SAND2021-10534C

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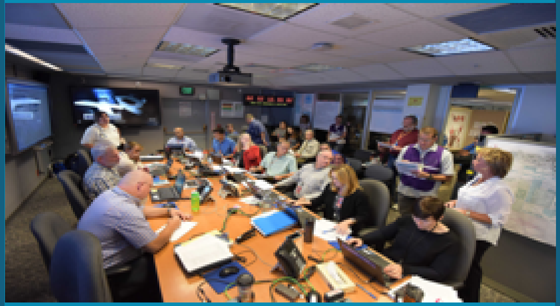
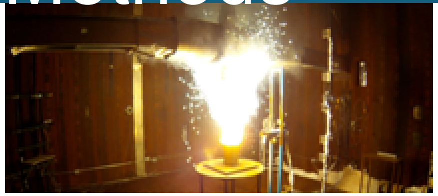
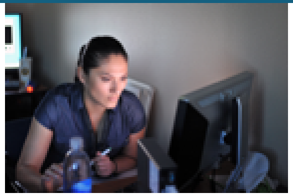
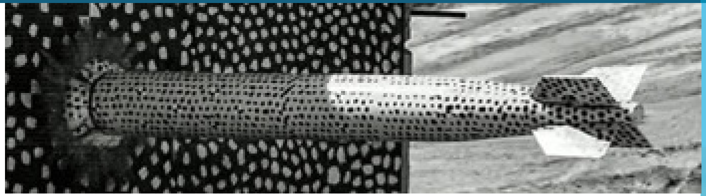
Relativistic Two-Fluid Electrodynamics Using Implicit-Explicit Discontinuous-Galerkin Methods









PRESENTED BY

Forrest W. Glines, Kris Beckwith

With contributions from the EMPIRE Team: Joshua R. Braun, Eric C. Cyr, Curtis C. Ober, Matthew Bettencourt, Keith L. Cartwright, Sidafa Conde, Sean T. Miller, Nicholas Roberds, Nathan V. Roberts, Matthew S. Swann, Roger Pawlowski, Thomas M. Smith, Nathaniel D. Hamlin

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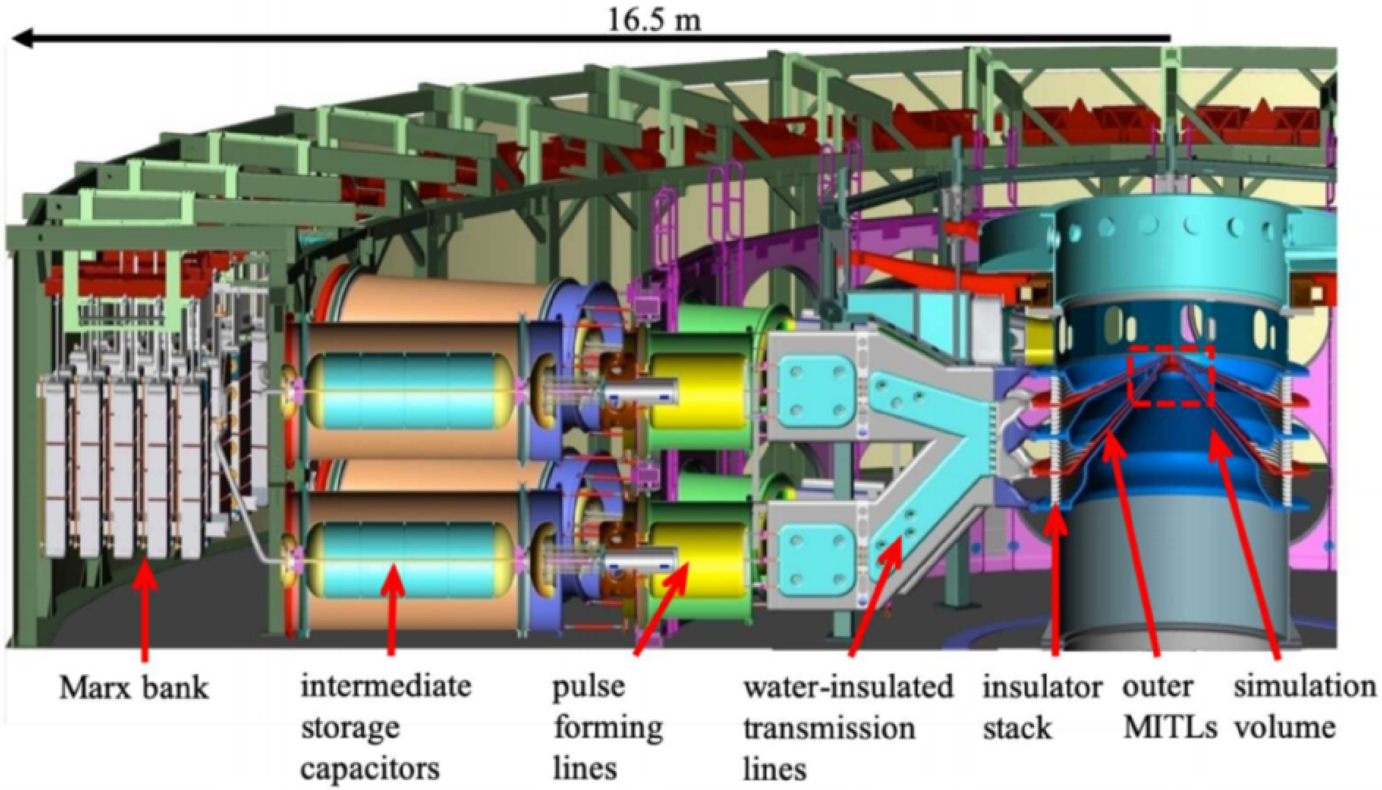
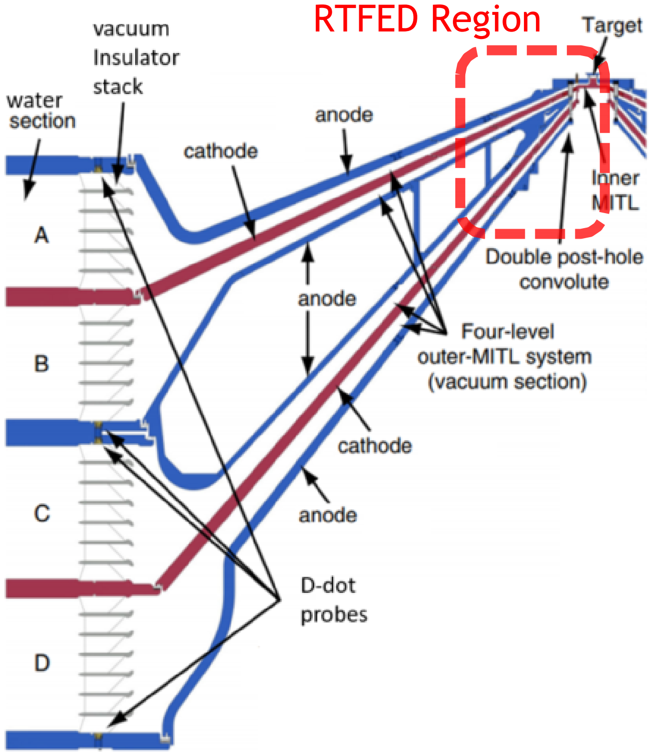
Terrestrial Application: Z Power Flow

- Z power flow
  - Magnetically-Insulated Transmission Line inside Z Pulsed Power Machine
  - Conical geometry, ~2-4 MV, gap ~10mm ⇒ $\gamma \approx 5 - 9$ (Gomez et al., 2017)
  - Electron models that don't account for relativity can possess unphysical super-luminal velocities





Astrophysical Application: Magnetized Relativistic Jets

- Active Galactic Nuclei launch relativistic jets ($\gamma \approx 2 - 10$) out to kiloparsecs
- How are cosmic rays from jets accelerated?
  - Relativistic non-thermal ions accelerated from AGN
  - Across shocks? Magnetized Turbulence? Magnetic Reconnection?
- Are the AGN jets comprised of ion-electrons or positrons-electrons?
- Multi-fluid relativistic plasma methods are needed

Centaurus A

Credits: X-ray: NASA/CXC/SAO; optical: Rolf Olsen; infrared: NASA/JPL-Caltech; radio: NRAO/AUI/NSF/Univ.Hertfordshire/M.Hardcastle

Relativistic Two-Fluid Electrodynamics

Relativistic Hydrodynamics

Coupling Source Terms

$$\frac{\partial}{\partial t}(\rho_s\gamma_s) + \nabla\cdot(\rho_s\mathbf{u}_s) = 0$$

$$\frac{\partial}{\partial t}\left(\frac{w_s}{c^2}\gamma\mathbf{u}_s\right) + \nabla\cdot\left(\frac{w_s}{c^2}\mathbf{u}_s\mathbf{u}_s + p_s\mathbf{I}\right) = \mu_s\gamma_s\rho_s\left(\mathbf{E} + \frac{\mathbf{u}_s}{\gamma_s c}\times\mathbf{B}\right) + \mathbf{R}_s$$

$$\frac{\partial}{\partial t}(w_s\gamma_s^2 - p_s) + \nabla\cdot(w_s\gamma_s\mathbf{u}_s) = \mu_s\rho_s\mathbf{u}_s\cdot\mathbf{E} + R_s^0$$

$$\frac{1}{c}\frac{\partial}{\partial t}\mathbf{E} - \nabla\times\mathbf{B} = -\frac{\mathbf{J}}{c}$$

$$\frac{1}{c}\frac{\partial}{\partial t}\mathbf{B} + \nabla\times\mathbf{E} = 0$$

$$\nabla\cdot\mathbf{E} = \varrho$$

$$\nabla\cdot\mathbf{B} = 0$$

Electrodynamics

- Two (or more) charged fluids
  - Relativistic velocities and/or temperatures
  - Coupled together via Maxwell's equations
- Conservation, stability, robustness is crucial

Discontinuous Galerkin Method

- Discontinuous Galerkin Method
  - Domain decomposed into cells
  - Variables approximated via polynomial basis over each cell

$$\mathbf{U}(\mathbf{x}) \approx \mathbf{U}^h(\mathbf{x}) = \sum_{i=1} \mathbf{U}_i \phi_i(\mathbf{x}) \quad \mathbf{x} \in \Omega_k$$

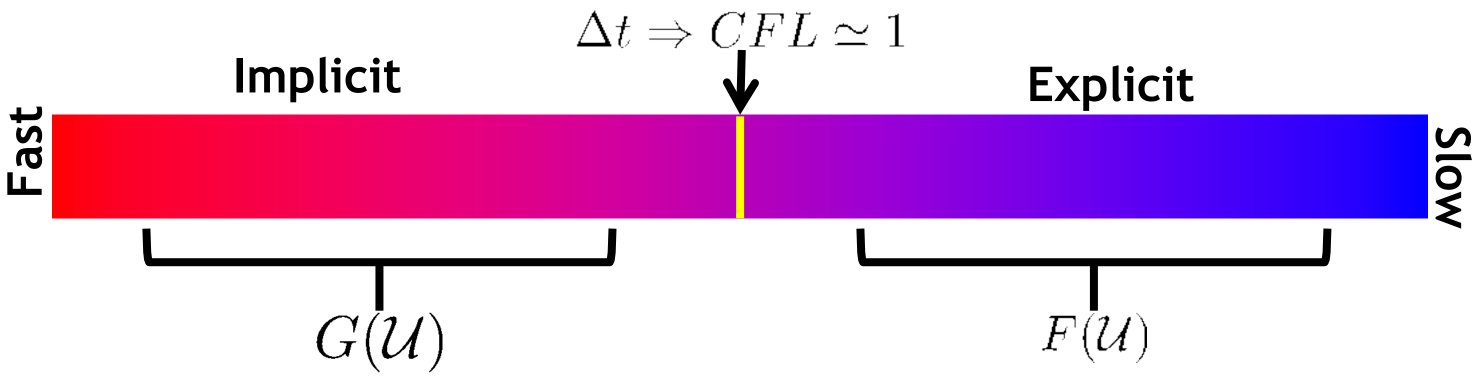
$$\phi_i(\mathbf{x}_j) = \delta_{ij}$$

$$\int_{\Omega_k} \frac{\partial \mathbf{U}^h}{\partial t} \phi(\mathbf{x}) d\mathbf{x} + \oint_{\partial \Omega_k} \overline{\mathcal{F}[\mathbf{W}^h(\mathbf{U})] \cdot \mathbf{n}} \phi(\mathbf{x}) ds - \int_{\Omega_k} \mathcal{F}[\mathbf{W}^h(\mathbf{U})] \cdot \nabla \phi(\mathbf{x}) d\mathbf{x} = 0, \quad \forall \phi \in \{\phi_i\}$$

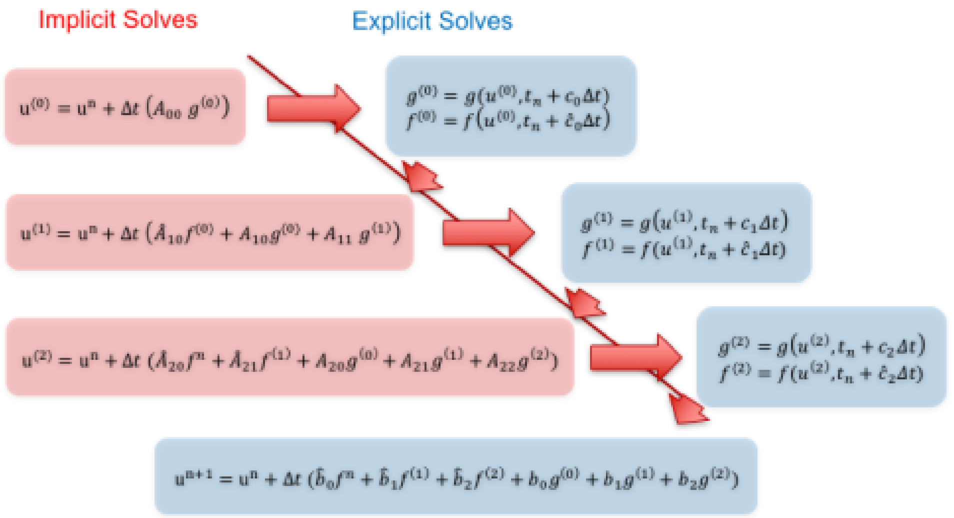
- Surface integrals imply Riemann Solvers to compute fluxes
- Nodal basis for fluid, edge basis for electric fields, face basis for magnetic fields
  - Gauss's Law and divergence free magnetic field enforced to machine precision by polynomial bases

Implicit-Explicit (IMEX) Time Integration & stiff modes in relativistic plasmas

- IMEX methods split fast and slow modes
  - Implicit terms solve for stiff modes (plasma oscillation, collisions, cyclotron frequency)
  - Explicit terms are accurately resolved (all of CoM physics)
  - IMEX assumes an additive decomposition: $\dot{\mathcal{U}} + F(\mathcal{U}) + G(\mathcal{U}) = 0$



3 Stage IMEX-RK Algorithm



$$\frac{\partial}{\partial t}\left[\sum_s \mu_s\rho_s\gamma_s\right] + \nabla\cdot\left[\sum_s \mu_s\rho_s\mathbf{u}_s\right] = 0$$

$$\frac{\partial}{\partial t}\left[\sum_s \mu_s\rho_s h_s\gamma_s\mathbf{u}_s\right] + \nabla\cdot\left[\sum_s\left(\mu_s\rho_s h_s\mathbf{u}_s\mathbf{u}_s + \mu_s P_s\mathbb{I}\right)\right] = \omega_P^2\left(\gamma\mathbf{E} + \frac{\mathbf{u}}{c}\times\mathbf{B}\right) + \sum_s \mu_s\mathbf{R}_s$$

$$\frac{\partial}{\partial t}\left[\sum_s\left(\mu_s\rho_s h_s\gamma_s^2 - \mu_s P_s\right)\right] + \nabla\cdot\left[\sum_s \mu_s\rho_s h_s\gamma_s\mathbf{u}_s\right] = \omega_P^2\mathbf{u}\cdot\mathbf{E} + \sum_s \mu_s R_s^0$$

$$\frac{\partial\mathbf{E}}{\partial t} = c\nabla\times\mathbf{B} - \sum_s \mu_s\rho_s\mathbf{u}_s$$

Stiff Modes:

- Plasma., Oscillation (blue)
- Collisions (green)
- Cyclotron frequency (red)

Non-relativistic vs. Relativistic Hydrodynamics

Non-Relativistic Conserved Variables:

$$\boldsymbol{U} = \begin{bmatrix} D \\ \boldsymbol{M} \\ E \end{bmatrix} = \begin{bmatrix} \rho \\ \rho \boldsymbol{v} \\ \rho\left(\frac{1}{2}|\boldsymbol{v}|^2 + e\right) \end{bmatrix}$$

Relativistic Conserved Variables:

$$\boldsymbol{U} = \begin{bmatrix} D \\ \boldsymbol{M} \\ E \end{bmatrix} = \begin{bmatrix} \gamma\rho \\ \gamma^2 \rho h \boldsymbol{v}/c^2 \\ \gamma^2 \rho h - P \end{bmatrix}$$

$$\gamma = \frac{1}{\sqrt{1 - |\boldsymbol{v}/c|^2}} \qquad h = \frac{e + P}{\rho}$$

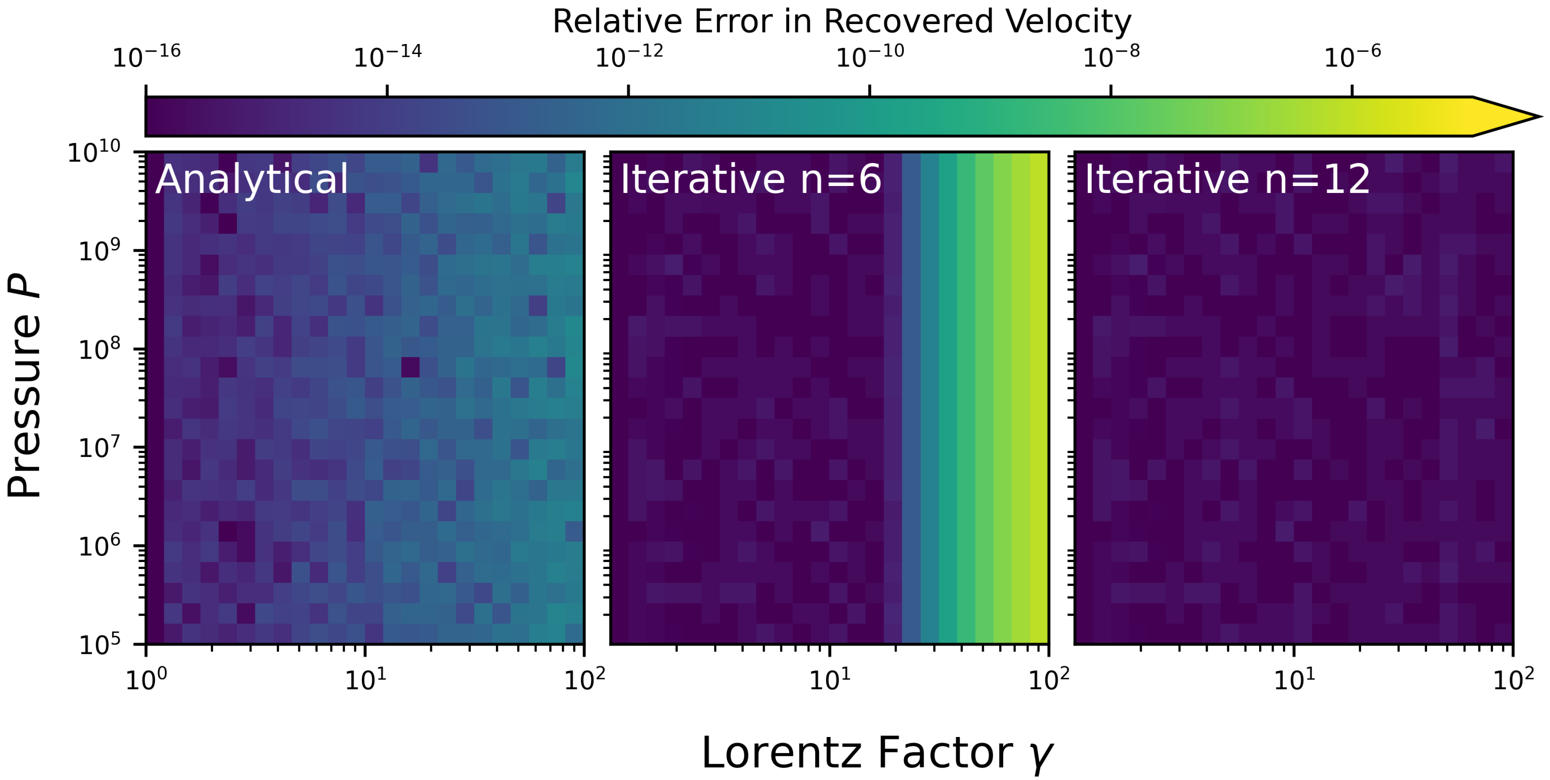
- Non-relativistic Hydrodynamics
  - Density, Momentum Density, Energy Density
  - Coupling between velocity and momentum is linear
- Relativistic Hydrodynamics
  - Relativistic mass density, relativistic momentum density, Energy Density (including rest mass)
  - Everything coupled though non-linear Lorentz factor
- Challenges:
  - I. Conserved to Primitive conversion is error-sensitive
    - High Lorentz factors => Velocity asymptotes to speed of light
    - Small errors in velocity lead to larger error in Lorentz factor or breaking of causality
  - II. Not all conserved variables are physical
    - Velocity must be subluminal, pressure and density positive
    - Not all conserved states have a primitive state
- High Lorentz factors and relativistic temperatures require robust methods

Conserved to Primitive: Solving Analytically vs. Iteratively

Convert $D, \boldsymbol{M}, E \rightarrow \rho, \boldsymbol{v}, P$

$$\boldsymbol{U} = \begin{bmatrix} D \\ \boldsymbol{M} \\ E \end{bmatrix} = \begin{bmatrix} \gamma\rho \\ \gamma^2 \rho h \boldsymbol{v}/c^2 \\ \gamma^2 \rho h - P \end{bmatrix}$$

$$\gamma = \frac{1}{\sqrt{1 - |\boldsymbol{v}/c|^2}} \quad h = \frac{e + P}{\rho}$$



Solution 1: Solve quartic polynomial analytically for velocity

- Square roots, inverse trigonometry
- Machine precision can lead to superluminal velocities
- Small errors in velocity translate into large errors in Lorentz factor, other primitives

Solution 2 Solve quartic polynomial iteratively with Newton-Raphson for

$$v = c\frac{2w}{1 + w^2}$$

- Arbitrary accuracy with enough iterations
- Robust and accurate without square roots and inverse trigonometry
- Simplicity leads to faster execution, especially for non-relativistic flows

Physicality Enforcing Operator

- Not all conserved states are physical:
  - Second order methods and shocks can lead to non-physical conserved states
  - Conserved variables must satisfy:

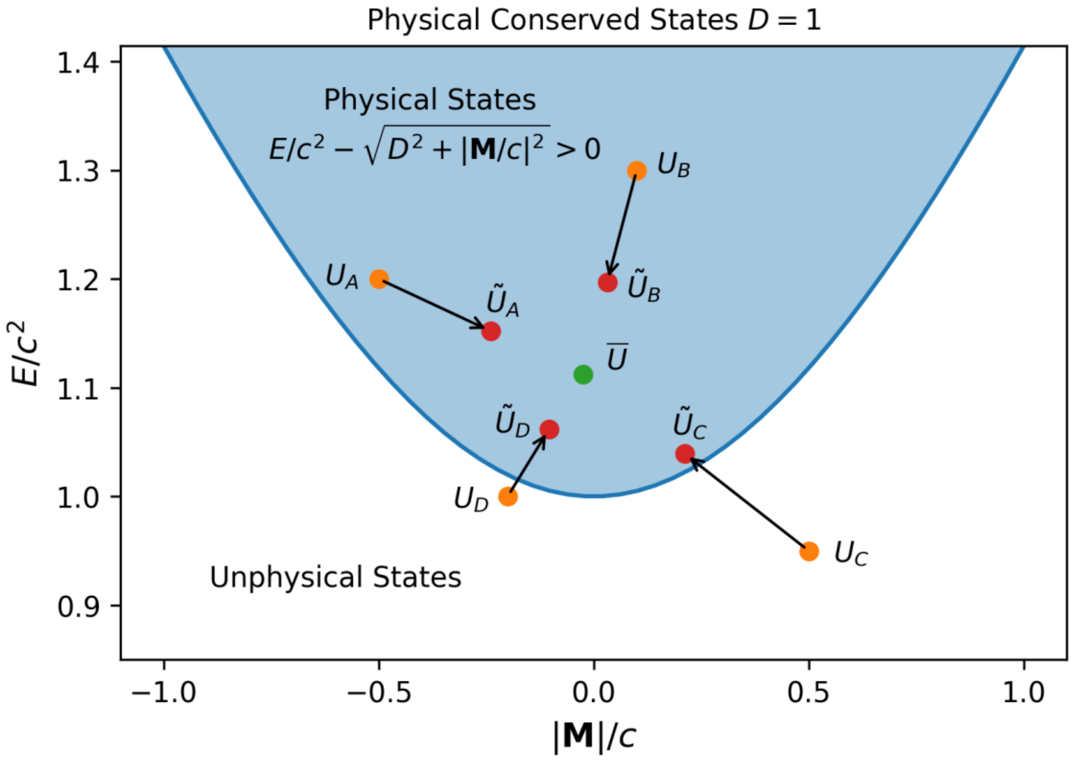
$$E > 0, \qquad D > 0, \qquad E/c^2 - \sqrt{D^2 + |\boldsymbol{M}/c|^2} > 0$$

- Set of physical conserved states is convex
  - If the cell volume average is physical, unphysical nodal points can be smoothed towards average

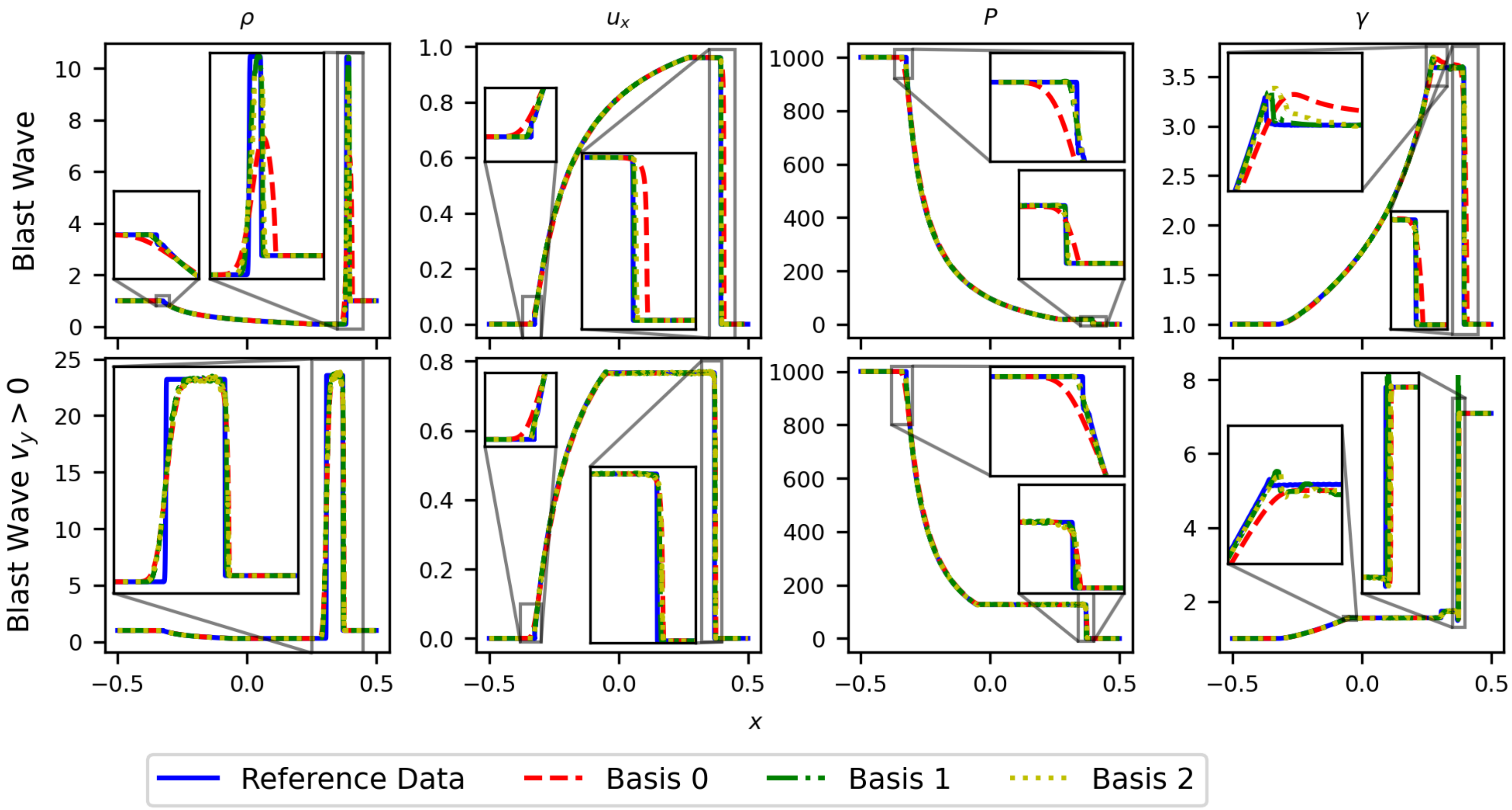
Physicality Enforcing Operator

1. Cells with unphysical nodal points are flagged
2. For each unphysical nodal point, we compute the least amount of averaging required
3. For each flagged cell, the least amount of averaging required for all points is applied

- Preserves volume average of conserved state
- Does not affect physical cells



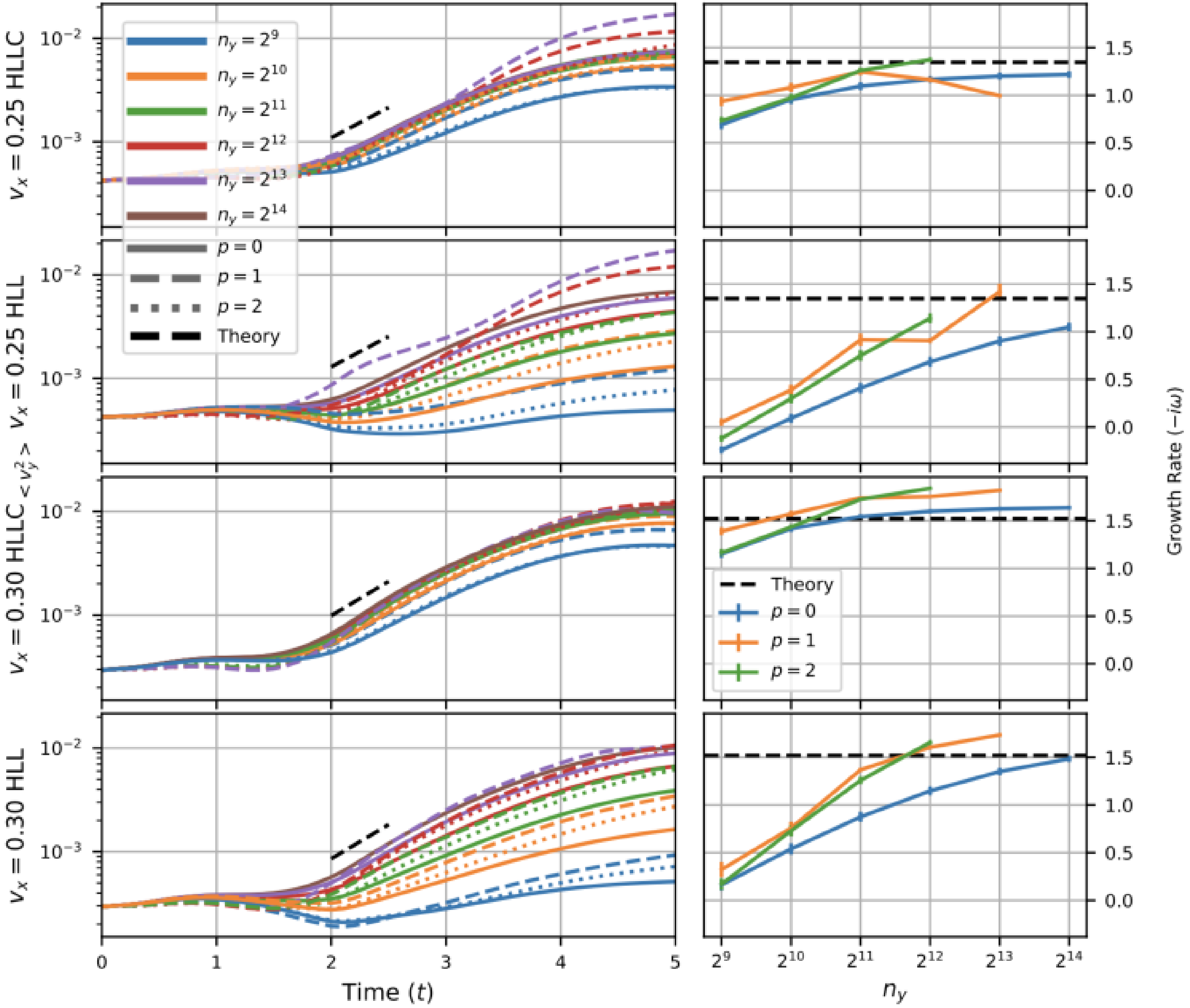
Single Fluid 1D Shocks



- Transverse velocity changes Lorentz factor, density, and pressure
- Conserved to Primitive solver enables high Lorentz factor
- Physicality Enforcing Operator handles low pressures

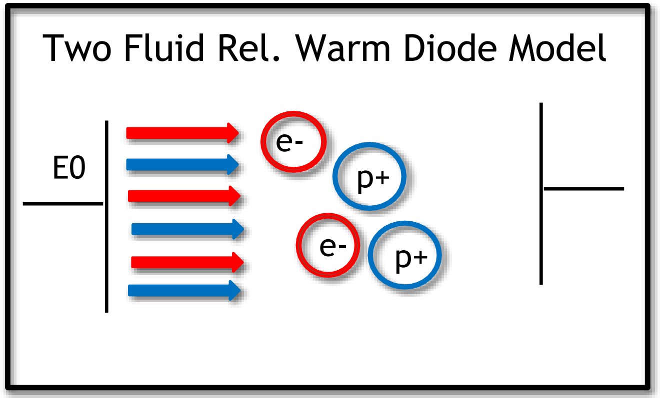
Relativistic Kelvin-Helmholtz Instability

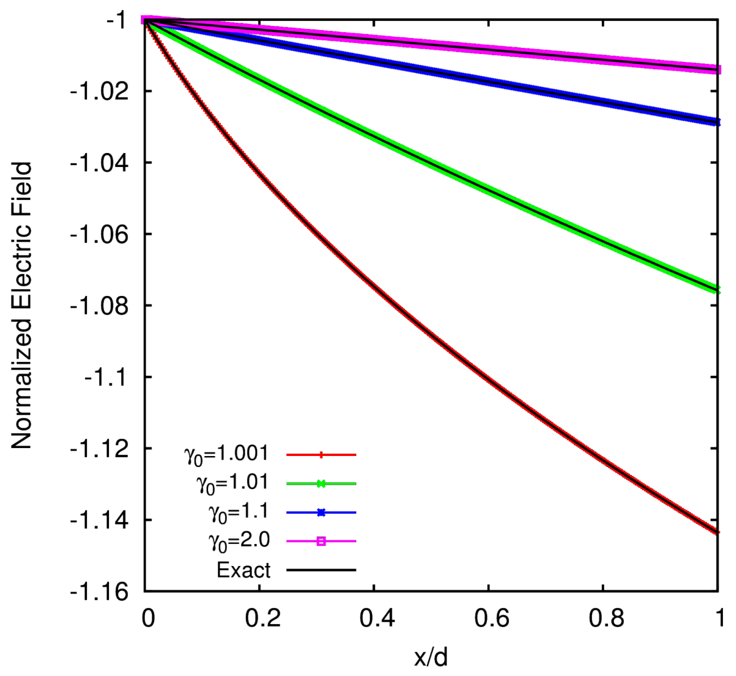
- Suite of Kelvin-Helmholtz simulations (using Bodo 2004)
  - Probing resolution, method order, Reimann solver
  - For different shear velocities
- Compared to analytic growth rate
- Riemann solver makes the biggest difference

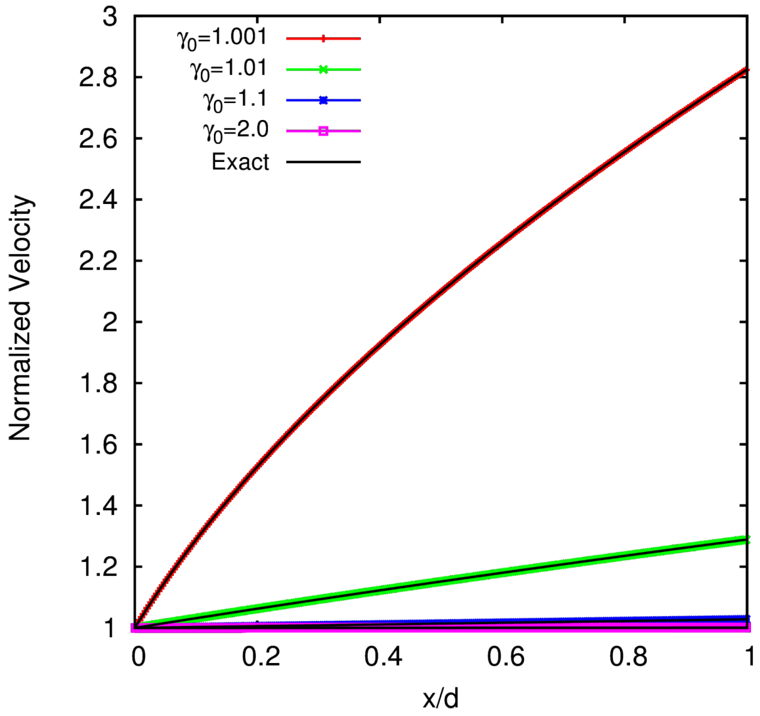


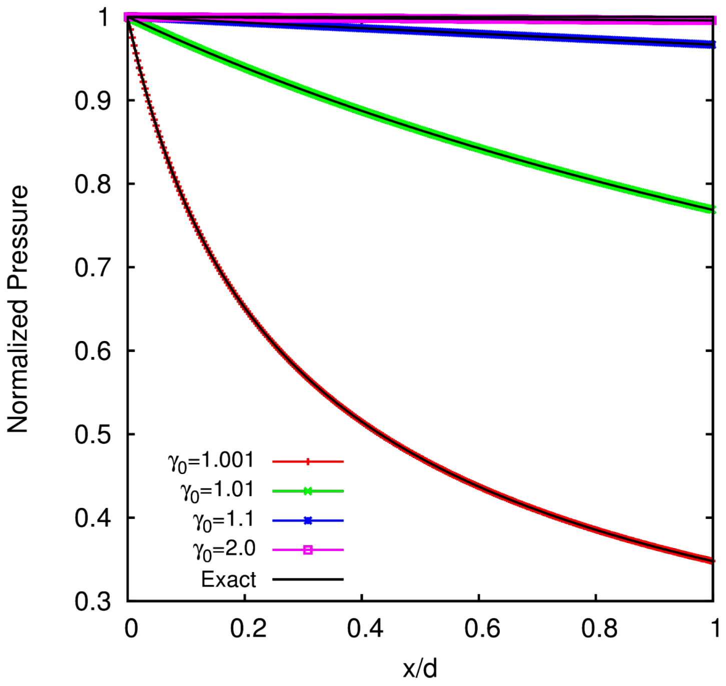
Two Fluid Test: Two-Fluid Relativistic Warm Diode (N.D. Hamlin)

- Two-Fluid Warm Diode
  - 1D Electrostatic Problem with analytic solution
  - Warm beam of charged particles with relativistic velocity from one side
  - More detail in presentation "Using Diode Simulations to Verify Plasma Physics Codes," T.M. Smith and K. L. Cartwright
    - Wednesday 10:30am in Computational Plasma Physics #403 7O-A-03









Current Work

- Reimplementing implicit step to enable multiple ions and minimize primitive conversions to improve accuracy
- Apply relativistic two-fluid electrodynamics methods to astrophysical relativistic jets and terrestrial power flows

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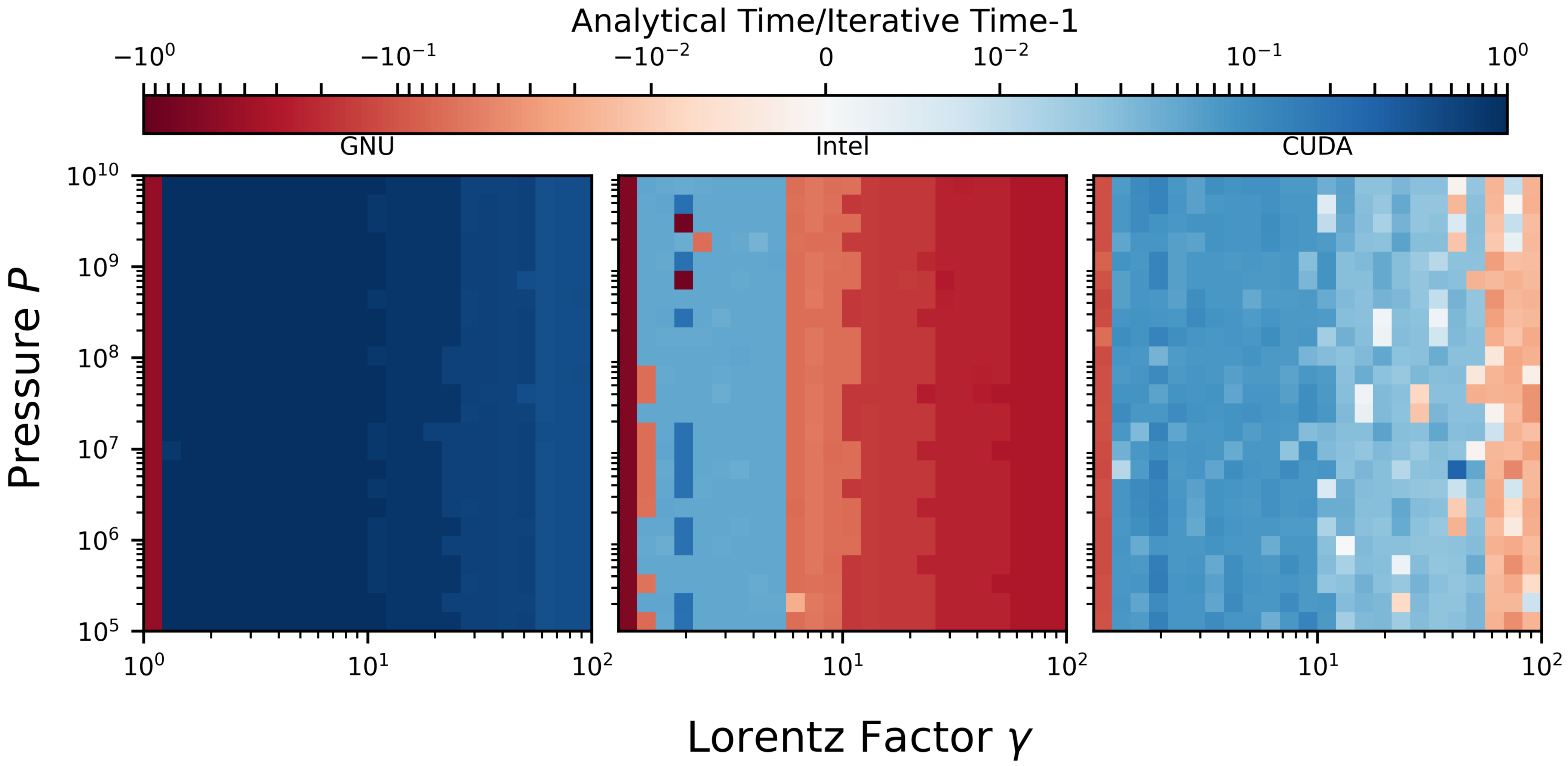
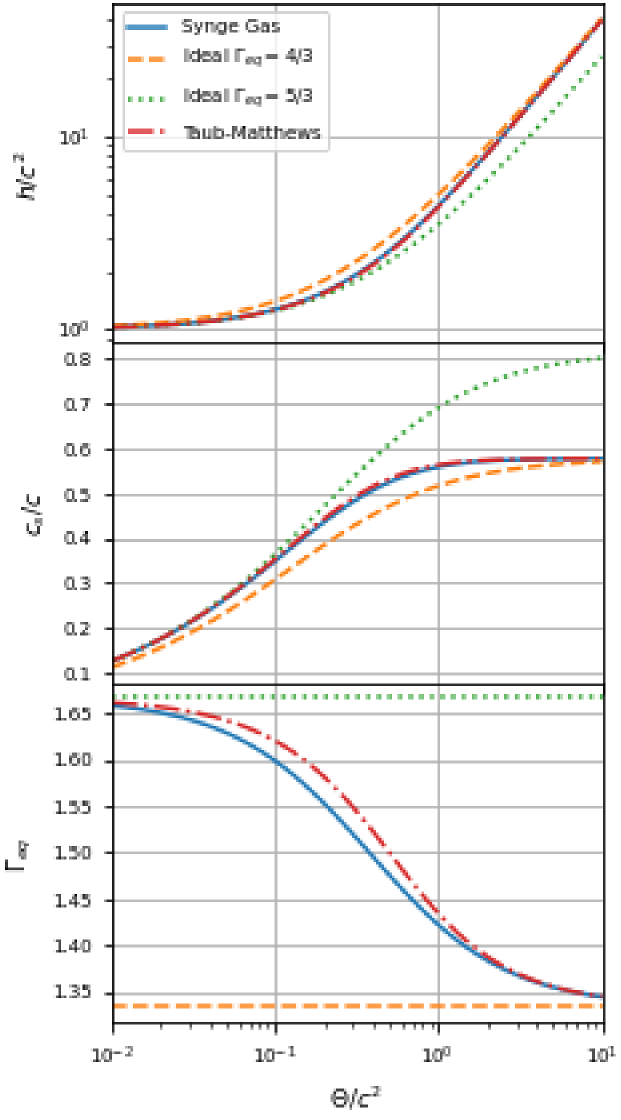
Backup Slides

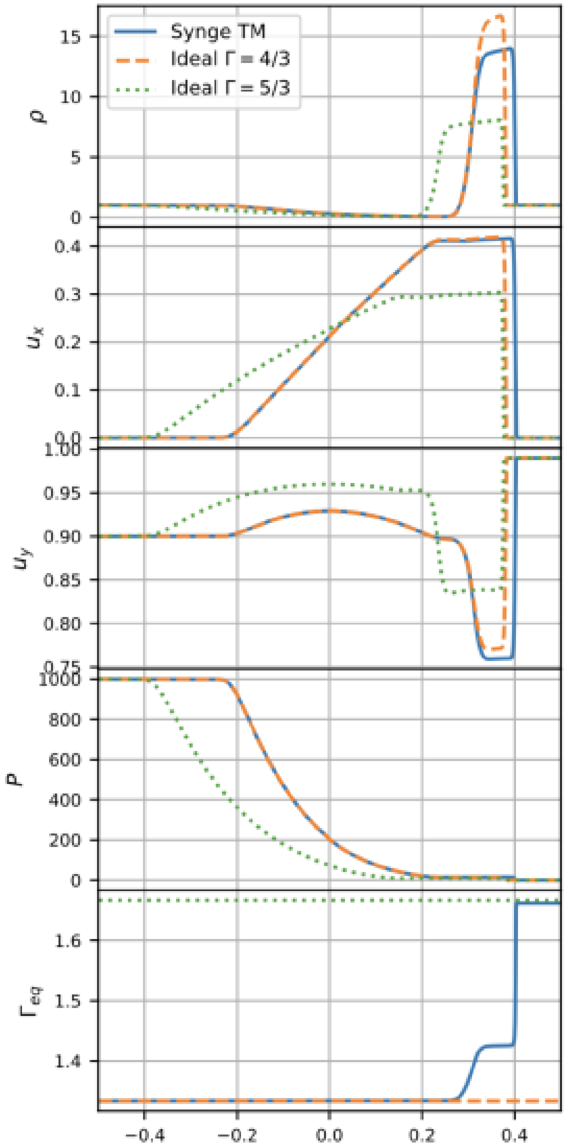
Iterative method can be faster than analytical method



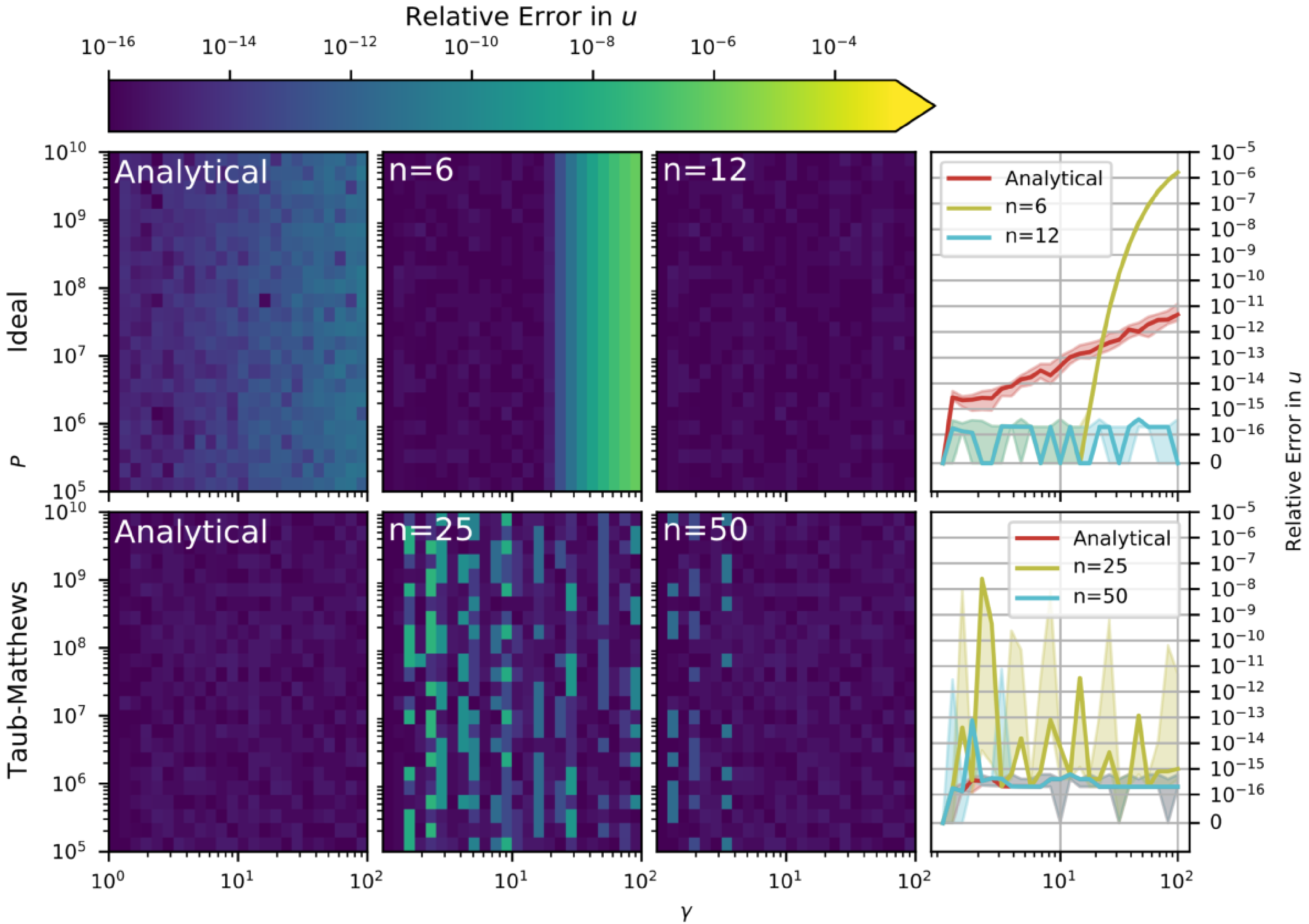
Synge Gas

- Adiabatic index of a perfect gas varies from 5/3 to 4/3 for sub-relativistic to relativistic temperatures
- Synge gas correctly models perfect gas
  - Requires Bessel functions, Inverse Bessel functions
- Taub-Matthews approximates Synge Gas

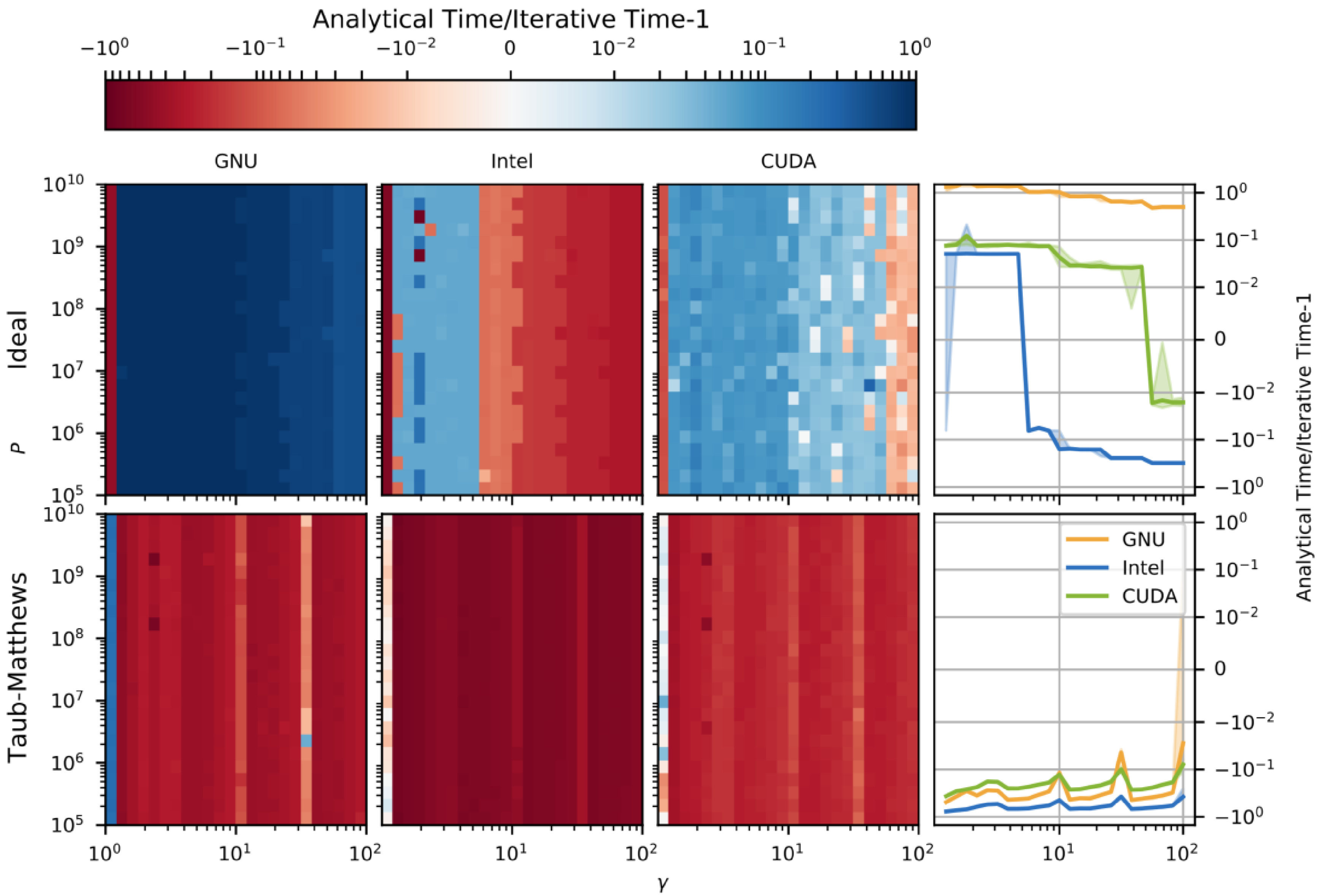




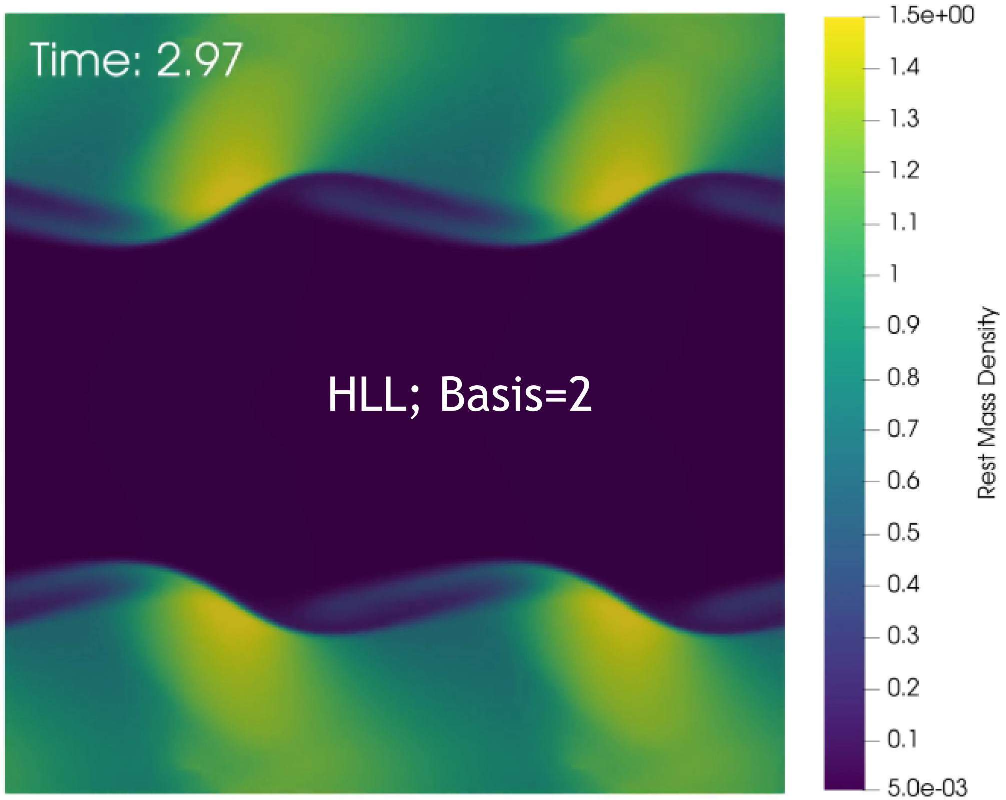
Ideal and Taub-Matthews Solver Accuracy

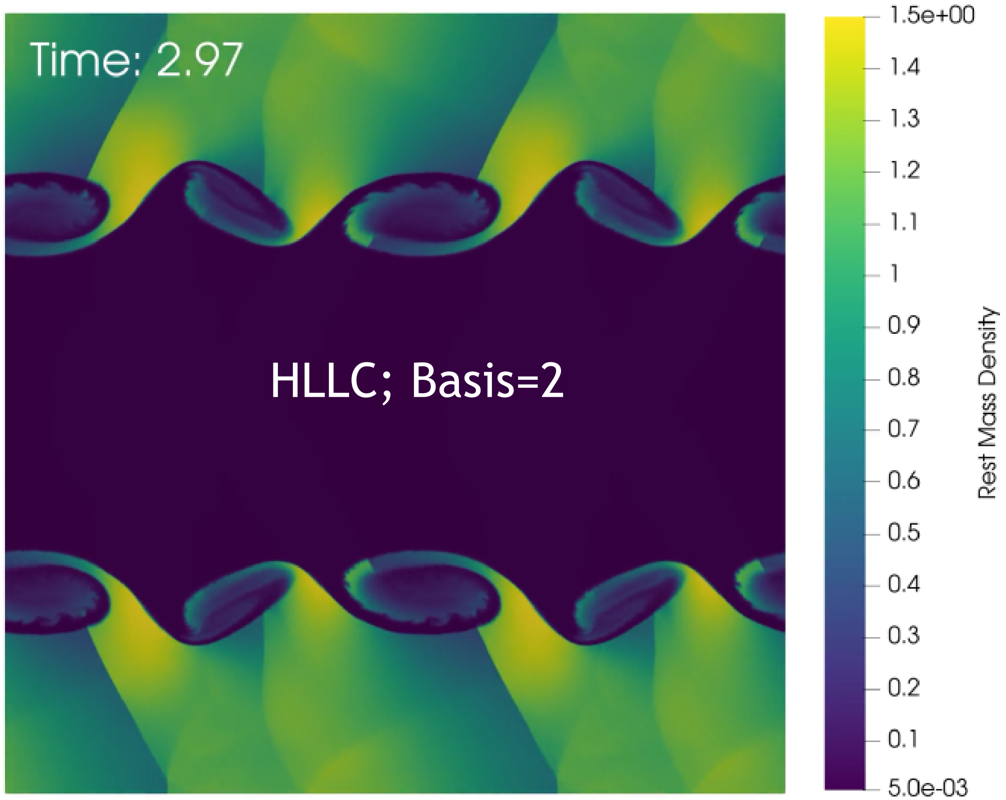


Synge Gas Performance



Kelvin Helmholtz Instability





2D Riemann Problems