SAND2021-11094C

Sandia National Laboratories

Exceptional service in the national interest

Simulation of a relativistic magnetron using a fluid electron model

Nicholas Roberds, Andrew Sandoval, Keith Cartwright and Kris Beckwith

ICOPS 2021

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia LLC, a wholly owned subsidiary of Honeywell International Inc. for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

Motivation for using a fluid model

- Computer simulation of magnetrons has usually been accomplished with PIC
  - Has seen good success in quantitatively reproducing experimental features [1]
- Theoretical work has largely been accomplished by analysis of cold fluid equations
  - Hull cutoff and Buneman-Hartree threshold [2]
  - Nonlinear diffusion of the electron density [2,3]
    - Nonlinear diffusion of electrons (nominally magnetically confined) has also been described kinetically [4]
  - Linear instabilities of the electron beam e.g. magnetron instability and diocotron instability.

- **It therefore seems natural to consider numerical solutions to a fully non-linear fluid model for studying magnetron operation**
  - This has not been previously attempted as far as we know

[1] Lemke, R. W., T. C. Genoni, and T. A. Spencer. "Three-dimensional particle-in-cell simulation study of a relativistic magnetron." *Physics of Plasmas* 6.2 (1999): 603-613.
[2] Davidson, Ronald C. *Physics of nonneutral plasmas*. World Scientific Publishing Company, 2001.
[3] Kaup, D. J. "Theoretical modeling of an A6 relativistic magnetron." *Physics of Plasmas* 11.6 (2004): 3151-3164.
[4] Desjarlais, M. P., and R. N. Sudan. "Electron diffusion and leakage currents in magnetically insulated diodes." *The Physics of fluids* 30.5 (1987): 1536-1552.

Relativistic fluid model

Conserved quantities of the relativistic fluid:

$$\boldsymbol{u}(\rho, v, P) = \begin{cases} D = \gamma\rho \\ \boldsymbol{M} = \gamma w \boldsymbol{v}/c^2 \\ E = \gamma^2 w - P \end{cases}$$

The fluid state evolves according to,

$$\partial_t \boldsymbol{u} = -\nabla \cdot \boldsymbol{F} + \boldsymbol{Q}(\boldsymbol{u}, \boldsymbol{E}, \boldsymbol{B})$$
$$\boldsymbol{\partial_t B} = -\nabla \times \boldsymbol{E}$$
$$\boldsymbol{\partial_t E} = \nabla \times \frac{\boldsymbol{B}}{\mu_0 \epsilon_0} - \frac{\boldsymbol{j}}{\epsilon_0}$$
$$\boldsymbol{F} = \begin{cases} D\boldsymbol{v} \\ \boldsymbol{M}\boldsymbol{v} + P\boldsymbol{I} \\ \boldsymbol{M}c^2 \end{cases}$$

- ρ is the proper mass density (so D is the density in the lab frame)
- γ is the Lorentz factor
- P is the proper pressure
- w is the enthalpy density
- $\boldsymbol{j}$ is the electric current
- $\boldsymbol{Q}$ is the Lorentz force source term
- $\boldsymbol{B}$ is the magnetic field
- $\boldsymbol{E}$ is the electric field

For details and the numerical methods used, see Glines' ICOPS presentation: *20-A-3, "Relativistic Two-Fluid Electrodynamics Using Implicit-Explicit Discontinuous-Galerkin Methods*

Fluid SCL Boundary condition to account for cathode electron emission

Boundary conditions prescribe the value of the flux $\boldsymbol{F}$ at quadrature points on the domain boundary

$\boldsymbol{F}$ must account for the outgoing characteristics (see [5]). We therefore compute $\boldsymbol{F}$ with an approximate solution to a Riemann problem:

$$\boldsymbol{F} = \boldsymbol{F}_{RI}(\boldsymbol{u}_V, \boldsymbol{u}_I)$$

- $\boldsymbol{u}_V$ is a "virtual state"
- $\boldsymbol{u}_I$ is the interior state

The SCL boundary condition prescribes the flux so that the influx of electrons is proportional to the electric field:

$$\boldsymbol{F}_{\mathbf{1}} = \mathrm{D}\mathbf{v} = \Gamma\boldsymbol{n}$$

$$\Gamma = (\gamma_0 \boldsymbol{E}\cdot\boldsymbol{n} + \gamma_1 \partial_t \boldsymbol{E}\cdot\boldsymbol{n})\tanh\left(t/\tau_{ramp}\right)$$

Where $\boldsymbol{n}$ is the unit vector outward normal to the boundary

To prescribe an appropriate virtual state $\boldsymbol{u}_V$ so that the injection flux is exactly $\boldsymbol{F}_{\mathbf{1}} = \Gamma\boldsymbol{n}$,

1. A value for the "injection velocity" v_{inj} is prescribed (this is a user-adjustable parameter)
2. A value for the "injection temperature" T_{inj} is prescribed (this is a user-adjustable parameter)
3. Finally, we solve an inverse problem using Brent's method to determine the value of the virtual state density

- We also use this strategy for a fluid thermal desorption boundary condition, which requires precise control of the injected quantity.

[5] Mengaldo, Gianmarco, et al. "A guide to the implementation of boundary conditions in compact high-order methods for compressible aerodynamics." *7th AIAA Theoretical Fluid Mechanics Conference*. 2014.

2D diode benchmark problem for the fluid SCL boundary condition

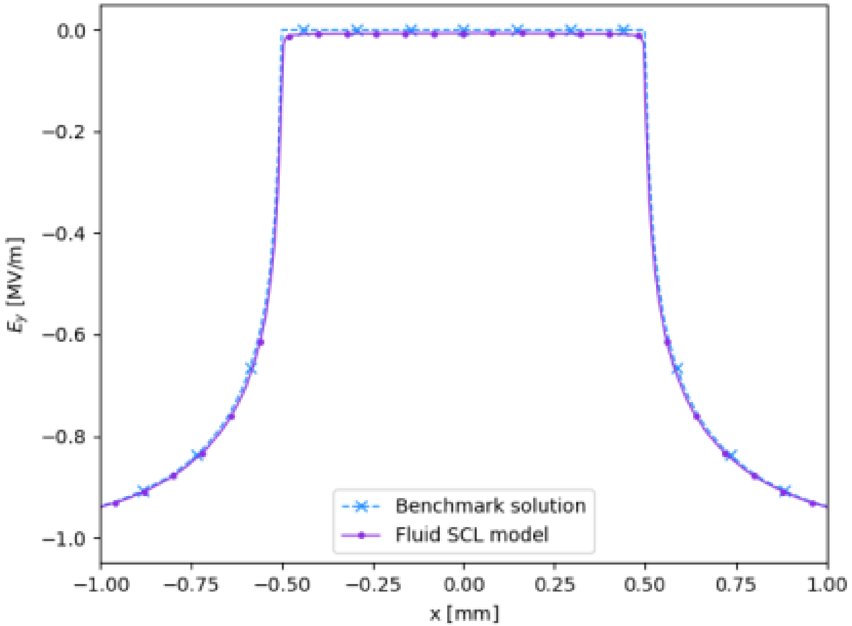
A similar 2D diode problem has been used to benchmark PIC SCL algorithms [6,7]

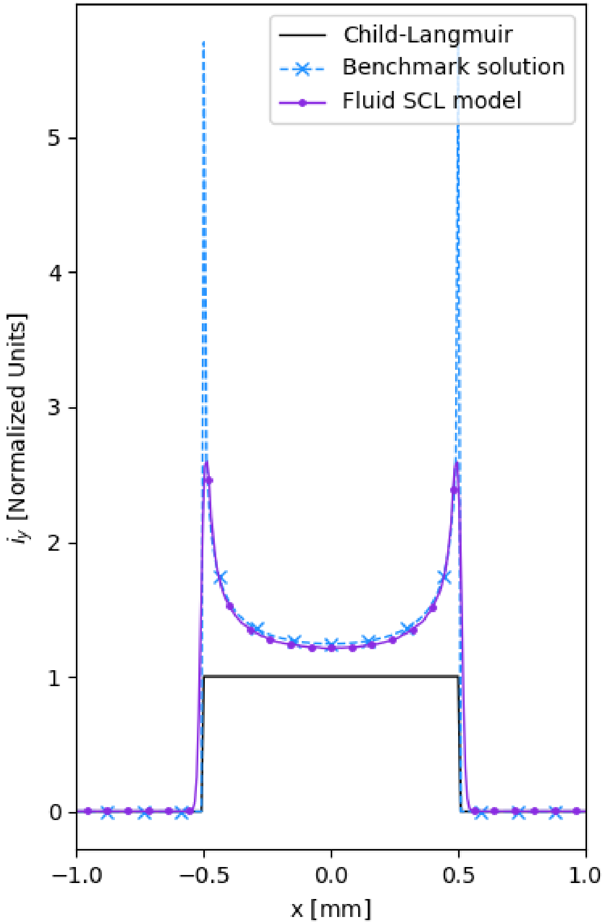
- Anode is located at $y = 1$ mm
- Cathode is located at $y = 0$ mm
- SCL emission occurs on the cathode for $-\frac{w}{2} < x < \frac{w}{2}, w = 1$ mm
- A strong, uniform magnetic field $B_y = 0.5$ T keeps the electron beam from spreading
- Applied voltage, V = 1000 V

SCL parameters,

- $\gamma_0 = 10\epsilon_0$
- $\gamma_1 = \epsilon_0^2$
- $\tau_{\text{ramp}} = 10$ ps
- $v_{inj} = 5.93 \times 10^5$ m/s (corresponds to 1 eV)
- $T_{inj} = 5000$ K

The benchmark solution was computed using the method of Luginsland et. al [8]. PIC codes typically see a factor of 3 enhancement in the current density at the emission edges $x = \pm\frac{w}{2}$, similar to our result here.





Plots of the electric field (above) and current density (right) for the relativistic fluid solution, the benchmark solution and 1D Child-Langmuir. Note that, as shown by Rokhlenko [9], the exact solution to the benchmark solution has singularities in the current density at $x = \pm w/2$.

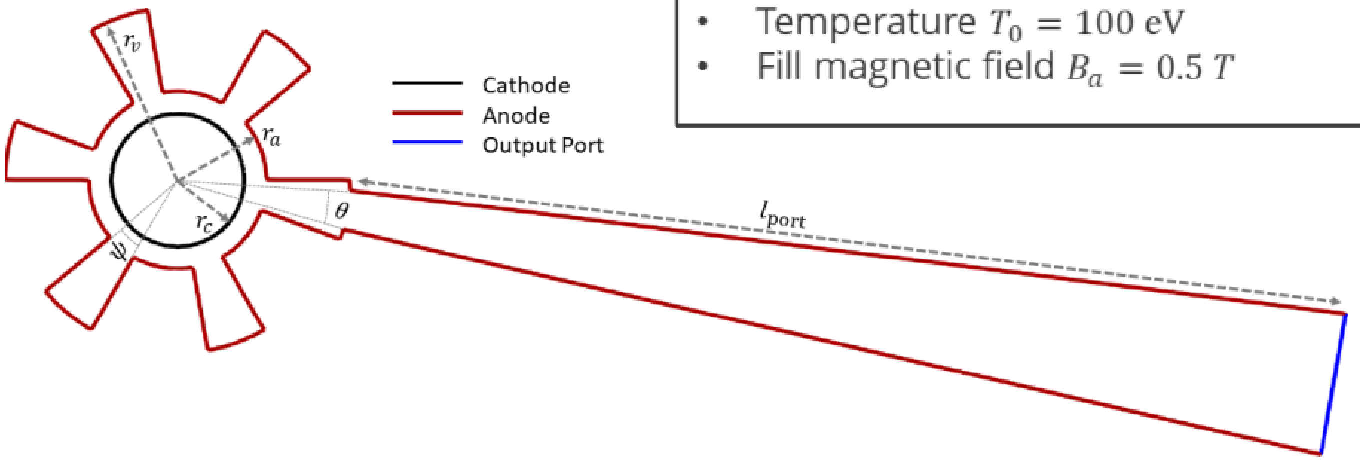
[6] Watrous, J. J., J. W. Lugisland, and G. E. Sasser III. "An improved space-charge-limited emission algorithm for use in particle-in-cell codes." *Physics of Plasmas* 8.1 (2001): 289-296.

[7] Stoltz, Peter H., et al. "A new simple algorithm for space charge limited emission." *Physics of Plasmas* 27.9 (2020): 093103.

[8] Luginsland, J. W., et al. "Beyond the Child–Langmuir law: A review of recent results on multidimensional space-charge-limited flow." *Physics of Plasmas* 9.5 (2002): 2371-2376.

[9] Rokhlenko, A. "Space charge limited flow in a rectangular region: Profile of the current density." *Journal of applied physics* 100.1 (2006): 013305.

Problem specification: Two-dimensional A6 magnetron with an output port



Initial Conditions

- Electron number density $n_0 = 10^{17}$
- Velocity $\boldsymbol{v} = 0$
- Temperature $T_0 = 100$ eV
- Fill magnetic field $B_a = 0.5\ T$

Geometric Parameters

- $\theta = 10^\circ$
- $\psi = 20^\circ$
- $r_a = 2.11$ cm
- $r_c = 1.58$ cm
- $r_v = 4.11$ cm
- $l_{port} = 23.8$ cm

Fluid Boundary Conditions

- **Anode:** Farfield boundary
  - Electron number density $n_0 = 10^{17}$
  - Velocity $\boldsymbol{v} = 0$
  - Temperature $T_0 = 100$ eV
- **Output Port:** (Same as Anode boundary condition)
- **Cathode:** SCL boundary condition
  - $\gamma_0 = 100\epsilon_0$
  - $\gamma_1 = 0$
  - $\tau_{ramp} = 0.1$ ns

Electromagnetic Boundary Conditions

- **Anode, Cathode:** Perfect electrical conductor
  - $E_\parallel = 0$
  - $B_\perp = 0$
- **Output Port:** Impedance boundary
  - $\boldsymbol{B}_\perp = \sqrt{\mu_0 \epsilon_0}\, \boldsymbol{n} \times \boldsymbol{E}_\parallel$

Ghost current (to energize AK gap)

- 16 kA/(7.2 cm length of cathode)

Hull cutoff and Buneman-Hartree threshold

Most fundamental magnetron criterion:

The applied voltage must lie,

- Below the Hull cutoff V_H
- Above the Buneman-Hartree threshold V_{BH}^n

$$V_H = \frac{mc}{cd_e}\sqrt{\frac{2\text{ eV}}{mc^2} + \left(\frac{1\text{ eV}}{mc^2}\right)^2}$$

$$V_H = 350\ kV$$

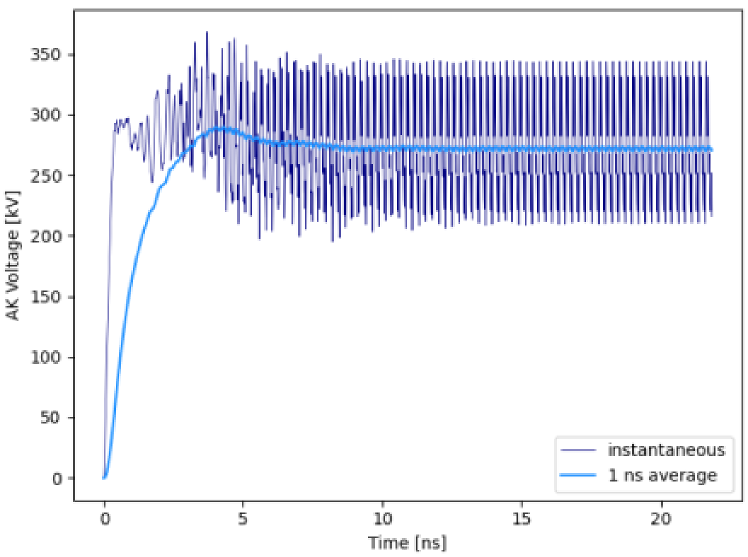
$$V_{BH}^n = \frac{B_a\omega_n}{n} r_a d_e - \frac{mc^2}{e}\left(\sqrt{1 - \left(\frac{r_a\omega_n}{cn}\right)^2} - 1\right)$$

For the 2π mode $(n = 6)$ and the π $(n = 3)$ modes,

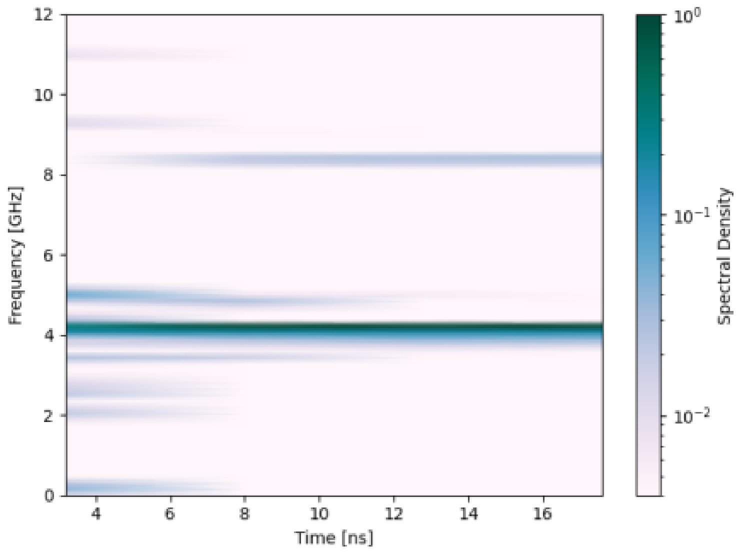
$$V_{BH}^{2\pi} = V_{BH}^{\pi} = 195\text{ kV}$$

Magnetron simulation result (grid resolution dx=0.125 mm)

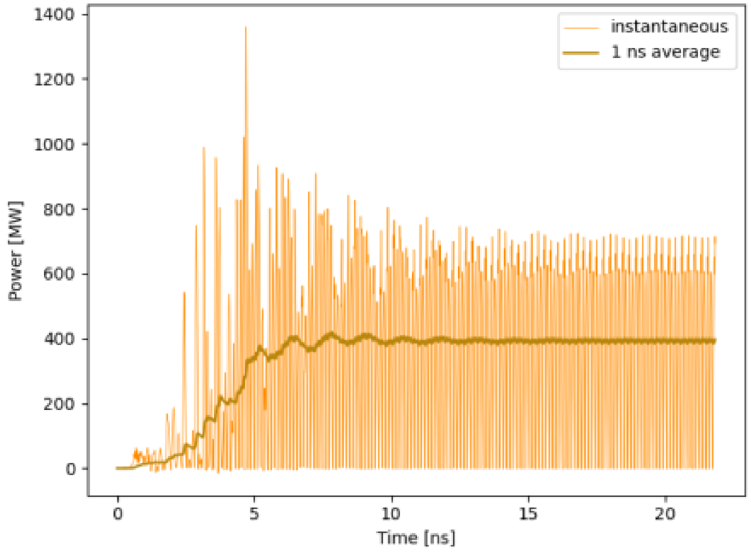
Electromotive force along $\theta = 5^{\circ}$



Spectrogram (6.4 ns window)

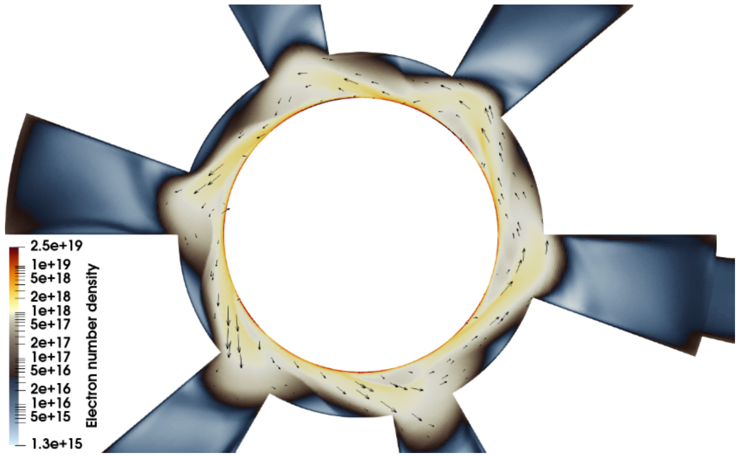


Output RF power



- Bunching of the electrons and corresponding excitation of the 2π cavity mode occurs shortly after the AK voltage exceeds the Buneman-Hartree threshold.
- Electron spokes break the magnetic confinement, allowing current across the gap. The time averaged AK voltage equilibrates to ~270 kV.
- Average output RF power settles to ~390 MW / (7.2 cm length of cathode)
- Spectrograph confirms that most of the energy is in the 2π mode (4.3 GHz). The band at 8.6 GHz is due to the large amplitude excitation of the 2π mode.

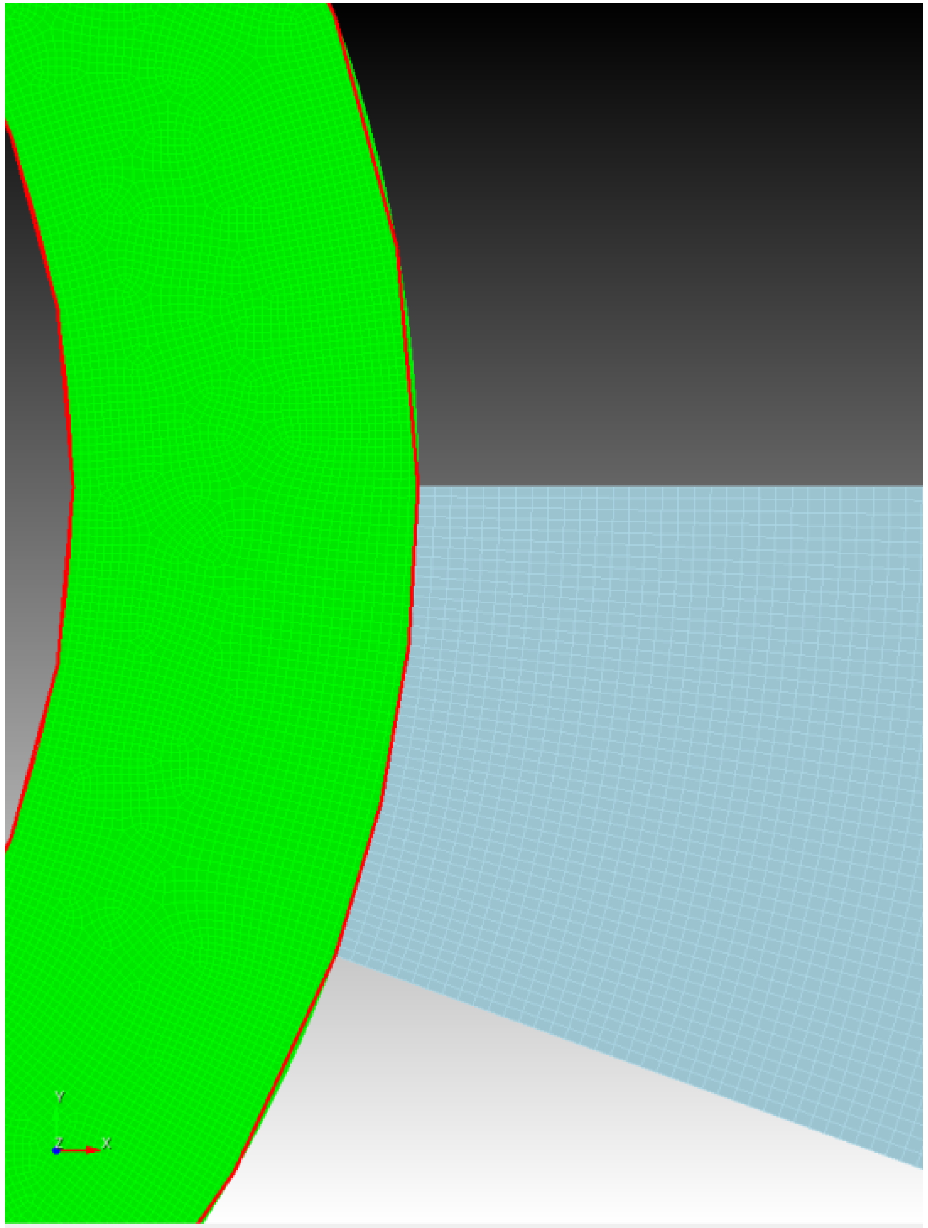
Electron density (arrows indicate electric current)



Numerical parameters

- A non-uniform mesh is used. Grid spacing is,
  - dx in the interaction space, $r_c < r < r_a$
  - $2\ dx$ in the vanes, $r_a < r < r_c$
  - $\frac{1}{64}$ of a $\omega = 4.3\ \text{GHz}$ free space wavelength in the output waveguide
  - Results shown on the previous slide use, $dx = 0.125\ \text{mm}$

- Adaptive time stepping
  - Electron fluid CFL = 0.85



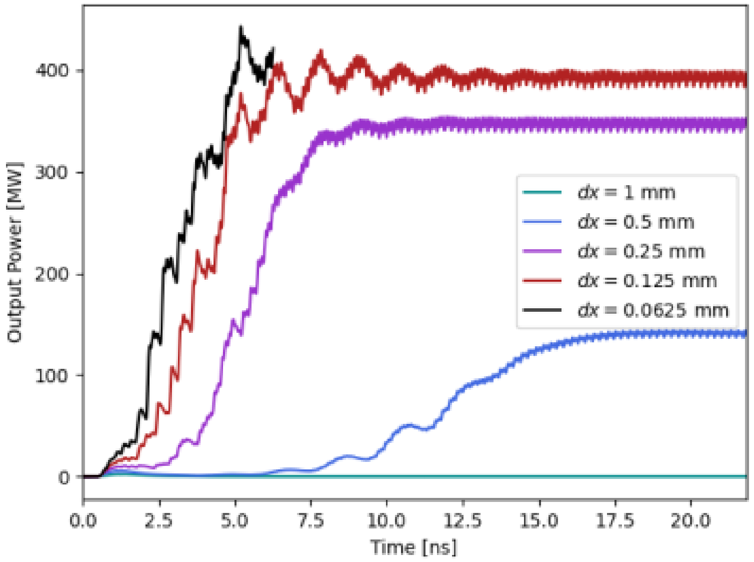
Spatial convergence

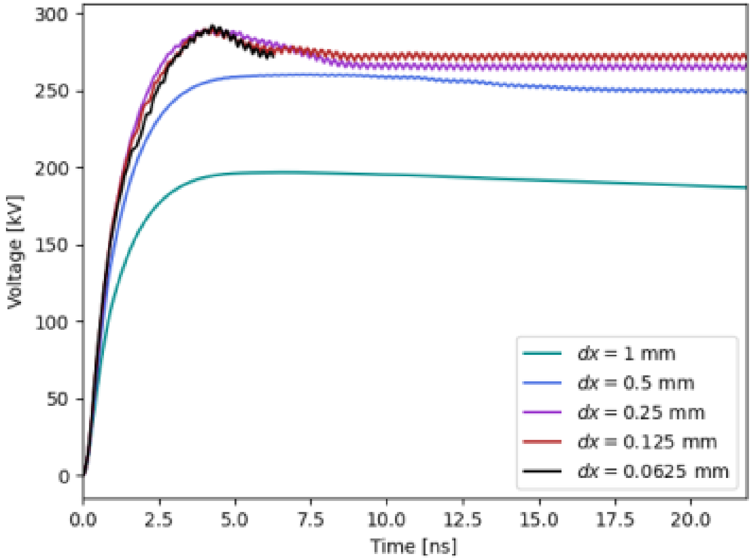
A relatively high cross-sectional resolution of the mesh is required for accurate results.

- A classic two-dimensional PIC calculation for the A6 magnetron required only 3000 mesh cells and 30000 macroparticles [10].
- The present, dx=0.125 mm, grid has 133042 elements. dx=0.0625 mm has 503456 elements. Note: A periodic slice is actually used that is 2 elements wide, not a true 2D grid.

The magnetron perveance, K, decreases as grid resolution is refined.

$$K = I/V^{\frac{3}{2}}$$





[10] Chan, Hei-Wai, Chiping Chen, and Ronald C. Davidson. "Numerical study of relativistic magnetrons." Journal of applied physics 73.11 (1993): 7053-7060.

Summary and Conclusions

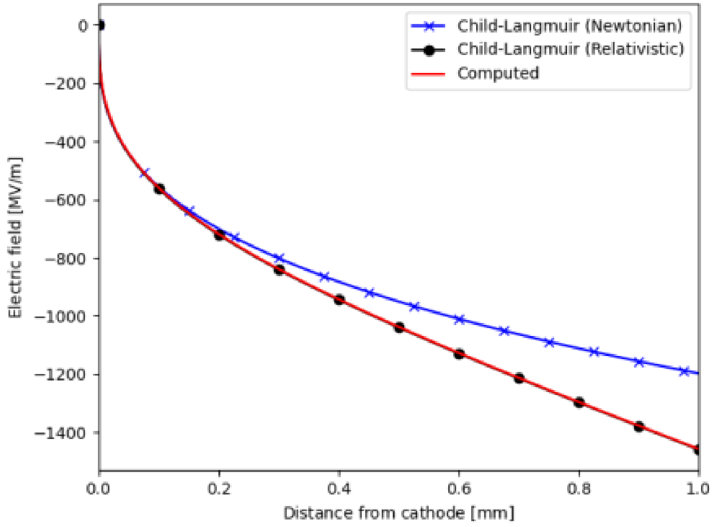
- A fully non-linear, relativistic fluid model was used to model a 2D A6 magnetron
  - Most magnetron theory has made use of fluid models
  - Results indicate that this approach, or a similar approach, could be a useful
  - A much higher spatial resolution is required compared to a PIC simulation to obtain a converged result

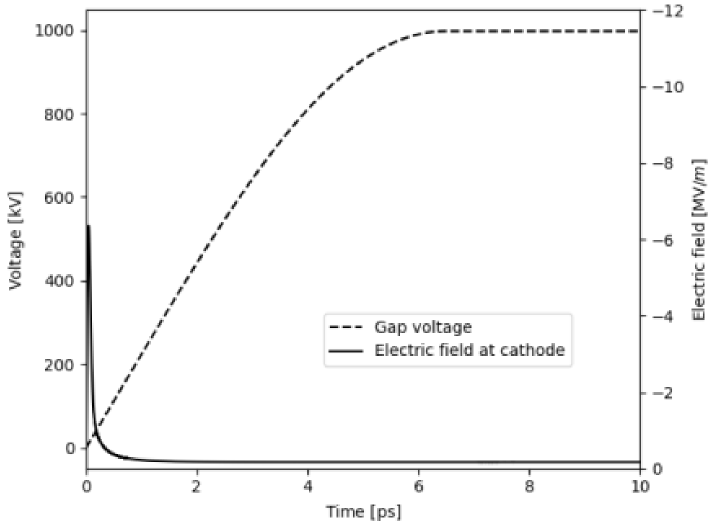
- An SCL emission boundary condition for the fluid model was developed
  - Benchmarking results indicate similar behavior compared to PIC SCL boundary conditions
  - The methods used also enable a fluid thermal desorption boundary condition for study of electrode impurities, to be presented in future works
  - Many other applications besides magnetrons should be possible

1D diode benchmark problem for the fluid SCL boundary condition

We applied the fluid SCL boundary condition to the 1D relativistic Child-Langmuir diode. A development and analysis of this canonical problem can be found in the treatise by Davidson [2].

- Gap distance $d = 1\ \mathrm{mm}$
- Constant AK circuit current, $I = 1.9852 \times 10^9\ \mathrm{A/m^2}$
- SCL parameters,
  - $\gamma_0 = 7500\epsilon_0$
  - $\gamma_1 = 0$
  - $\tau_{\mathrm{ramp}} = 1\ \mathrm{ps}$
  - $v_{inj} = 5.93 \times 10^5\ \mathrm{m/s}$ (corresponds to 1 eV)
  - $T_{inj} = 5000\ \mathrm{K}$





On the left, a comparison between the fluid SCL electric field result is plotted against relativistic and non-relativistic theory. On the right, the transient evolution of the electric field at the cathode and the gap voltage is plotted.