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A MACHINE LEARNING FRAMEWORK FOR ALLEVIATING BOTTLENECKS OF PROJECTION-BASED REDUCED ORDER MODELS

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Problem Statement

Physics-Based Reduced Order Modelling (ROM)

Real life system

Complex Dynamics

Nonlinear behaviour

Parametric dependency on:

- *Geometric features*
- *Material properties*
- *Environmental conditions*
- *Operational conditions*
- *Excitation*

High fidelity Finite Element model

Capture underlying dynamics

Reproduce physical behaviour

Retain parametric dependencies

Efficient evaluation

Virtual Representation

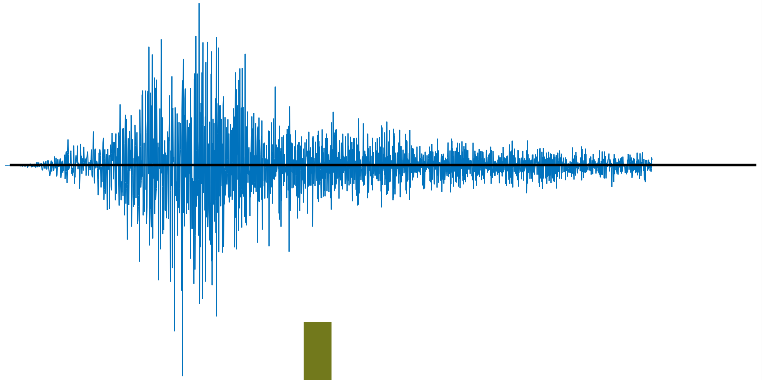
Reduced Order Model (ROM)

Problem Statement

Physics-Based Reduced Order Modelling (ROM)

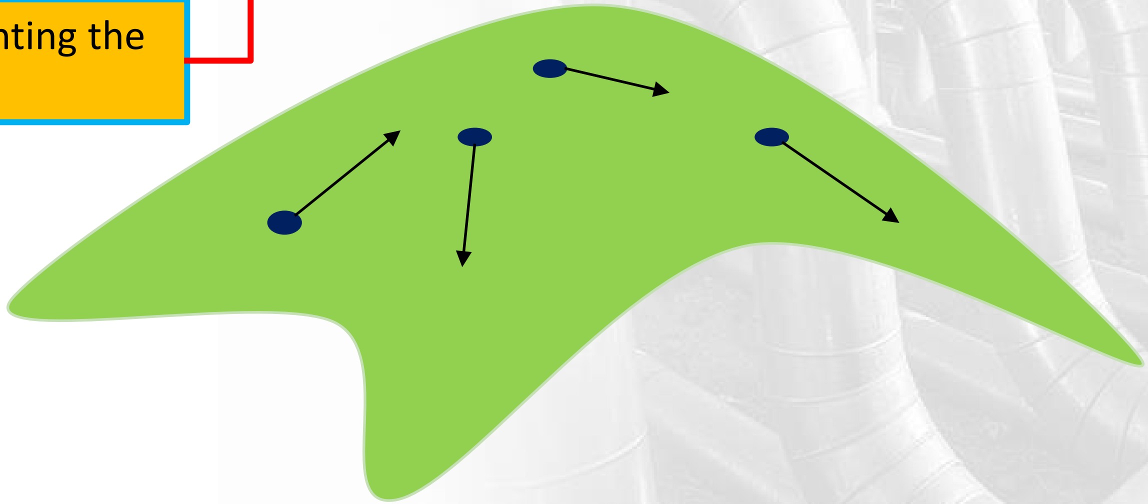
Real life system

Dynamic structural response



Try to extract components representing the **solution manifold S**

Physical Interpretation:
Dynamic response under any parametric state spans low-dimensional subspace ***S***



Problem Statement
Projection-Based Reduction

Assuming a general a nonlinear, parametric, dynamical system:

$$\mathbf{M}(\mathbf{p})\ddot{\mathbf{u}}(t) + \mathbf{g}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t), \mathbf{p}\right) = \mathbf{f}(t, \mathbf{p})$$

$$\mathbf{u}(t) \in \mathbb{R}^n, \mathbf{M}(\mathbf{p}) \in \mathbb{R}^{n \times n}, \mathbf{f}(t, \mathbf{p}) \in \mathbb{R}^n, \mathbf{g}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t)\right) \in \mathbb{R}^n$$

Parametric dependency on *k* parameters denoted by: $\mathbf{p} = [p_1, ..., p_k]^T \in \Omega \subset \mathbb{R}^k$

Relevant notation:

M is the system mass matrix

f is the vector of external loads

u is the response time history

g are the nonlinear, state-dependent internal forces

Problem Statement

Projection-Based Reduction

*The **goal of parametric ROM is to generate a low-dimensional, equivalent system** such that the underlying physics along with the parametric dependencies of interest are further retained.*

$$\mathbf{M}_{\mathrm{r}}(\mathbf{p}_j)\ddot{\mathbf{u}}_{\mathrm{r}}(t) + \mathbf{g}_{\mathrm{r}}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t), \mathbf{p}_j\right) = \mathbf{f}_{\mathrm{r}}(t, \mathbf{p}_j)$$

$$\mathbf{M}_{\mathrm{r}}(\mathbf{p}_j) \in \mathbb{R}^{r\times r}, \mathbf{g}_{\mathrm{r}}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t), \mathbf{p}_j\right) \in \mathbb{R}^{r}, \mathbf{f}_{\mathrm{r}}(t, \mathbf{p}_j) \in \mathbb{R}^{r}$$

$$r \ll n$$

Galerkin Projection Basis

$$\mathbf{u}(t) = \mathbf{V}(\mathbf{p}_j)\mathbf{u}_{\mathrm{r}}(t)$$

$$\mathbf{M}_{\mathrm{r}}(\mathbf{p}_j) = \mathbf{V}(\mathbf{p}_j)^T\mathbf{M}(\mathbf{p}_j)\mathbf{V}(\mathbf{p}_j)$$

$$\mathbf{f}_{\mathrm{r}}(\mathbf{p}_j) = \mathbf{V}(\mathbf{p}_j)^T\mathbf{f}(t, \mathbf{p}_j)$$

$$\mathbf{g}_r(\mathbf{p}_j) = \mathbf{V}(\mathbf{p}_j)^T\mathbf{g}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t), \mathbf{p}_j\right)$$

Problem Statement

Algorithmic Framework

Step 1: Parametric input states

$$\forall \mathbf{p}_k, k \in [1, N_s]$$

Step 2: Time Integration of Full Model

$$\forall t_i, i \in [0, N_t]$$

Notation:

- n : Full-order dimension
- N_s : Number of training samples
- N_t : Number of simulated timesteps
- $\mathbf{M}$: Mass matrix
- $\mathbf{f}$: External forcing
- $\mathbf{u}$: Response solution

For each parametric state:

- Assemble **system matrices**
  stiffness K / mass M / damping C / ***Excitation f***
- Evaluate the **time domain response** (integration)

The full-order, high fidelity finite element model **depends on a parametric input state.**

The parametric states are first sampled. The respective **parameters may represent**:

- ***system properties:*** *yield stress, hysteretic damping coeffs.*
- ***excitation traits:*** *amplitude of ground motion, frequency content*

Problem Statement

Algorithmic Framework

Training / Offline Phase

Step 1: Parametric input states

$$\forall \mathbf{p}_k, k \in [1, N_s]$$

Step 2: Time Integration of Full Model

$$\forall t_i, i \in [0, N_t]$$

Step 3: Assemble matrices and evaluate Equations of Motion

$$\mathbf{M}(\mathbf{p})\ddot{\mathbf{u}}(t_i) + \mathbf{g}_i(\mathbf{u}(t_i), \dot{\mathbf{u}}(t_i), \mathbf{p}) - \mathbf{f}(\mathbf{t_i}, \mathbf{p}) = \mathbf{0}$$

$$\mathbf{u}(t) \in \mathbb{R}^n, \mathbf{M}(\mathbf{p}) \in \mathbb{R}^{n \times n}, \mathbf{f}(t, \mathbf{p}) \in \mathbb{R}^n, \mathbf{g}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t)\right) \in \mathbb{R}^n$$

Nonlinear terms *includes stiffness and damping*

Step 4: Compute Residual on Equations and "predict" correction

$$if \quad \mathbf{R}_i(\mathbf{u}_r(t_i)) > tol => \tilde{\mathbf{u}}_r(t_i)$$

Step 5: Employ updated solution to re-assemble nonlinear terms & matrices

$$\tilde{\mathbf{u}}(t_i) => \mathbf{g}_i(\tilde{\mathbf{u}}(t_i), \dot{\tilde{\mathbf{u}}}(t_i), \mathbf{p})$$

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- N_s: Full-order dimension
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Problem Statement

Algorithmic Framework

POD - Projection-based Reduction

Assemble POD Basis

Proper Orthogonal Decomposition

$$\mathbf{u}(t) = \mathbf{V}(\mathbf{p})\mathbf{u}_{\mathrm{r}}(t) \qquad \mathbf{U} \equiv [\mathbf{u}(t_1)...\mathbf{u}(t_N)] = \mathbf{W}\boldsymbol{\Sigma}\mathbf{R}^T$$

$$\mathbf{V} \equiv \mathbf{W}_k = \mathbf{W}(:, 1:\mathrm{k})$$

Each training point has a *projection basis.*

principal components

As a next step, to evaluate the pROM :

- Assemble the ***full-order matrices*** of the validation parametric state
- Assemble a ***suitable basis*** representing the solution subspace (***Interpolation / Clustering***)
- ***Project*** the system matrices on the reduced dimension
- ***Integrate the reduced order matrices*** forward in time to evaluate the dynamic response
- ***Map*** the reduced response ***back*** to full coordinates

Problem Statement

Bottlenecks/Limitations

POD - Projection-based Reduction

Assemble POD Basis

Proper Orthogonal Decomposition

$$\mathbf{u}(t) = \mathbf{V}(\mathbf{p})\mathbf{u}_{\mathrm{r}}(t) \qquad \mathbf{U} \equiv [\mathbf{u}(t_1)...\mathbf{u}(t_N)] = \mathbf{W\Sigma R}^T$$

$$\mathbf{V} \equiv \mathbf{W}_k = \mathbf{W}(:, 1:\mathrm{k})$$

Limitations:

- **POD is a linear operator**
  Linearization in neighbourhood of stable points is assumed to address nonlinearities
- **Accuracy** for new parametric states *relies on clustering or interpolation between POD bases*

Problem Statement

Algorithmic Framework

ROM Evaluation / Online Phase

Step 1: Parametric input states

$$\exists \mathbf{p}_v, v \notin [1, N_s]$$

Step 2: Time Integration of ROM

$$\forall t_i, i \in [0, N_t]$$

Step 3: Assemble matrices and evaluate Equations of Motion

$$\mathbf{M}_r(\mathbf{p})\ddot{\mathbf{u}}_r(t_i) + \mathbf{g}_{ri}(\mathbf{u}(t_i), \dot{\mathbf{u}}(t_i), \mathbf{p}) - \mathbf{f}_\mathrm{r}(\mathbf{t_i}, \mathbf{p}) = \mathbf{0}$$

$$\mathbf{M}_\mathrm{r}(\mathbf{p}_j) \in \mathbb{R}^{r\times r}, \mathbf{g}_\mathrm{r}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t), \mathbf{p}_j\right) \in \mathbb{R}^r, \mathbf{f}_\mathrm{r}(t, \mathbf{p}_j) \in \mathbb{R}^r$$

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$$\mathbf{g}_{ri} = \mathbf{V}^T\mathbf{g}_i$$

Problem Statement

Bottlenecks/Limitations

ROM Evaluation / Online Phase

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Nonlinear terms
*still **scale** with **full dimension***

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Bottlenecks/Limitations

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$$\mathbf{g}_{ri} = \mathbf{V}^T\mathbf{g}_i$$

- The evaluation of the nonlinear terms still ***scales with the full order dimension.***
- For every solution increment we need to:
  - ***Project*** displ./vel. ***back to full-order***
  - Evaluate nonlinear terms
  - ***Update*** forces and stiffness matrix
  - **Project** updated matrices ***back to reduced-order*** coordinates.

*This **back-and-forth projection is a major computational bottleneck**.*

Especially in large scale systems
where time integration savings cannot outweigh
the projection & evaluation.

*To address this, we rely on **hyper-reduction, a second-tier approximation** of the nonlinear contributions.*

Problem Statement

Bottlenecks/Limitations

- **Back & forth projection** to update nonlinear terms **compromises efficiency**

- Hyper-reduction is introduced

  ⇒*Several alternatives available (ECSW, DEIM, GNAT, EQM)*

  ⇒***ECSW**: Sparse evaluation of full-order nonlinear terms based on energy contributions coupled with weighting scheme*

  ⇒***DEIM:** Additional POD-based reduction coupled with selection scheme*

  ✓ Hyper-reduction **is essential for efficiency**

❖ Introduces an additional source of error that usually outweighs the POD projection error

=> Accuracy bottleneck/threshold for the ROM

Step 5: Employ updated solution to re-assemble nonlinear terms & matrices

$$\tilde{\mathbf{u}}(t_i) = \mathbf{V}\tilde{\mathbf{u}}_\mathbf{r}(t_i)$$

$$\tilde{\mathbf{u}}(t_i) => \mathbf{g}_i(\tilde{\mathbf{u}}(t_i), \dot{\tilde{\mathbf{u}}}(t_i), \mathbf{p})$$

$$\mathbf{g}_{ri} = \mathbf{V}^T\mathbf{g}_i$$

Working Approaches

Reconstruction Error Minimization

Tackle projection-based reduction error

=> Replace POD with AE-driven process

Argumentation:
POD is a linear operator / Accuracy relies on linearization assumptions

- *Employ mapping techniques based on nonlinear (feature) transformations*
- *Explore potential nonlinear kernels*
- *Check if orthogonality conditions need to be retained*

=> Improve pROM accuracy

- Initial approaches yield questionable results
- Additional bottleneck on how to propagate ROM dynamics after projection

=> Further and deeper research is needed

Working Approaches

Data-driven Mapping for Hyper-Reduction

Address ROM's nonlinear mapping limitations

⇒ ***Replace* Hyper-Reduction with *data-driven surrogates***

Argumentation:
Hyper-Reduction guarantees efficiency however is the largest error source of the pROM

- *Couple POD basis assembly process with data-driven method to learn the nonlinear mapping directly in ROM coordinates*
- *Every iteration of the training state contributes nonlinear mapping training data*
  => Thousands of data available for a single training realization
- *Parametric dependencies on the nonlinear law might be explored as a next step*

+ Potentially substantial improvement in efficiency
+ Achieve real-time evaluations
- Potential trade-off with accuracy
- Parametric dependency is challenging

Numerical Validation

Case study description

Two-story shear frame with nodal connections modeled employing hysteretic links

Multisine **stochastic ground motion excitation**

Hysteretic links response model

- *Total restoring force:*

$$\mathbf{R} = \mathbf{R}_{linear} + \mathbf{R}_{hysteretic} = \alpha k \mathbf{u} + (1-\alpha) k \mathbf{z}$$

- *Bouc-Wen equation with degradation/deterioration effects:*

$$\dot{\mathbf{z}} = \frac{A\dot{\mathbf{u}} - \nu(t)(\beta|\dot{\mathbf{u}}|\mathbf{z}|\mathbf{z}|^{w-1} - \gamma\dot{\mathbf{u}}|\mathbf{z}|^{w})}{\eta(t)}$$

$$\nu(t) = 1.0 + \delta_\nu \epsilon(t), \quad \eta(t) = 1.0 + \delta_\eta \epsilon(t), \quad \epsilon(t) = \int_0^t \mathbf{z}\dot{\mathbf{u}} \delta t$$

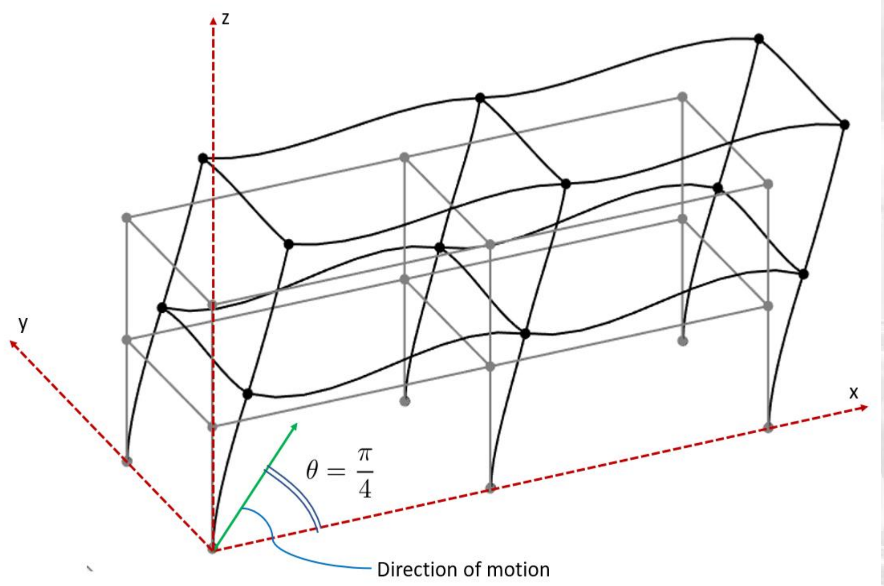
Characteristics of the Bouc-Wen links:

β, γ, A, w : *Smoothness and shape of hysteresis curve*

δ_ν, δ_η : *Degradation/Deterioration effects*

a, k : *Linear/Hysteretic contribution weighting*

=> Parametric dependencies of the hysteretic links



Benchmark example featured in:

- Vlachas K. et al. "A local basis approximation approach for nonlinear parametric model order reduction." *Journal of Sound and Vibration* 502 (2021): 116055.
- Vlachas K. et al. " Two-story frame with Bouc-Wen hysteretic links as a multi-degree of freedom nonlinear response simulator." Workshop on non-linear system identification benchmarks (2021)

Numerical Validation

Data-driven Mapping for Hyper-Reduction

Address ROM's nonlinear mapping limitations
$\Rightarrow$ ***Replace* Hyper-Reduction with *data-driven surrogates***

Argumentation:
Hyper-Reduction guarantees efficiency however is the largest error source of the pROM

Detailed Task Formulation

Input:
- *Reduced-Order Displacements in current iteration (and previous ones)* $\rightarrow \mathbf{U} \in \mathrm{R}^{16,(t-k):t}$
- *Reduced-Order Force terms in previous iteration(s)* $\rightarrow \mathbf{F} \in \mathrm{R}^{16,(t-k):(t-1)}$
- *Reduced-Order Stiffness terms in previous iteration(s)* $\rightarrow \mathbf{K} \in \mathrm{R}^{16\times16,(t-k):(t-1)}$

Output:
- *Reduced-Order Force terms in current iteration* $\rightarrow \mathbf{F} \in \mathrm{R}^{16,1}$
- *Reduced-Order Stiffness terms in current iteration* $\rightarrow \mathbf{K} \in \mathrm{R}^{16\times16,1}$

Numerical Validation

Network details and potential improvements

Neural Network Details

- 1D (Temporal) Convolution layers
- Spatial local correlations ignored => Assumes only time correlations
- Activation function: CELU, Tanh
- Train / Validation ratio: 0.75
- Temporal width: 8
- Dropout: 0.01
- Scaling: Normalization/ Minimum=0, Maximum=1
- Kernel size = 3
- Batch size = 64 / Epochs = 10000
- Adaptive learning rate with overfitting patience enabled
- Network size: ~500,000 parameters

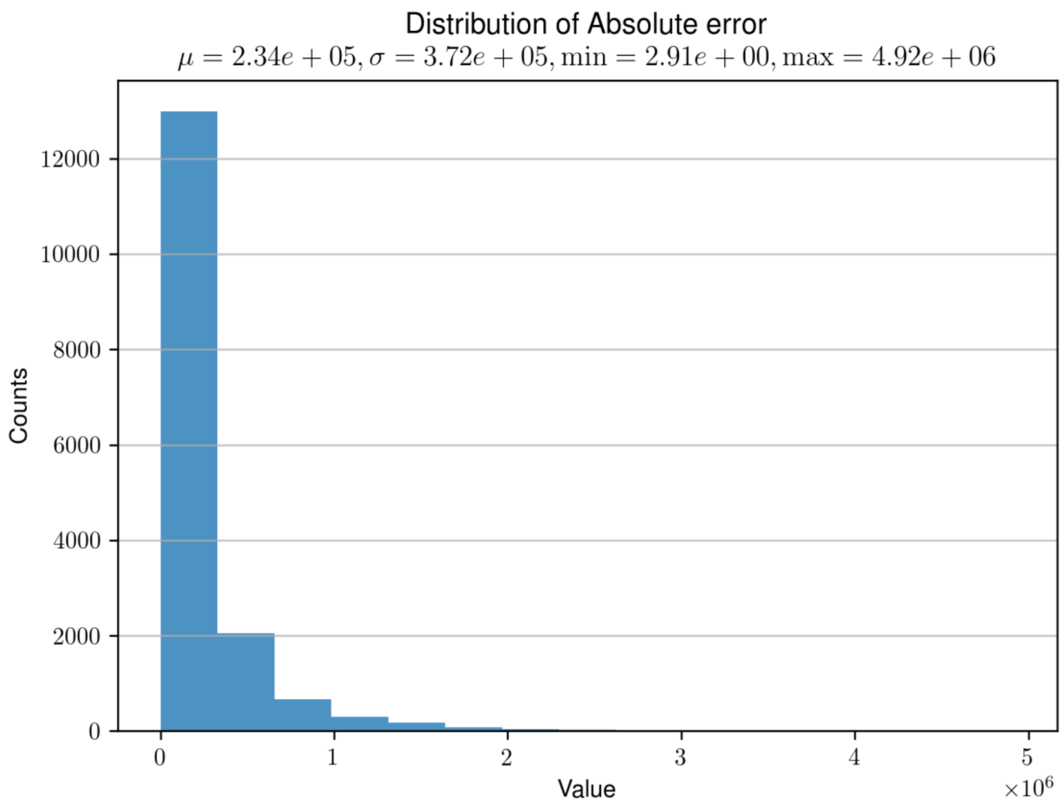
Potential improvements:

- ✓ Increase temporal width parameter
- ✓ Improve post-processing and error diagnostics
- ✓ Explore the trade-off between deep CNNs and efficiency
- ✓ Explore MLP network potential

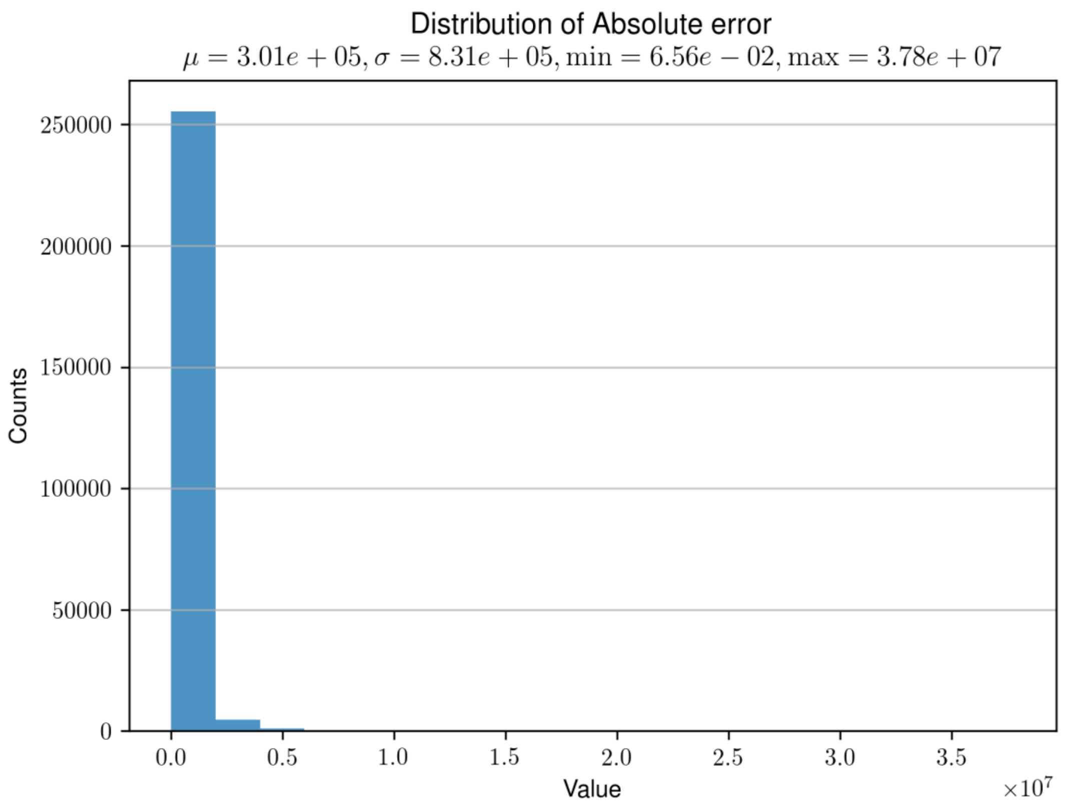
Numerical Validation

Absolute errors distribution

Internal Forces F



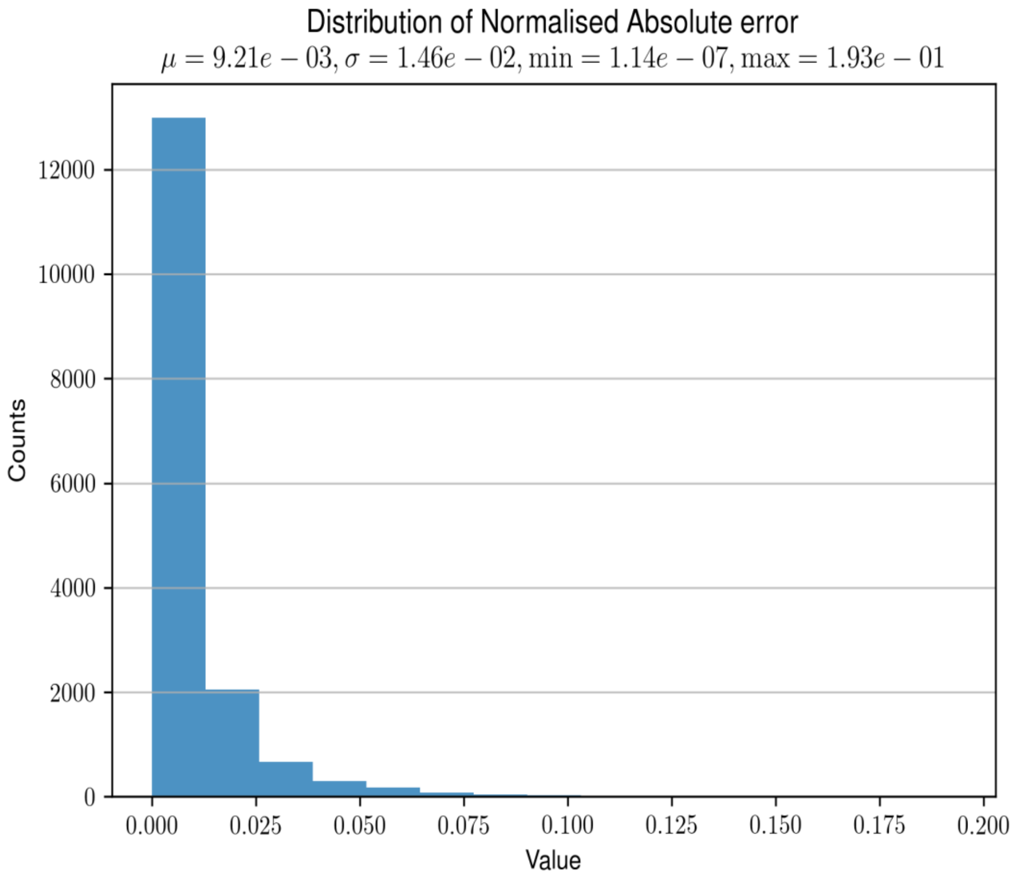
Stiffness Matrix K



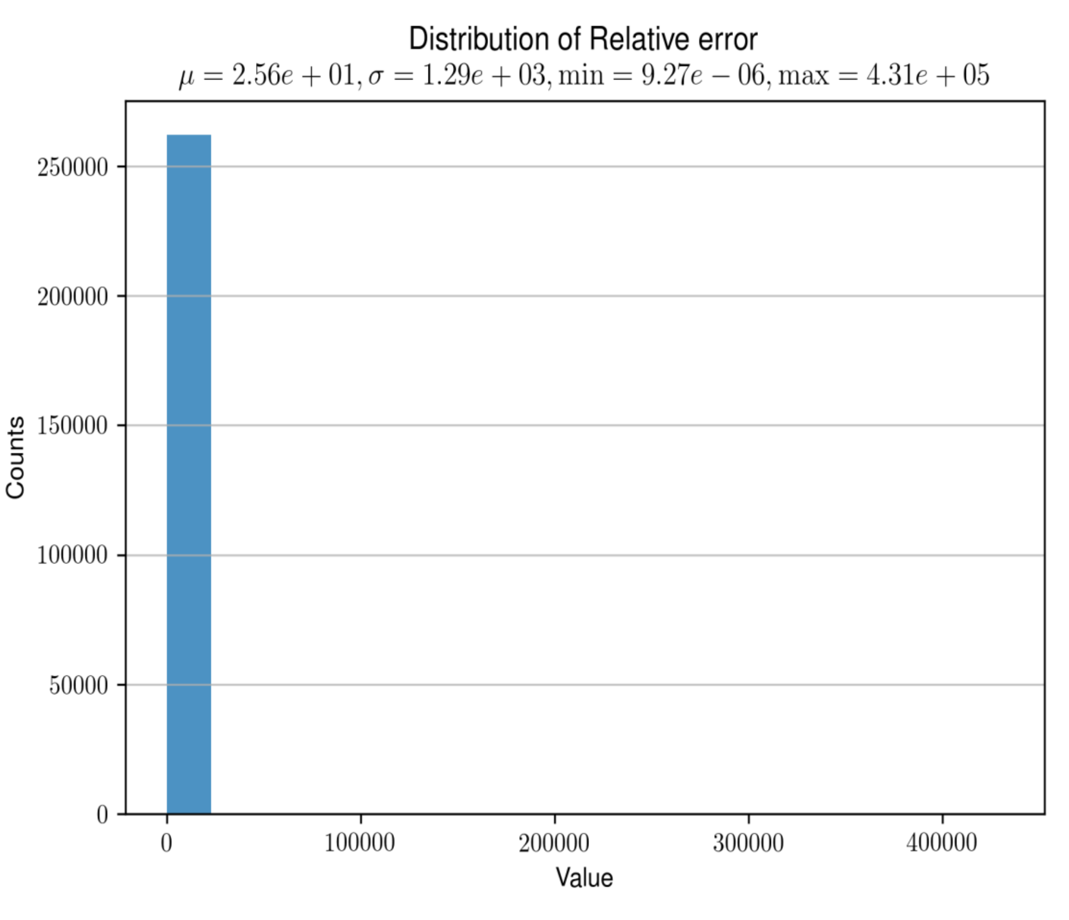
Numerical Validation

Relative errors distribution

Internal Forces F

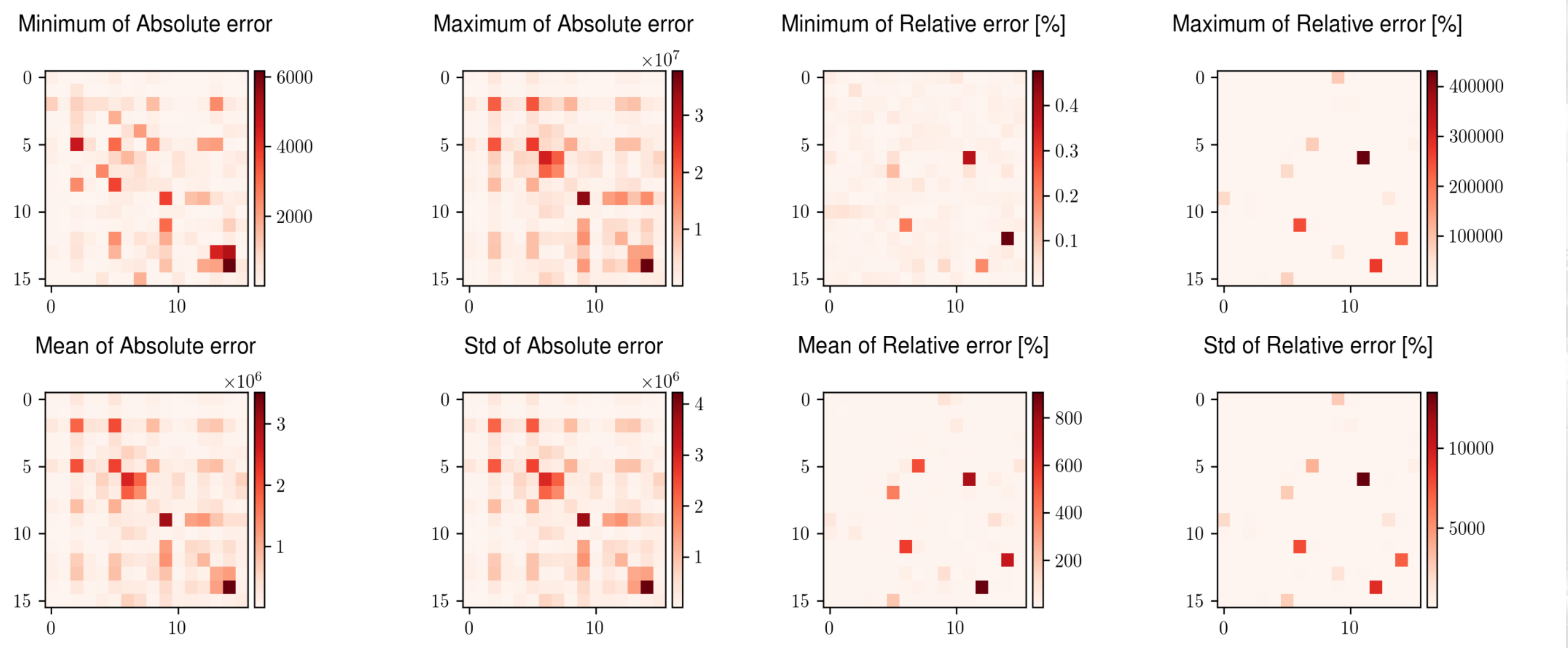


Stiffness Matrix K



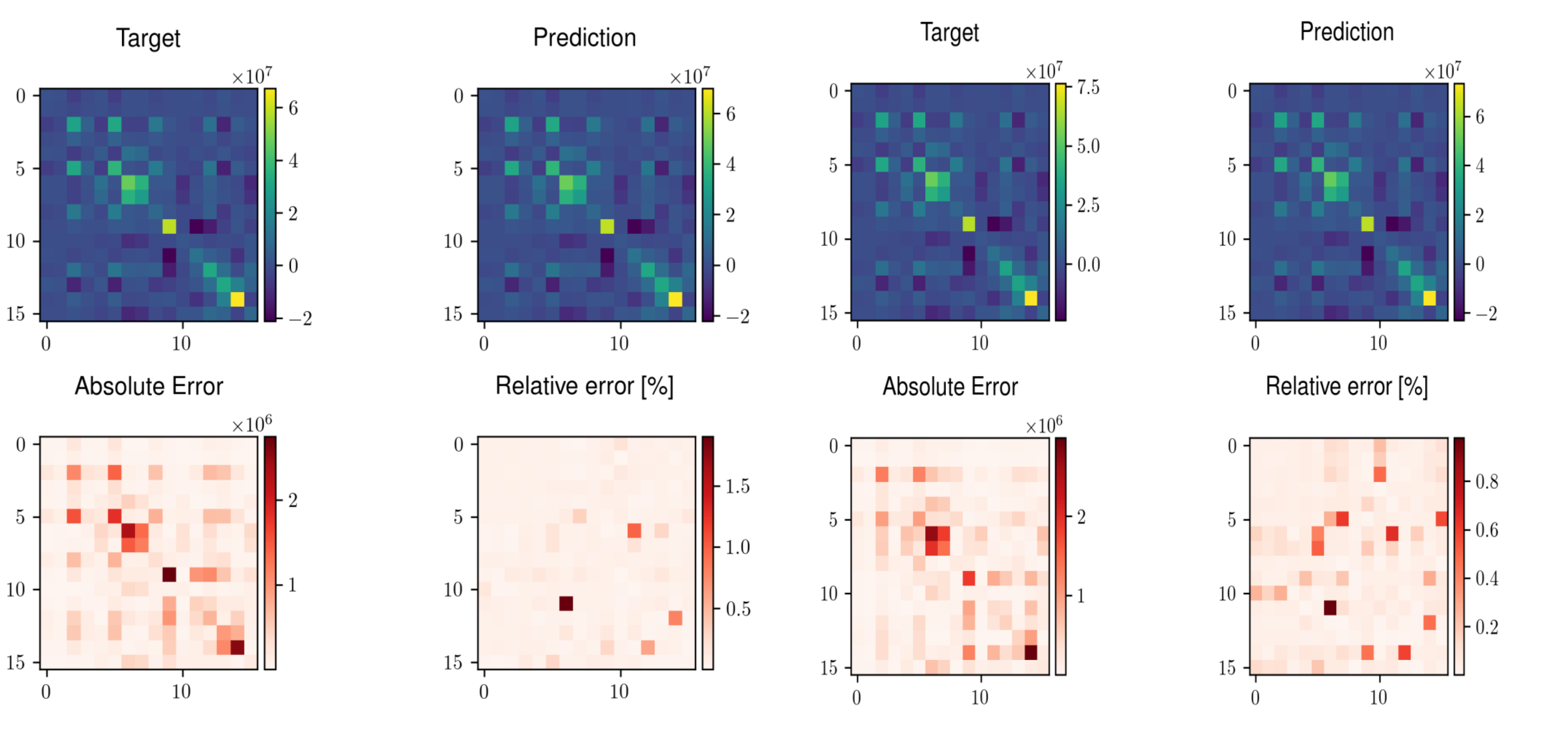
Numerical Validation

Error visualization in stiffness matrix



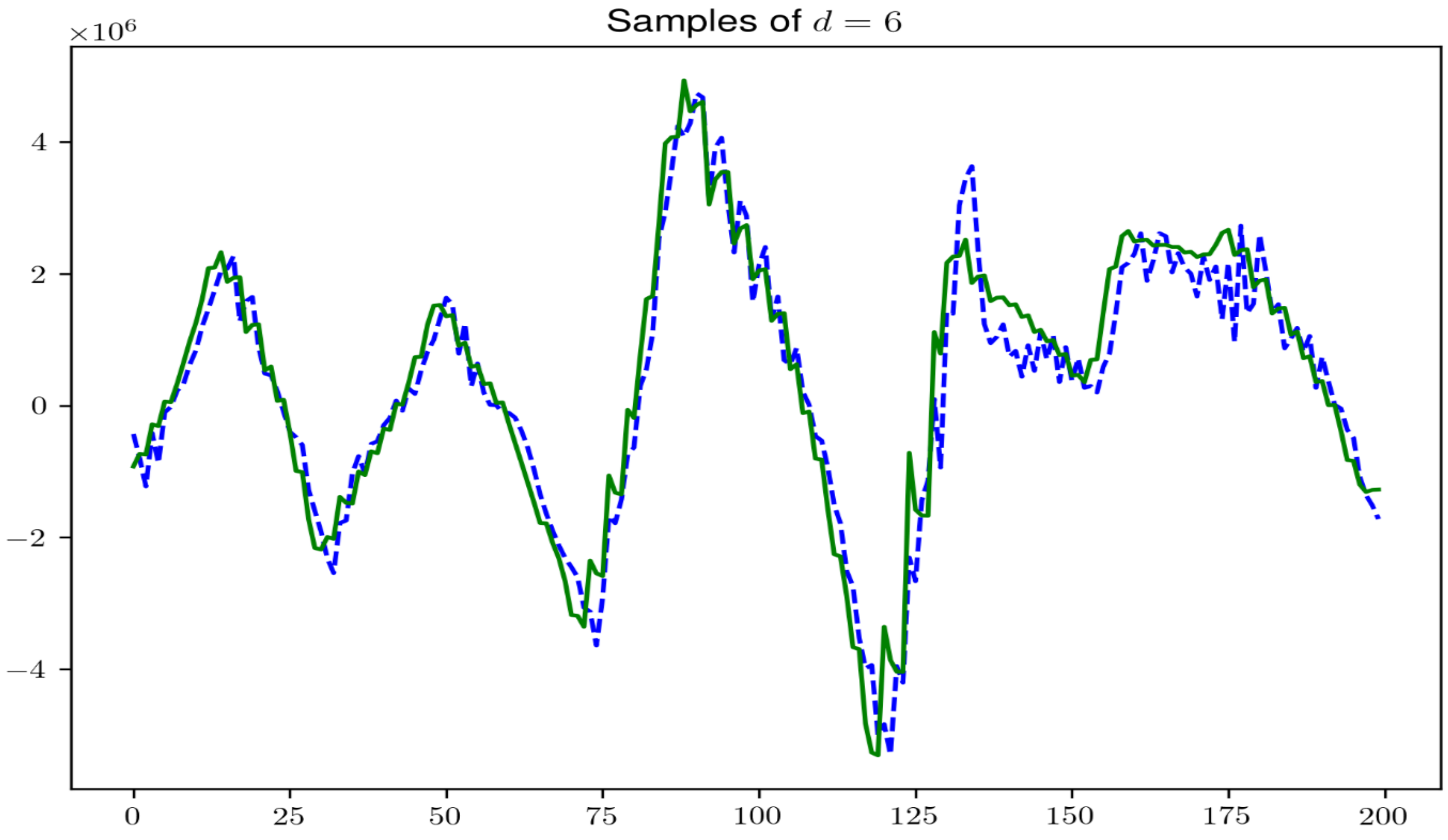
Numerical Validation

Prediction visualization in stiffness matrix



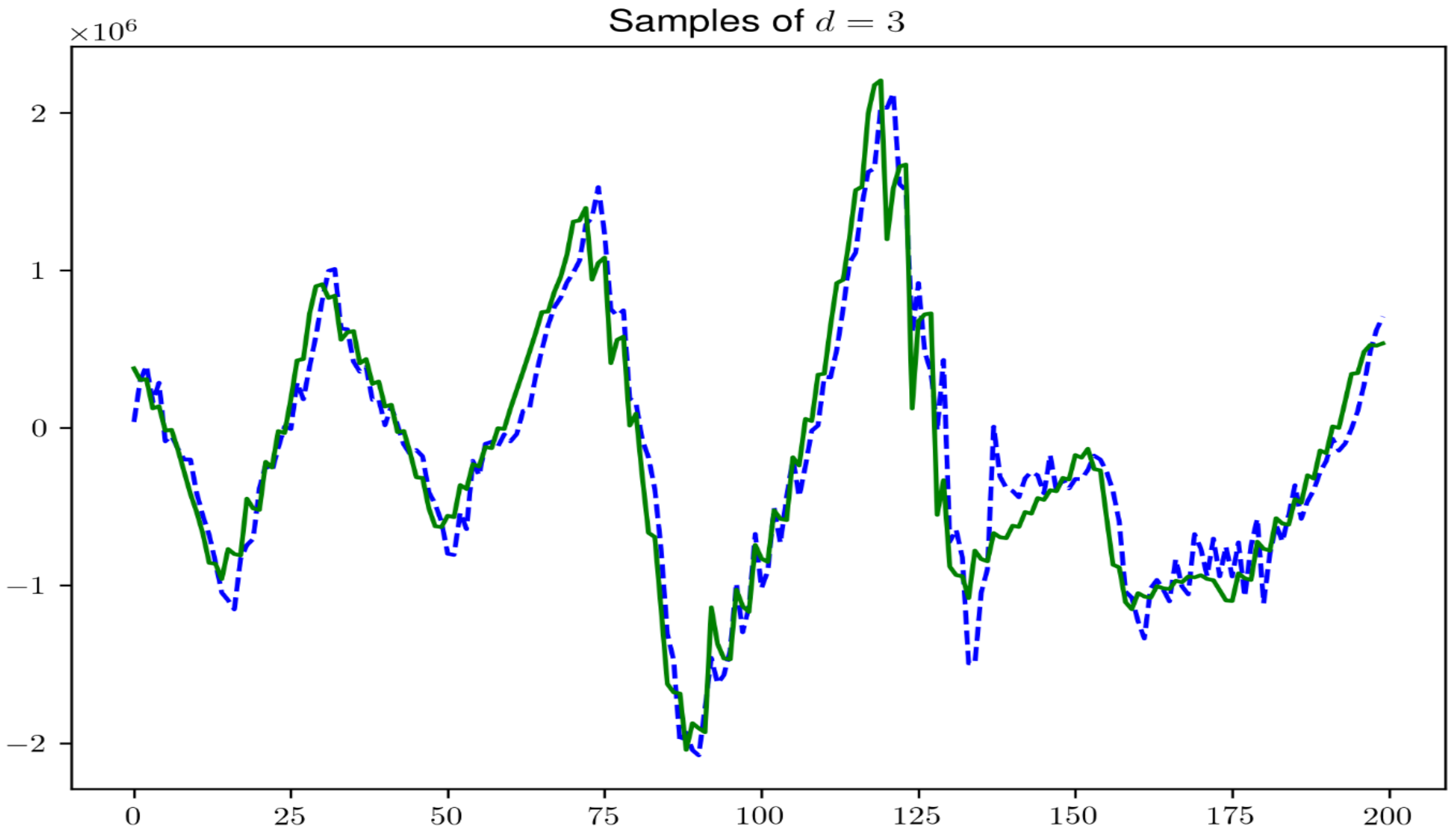
Numerical Validation

Prediction visualization in internal forces



Numerical Validation

Prediction visualization in internal forces



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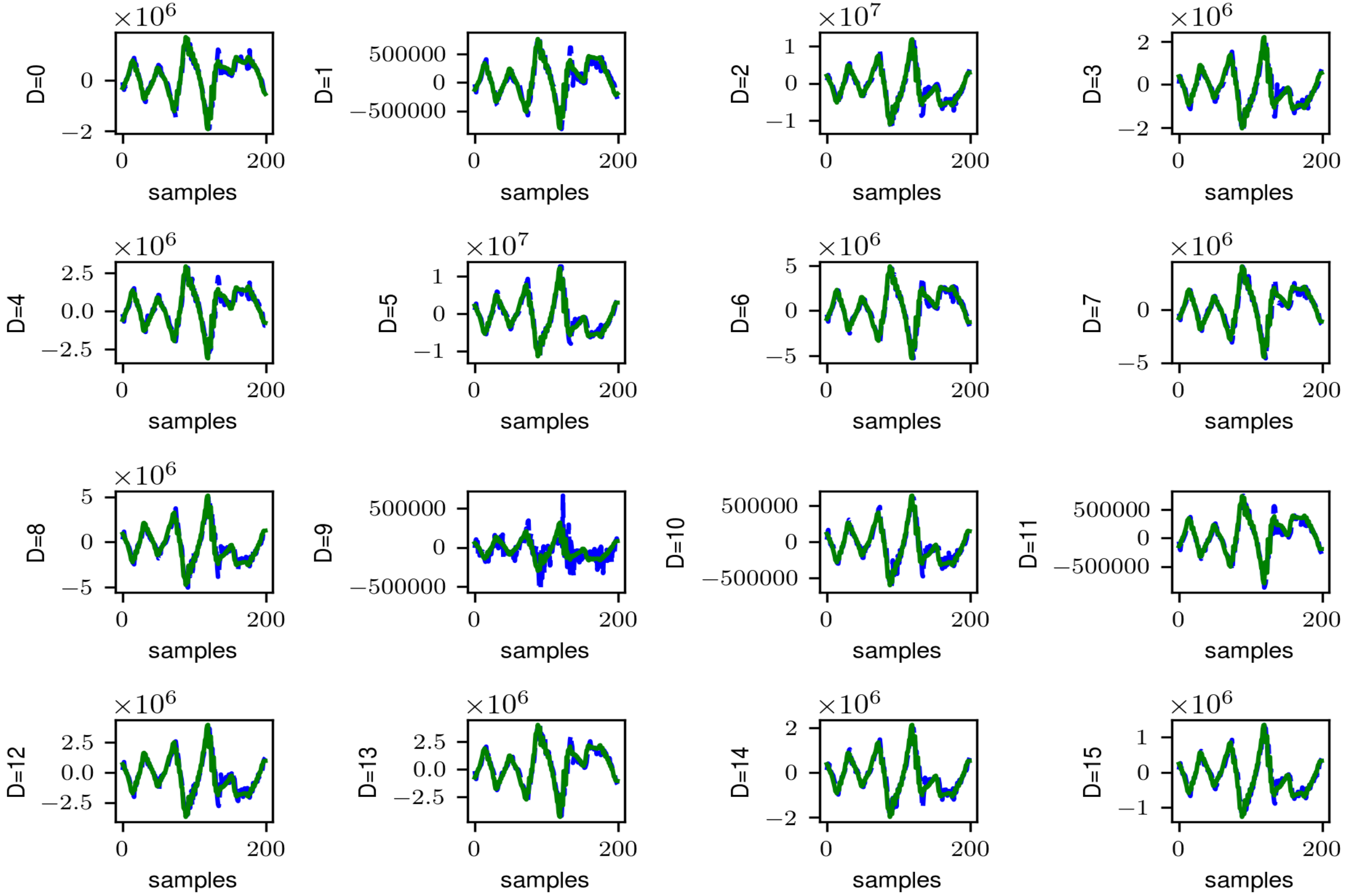

Concluding Remarks

ROM Performance and Generalization / Outlook

Hyper-Reduction Based POD-ROM vs Data-driven assisted POD-ROM

Accuracy performance of the pROM

*Median Error = **~2%** vs **~20%***

*Max Error = **5%** vs **25%***

Efficiency performance of the pROM (Evaluation in GPU)

*Average speed-up factor = **~7** vs **~50***

Parametric dependency treatment

***One hyper-reduction scheme** per POD cluster / Approximately **3 NN models** per cluster*

Suffers from ***error propagation***

=> Improvement by training in sequences

Remarkably ***greater efficiency***

=> Explore deeper networks

Generalization needs improvement

=> Include dependency as input variable

=> Ongoing Research