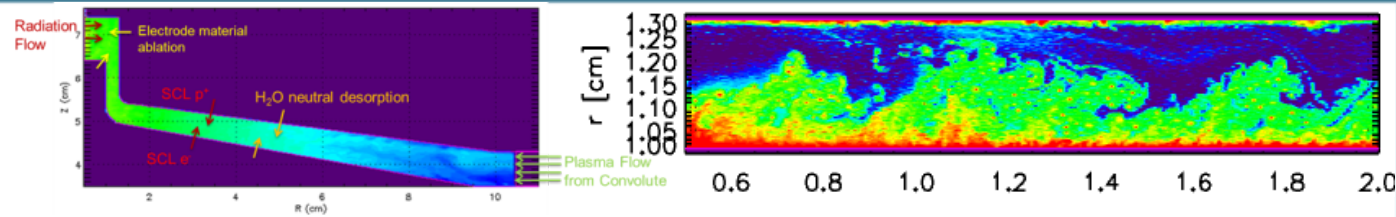


Modeling magnetized transport



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Z Fundamental Sciences Workshop

August 9, 2021

Albuquerque, NM

1. Particle transport in highly magnetized systems, such as at the Z accelerator requires consideration of the complete equations of motion.
2. Modeling demonstrates a Hall-term current across the inner transmission line.
3. This modeling may be included in the shot design stage to aid in predicting current delivery to the load.

In multi-MA accelerators, current is inhibited perpendicular to the magnetic field.



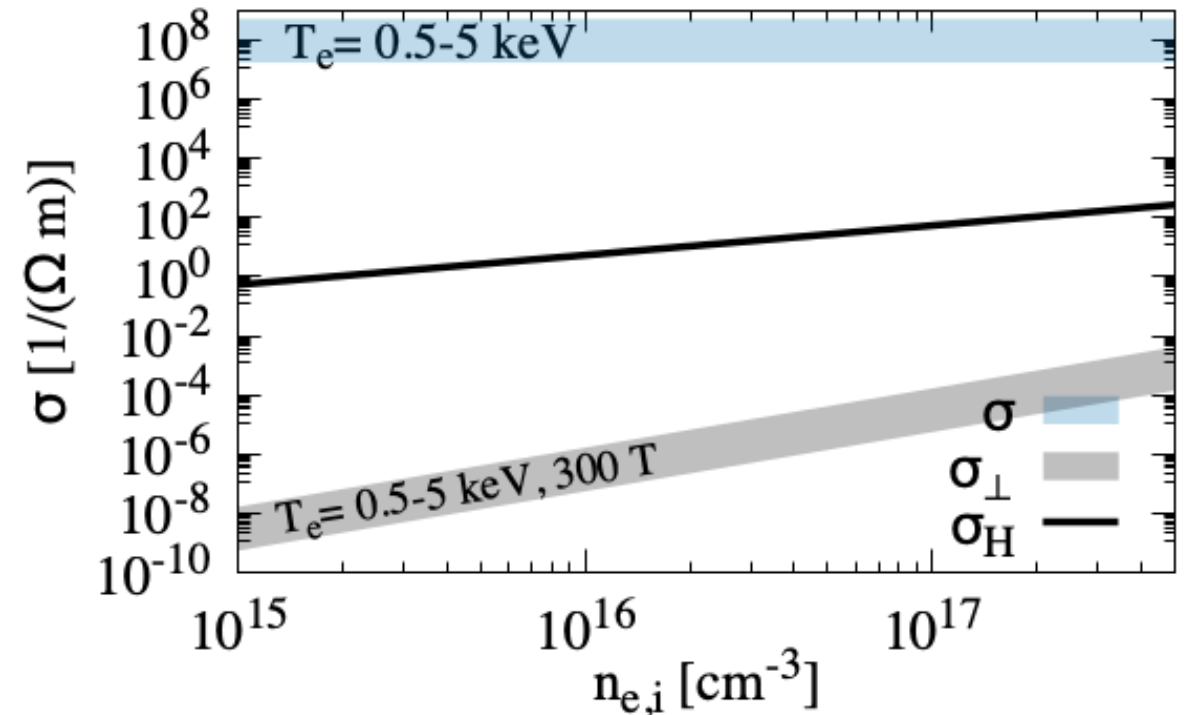
$$\mathbf{j} = \boldsymbol{\sigma} \cdot \left[\underbrace{\mathbf{E} + \mathbf{v} \times \mathbf{B} + \frac{1}{en_e} \nabla(p_e)}_{\text{effective } \mathbf{E} \text{ in the ion inertial frame}} \right]$$

effective \mathbf{E} in the ion inertial frame

for $\mathbf{B} = B\hat{\theta}$

$$\boldsymbol{\sigma} = \frac{e^2 n_e}{m_e \nu_c} \begin{pmatrix} \frac{1}{1+\omega_c^2/\nu_c^2} & 0 & \frac{-\omega_c/\nu_c}{1+\omega_c^2/\nu_c^2} \\ 0 & 1 & 0 \\ \frac{\omega_c/\nu_c}{1+\omega_c^2/\nu_c^2} & 0 & \frac{1}{1+\omega_c^2/\nu_c^2} \end{pmatrix}$$

Elements of the conductivity tensor differ by orders of magnitude at 300 T.



When the diagonal elements are insufficient, look at the off-diagonal.



$$\left. \begin{aligned} \sigma &= \frac{ne^2}{m\nu_c} \\ \sigma_{\perp} &= \frac{\sigma}{1 + \frac{\omega_c^2}{\nu_c^2}} \\ \sigma_H &= \sigma_{\perp} \frac{\omega_c}{\nu_c} \end{aligned} \right\} \mathbf{j} = \underbrace{\sigma \mathbf{E}'_{\parallel}}_{\text{not relevant}} + \underbrace{\sigma_{\perp} \mathbf{E}'_{\perp}}_{\text{not happening}} + \underbrace{\sigma_H (\mathbf{b} \times \mathbf{E}'_{\perp})}_{\text{possible with sufficient } n_e, E, \text{ and area}}$$

Considering all off-diagonal elements in the derivation of the diffusion coefficients yields a similar Hall diffusion.*

$$D_{\parallel} = \frac{k_B T}{m\nu_c}$$

$$D_{\perp} = \frac{k_B T}{m\nu_c \left(1 + \frac{\omega_c^2}{\nu_c^2}\right)}$$

$$\begin{aligned} D_H &= \frac{k_B T \omega_c}{m\nu_c^2 \left(1 + \frac{\omega_c^2}{\nu_c^2}\right)} \\ &\simeq \frac{k_B T}{m\omega_c} = \frac{k_B T}{eB} \end{aligned}$$

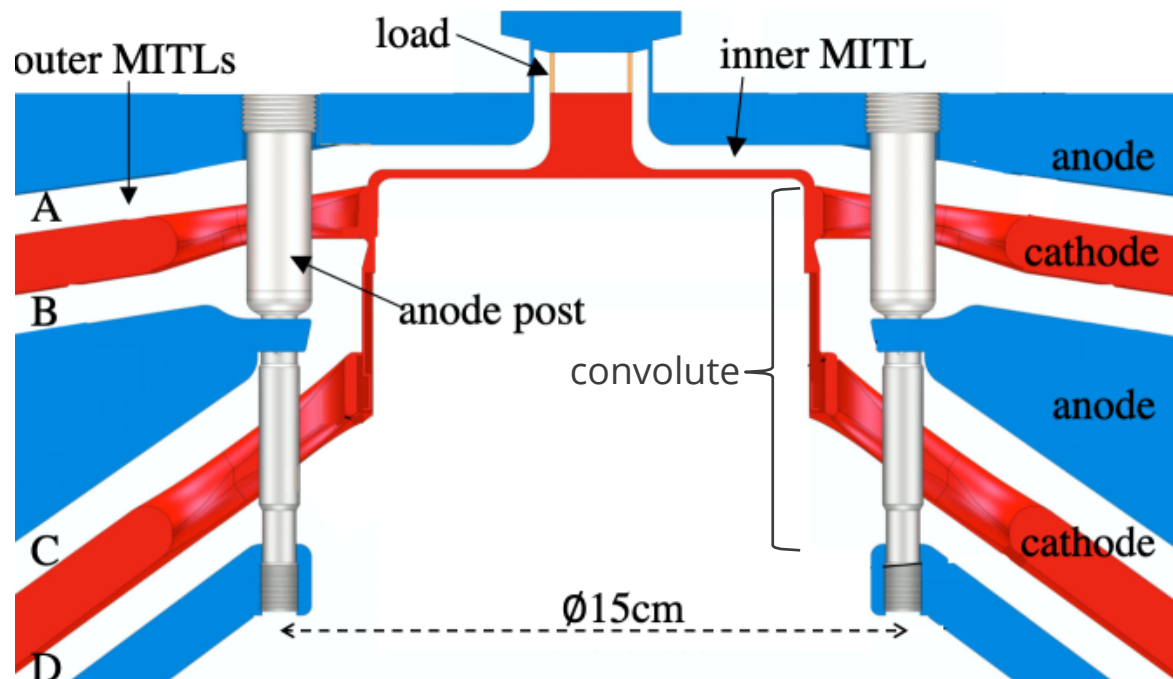
Bohm diffusion

On Z, measurable current is diverted from the load (“ I_{loss} ”).

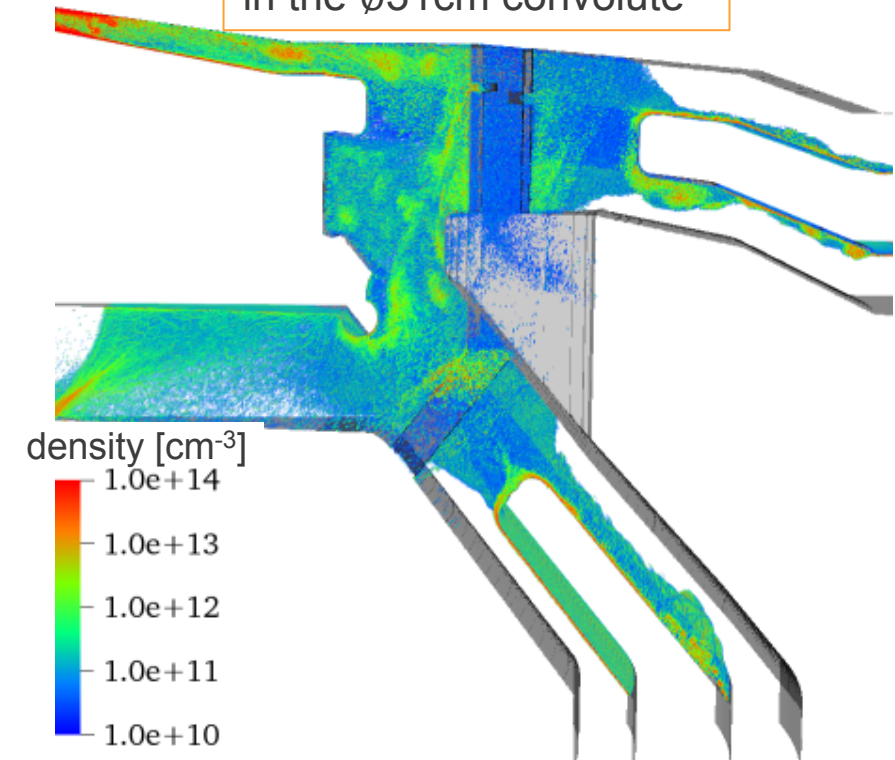
Characteristics of loss current:

- ubiquitous, although ranges from 10^5 to 10^6 A
- may be distributed between the current-adder region and the inner MITL
- not large enough to prevent the z-pinch
- not collisional! $\omega_c/\nu_c > 10^4$

Z Accelerator: current adder to the load



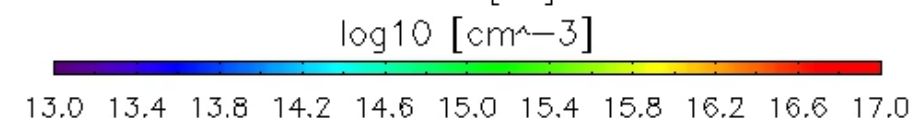
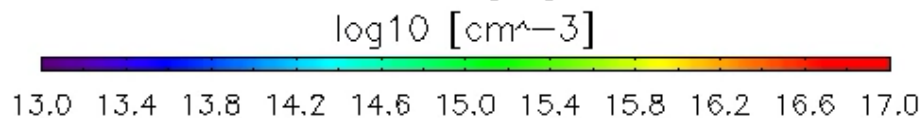
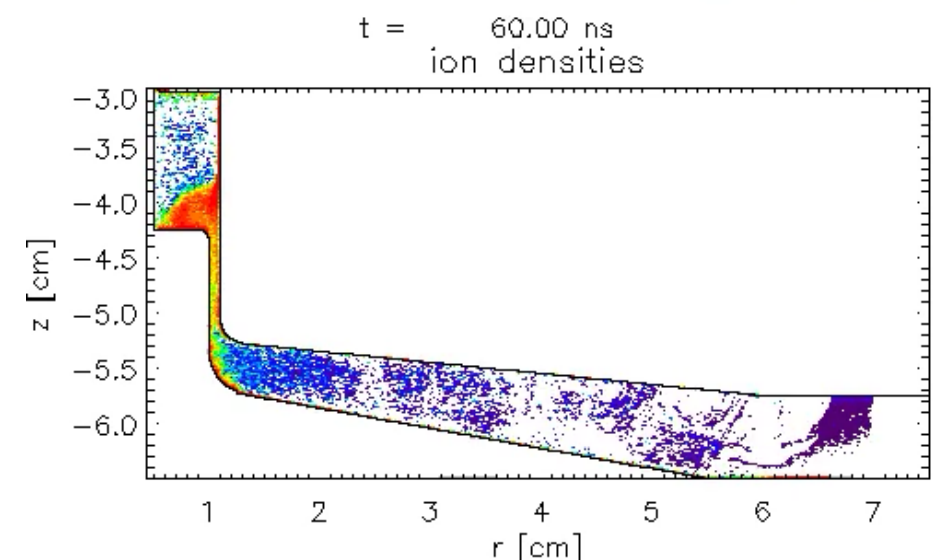
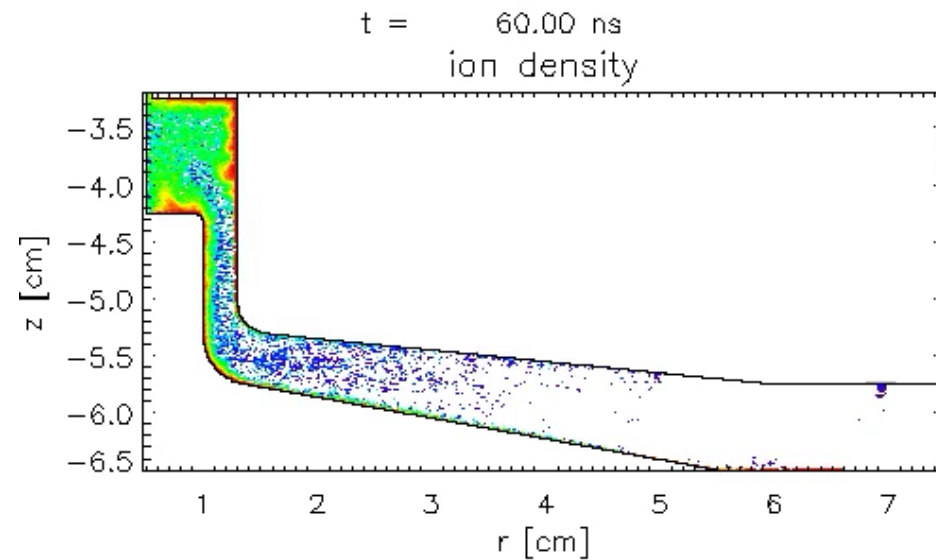
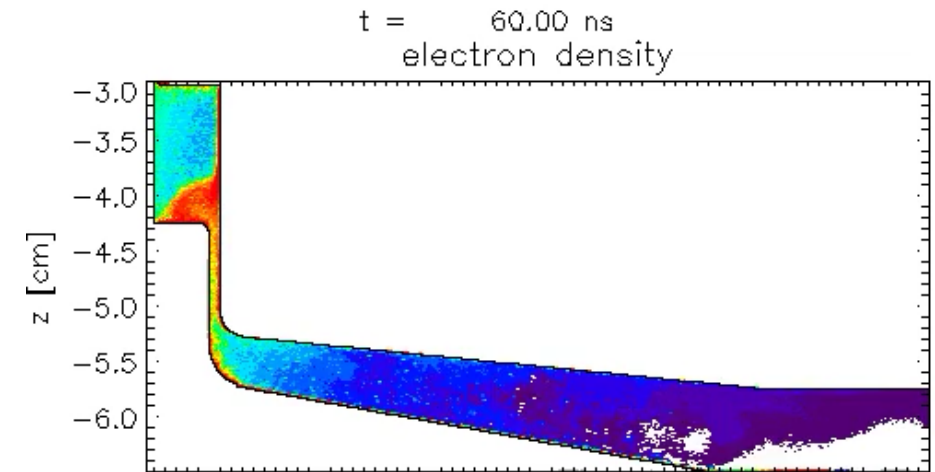
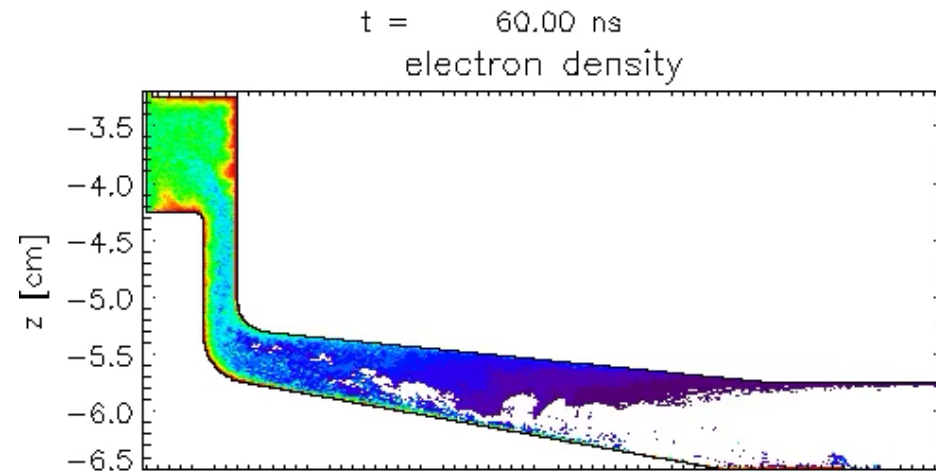
electron density at 80 ns
in the $\varnothing 31\text{cm}$ convolute



Current loss occurs in models of the inner MITL and convolute only when plasma formation is included.



- Plasma that is insulated from crossing the gap without Hall, drifts downstream and increases the local density closer to the can.
- If gap-spanning plasma carries enough j in a small region, it may $j \times B$ downstream.



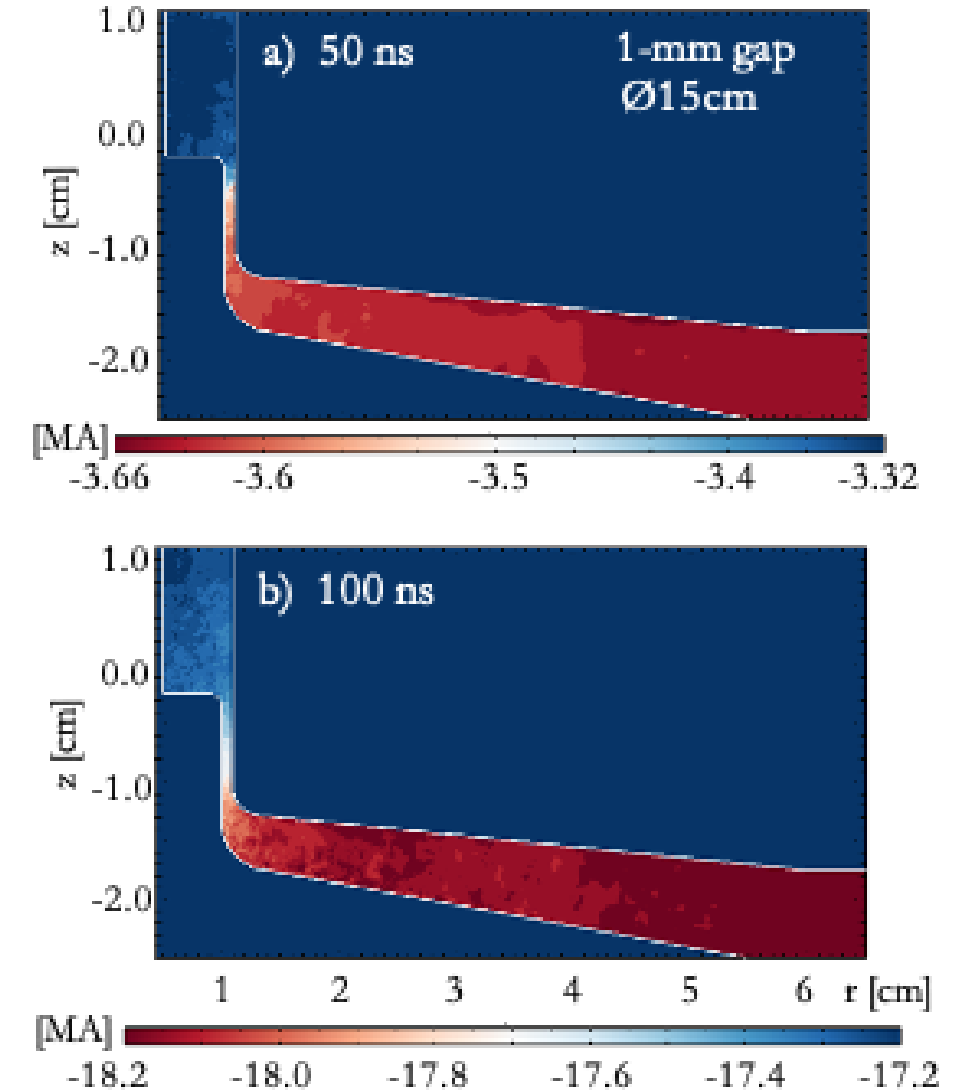
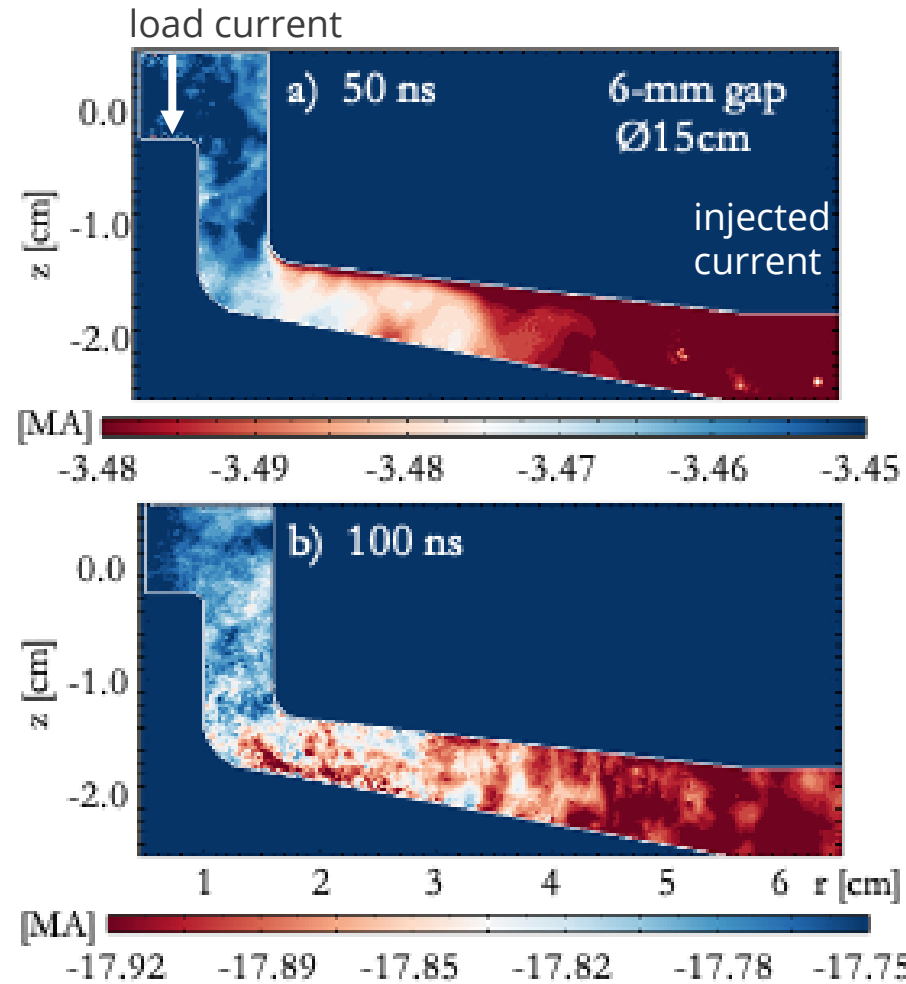
Current loss occurs where plasma accumulates.



Simulated Iloss is the difference between the injected and load currents.

This effect is captured in kinetic and multi-fluid treatments.

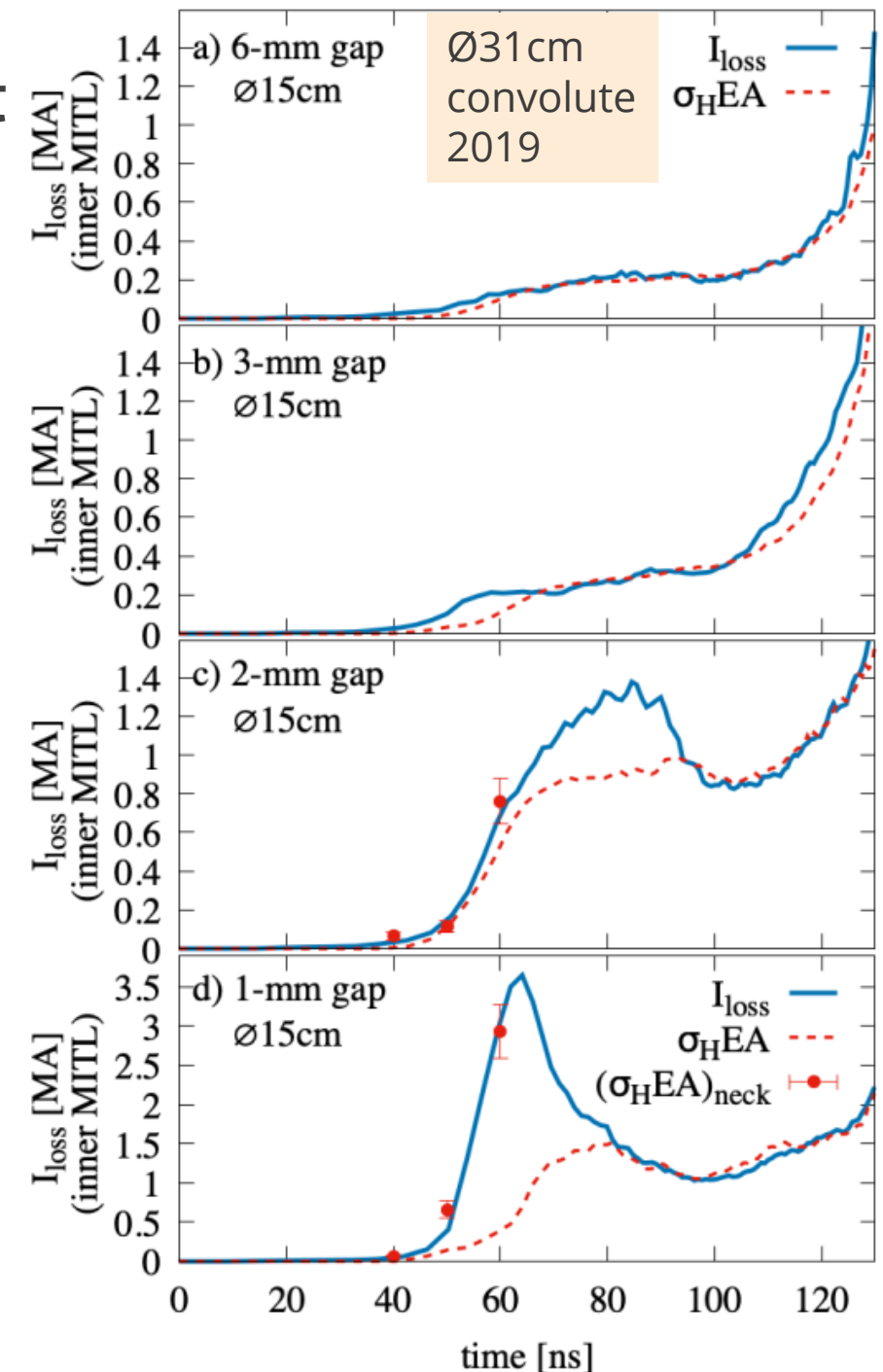
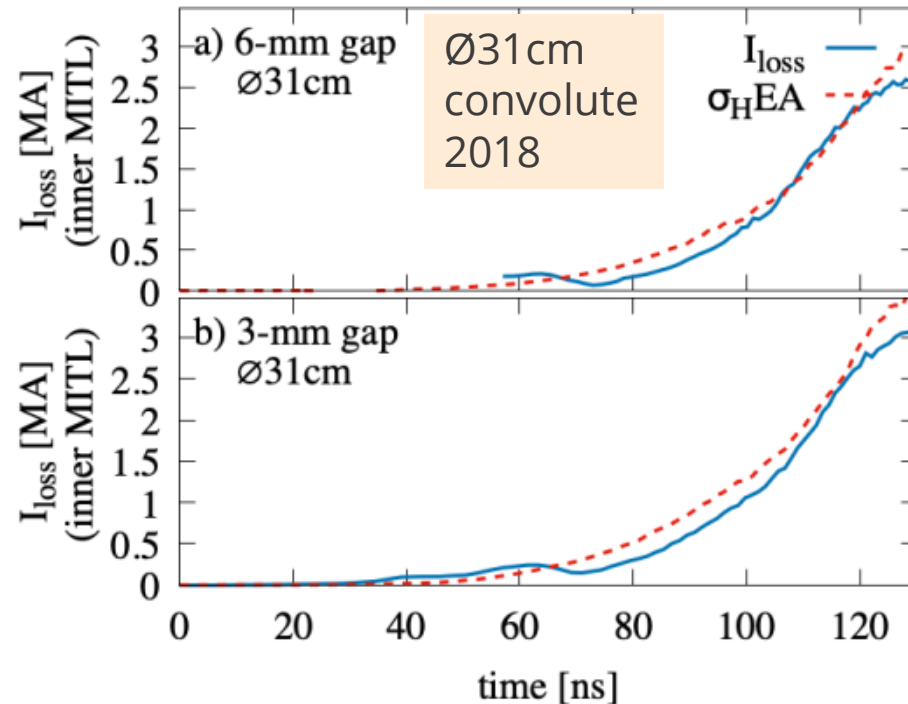
However, kinetic may include additional surface physics that impacts surface plasma formation.



A calculated Hall-term current ($I_{\text{loss}} = \sigma_H EA$) agrees with the current diverted in the inner MITL.

Calculating I_{loss} using values of n_e , B , T_e , and E in the higher plasma region (A).

$$\sigma_H = \frac{ne^2}{m\nu_c} \left(\frac{\frac{\omega_c}{\nu_c}}{1 + \frac{\omega_c^2}{\nu_c^2}} \right)$$

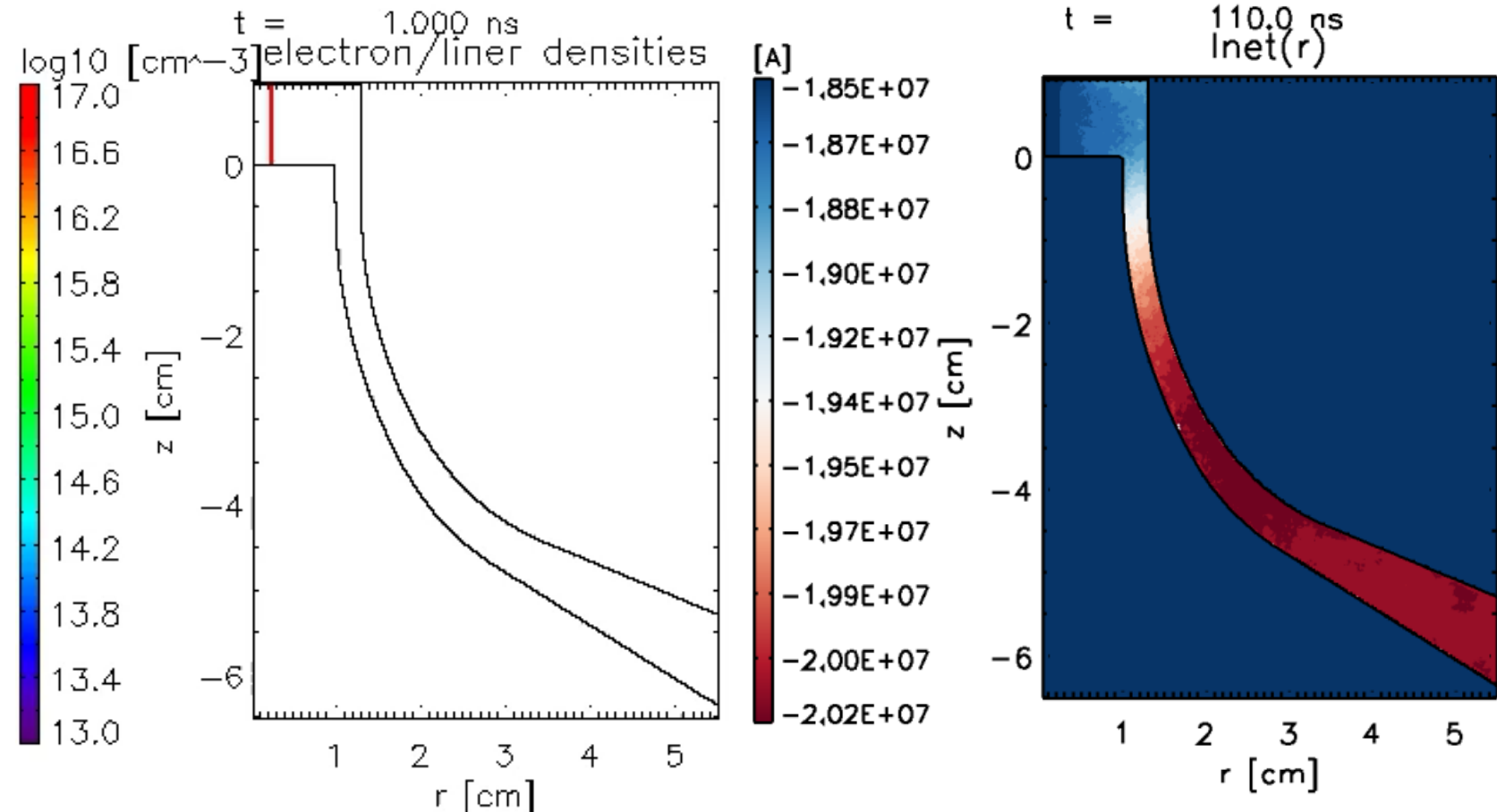


Improvements to simulation run time and load inductance modeling have made power flow simulations valuable tools for Z shot designs.



MagLIF* type geometry without an applied B_z field has current loss co-located with higher n_e and E .

- Multi-fluid simulations provide predictions relatively rapidly. (This depends on the hardware design and the computing resources available.)
- The “moving liner” model is being applied to studies conducted by M. Gomez and D. Zimmer. This provides a robust simulation of the inductive E-filed during implosion.



Current loss in Z may be studied /predicted using kinetic of multi-fluid models that include surface plasma formation.



1. Kinetic modeling includes the complete equations of motion. Multi-fluid should also allow for charge separation and space-charge-generated electric fields.
2. Cross-gap currents can manifest as continued losses over a region of the transmission line. They may also accumulate enough current density to $j \times B$ downstream.
3. Multi-fluid modeling is sufficiently fast to be included in the shot design stage, complementing circuit-model predictions of current delivery to the load. (See N. Bennett, M. Hess, and E. Estatiev.)