



# Temperature and density analysis and their uncertainties

*PRESENTED BY*

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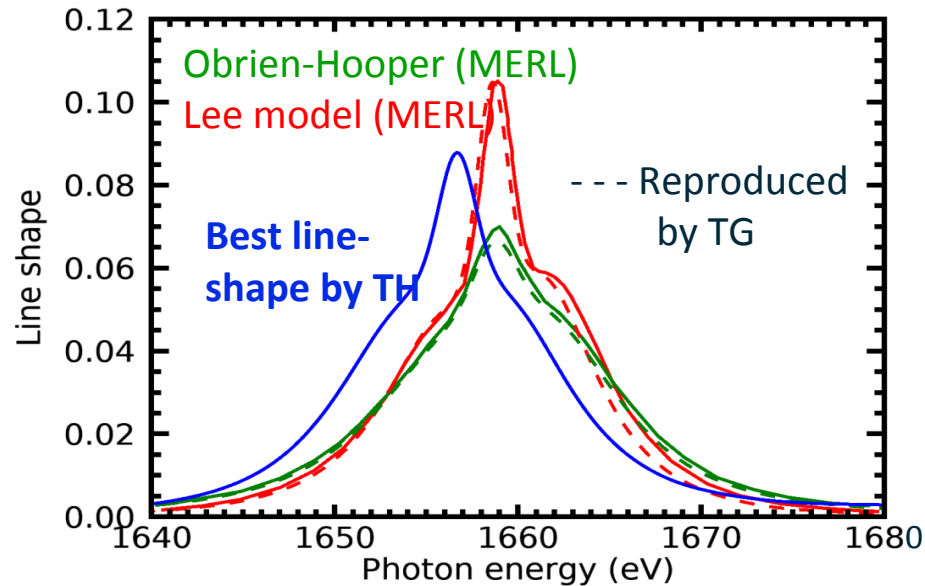
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# Sample temperature and density need to be reanalyzed due to recent refinements in line shapes and background inference



## Line-shape refinements [1,2]

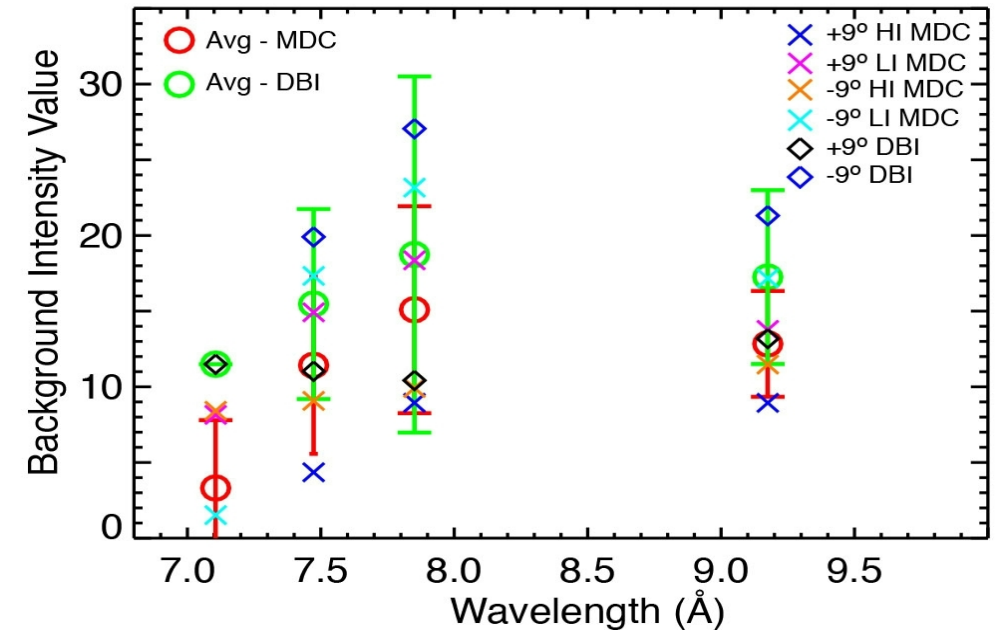
- Electron capture
- Removing 3 approximations



*Affects  $n_e$  by up to 30%*  
*Affects  $T_e$  by a few %*

## Background refinements [3]

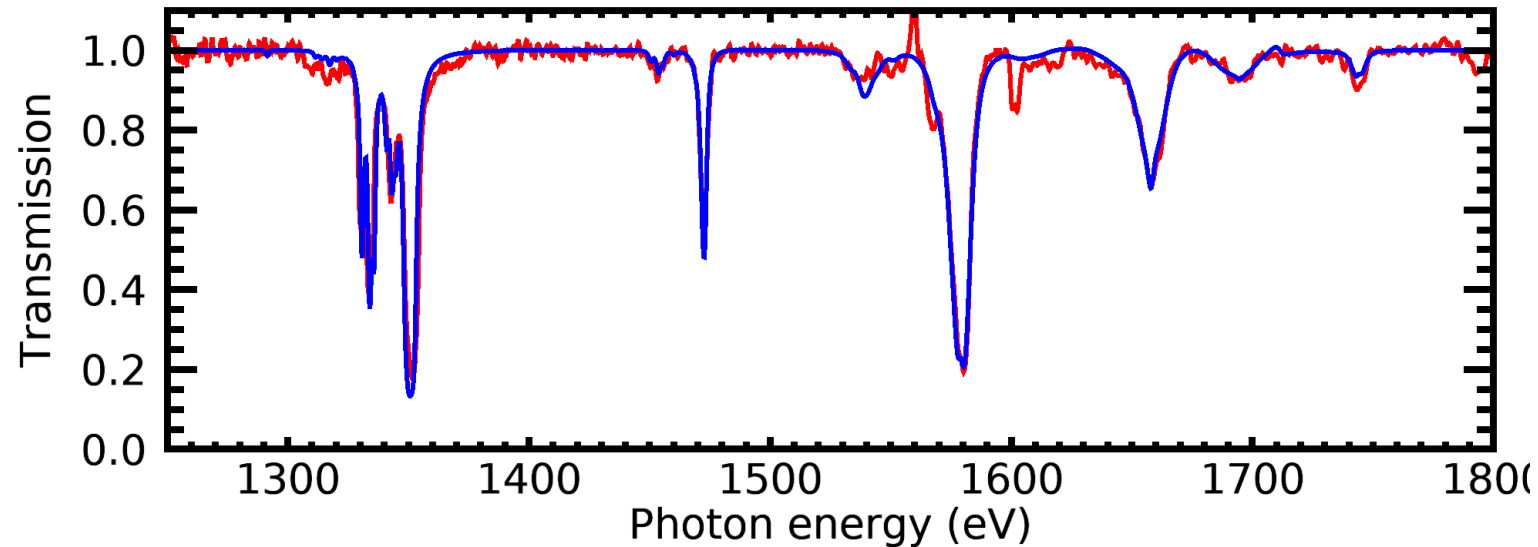
- Method1: Dual backlight intensity (DBI)
- Method2: Modal-data comparison (MDC)



*Affect both  $n_e$  and  $T_e$  for strong lines (e.g.,  $\text{He}\beta$ )*

**Need to revisit  $T_e$  and  $n_e$  analysis, but let's revisit analysis method itself**

I've been analyzing all lines simultaneously and infer  $T_e$ ,  $n_e$ ,  $\rho L$ , and background simultaneously



z2624  
Fe at anchor2

$$T_e = 179.7 \pm 0.5 \text{ eV (0.2\%)}$$

$$n_e = (3.53 \pm 0.05) \times 10^{22} \text{ eV (1.4\%)}$$

$$\rho L = (6.2 \pm 0.1) \times 10^{-5} \text{ g/cm}^2 \text{ (1.6\%)}$$

$$\text{background} = 6.0 \pm 0.6 \text{ J/sr/\AA (10\%)}$$

Final fits look great, but this analysis produces too small uncertainties, raising concern in uncertainty quantification

Uncertainties inferred from  $\chi^2$  analysis (or Bayesian analysis) are often too small when *assumptions* are inappropriate

### *Assumptions*

- Experiment and data reduction are perfect
- Background treatment is perfect
- Uncertainty is dominated by random noise
- Plasma condition is spatially and temporally uniform
- Spectral model is perfect
  - Atomic data are perfect
  - Line-broadening model is perfect
  - Continuum lowering is perfect

**Whenever the assumptions are invalid, the analysis shows inconsistency.**

**→ This inconsistency artificially reduce uncertainty.**

# Uncertainties inferred from $\chi^2$ analysis (or Bayesian analysis) are often too small when *assumptions* are inappropriate

## *Assumptions*

- | <i>Assumptions</i>                                     |   | <i>Reality</i>  |
|--|---|---|
| - Experiment and data reduction are perfect            | → | We know they are not perfect  |
| - Background treatment is perfect                      | → | We don't know where background is from                                  |
| - Uncertainty is dominated by random noise             | → | There are systematic uncertainties                                      |
| - Plasma condition is spatially and temporally uniform | → | There is axial gradient that changes with time                          |
| - Spectral model is perfect                            | → | Different spectral model gives different answers → Nagayama HEDP (2016) |
| - Atomic data are perfect                              |   |   |
| - Line-broadening model is perfect                     |   |   |
| - Continuum lowering is perfect                        |   |   |

**Whenever the assumptions are invalid, the analysis shows inconsistency.**

**→ This inconsistency artificially reduce uncertainty.**

Jim took more conservative approach in 2008\*



Different lines suggest different  $n_e$

$$\left\{ \begin{array}{l} \text{He}\beta \rightarrow 9.6 \times 10^{21} \text{ cm}^{-3} \\ \text{He}\gamma \rightarrow 6.1 \times 10^{21} \text{ cm}^{-3} \\ \text{He}\delta \rightarrow 4.9 \times 10^{21} \text{ cm}^{-3} \end{array} \right. \rightarrow (6.9 \pm 1.7) \times 10^{21} \text{ cm}^{-3}$$

Different line ratios suggest different

$$\begin{array}{l} \overline{T_e} \left\{ \begin{array}{l} \text{Ly}\alpha/\text{He}\beta \quad \text{Ly}\alpha/\text{He}\gamma \quad \text{Ly}\alpha/\text{He}\delta \\ \text{Ly}\beta/\text{He}\beta \quad \text{Ly}\beta/\text{He}\gamma \quad \text{Ly}\beta/\text{He}\delta \end{array} \right. \\ \rightarrow 156 \pm 6 \text{ eV} \end{array}$$

Since they give inconsistent results, he averaged them together.

**Key question: Don't we get more accurate result if we analyze them together?**

**→ Answer depends on:**

- If they are giving consistent answers
- If not, why they are giving inconsistent answers

All line analysis under this circumstance results in *some average* with very small uncertainties



Individual analysis:

Heb:  $(3.55 \pm 0.05) \times 10^{22}$  (1.4%)

Heg:  $(4.21 \pm 0.10) \times 10^{22}$  (2.4%)

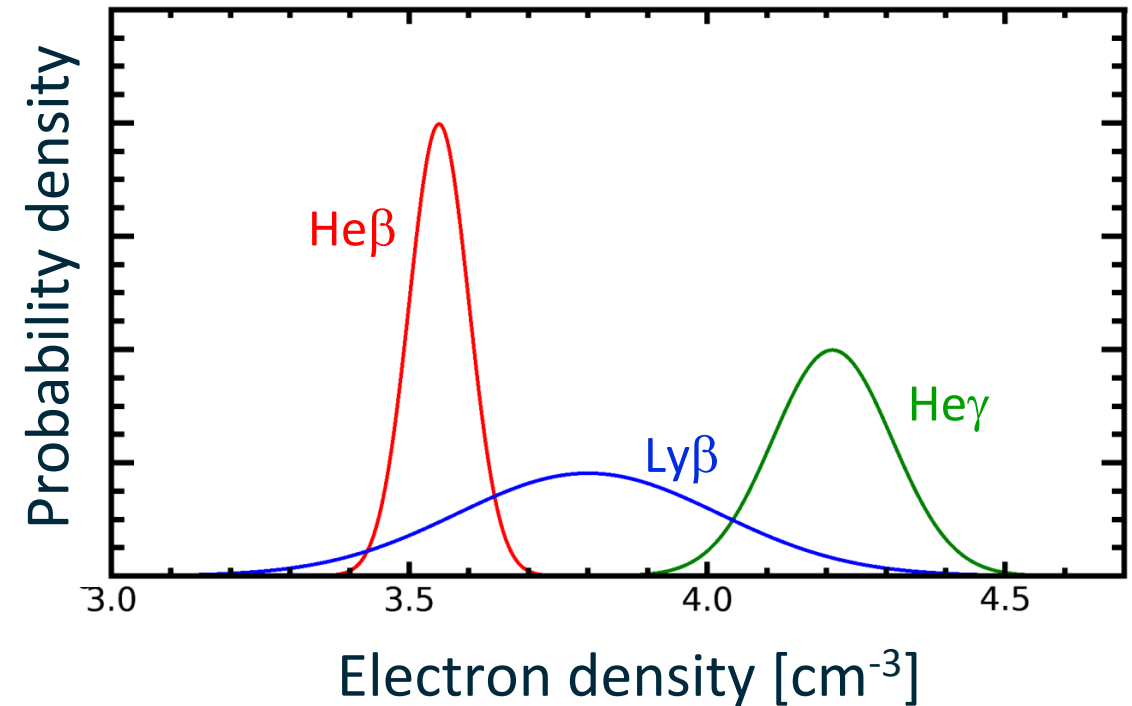
Lyb:  $(3.80 \pm 0.22) \times 10^{22}$  (5.8%)

What's  
happening?



Simultaneous analysis:

Heb, Heg, Lyb together  
 $(3.68 \pm 0.04) \times 10^{22}$  (1.1%)  $\text{cm}^{-3}$



Both  $\chi^2$  and Bayesian analyses use the likelihood and thus can introduce this artificial underestimate of uncertainty *when model is inappropriate*

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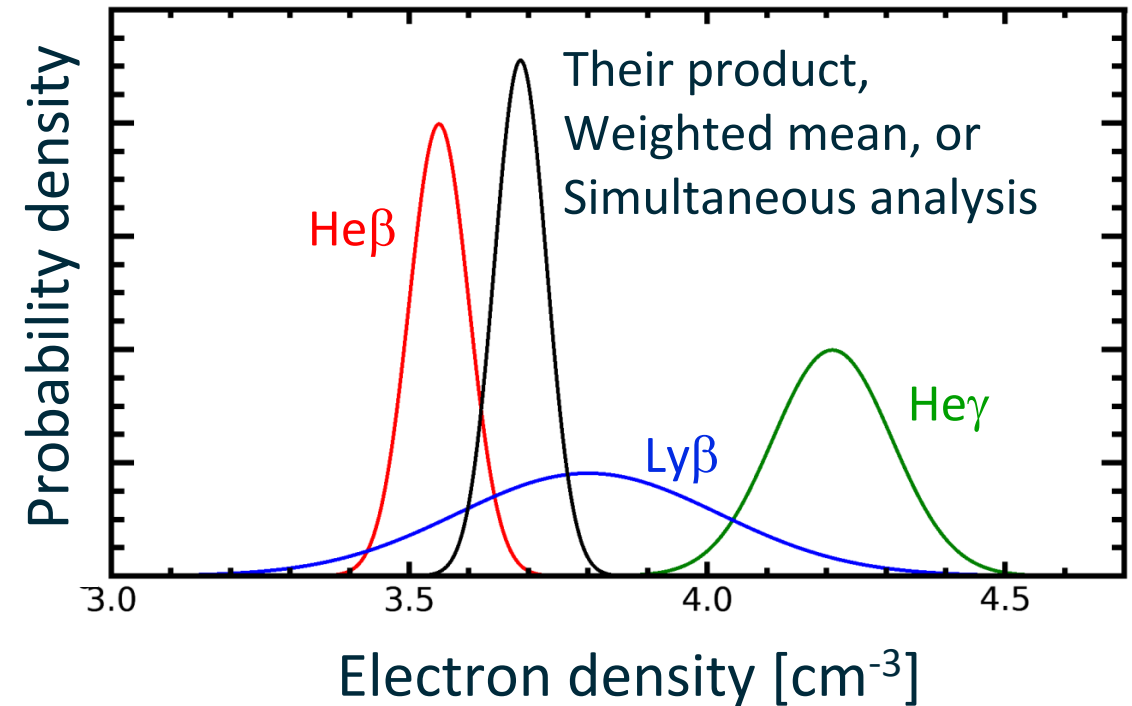
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Our new approach quasi-isolate various dependencies,  
include inconsistencies into the parameter uncertainties



**Step1: Background:** Determine and subtract background from the data

**Step2:  $n_e$ :** Analyze Mg He $\beta$ , He $\gamma$ , Ly $\beta$  line widths

**Step3:  $\rho_L$ :** Analyze Mg He $\beta$  and He $\gamma$  line depths

**Step4:  $T_e$ :** Analyze 11 line ratios

**Advantage:**

- This accounts for various uncertainties including uncertainty due to assumptions
- We may be able to identify issues from too large uncertainties

# Pros and cons for each approach



## Individual analysis:

### Pros:

- Physics we rely on is clear --> More insightful
- Account for inconsistencies into uncertainty

### Cons:

- Harder to account correlation between parameters and between objectives

## Simultaneous analysis

### Pros:

- Account for correlation between parameters and between objectives
- Statistically more accurate if dominant source of uncertainty is statistical noise

### Cons:

- Physics we rely on is less clear --> Less insightful
- Cannot account for inconsistencies into uncertainties