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Temperature and density analysis and their uncertainties

PRESENTED BY

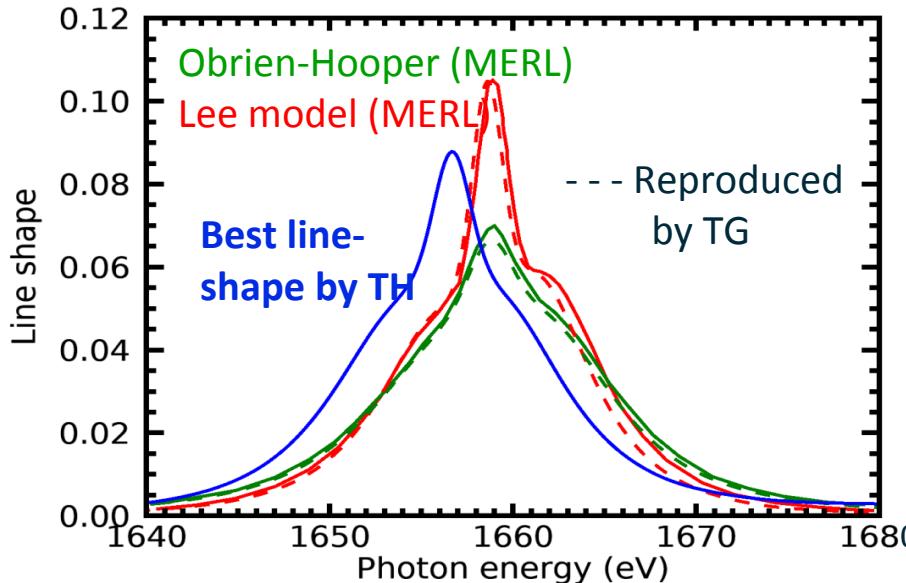
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Sample temperature and density need to be reanalyzed due to recent refinements in line shapes and background inference



Line-shape refinements [1,2]

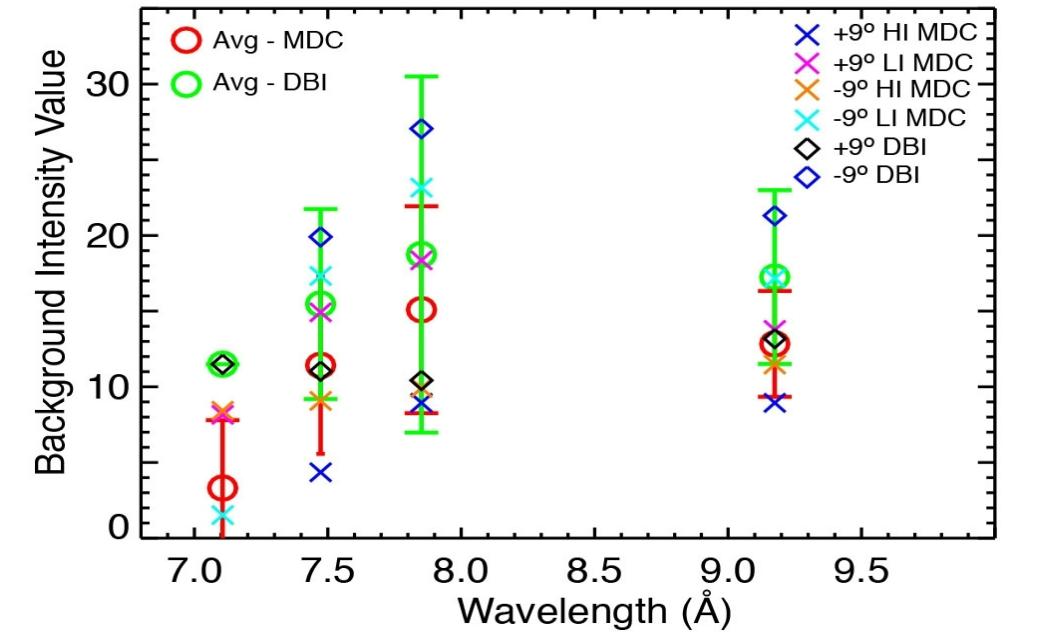
- Electron capture
- Removing 3 approximations



Affects n_e by up to 30%
Affects T_e by a few %

Background refinements [3]

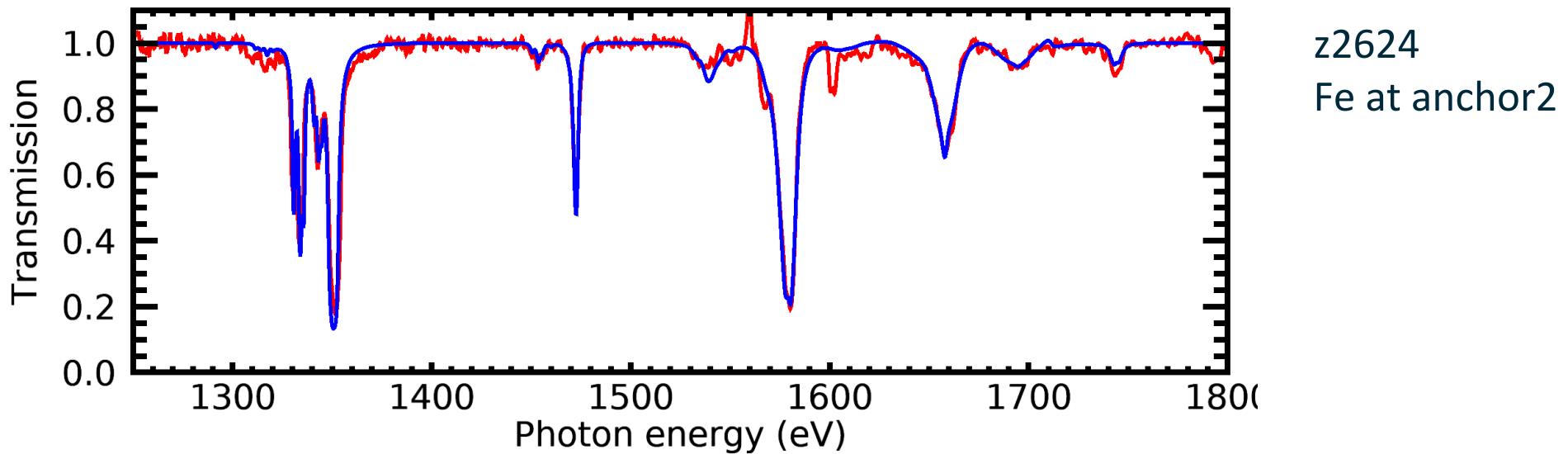
- Method1: Dual backlight intensity (DBI)
- Method2: Modal-data comparison (MDC)



Affect both n_e and T_e for strong lines (e.g., $He\beta$)

Need to revisit Te and ne analysis, but let's revisit analysis method itself

I've been analyzing all lines simultaneously and infer T_e , n_e , ρL , and background simultaneously



$$T_e = 179.7 \pm 0.5 \text{ eV (0.2\%)}$$

$$n_e = (3.53 \pm 0.05) \times 10^{22} \text{ eV (1.4\%)}$$

$$\rho L = (6.2 \pm 0.1) \times 10^{-5} \text{ g/cm}^2 \text{ (1.6\%)}$$

$$\text{background} = 6.0 \pm 0.6 \text{ J/sr/Å (10\%)}$$

Final fits look great, but this analysis produces too small uncertainties, raising concern in uncertainty quantification

Uncertainties inferred from χ^2 analysis (or Bayesian analysis) are often too small when *assumptions* are inappropriate

Assumptions

- Experiment and data reduction are perfect
- Background treatment is perfect
- Uncertainty is dominated by random noise
- Plasma condition is spatially and temporally uniform
- Spectral model is perfect
 - Atomic data are perfect
 - Line-broadening model is perfect
 - Continuum lowering is perfect

Whenever the assumptions are invalid, the analysis shows inconsistency.

→ This inconsistency artificially reduce uncertainty.

Uncertainties inferred from χ^2 analysis (or Bayesian analysis) are often too small when *assumptions* are inappropriate

Assumptions

- Experiment and data reduction are perfect → We know they are not perfect
- Background treatment is perfect → We don't know where background is from
- Uncertainty is dominated by random noise → There are systematic uncertainties
- Plasma condition is spatially and temporally uniform → There is axial gradient that changes with time
- Spectral model is perfect → Different spectral model gives different answers → Nagayama HEDP (2016)
 - Atomic data are perfect
 - Line-broadening model is perfect
 - Continuum lowering is perfect

Reality

**Whenever the assumptions are invalid, the analysis shows inconsistency.
→ This inconsistency artificially reduce uncertainty.**

Jim took more conservative approach in 2008*



Different lines suggest different n_e

$$\begin{aligned} \text{He}\beta &\rightarrow 9.6 \times 10^{21} \text{ cm}^{-3} \\ \text{He}\gamma &\rightarrow 6.1 \times 10^{21} \text{ cm}^{-3} \\ \text{He}\delta &\rightarrow 4.9 \times 10^{21} \text{ cm}^{-3} \\ &\rightarrow (6.9 \pm 1.7) \times 10^{21} \text{ cm}^{-3} \end{aligned}$$

Different line ratios suggest different T_e

$$\begin{aligned} &\text{Ly}\alpha/\text{He}\beta \quad \text{Ly}\alpha/\text{He}\gamma \quad \text{Ly}\alpha/\text{He}\delta \\ &\text{Ly}\beta/\text{He}\beta \quad \text{Ly}\beta/\text{He}\gamma \quad \text{Ly}\beta/\text{He}\delta \\ &\rightarrow 156 \pm 6 \text{ eV} \end{aligned}$$

Since they give inconsistent results, he averaged them together.

Key question: Don't we get more accurate result if we analyze them together?

→ Answer depends on:

- If they are giving consistent answers
- If not, why they are giving inconsistent answers

All line analysis under this circumstance results in *some average* with very small uncertainties



Individual analysis:

Heb: $(3.55 \pm 0.05) \text{e}22$ (1.4%)

Heg: $(4.21 \pm 0.10) \text{e}22$ (2.4%)

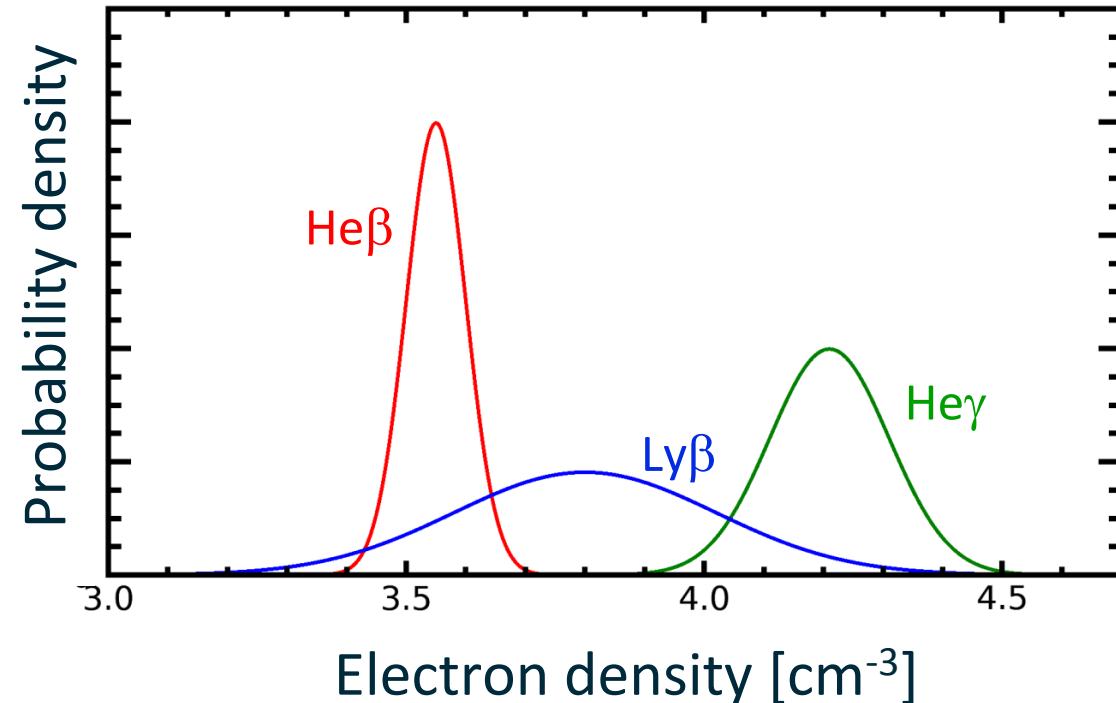
Lyb: $(3.80 \pm 0.22) \text{e}22$ (5.8%)

What's happening?

Simultaneous analysis:

Heb, Heg, Lyb together

$(3.68 \pm 0.04) \text{e}22$ (1.1%) cm^{-3}



Both χ^2 and Bayesian analyses use the likelihood and thus can introduce this artificial underestimate of uncertainty *when model is inappropriate*

All line analysis under this circumstance results in *some average* with very small uncertainties



Individual analysis:*

He β : $(3.55 \pm 0.05) \times 10^{22}$ (1.4%)

He γ : $(4.21 \pm 0.10) \times 10^{22}$ (2.4%)

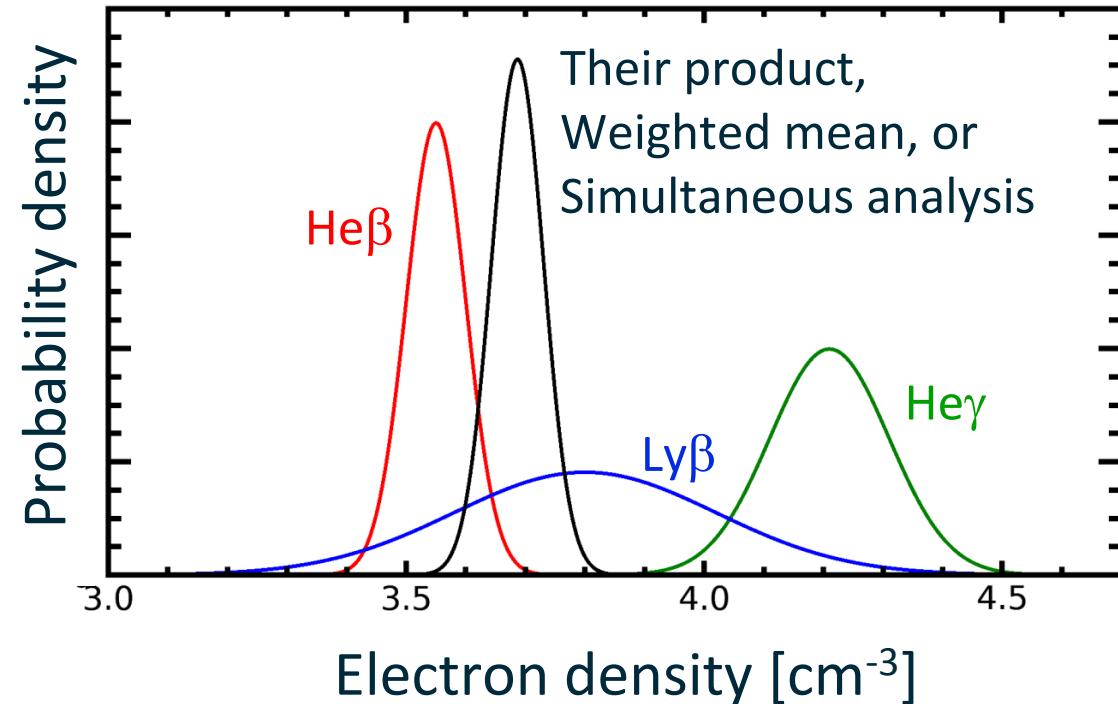
Ly β : $(3.80 \pm 0.22) \times 10^{22}$ (5.8%)

What's happening?

Simultaneous analysis:

He β , He γ , Ly β together

$(3.68 \pm 0.04) \times 10^{22}$ (1.1%) cm^{-3}



Both χ^2 and Bayesian analyses use the likelihood and thus can introduce this artificial underestimate of uncertainty *when model is inappropriate*

Our new approach quasi-isolate various dependencies, include inconsistencies into the parameter uncertainties



Step1: Background: Determine and subtract background from the data

Step2: n_e : Analyze Mg He β , He γ , Ly β line widths

Step3: ρL : Analyze Mg He β and He γ line depths

Step4: T_e : Analyze 11 line ratios

Advantage:

- This accounts for various uncertainties including uncertainty due to assumptions
- We may be able to identify issues from too large uncertainties

Pros and cons for each approach



Individual analysis:

Pros:

- Physics we rely on is clear --> More insightful
- Account for inconsistencies into uncertainty

Cons:

- Harder to account correlation between parameters and between objectives

Simultaneous analysis

Pros:

- Account for correlation between parameters and between objectives
- Statistically more accurate if dominant source of uncertainty is statistical noise

Cons:

- Physics we rely on is less clear --> Less insightful
- Cannot account for inconsistencies into uncertainties