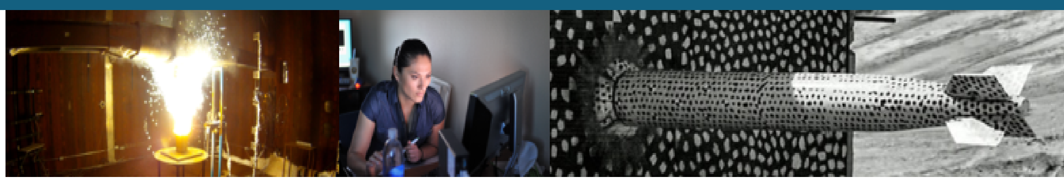
SAND2021-11095C

Sandia National Laboratories

Shock Response Spectrum Primer



91st Shock and Vibration Symposium

19 – 23 September 2021, Orlando, Florida

Carl Sisemore

Sandia National Laboratories

Albuquerque, NM

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Overview

Origins of the shock response spectrum

What is the shock response spectrum

How the SRS is calculated

Common features of shock response spectra

Common characteristics of classical shocks

Common characteristics of oscillatory shocks

Features of complex shocks

Uniqueness & non-uniqueness of the SRS

How to judge severity from a shock response spectrum

History & Theory

Shock Response Spectra History

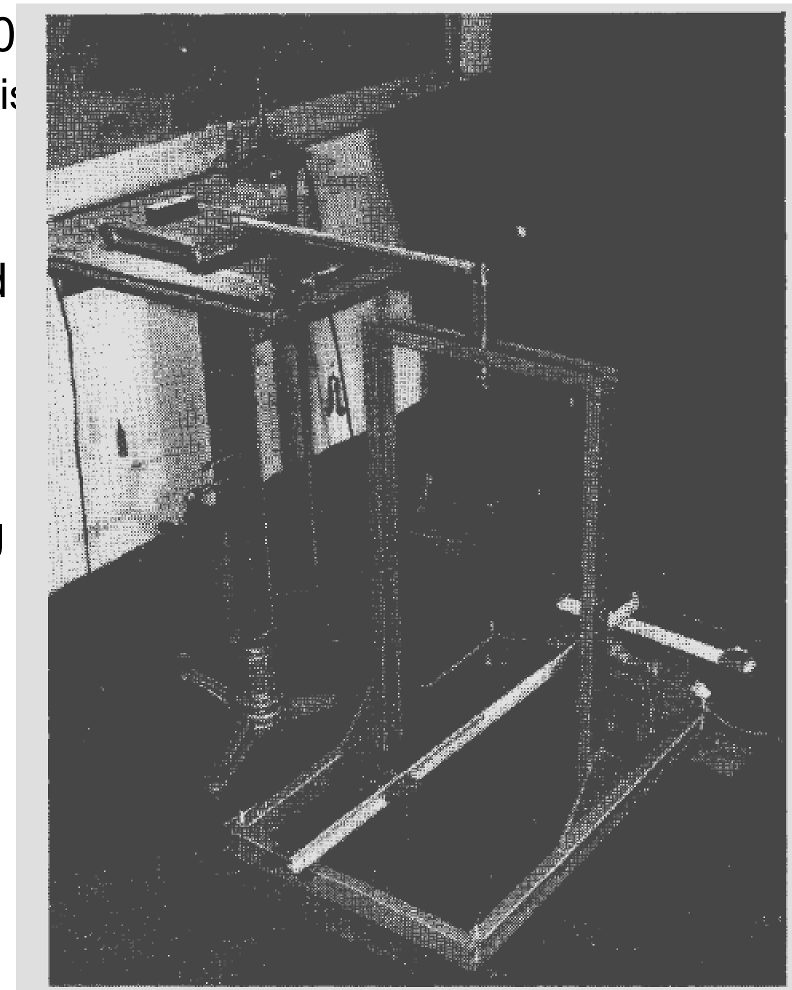
Concept was originally developed in the 1930

- Idea is generally credited to Maurice Biot from his 1933 Ph.D. Thesis
- No SRS plots are actually provided in his thesis

By 1940 shock spectra were being computed analog computers

- Eight hours to produce one spectrum plot
- And it only cost $40

By the early 1940's, SRS plots were showing lots of places



Biot, M. A., "A Mechanical Analyzer for the Prediction of Earthquake Stresses," Bulletin of the Seismological Society of America, Vol. 31, No. 2, April 1941

Shock Response Spectra History

SRS converts a transient excitation into a frequency domain representation of the response of a series of single degree-of-freedom oscillators (SDOF)

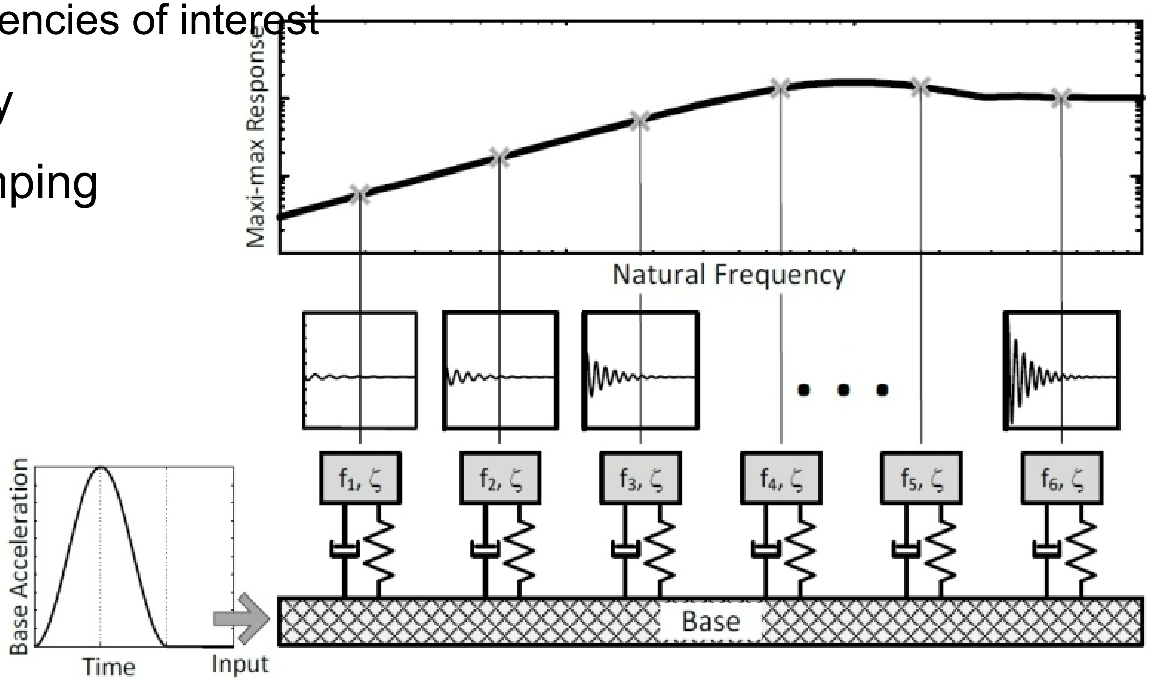
- Plot of an extremal response quantity of interest
- Acceleration, velocity, displacement, energy, or almost anything else

Significantly reduces the data complexity and number of data points

- You get to choose the frequencies of interest

Parameterized in frequency

Originally assumed no damping



Sisemore, SRS Primer, 91st Shock & Vibration Symposium, 19 – 23 Sept 2021

Shock Response Spectra History

SRS concept was derived from the Fourier series

- Any periodic function can be expressed by a series summation of sine and cosine terms

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

Each term in the Fourier series corresponds to a single frequency

The equation of motion for an undamped SDOF oscillator is

$$\ddot{y}(t) + \omega_n^2 y(t) = 0$$

And the free vibration response is given by

$$y(t) = A_1 \cos(\omega_n t) + B_1 \sin(\omega_n t)$$

Thus, the free vibration response of the SDOF oscillator is essentially one term in the Fourier series

Shock Response Spectra History

Fourier first published his theory in an essay to the French Academy of Science in 1812 entitled *Mathematical Theory of Heat*

- He won the academy prize but was criticized by the panel (Joseph Lagrange, Pierre Laplace, and Adrien Legendre) for a certain looseness with his reasoning
- Work was later published more formally in 1822 as the *Analytical Theory of Heat*
- Convergence of the series was not proven until 1829 by Dirichlet

The Fourier series is a “complete” transform because no data is lost and the transform is reversible.

The SRS is an incomplete transform and is not reversible

If the Fourier series was considered “loose” then the SRS may be considered “looser” still

SRS Background

Equation of motion for SDOF system subject to base excitation

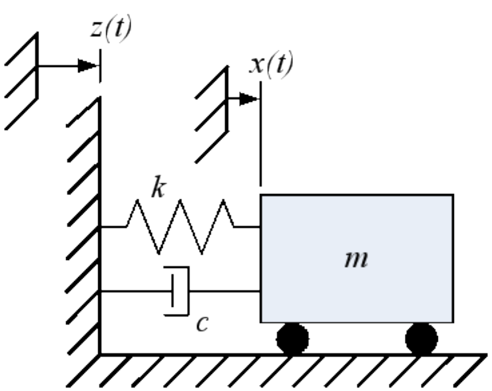
$$m\ddot{x}(t) + c\big(\dot{x}(t) - \dot{z}(t)\big) + k\big(x(t) - z(t)\big) = 0$$

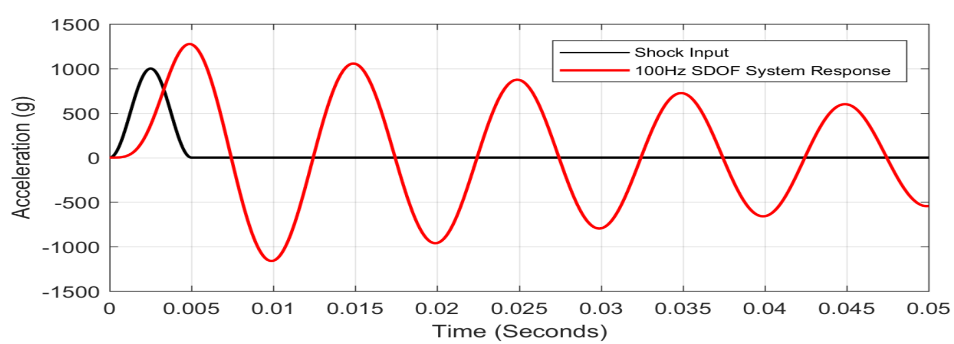
For relative motion between the mass and the base, substitute $y(t) = x(t) - z(t)$

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = -m\ddot{z}(t)$$

For shock analysis we typically assume the system starts at rest and the equation becomes

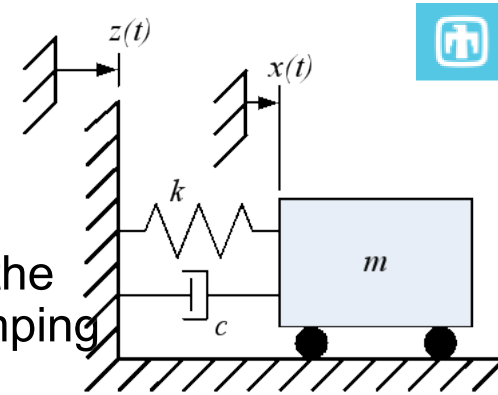
$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = -\ddot{z}(t)$$





$$\omega = \sqrt{k/m}$$

SRS Background



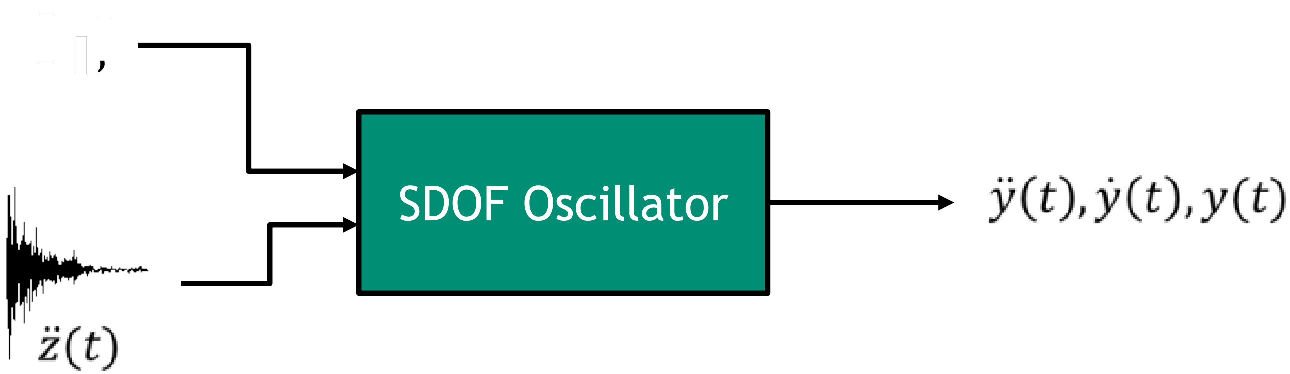
Since the SDOF equation of motion is mass normalized, the response only depends on the natural frequency and damping ratio of the SDOF oscillator plus the base input motion

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = -\ddot{z}(t)$$

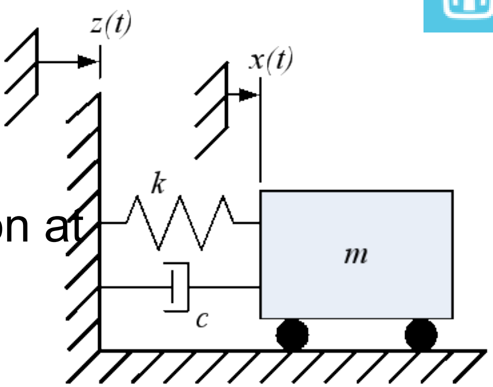
$$y(t) = x(t) - z(t)$$

Solution of the second-order differential equation is straightforward

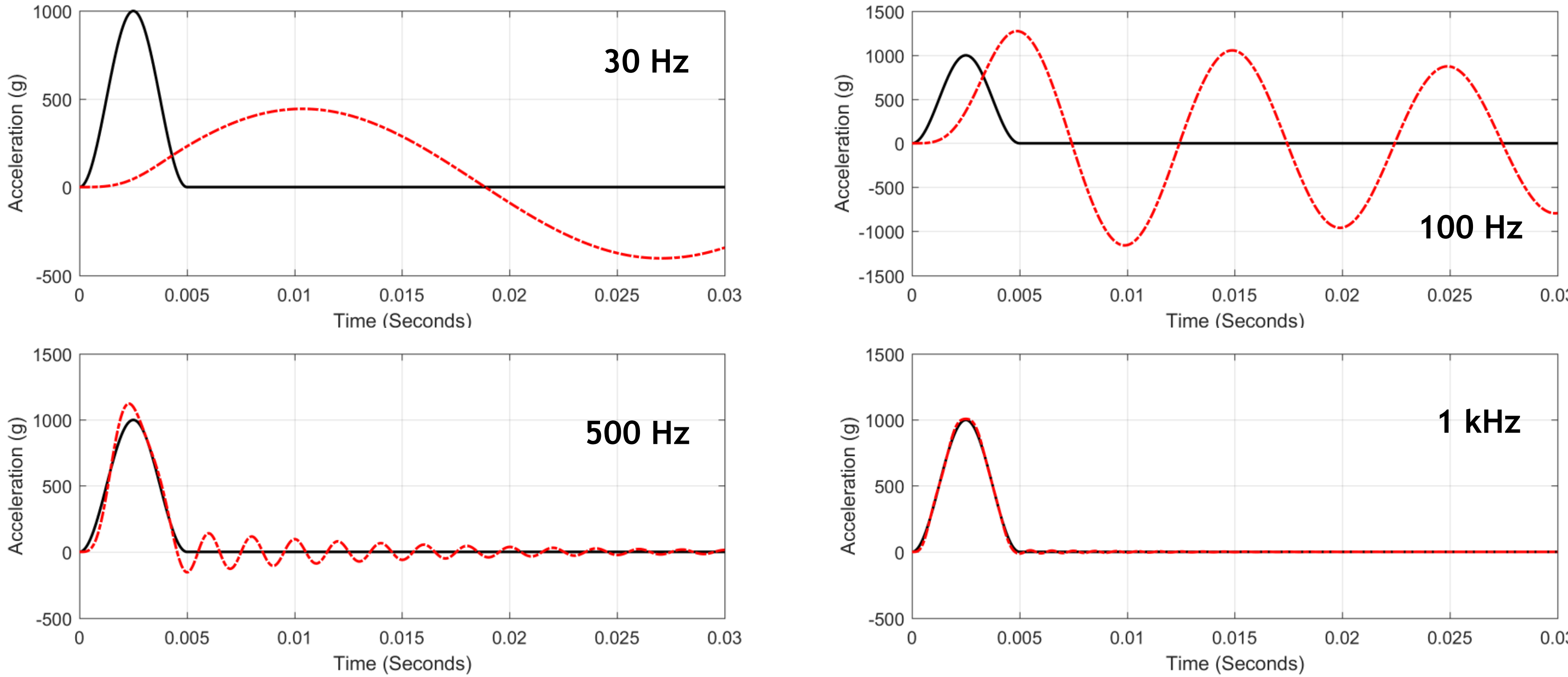
- Can be accomplished with any common ODE solver
- MATLAB ODE45, Newmark-beta, or any other personal favorite



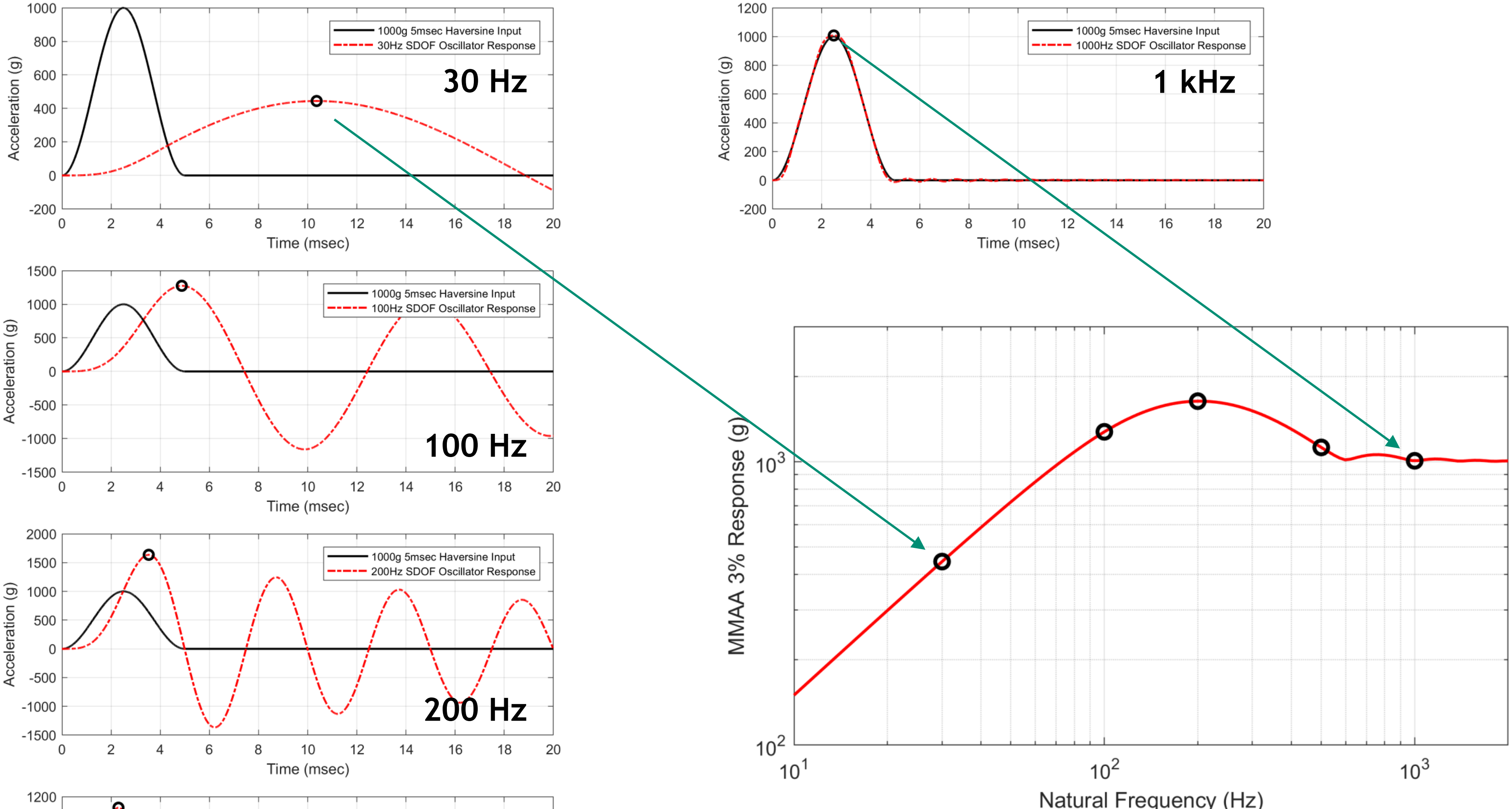
What does the SDOF Solution Look Like?

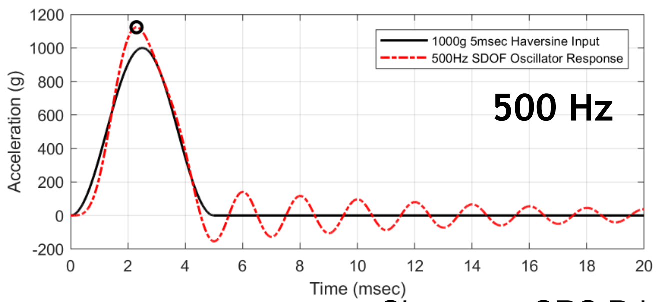


For a 1,000g 5msec classical haversine shock the solution at four different natural frequencies is given as:



How Do We Convert This Time History Data to an SRS?

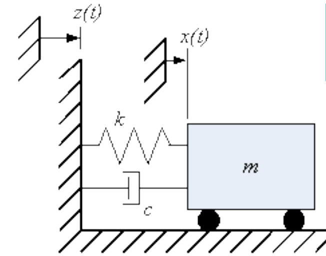


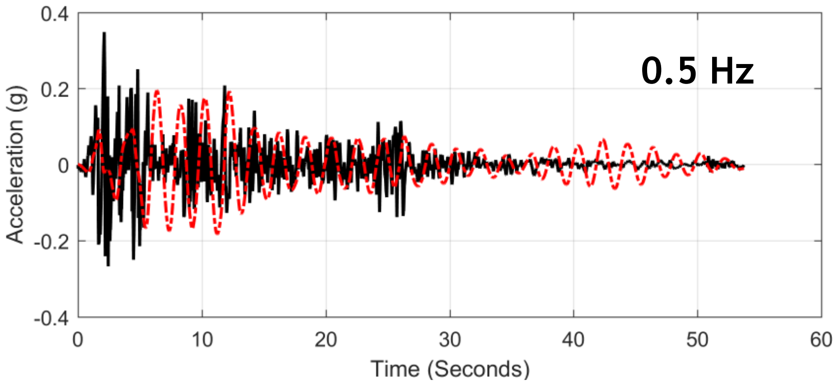


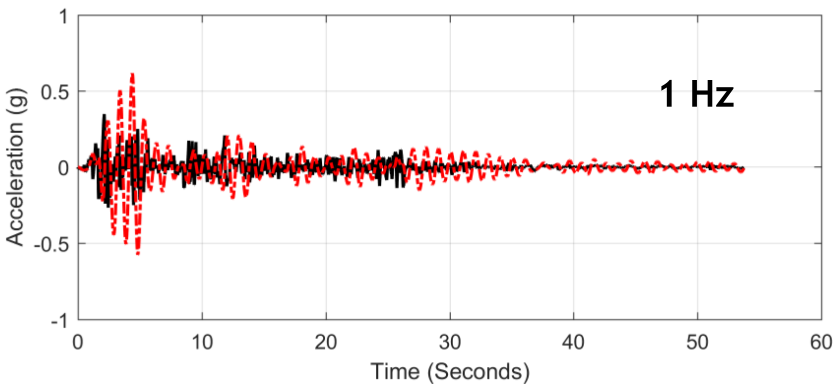
Generating the SRS is that simple

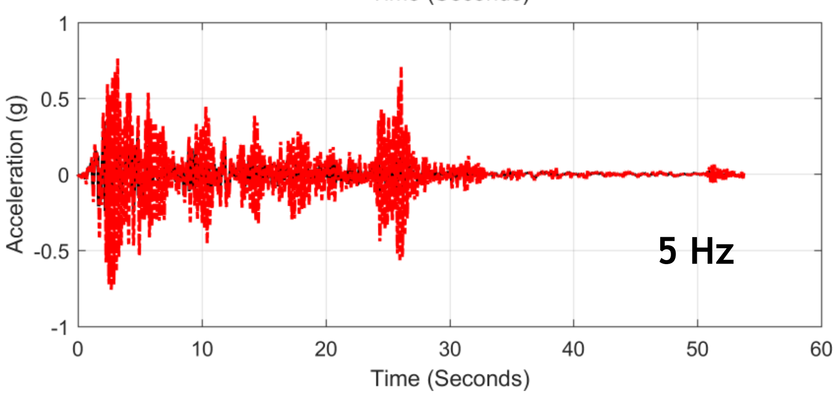
But there is a lot more to understanding it

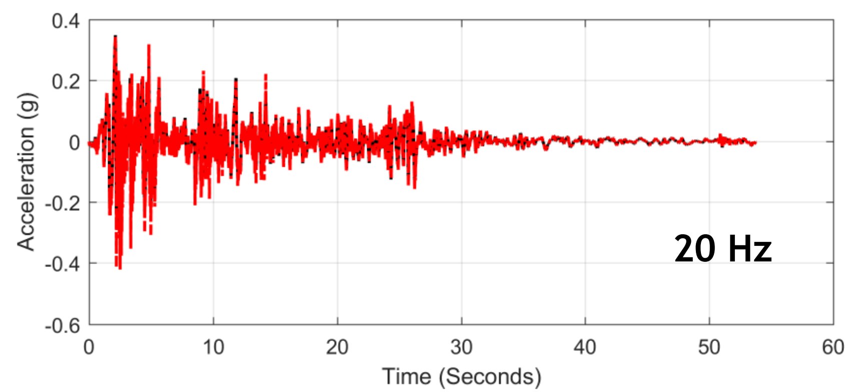
What does the SDOF Solution Look Like?



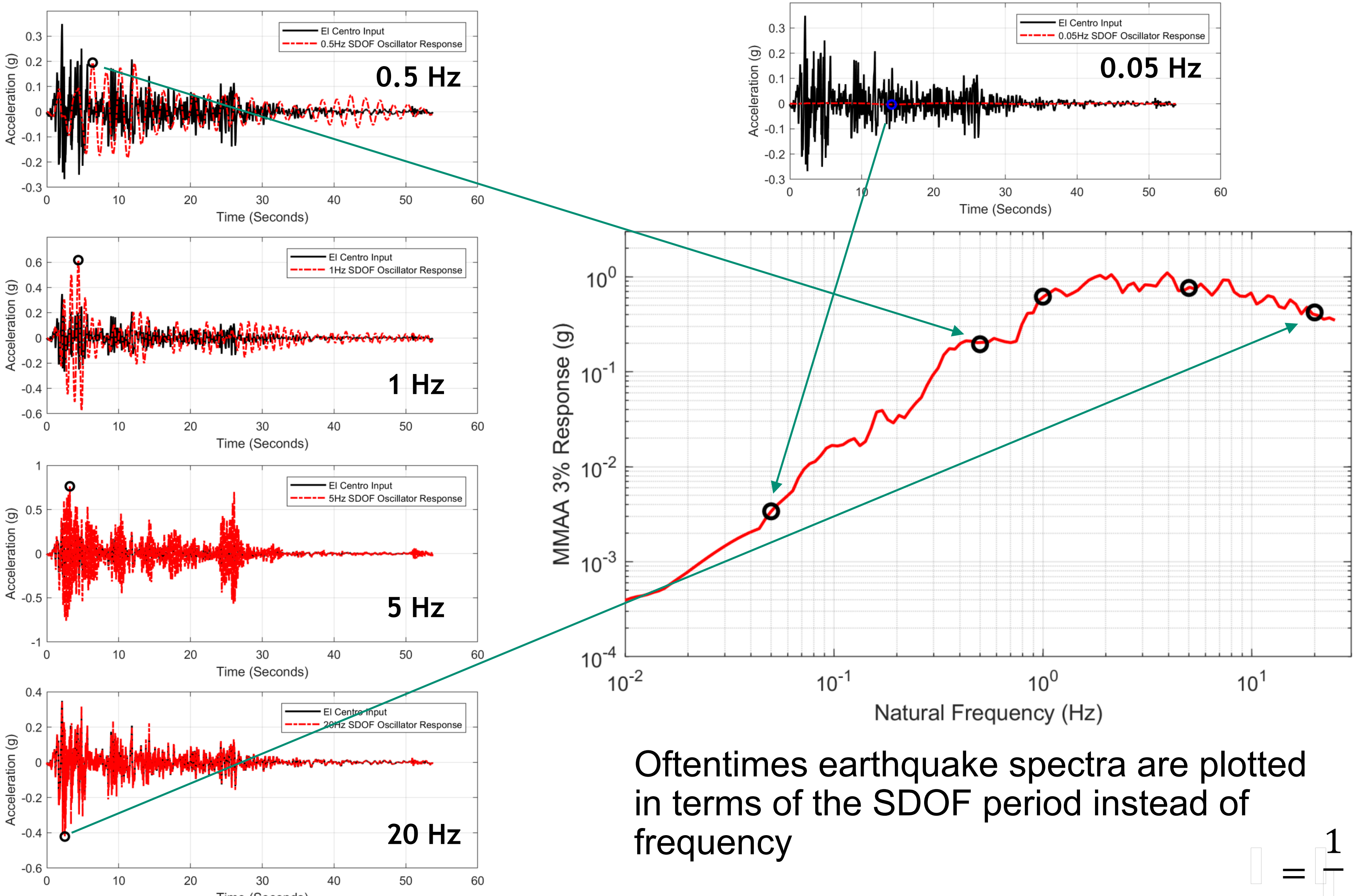








Converting El Centro Data to SRS



Oftentimes earthquake spectra are plotted in terms of the SDOF period instead of frequency

$$\square = \frac{1}{\square}$$

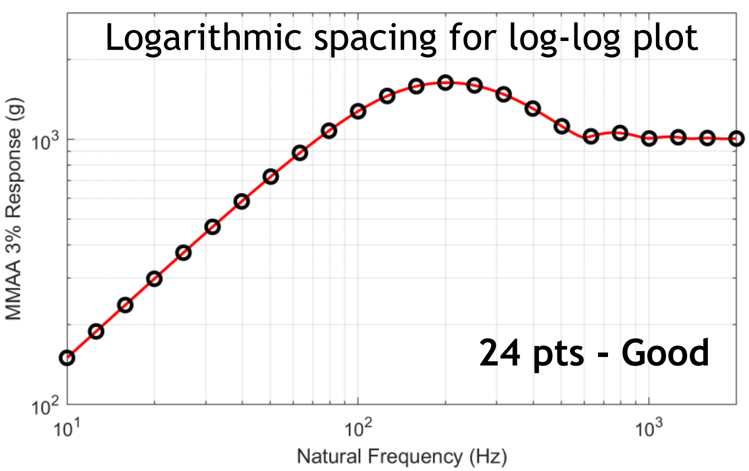
How Do We Pick SDOF Frequencies for Evaluation?

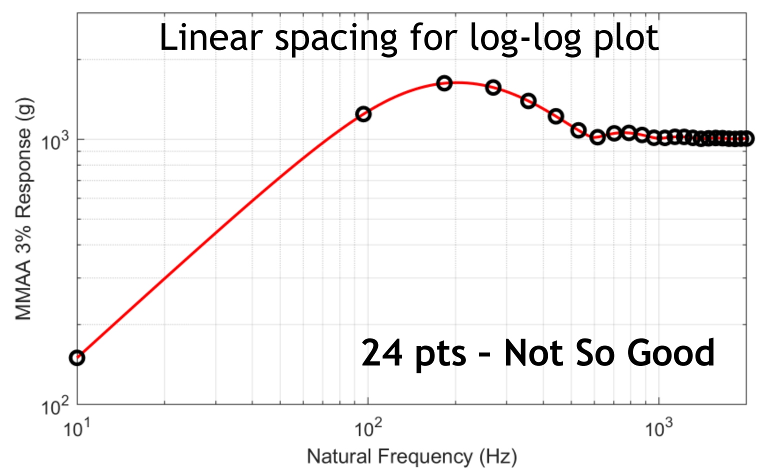
In theory any random set of SDOF frequencies can be used to calculate an SRS

The Fourier series is an infinite summation so any frequency is acceptable

In practice we want a reasonable set of frequencies

- What are the frequency ranges of interest?
- Assume that the SRS will be relatively smooth so we want a well-distributed



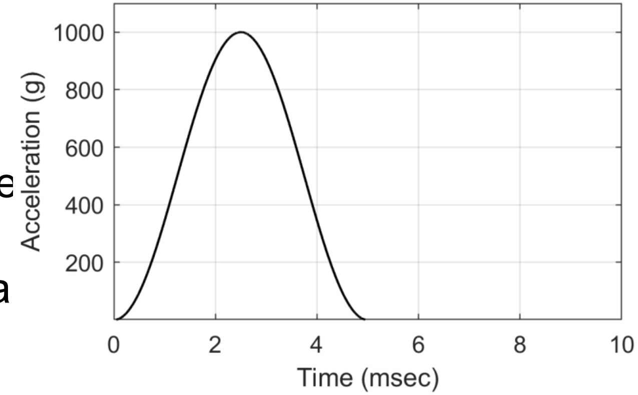


How Do We Pick SDOF Frequencies for Evaluation?

Want to evaluate SRS at the important frequencies

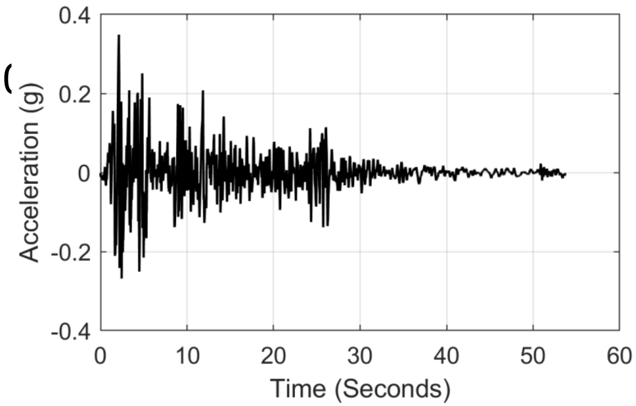
5msec haversine → 200Hz excitation

- SRS frequency band should cover frequencies above and below 200Hz
- May want to analyze down to low frequencies to obta an estimate of velocity change



Earthquake event

- 25 second duration → may need to analyze below 0.(
- Most energy here is in the 0.5 – 10Hz range



Pyroshock events

- May not care about frequencies below 100Hz

How Do We Pick SDOF Damping for Evaluation

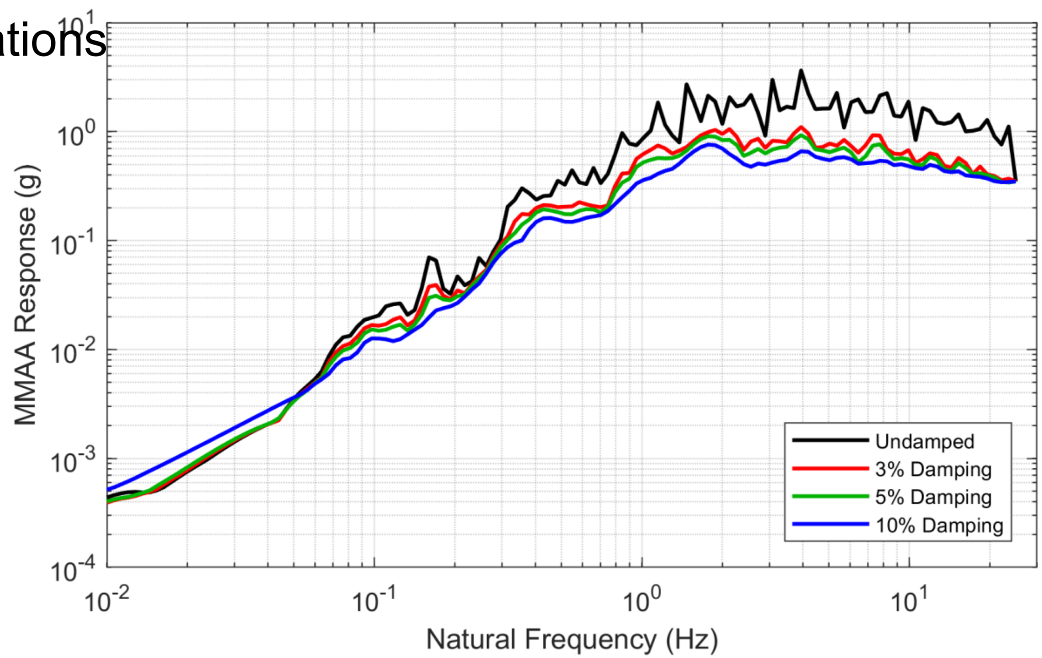
Biot's original work assumed an undamped SDOF oscillator

- Reason was that earthquakes are a short-time excitation and damping may not have time to play a significant impact in the response
- Calculation was also more conservative with zero damping

At the current time, nearly all SRS calculations assume some level of damping in the SDOF oscillator—no matter how short the shock duration

Generally assume some small damping value for most calculations

- Similar to expected structural damping level
- Assumes your system can be approximated by the SDOF oscillator
- 2 – 5% of critical damping
- Value may be set by program requirements or standards



Sisemore, SRS Primer, 91st Shock & Vibration Symposium, 19 – 23 Sept 2021

SRS Calculations

SRS Calculations

Analog computers are cool but not very practical for the type of shock analysis that we do today

Originally accomplished with numerical integration of the equation of motion for various frequencies

- Runge-Kutta, Newmark-Beta, ODE45, or similar
- Works very well but it is slow

Most commonly calculated using ramp invariant filters codified in ISO 18431-4

- Filter coefficients were developed by D. Smallwood
- Smallwood, D., *An Improved Recursive Formula for Calculating Shock Response Spectra*, 51st Shock and Vibration Symposium, October 1980
- Smallwood, D., *Derivation of the Ramp Invariant Filter for Shock Response Spectrum Calculations*, 76th Shock and Vibration Symposium, October 2005

INTERNATIONAL STANDARD

ISO 18431-4

First edition
2007-02-01

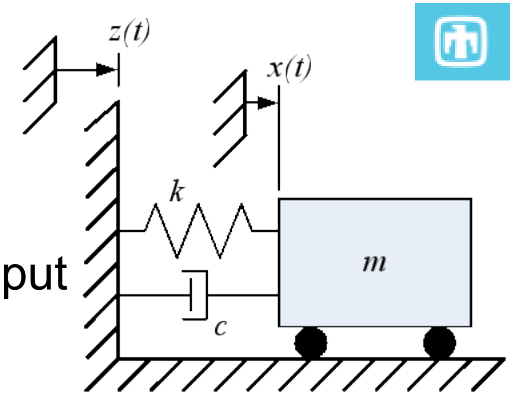
Mechanical vibration and shock — Signal processing —

Part 4:
Shock-response spectrum analysis

Vibrations et chocs mécaniques — Traitement du signal —
Partie 4: Analyse du spectre de réponse aux chocs

Reference number
ISO 18431-4:2007(E)

SRS Calculation Using Digital Filters



Treating the SDOF system as a filter through which the input is passed is the most computationally efficient method of calculating the output quantity of interest

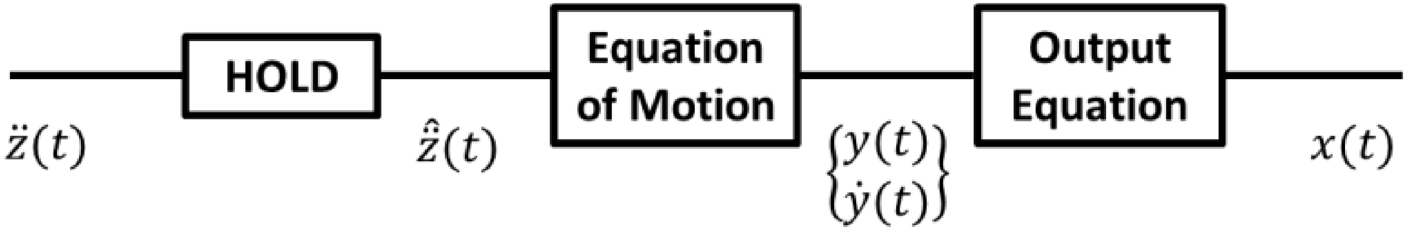
$$y(t) = x(t) - z(t)$$

However …

Loading is only sampled at discrete intervals, sampling rate

Input between samples is unknown

A hold must be used to fill in the missing information with an assumption

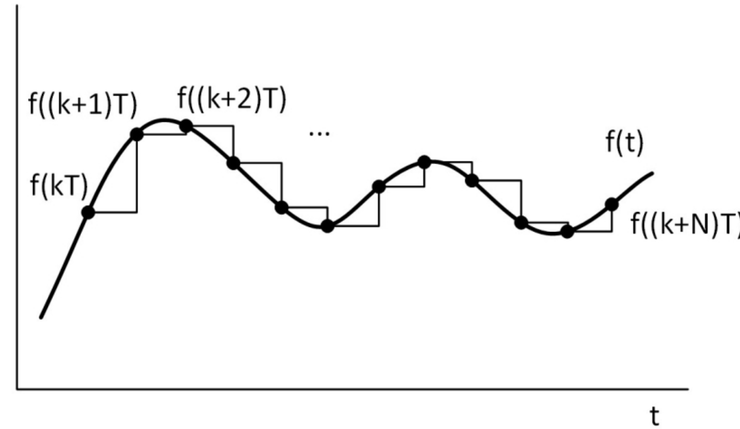


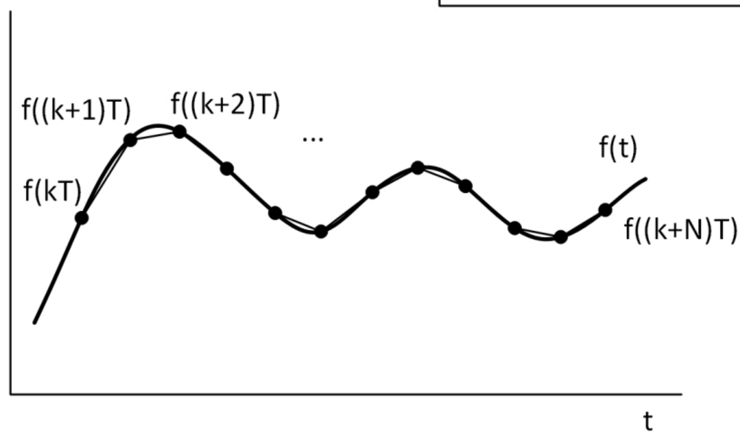
SRS Calculation Using Digital Filters

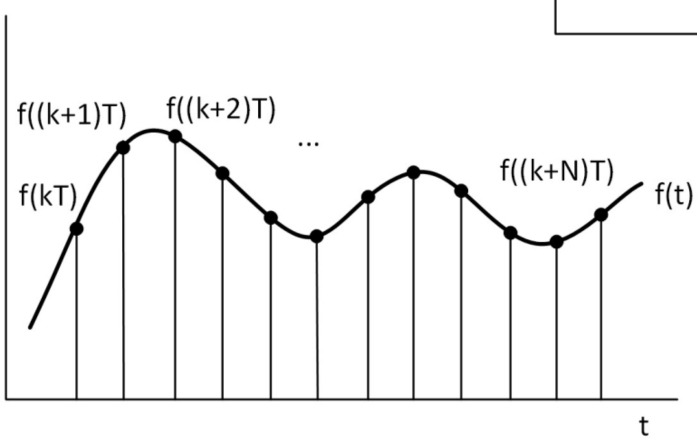
Several different types of holds are possible

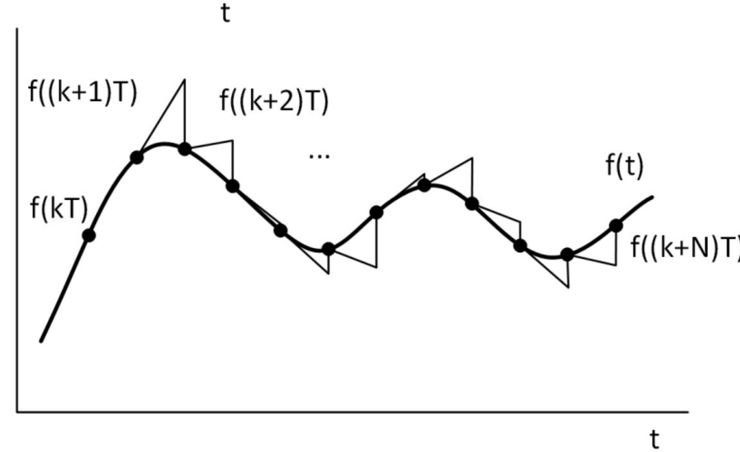
Zero-order hold, First-order causal hold, First-order non-causal hold, Impulse hold

The first-order non-causal hold is the basis for the ramp invariant filter









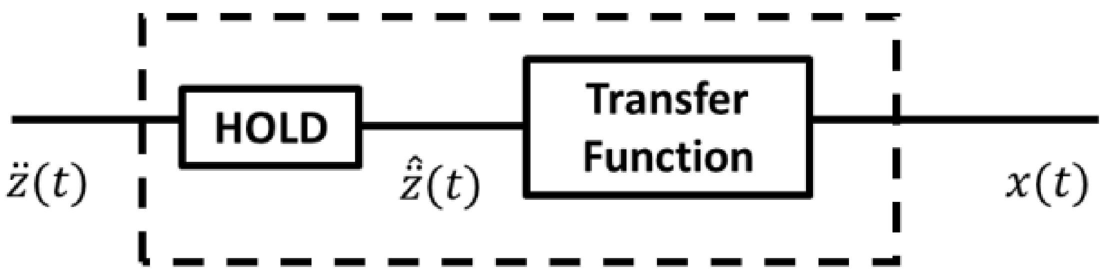
SRS Calculation Using Digital Filters

Can create an input-output model by combining the equation of motion and the output equation into a single transfer function

Transfer function relates the input to the output in the Laplace domain

$$H(s) = \frac{Y(s)}{F(s)} + z_0 + s\dot{z}_0$$

Typically we assume $z_0 = \dot{z}_0 = 0$



Discrete Time Transfer Function $H(z)$

SRS Calculations Using Digital Filters

The transfer functions with zero initial conditions is

$$H(s) = \frac{Y(s)}{F(s)}$$

The denominator polynomial is the characteristic equation, Laplace transform of the equation of motion, and is independent of input or output

$$F(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

The numerator equation is entirely dependent on the input and output

For the absolute acceleration case, the Laplace transform of the output equation is

$$Y(s) = (-\omega_n^2 - 2\zeta\omega_n s)Z(s)$$

And the transfer function from the base acceleration to the absolute acceleration of the SDOF oscillator mass is

$$H(s) = \frac{-\omega_n^2 - 2\zeta\omega_n s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

SRS Calculations Using Digital Filters

Since the data is discretized, we apply the z-transform to the continuous time transfer function

The discrete time transfer function of the SDOF oscillator is then:

$$H(z) = \frac{\beta_0 + \beta_1 z^{-1} + \beta_2 z^{-2}}{1 + \alpha_1 z^{-1} + \alpha_2 z^{-2}}$$

This can be written as a difference equation in terms of the delay operator

$$y_n = \beta_0 x_n + \beta_1 x_{n-1} + \beta_2 x_{n-2} - \alpha_1 y_{n-1} - \alpha_2 y_{n-2}$$

The coefficients α_1, α_2, β_0, β_1, and β_2 are provided in the ISO standard

Other filter coefficients could be derived but the algebra is tedious and not very fun

Absolute Acceleration Filter Coefficients

The absolute acceleration filter coefficients from ISO standard are

$$\square_0 = 1$$

$$\alpha_1 = -2e^{-\zeta\omega_n\Delta t}\cos\left(\omega_n\Delta t\sqrt{1-\zeta^2}\right)$$

$$\alpha_2 = e^{-2\zeta\omega_n\Delta t}$$

$$\beta_0 = 1 - e^{-\zeta\omega_n\Delta t}\frac{\sin\left(\omega_n\Delta t\sqrt{1-\zeta^2}\right)}{\omega_n\Delta t\sqrt{1-\zeta^2}}$$

$$\beta_1 = 2e^{-\zeta\omega_n\Delta t}\left[\frac{\sin\left(\omega_n\Delta t\sqrt{1-\zeta^2}\right)}{\omega_n\Delta t\sqrt{1-\zeta^2}} - \cos\left(\omega_n\Delta t\sqrt{1-\zeta^2}\right)\right]$$

$$\beta_2 = e^{-2\zeta\omega_n\Delta t} - e^{-\zeta\omega_n\Delta t}\frac{\sin\left(\omega_n\Delta t\sqrt{1-\zeta^2}\right)}{\omega_n\Delta t\sqrt{1-\zeta^2}}$$

Coefficients are given in terms of ζ, ω_n, and Δt

Other Filter Coefficients

ISO Standard provides β_0, β_1, and β_2 coefficients for

- Absolute acceleration response
- Relative velocity response
- Relative displacement response
- Pseudo-velocity response

α_1 and α_2 coefficients are the same for all SRS calculations

Can use MATLAB filter function or something similar with α, β, and the input time history to get the SDOF response

Again, other coefficients could be derived if you are inclined to do so

SRS Types

Types of Shock Response Spectra

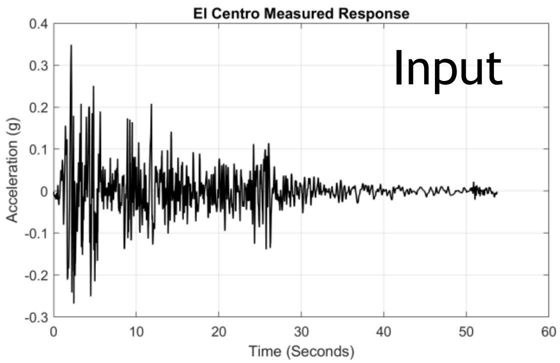
Spectra based on the output quantity of interest
- Acceleration
- Velocity
- Displacement
- Pseudo-velocity
- Energy
- Practically anything else of interest

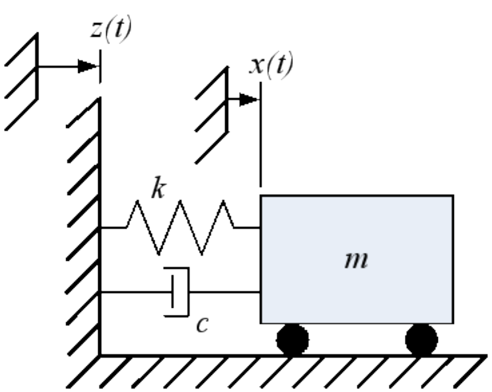
Spectra based on when the output quantity is collected
- Primary response
- Residual response
- Positive response
- Negative response
- Maxi-Max

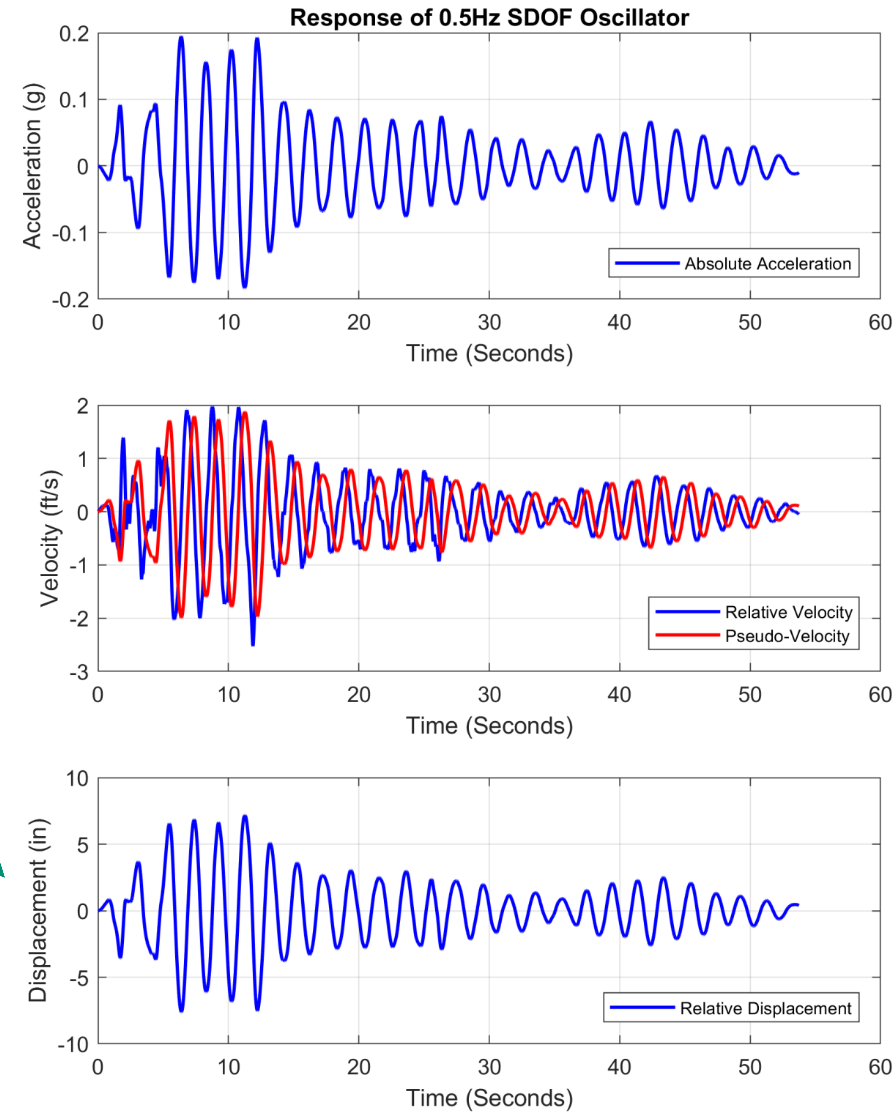
Acceleration, Velocity, or Displacement

The SRS is an extremal response but the response quantity is left to the discretion of the engineer or analyst

The most common choices are acceleration and pseudo-velocity







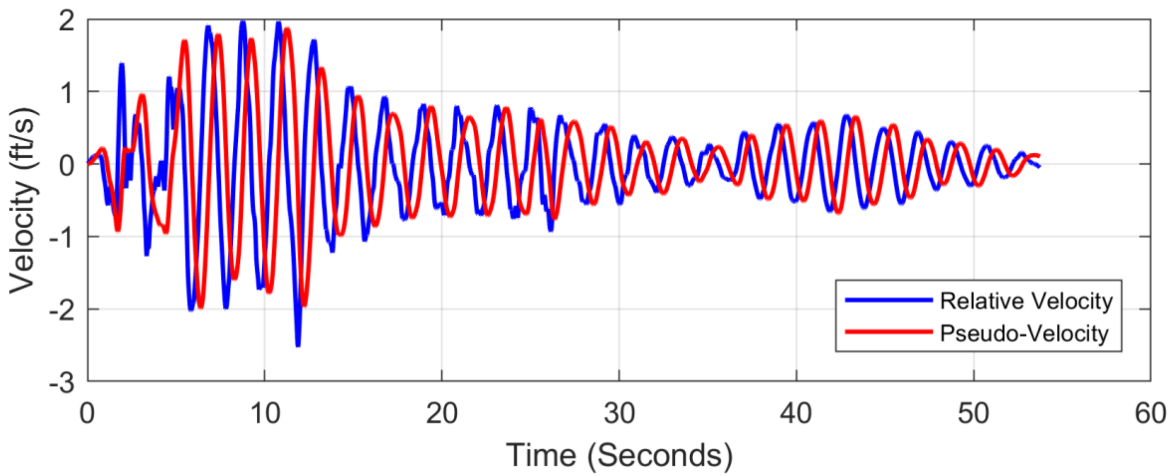
Pseudo-Velocity

Pseudo-velocity is the SDOF oscillator relative displacement scaled by the circular natural frequency

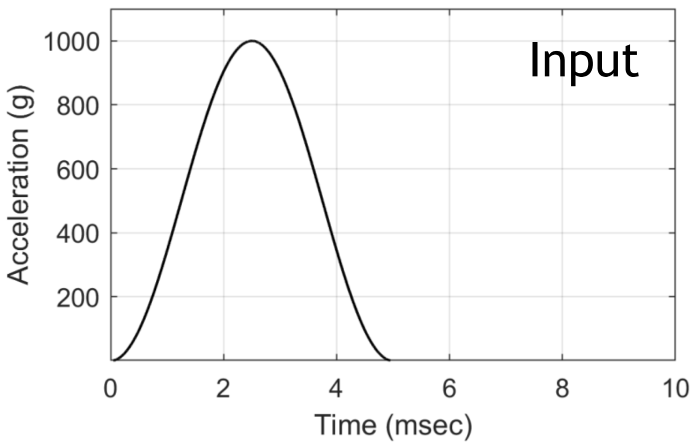
$$PV(t) = \omega_n y(t) = 2\pi f_n y(t)$$

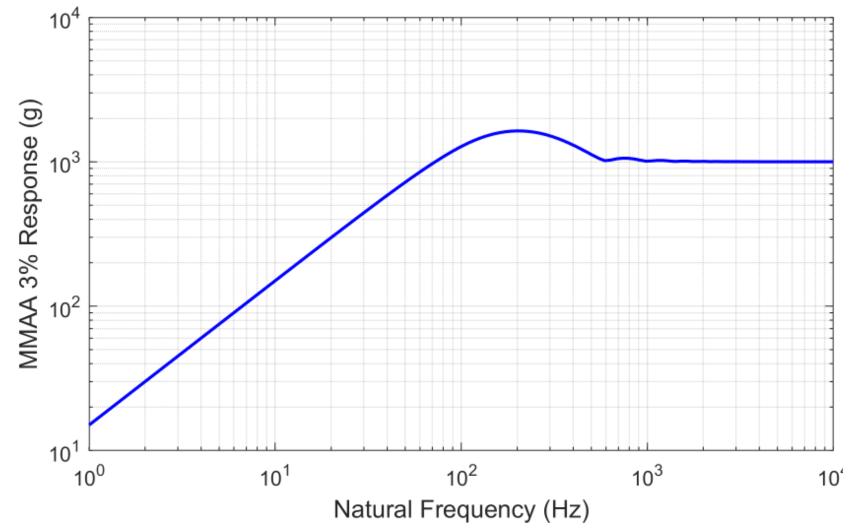
Pseudo-velocity has the units of velocity but is not actually velocity

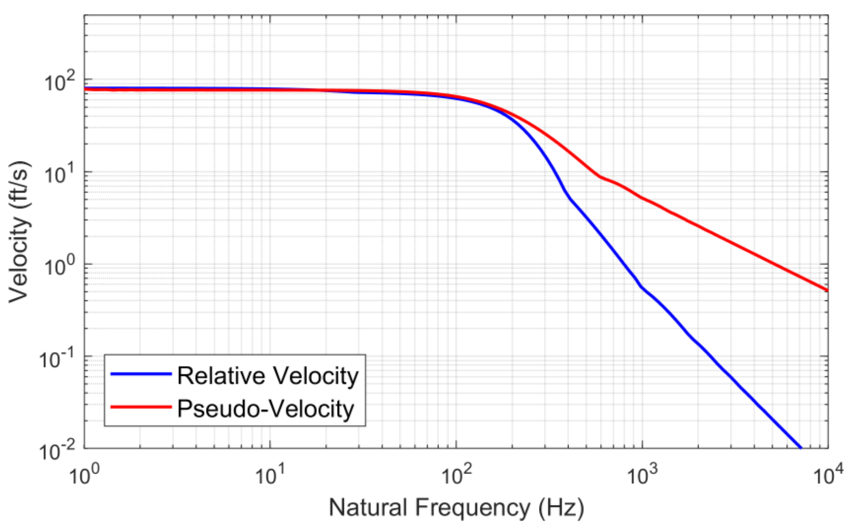
Proportional to the relative displacement and out-of-phase with the relative velocity

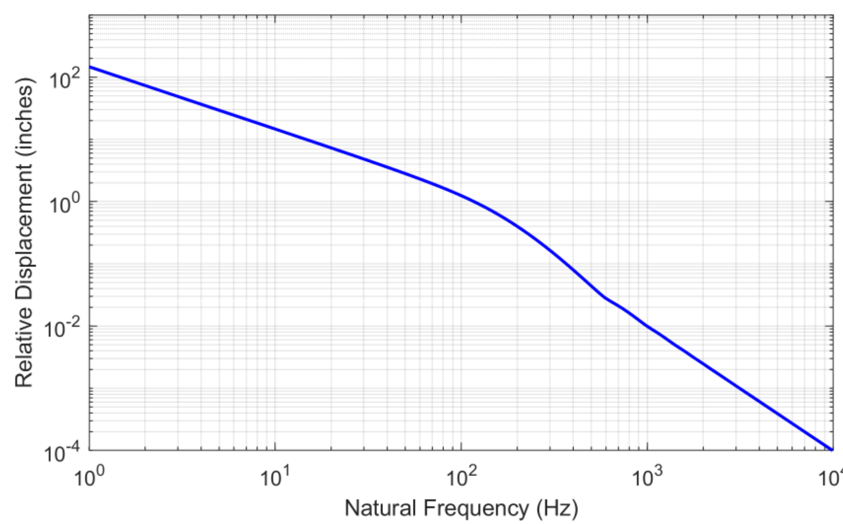


Types of Shock Response Spectra – Sample Drop Shock

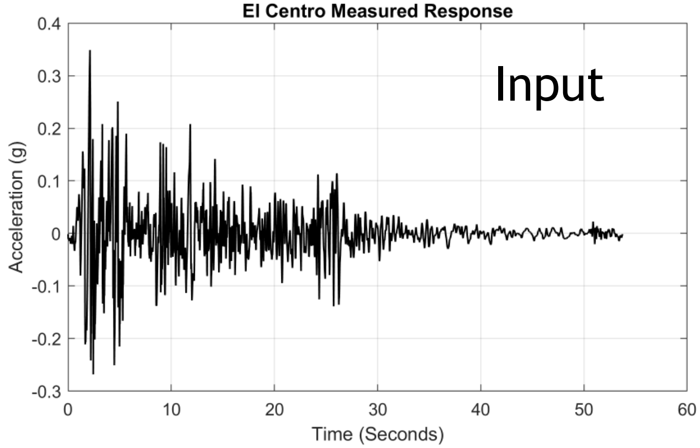


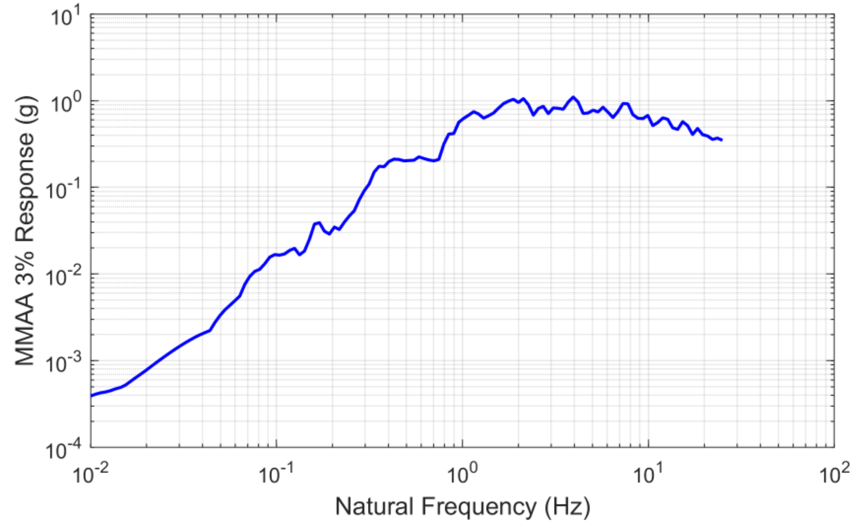


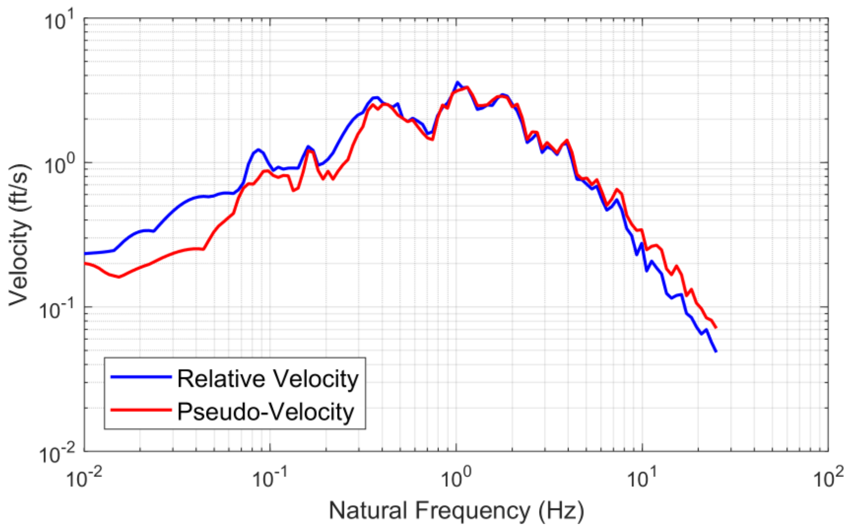


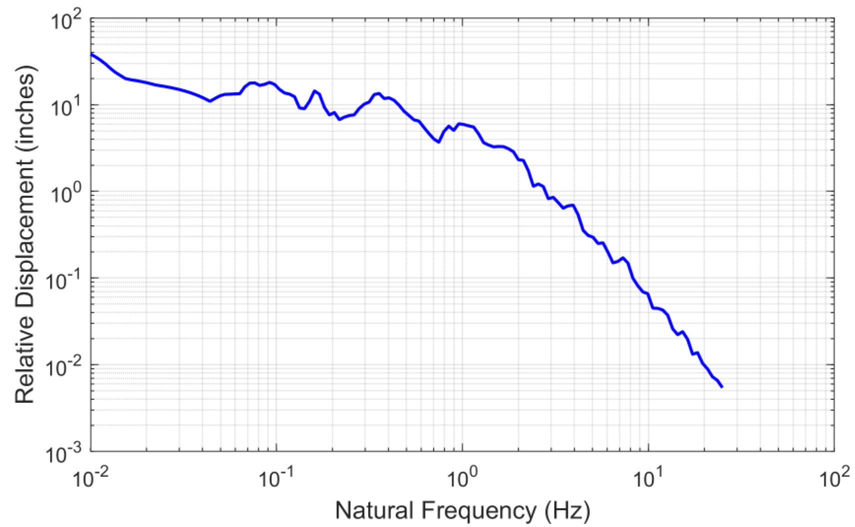


Types of Shock Response Spectra – Sample Earthquake Shock







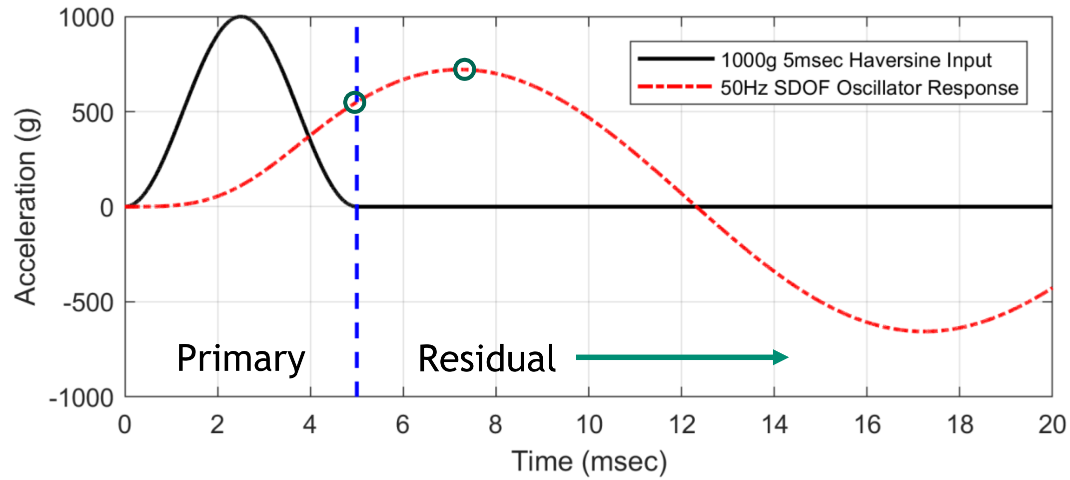


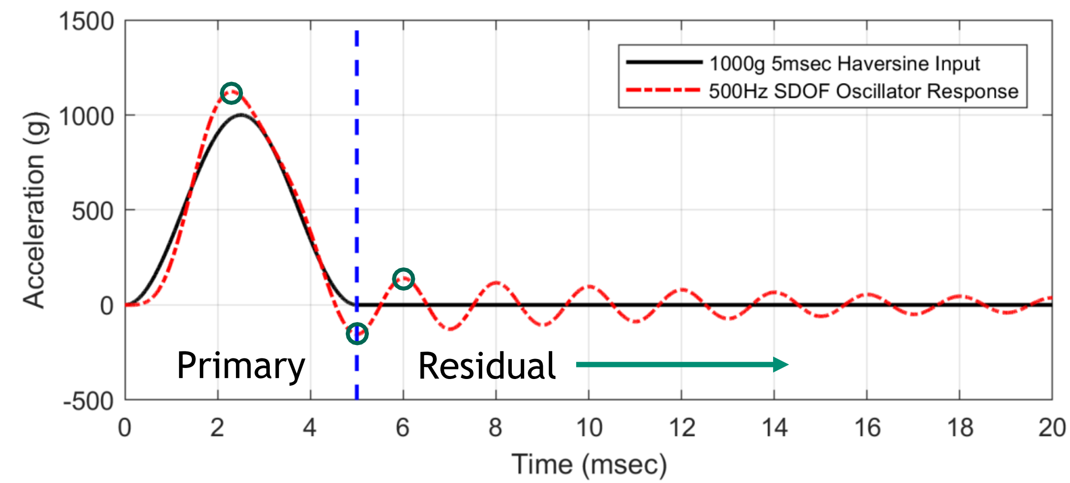
Primary & Residual Responses

Primary refers to the portion of the response that occurs while the transient is in progress

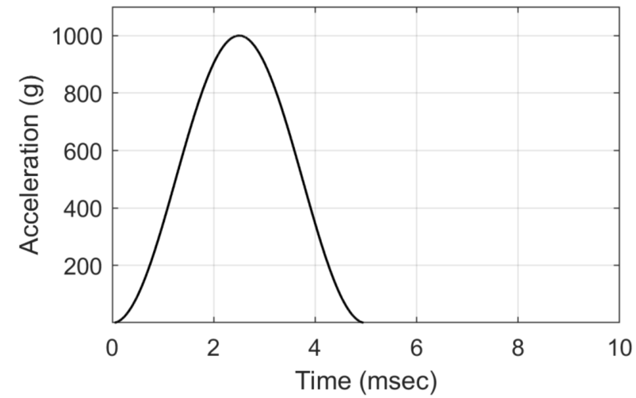
Residual refers to the portion of the response that occurs after the transient has passed

Typically looking for the extremes in both the primary and residual time windows





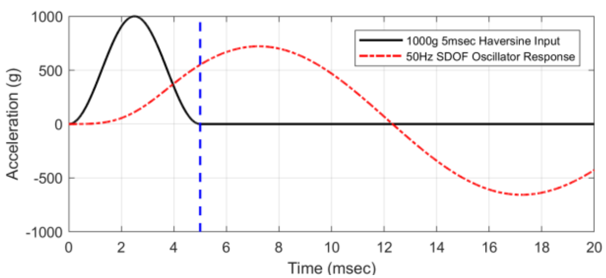
Primary & Residual SRS – Sample Drop Shock

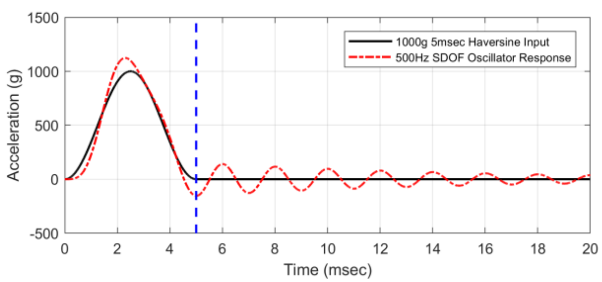


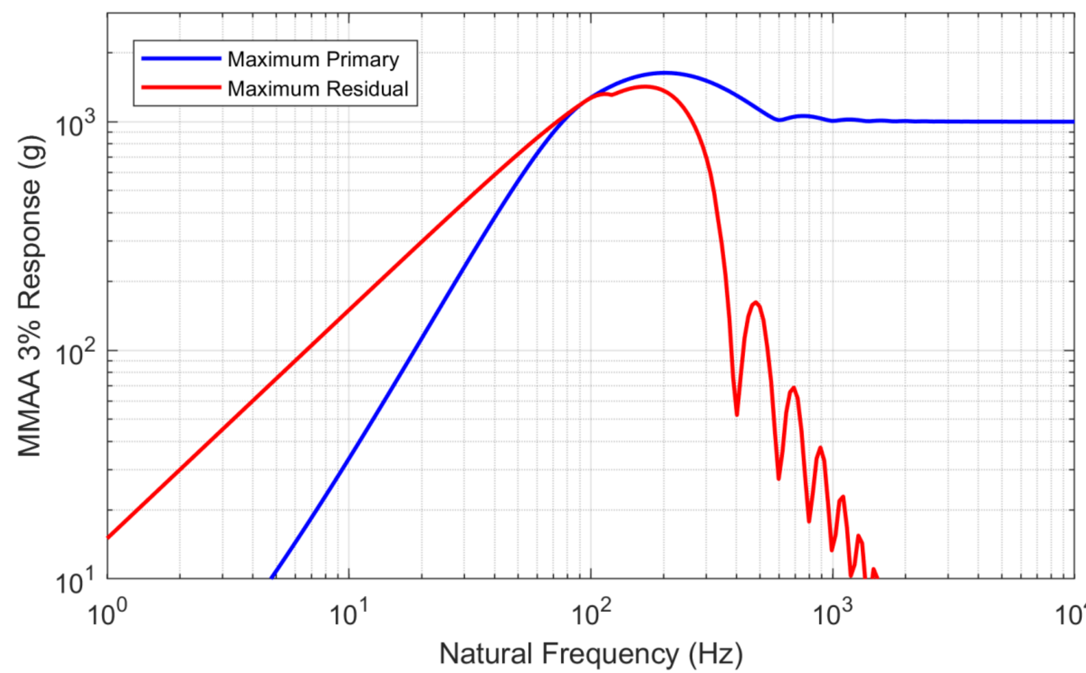
For the one-sided haversine pulse

- Primary response dominates the high-frequency SRS
- Residual response dominates the low-frequency SRS

Residual response decays to negligible levels at high frequency as SDOF sy respond dynamically to the



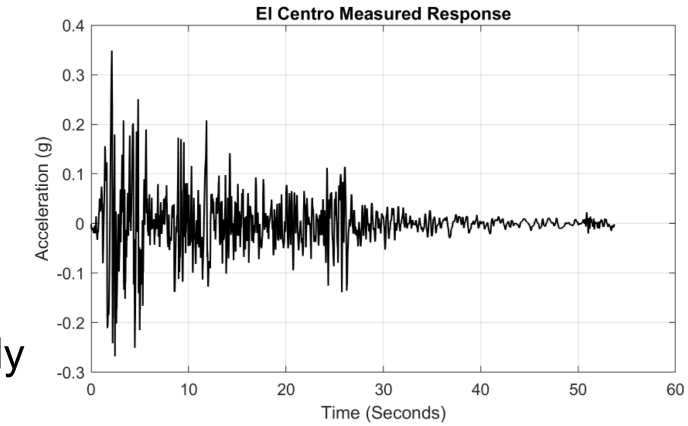


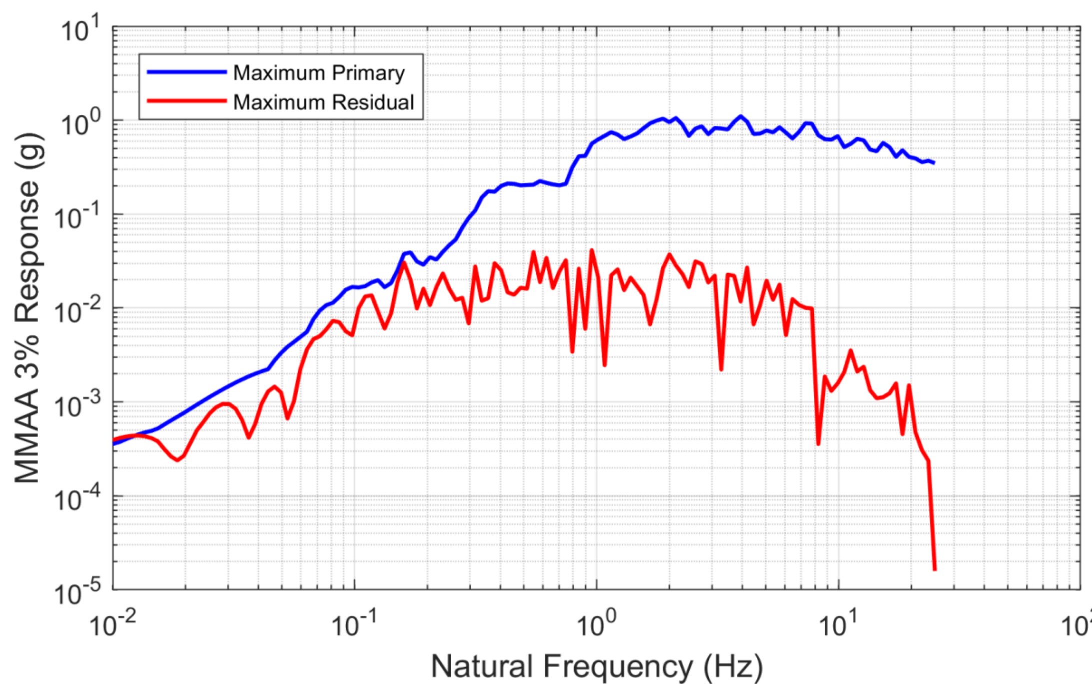


Primary & Residual SRS – Sample Earthquake Shock

Earthquake shock is fundamentally a two-sided shock

- All of the significant SDOF oscillator responses occur during the shock pulse
- Residual responses are significantly less and largely not concerning here

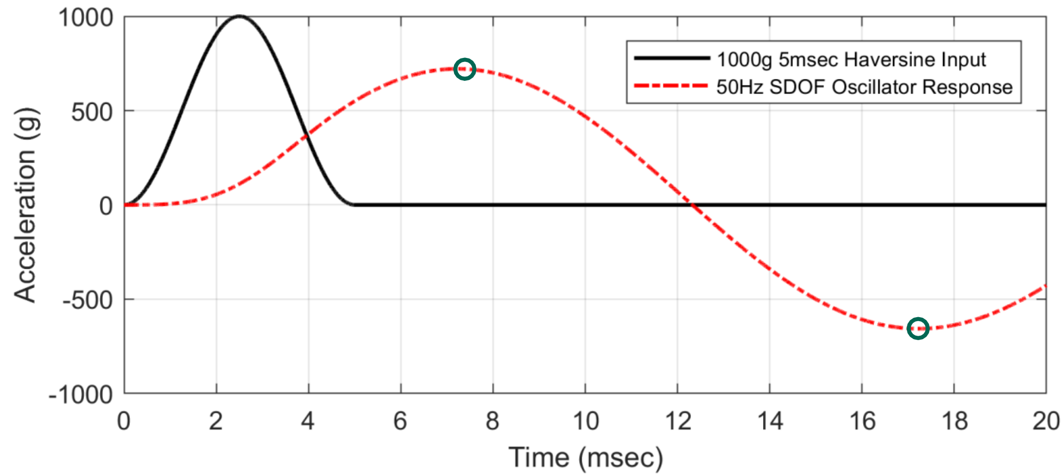




Positive & Negative Responses

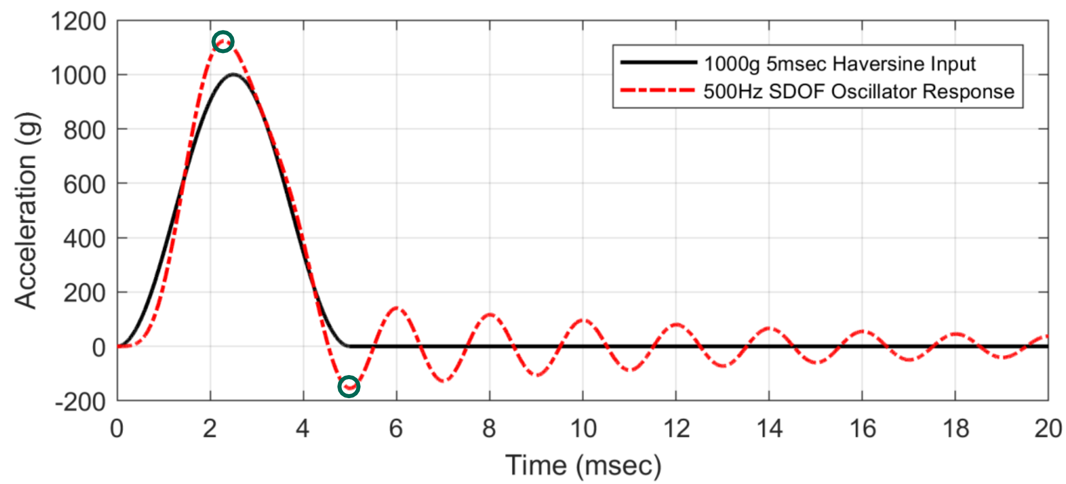
Positive refers to the portion of the response greater than zero

Negative obviously refers to the portion of the response less than zero



Here we are looking for the maximum and minimum extremes over all time

Can also look for maximum and minimum extremes in both the primary and residual time frames but this is not often necessary



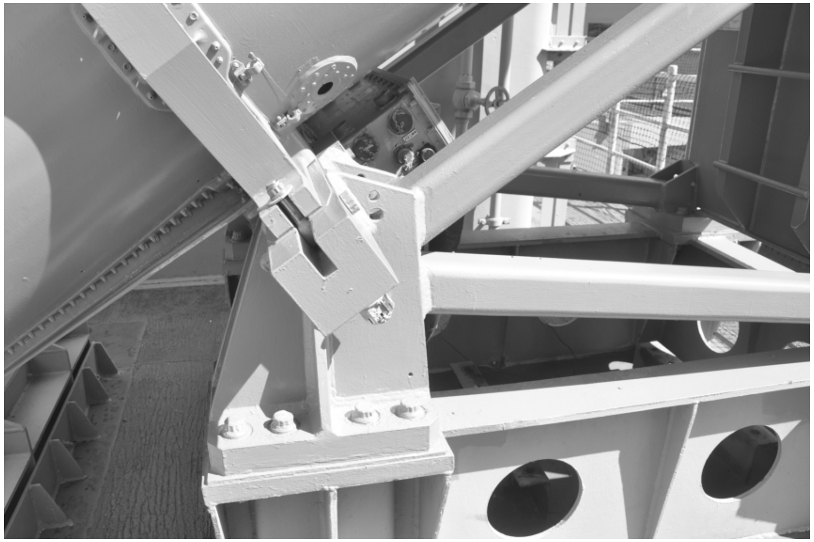
Positive & Negative SRS

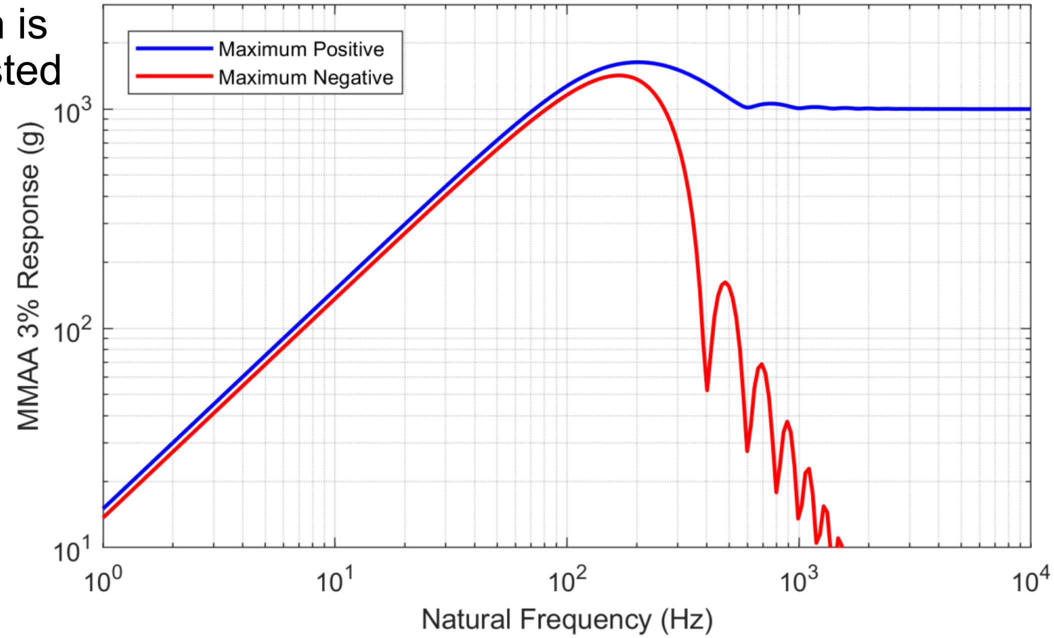
Sometimes there is little difference between positive and negative responses

Other times there can be more significant differences

Two common uses

- To ensure that a component is excited in both directions during a test
- If primary shock loading direction is known, then design can be adjusted accordingly





Sisemore, SRS Primer, 91st Shock & Vibration Symposium, 19 – 23 Sept 2021

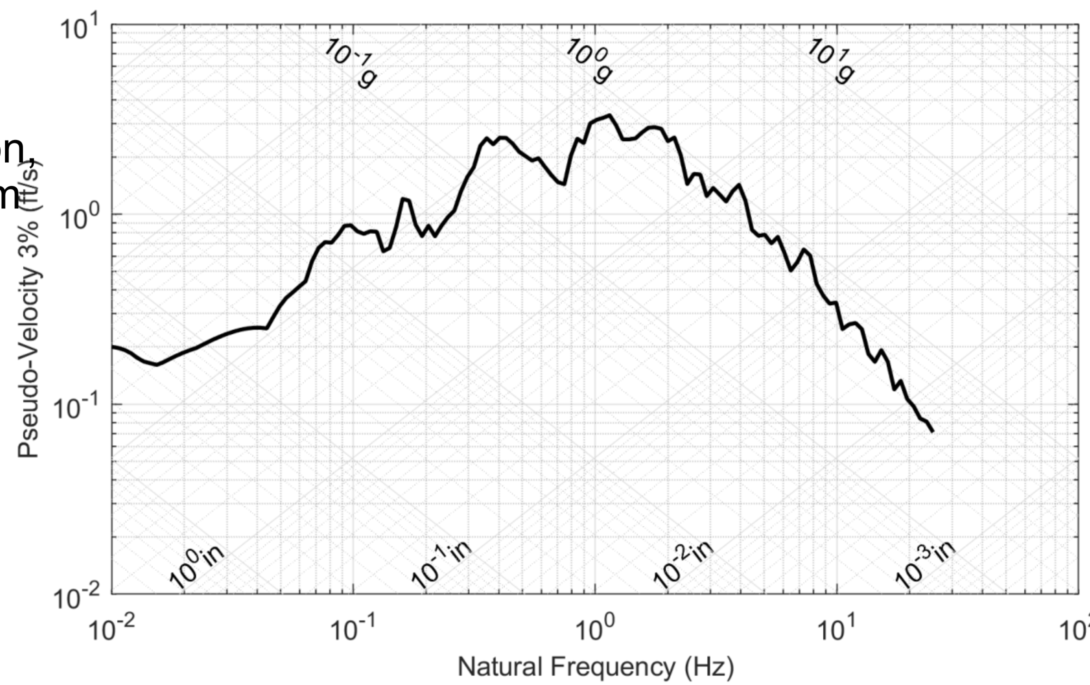
Velocity SRS on Tripartite paper

SRS are typically presented on log-log paper

- Log-log plots are convenient to the scales and distribution of data considered
- Log-log plots are also easy to make in most software packages

Tripartite paper is a plot with multiple log-log scales superimposed

- Scales for displacement, velocity, and acceleration
- Very nice presentation method
- Very hard to get a good plot from most software packages
- Can read velocity, acceleration, and displacement directly from one plot

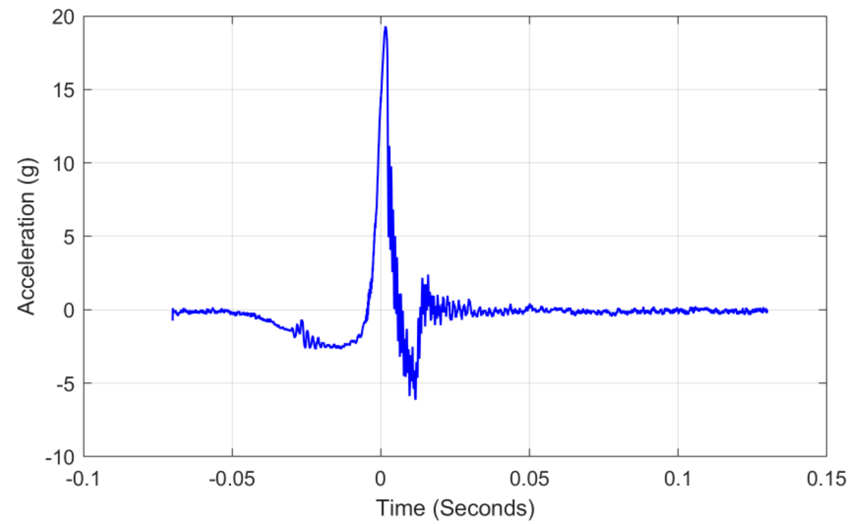


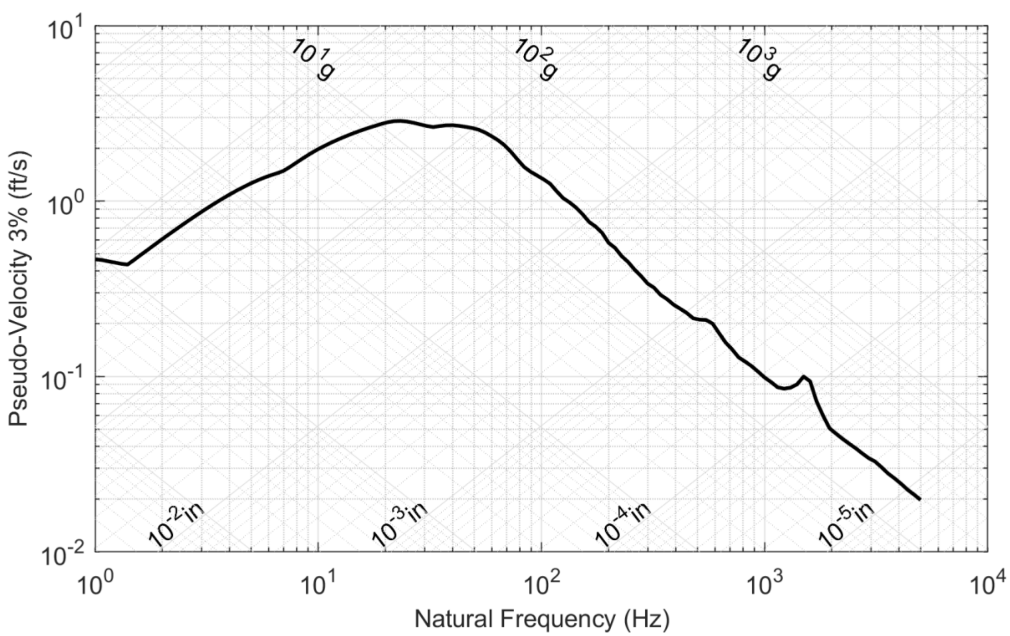
SRS on Tripartite Paper – Drop Shock Example

Another example of a low-level drop shock

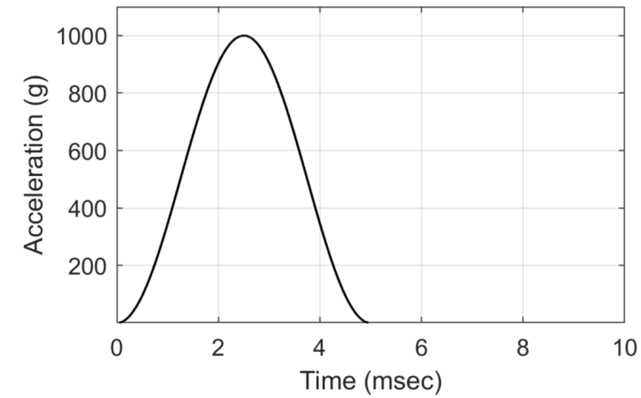
SRS shows:

- Peak velocity ~ 3 ft/s
- Peak acceleration ~ 20 g
- Peak displacement ~ 5/8 inch





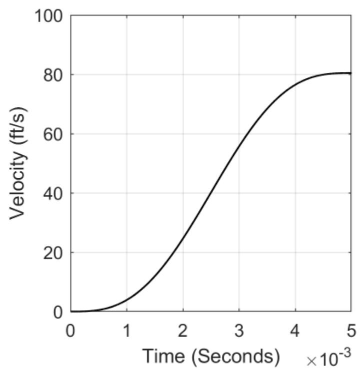
SRS on Tripartite Paper – Theoretical Haversine Example

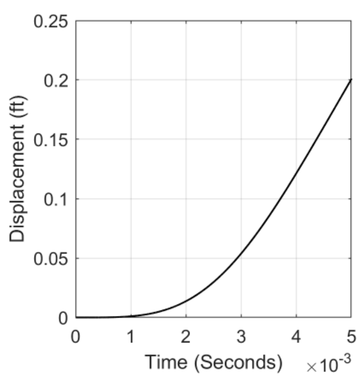


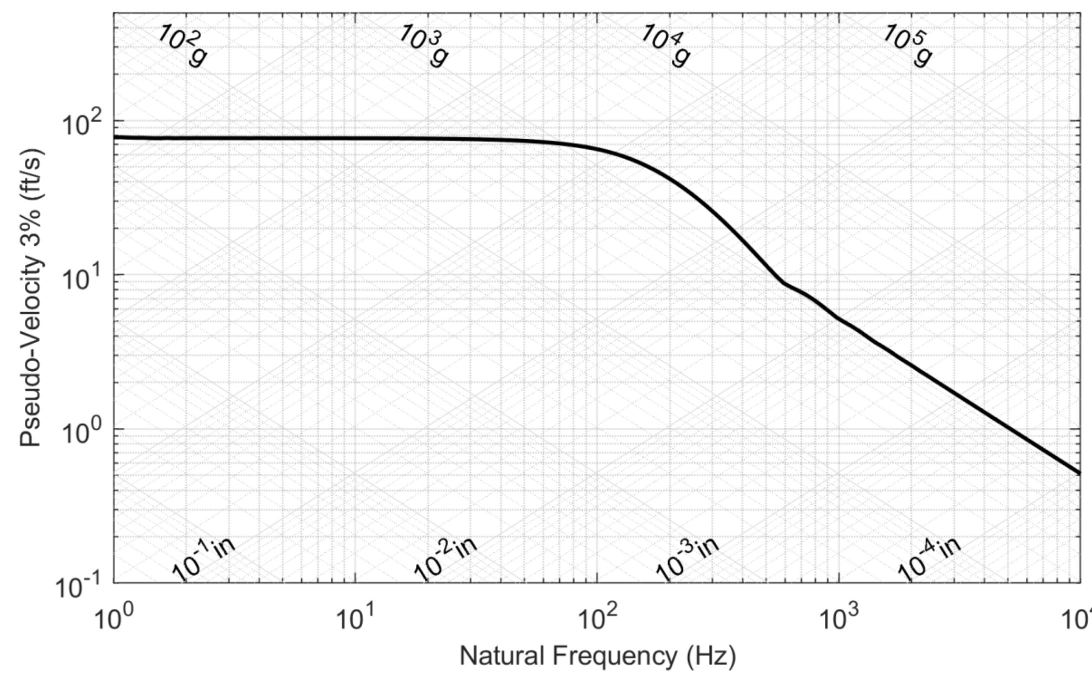
What is the displacement for the theoretical haversine?

SRS shows:

- Peak velocity ~ 80 ft/s
- Peak acceleration ~ 1,000 g
- Peak displacement ~ ?







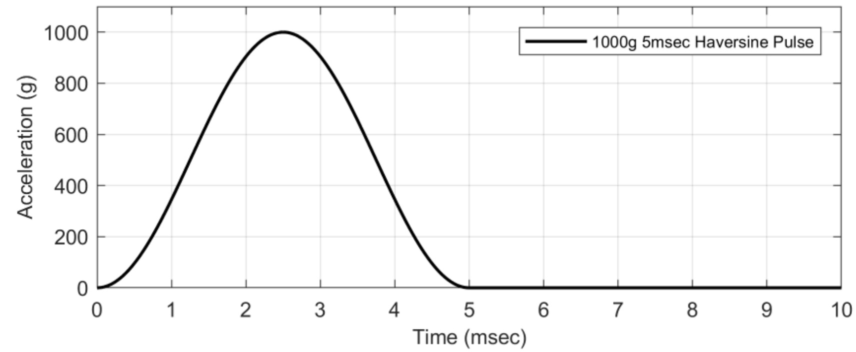
Classical Shocks

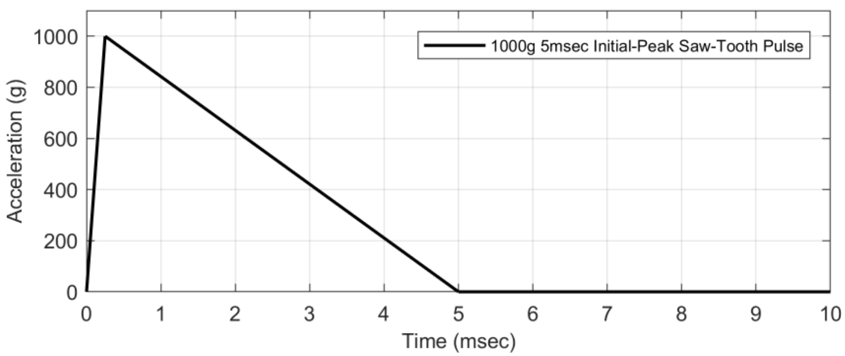
Five Classical Shocks

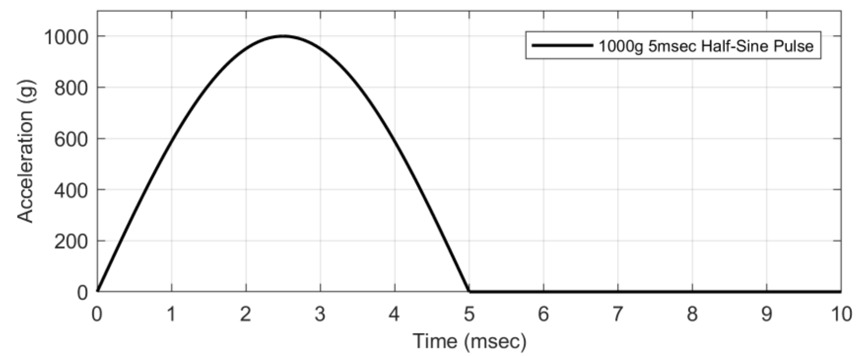
There are five basic classical shocks

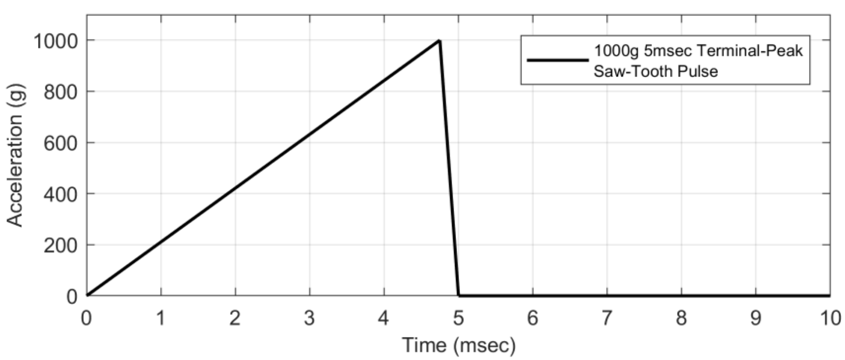
- Other variants can be created but are not appreciably different

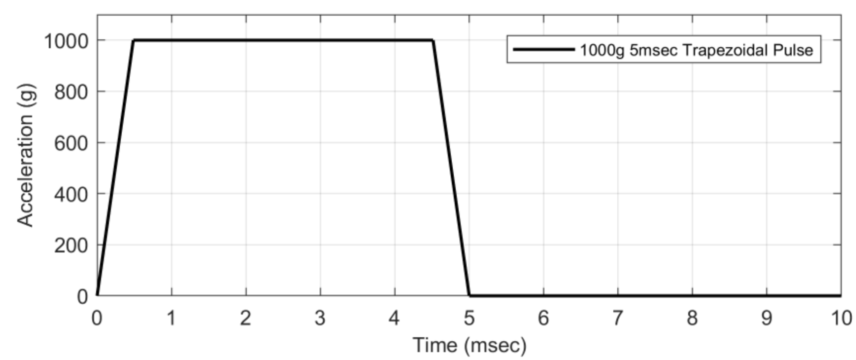
Will show that all the classical shocks are actually quite similar







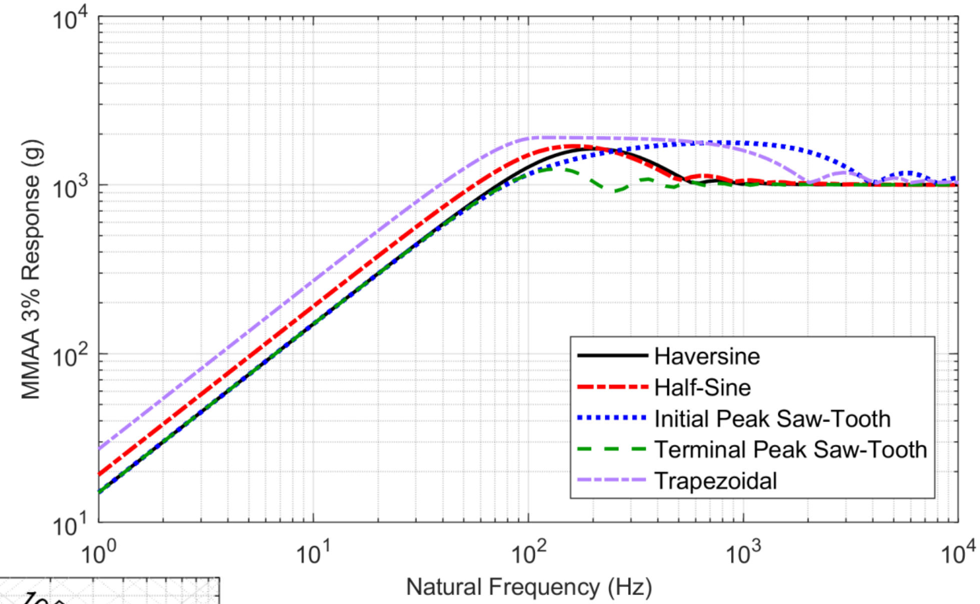


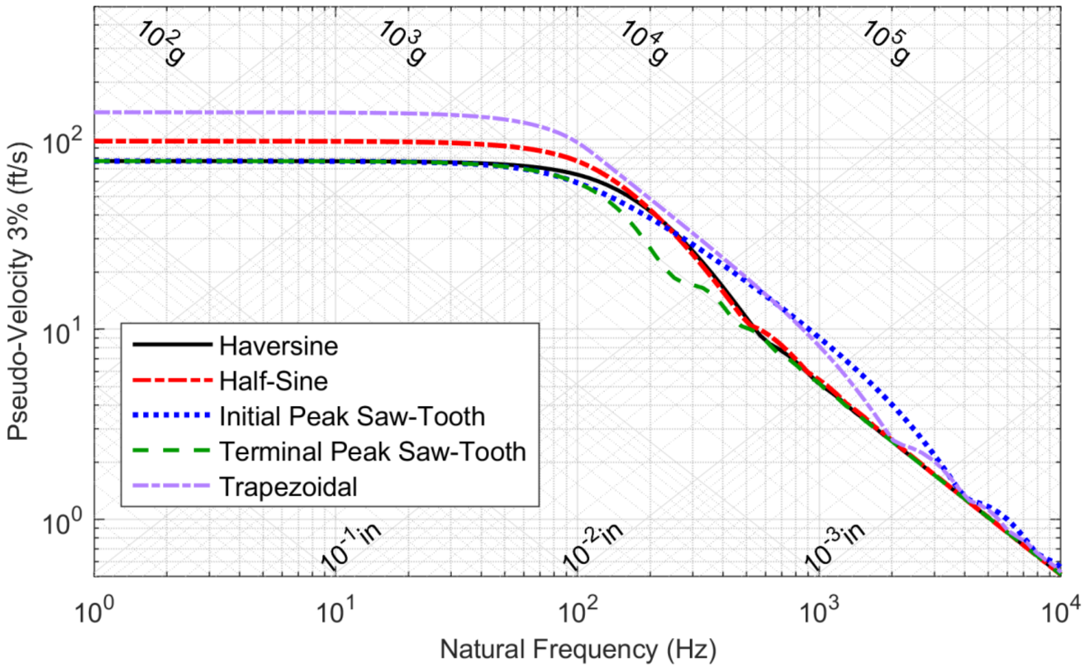


Five Classical Shocks

All five classical shocks give essentially the same SRS

Minor differences could be easily brought together with adjustments to amplitude and pulse duration





All classical shocks have a low-frequency slope of 6 dB/octave on the MMAA SRS

Why Are SRS Slopes Given in dB/Octave?

To confuse the uninitiated

An octave is a frequency interval defined by a doubling of the base frequency

- 1Hz – 2Hz is an octave, 400Hz – 800Hz is an octave, etc.

Number of octaves between any two frequencies is calculated by

$$N\ octaves = \log_2\left(\frac{f_2}{f_1}\right) = \frac{\log_{10}(f_2/f_1)}{\log_{10}(2)}$$

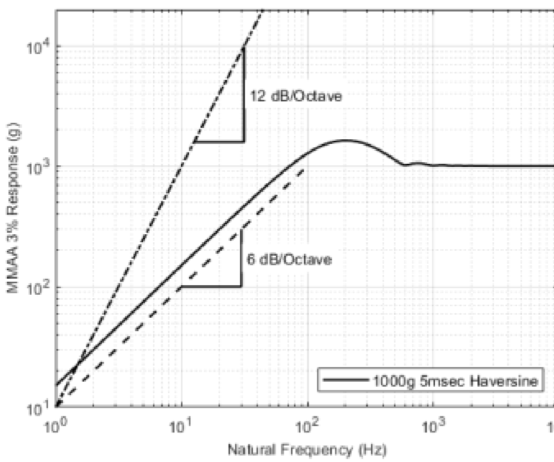
Likewise, the decibel (dB) is a logarithmic unit that defines a ratio between two quantities

- dB is a relative measure
- Usually expressed as a change from a baseline value

The difference between two SRS amplitudes in terms of dB is calculated by

$$N\ dB = 20\log_{10}\left(\frac{A_2}{A_1}\right)$$

Why Are SRS Slopes Given in dB/Octave?



The slope on an SRS plot is then

$$Slope = \frac{N\ dB}{N\ octaves} = \frac{20\log_{10}(A_2/A_1)}{\log_{10}(f_2/f_1)}\log_{10}(2)$$

A slope of 1 on log-log paper requires $A_2/A_1 = 10$ and $f_2/f_1 = 10$

In terms of dB/octave, that becomes

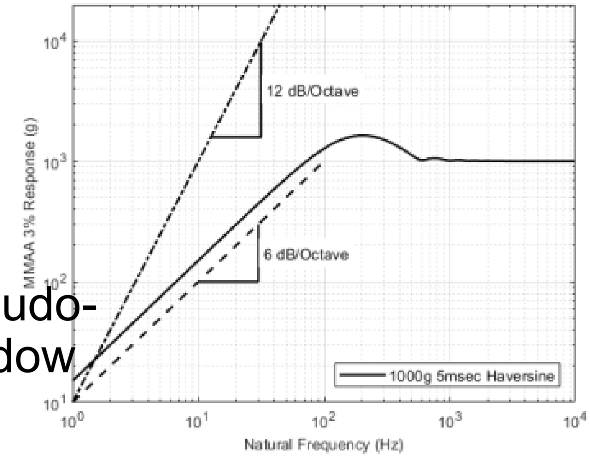
$$Slope = \frac{20\log_{10}(10)}{\log_{10}(10)}\log_{10}(2) = 20\log_{10}(2) = 6.02\ \text{dB/octave}$$

Which is always rounded down to 6 dB/octave

A slope of 2 on log-log paper ($A_2/A_1 = 100$, $f_2/f_1 = 10$) is 12 dB/octave

A slope of 3 on log-log paper ($A_2/A_1 = 1000$, $f_2/f_1 = 10$) is 18 dB/octave

Why the Low-Frequency Slope is 6dB/Octave



The MMAA low-frequency slope of a classical shock will always tend to 6 dB/octave, but why?

For an undamped system, the relative velocity and pseudo-velocity are the same in the residual vibration time window

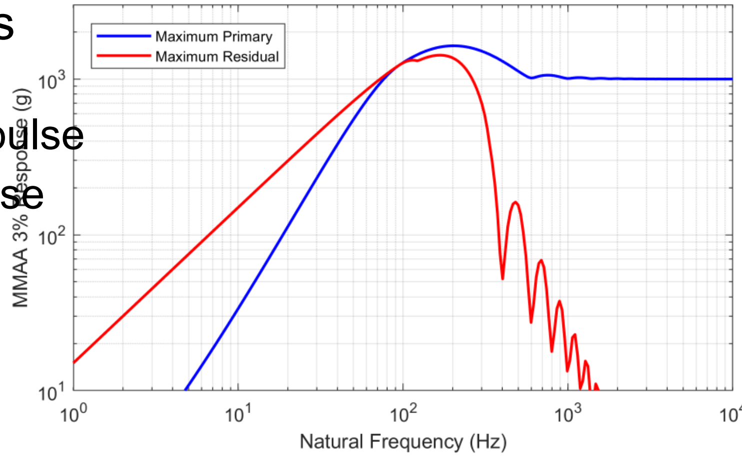
- This is true regardless of the classical shock form

The maximum velocity in the free vibration time window is equal to the velocity change imparted by the shock

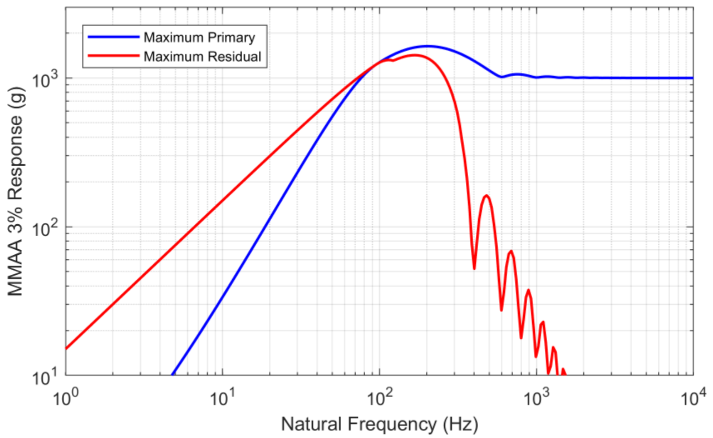
$$\Delta V = \int_0^T \ddot{z}(t)\, dt$$

The low-frequency portion of the MMAA SRS is defined by the residual spectra

- Equivalent to the free vibration response to an impulse
- Slope can be determined from the impulse response



Why the Low-Frequency Slope is 6dB/Octave



The relative displacement from the impulse response in the free vibration time is:

$$y(t) = \frac{\Delta V}{\omega_n} \sin(\omega_n t)$$

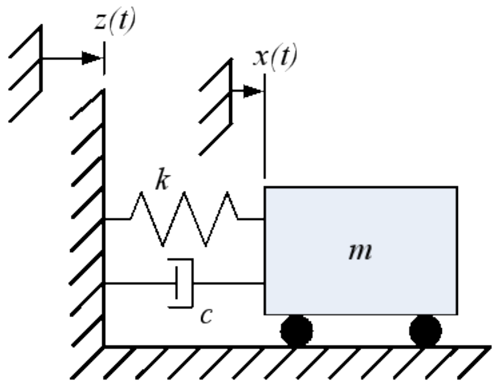
And the maximum is just: $$y(t)_{max} = \frac{\Delta V}{\omega_n}$$

And absolute acceleration is related to relative displacement by

$$\ddot{x}(t) = -\omega_n^2 y(t)$$

So the MMAA SRS is given by:

$$SRS_{MMAA} = \ddot{Y}(\omega_n) = \omega_n \Delta V$$



$$y(t) = x(t) - z(t)$$

Since ΔV is a constant, the MMAA SRS is linear in ω_n which gives the 6 dB/octave slope

Estimating Velocity from the SRS

Velocity change can be read directly from the velocity SRS if damping is low

- Slightly under-estimates the true velocity change

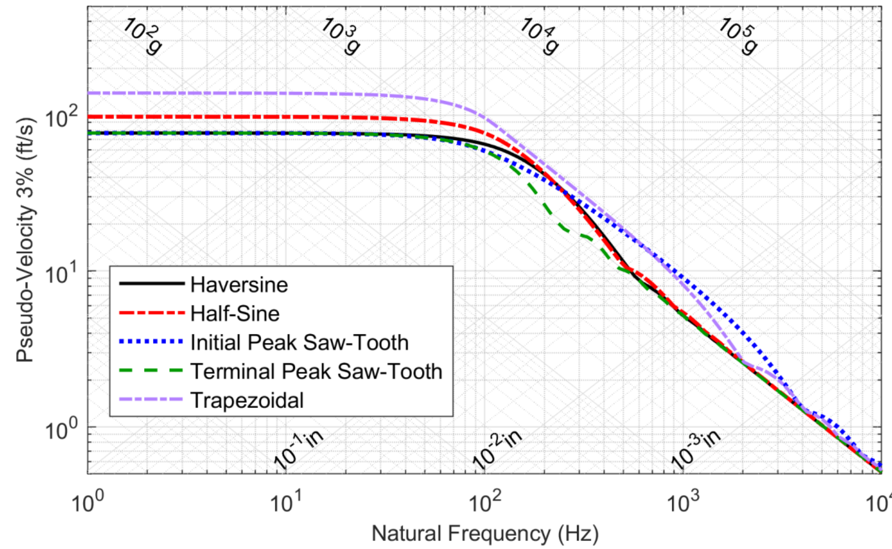
From an acceleration SRS, need to take a low-frequency point and convert to velocity

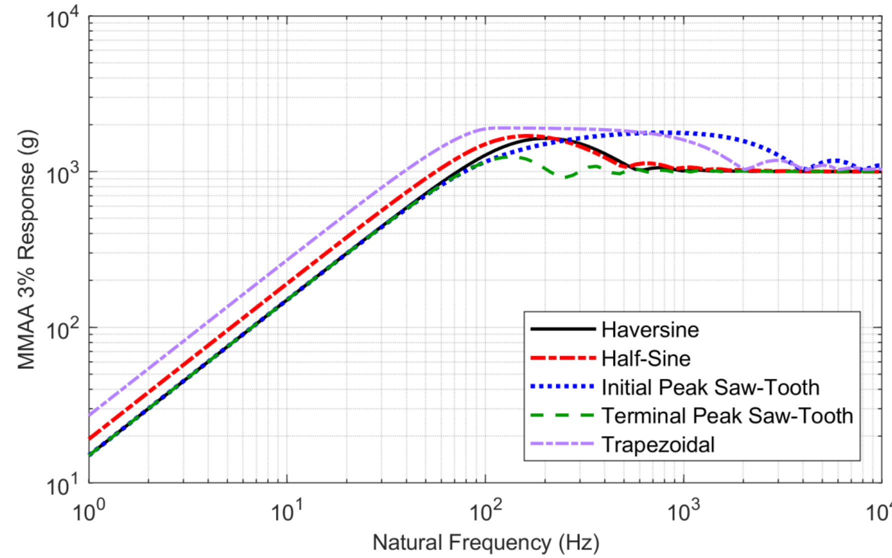
- Needs to be a point where the slope is at a nominal 6 dB/octave

$$\Delta V = \frac{A\,g}{2\pi f}$$

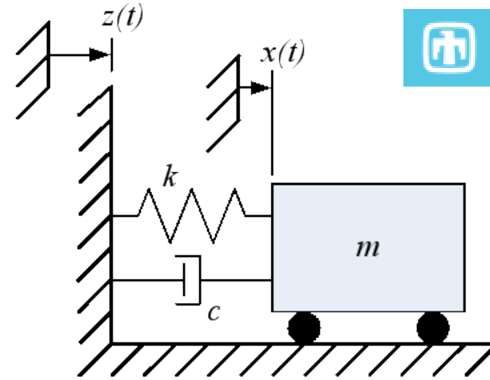
In this plot, the haversine SRS shows 15.02g at 1Hz. Using this gives:

$$\Delta V = \frac{(15.02)g}{2\pi(1)} = 77\,ft/s$$





The Effect of Damping on the SRS

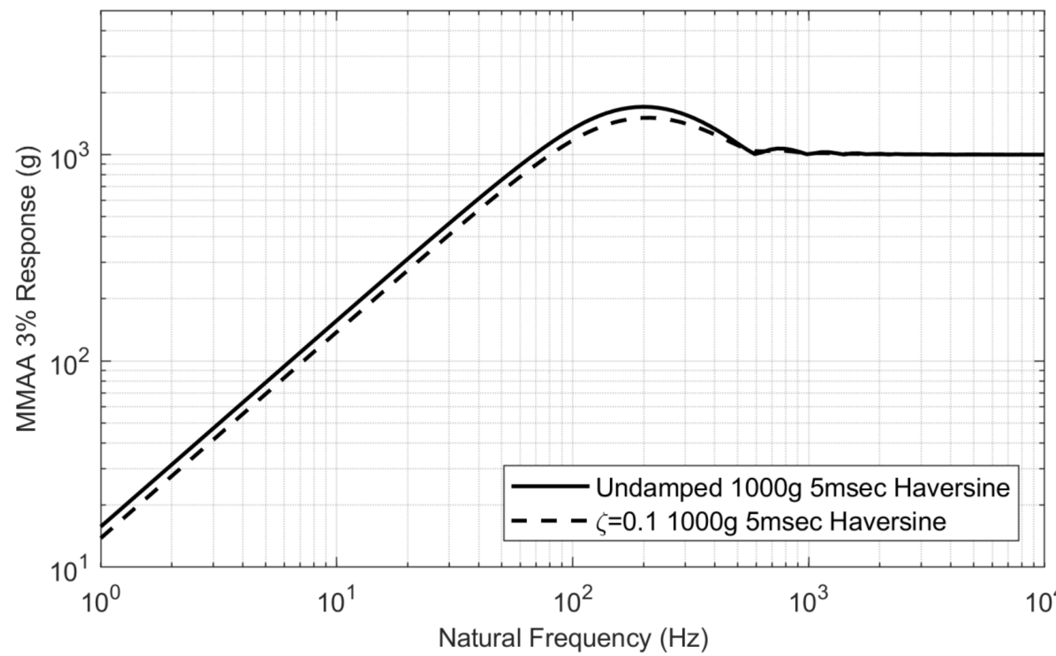


There are two kinds of damping to be considered

- Material damping in the part being tested
- Damping in the mathematical SDOF oscillator used to calculate the SRS
- We only control one of these

Increasing the SDOF oscillator damping artificially lowers the velocity change associated with the shock event

- After all, the SRS calculation occurs after the shock is over
- Damping is somewhat arbitrary for the SRS calculation
- Needs to be representative of the real system



Shock Bandwidth

The shock energy is not infinite

Acceleration SRS tends to obscure this fact with the infinite flat-line at the high-frequency end of the spectrum

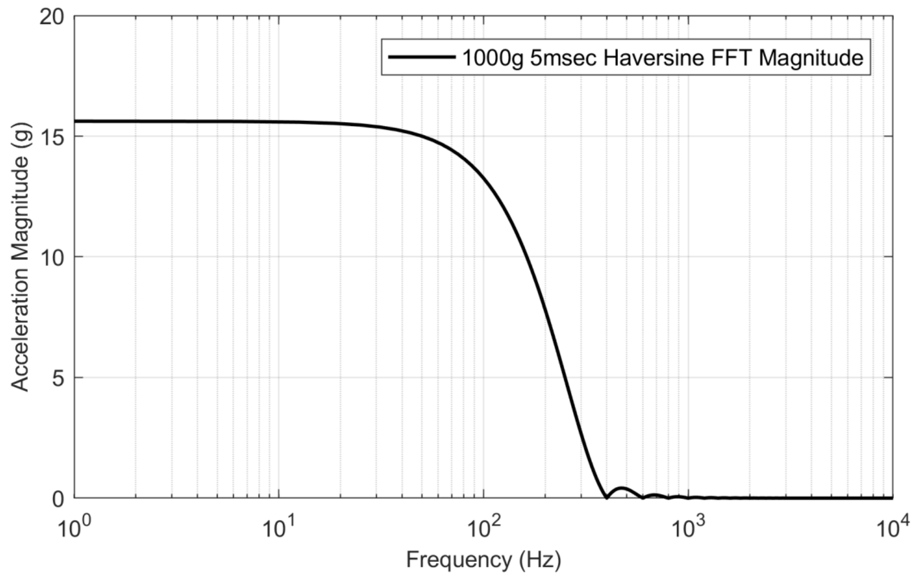
Parseval's identity states that

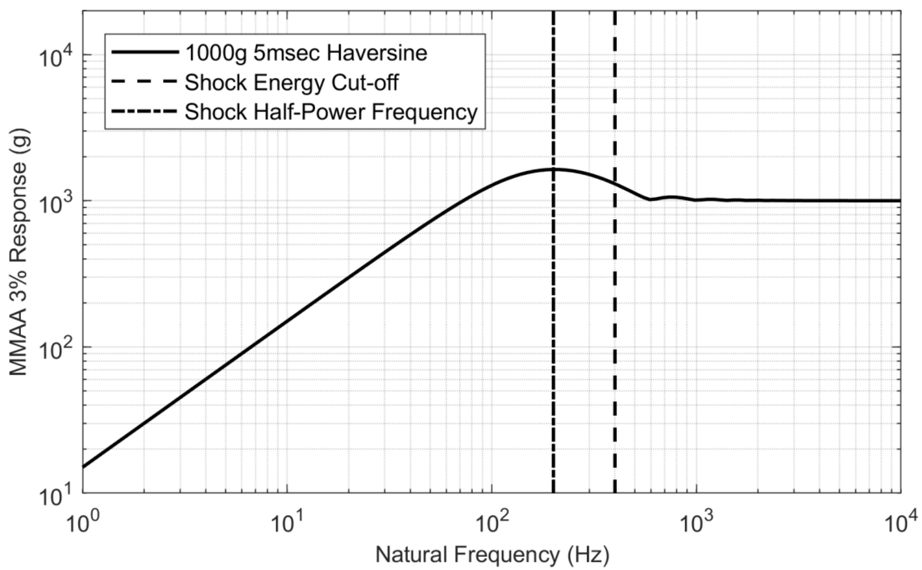
$$[x(t)]^2 = [X(\omega)]^2$$

Or the square of the energy in the time history equals the square of the energy in the frequency response

In this example, there is essentially no energy in the shock beyond 400Hz

Energy is falling off rapidly, even at the SRS peak



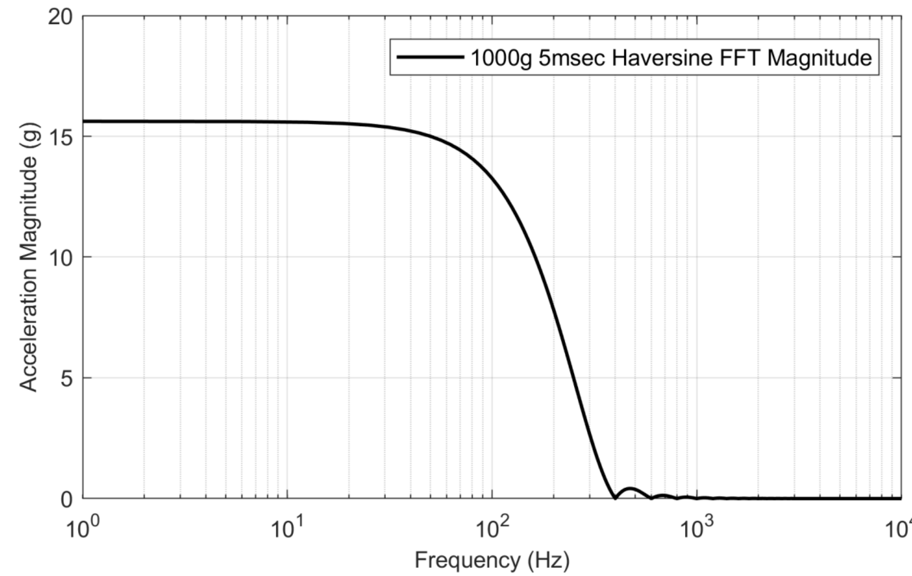


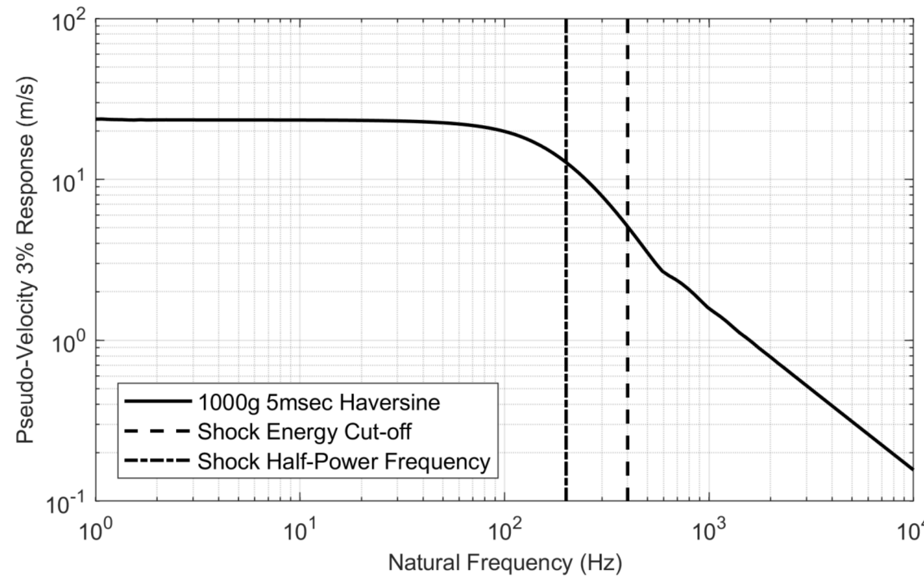
Shock Bandwidth

This is more clearly seen with a velocity SRS presentation

The roll-off in the shock energy closely parallels the roll-off in the velocity SRS

Same information but a difference in presentation style





Oscillatory Shocks

Oscillatory Shocks

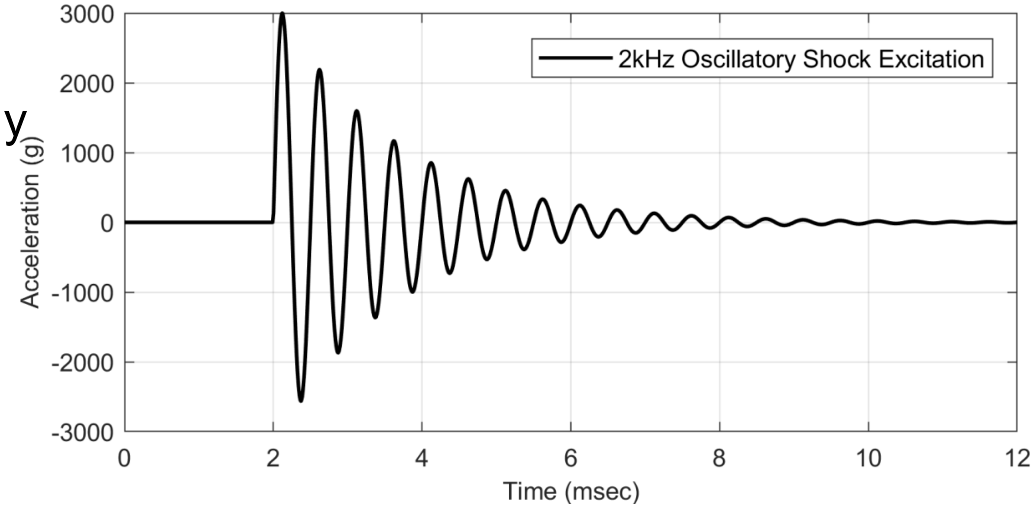
An oscillatory shock is essentially a two-sided transient event

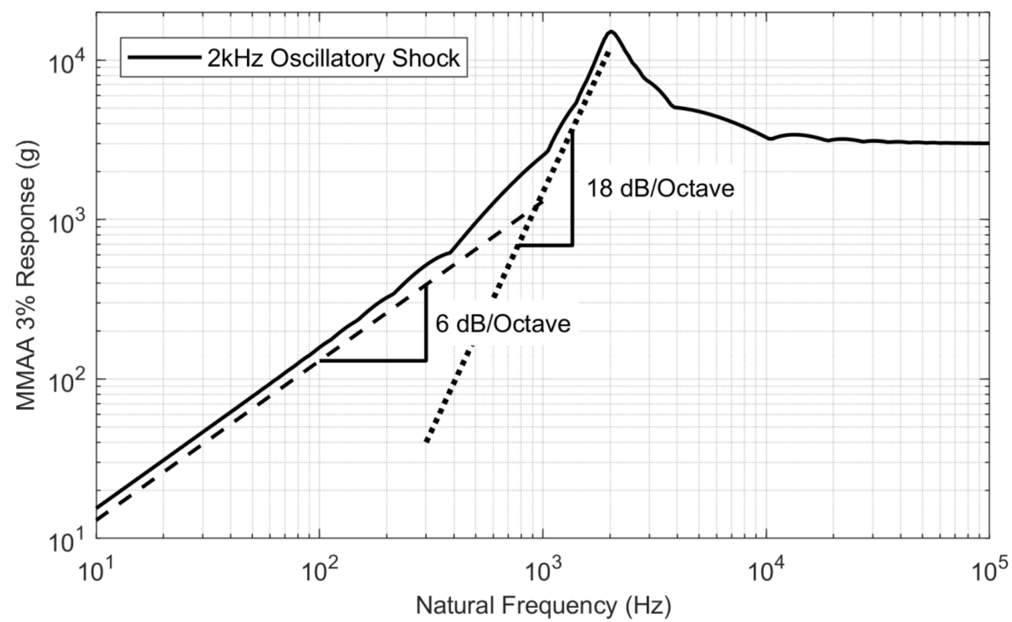
It could be a decaying harmonic as shown here, or something more random such as the earthquake time history shown previously

Differs from the classical shocks in that there is typically no net velocity change associated with the event

Examples include:

- Pyroshock
- Earthquake shock
- Shaker shock





SRS differs in many respects from the classical shock SRS

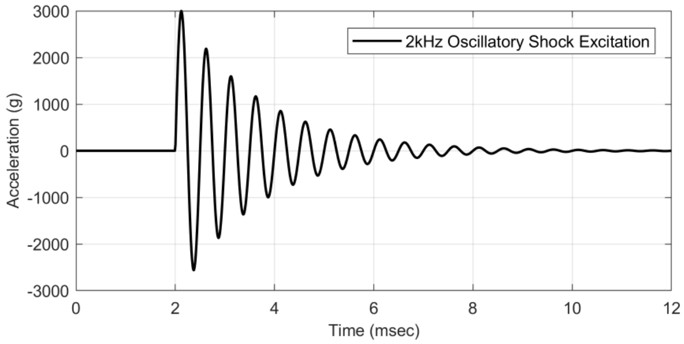
Oscillatory Shocks – Primary, Residual, Positive, Negative SRS

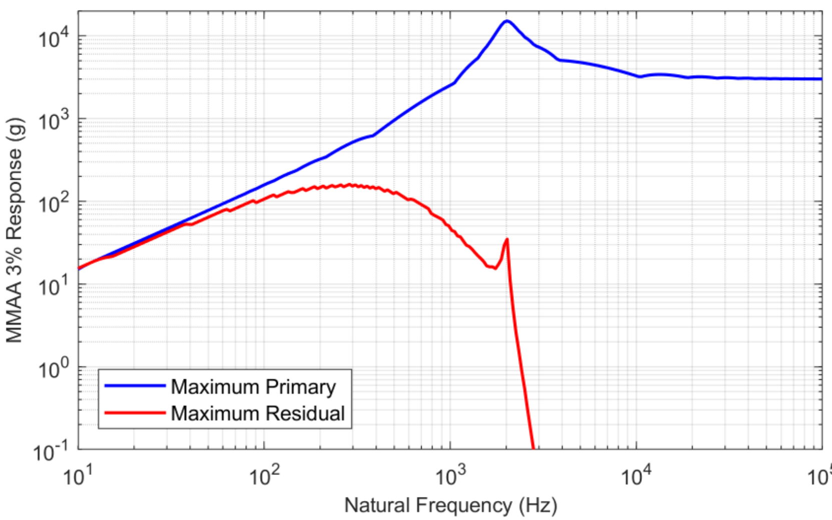
Primary and residual spectra are very different for oscillatory shocks

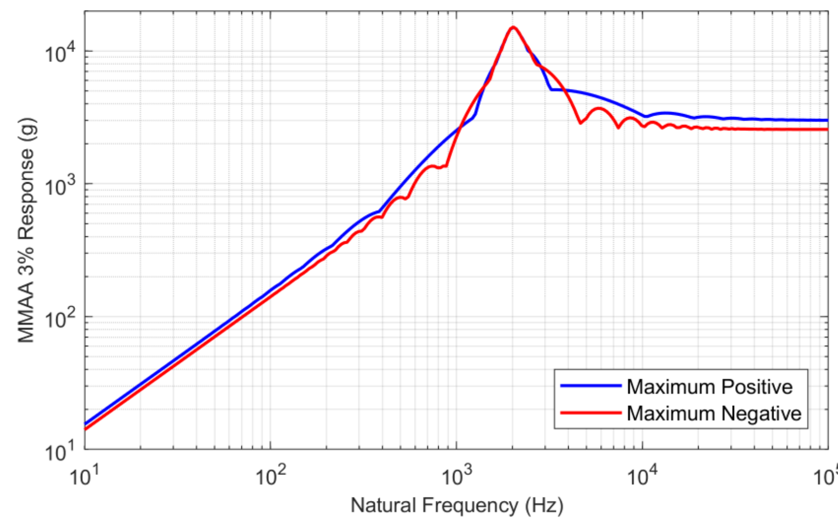
- Typically shock has decayed to near-zero in the “primary” window
- Results in little or no significant residual response

Positive and negative spectra are usually similar

- Only minor differences in peak positive and peak negative response







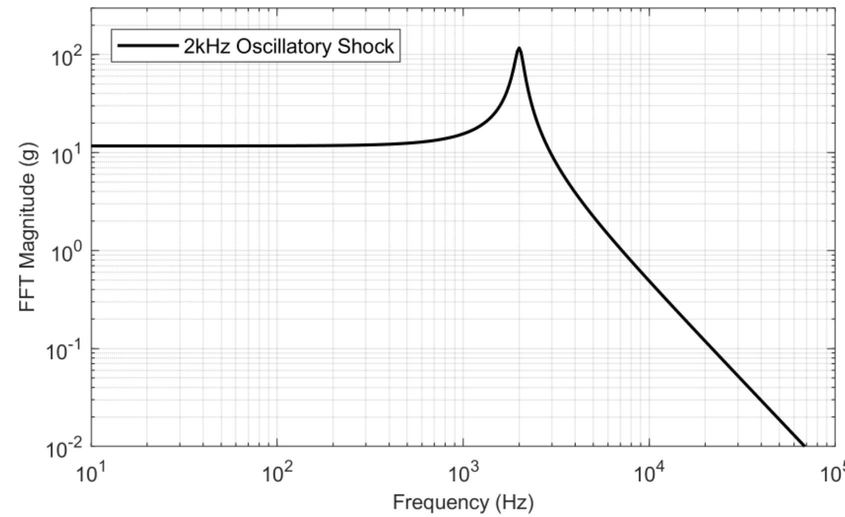
Oscillatory Shocks – Shock Bandwidth

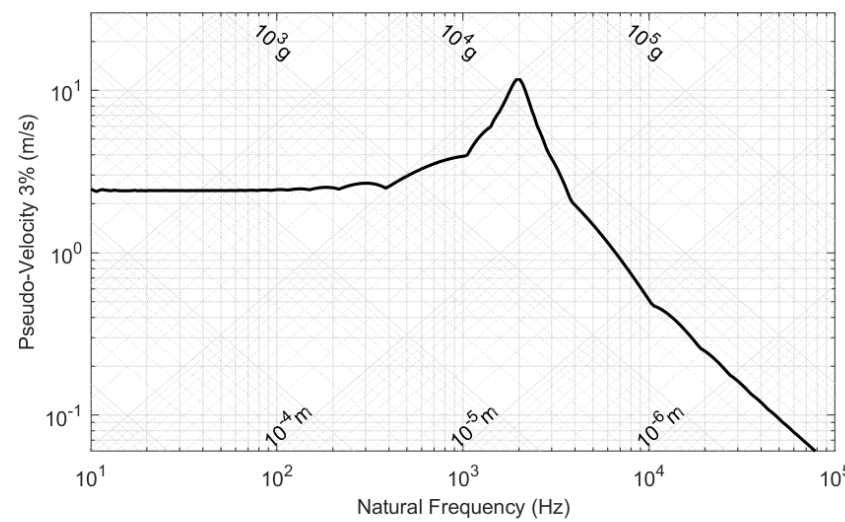
Energy contained in an oscillatory shock can be significantly narrower bandwidth

Like the classical shock, there is little energy beyond the primary frequency

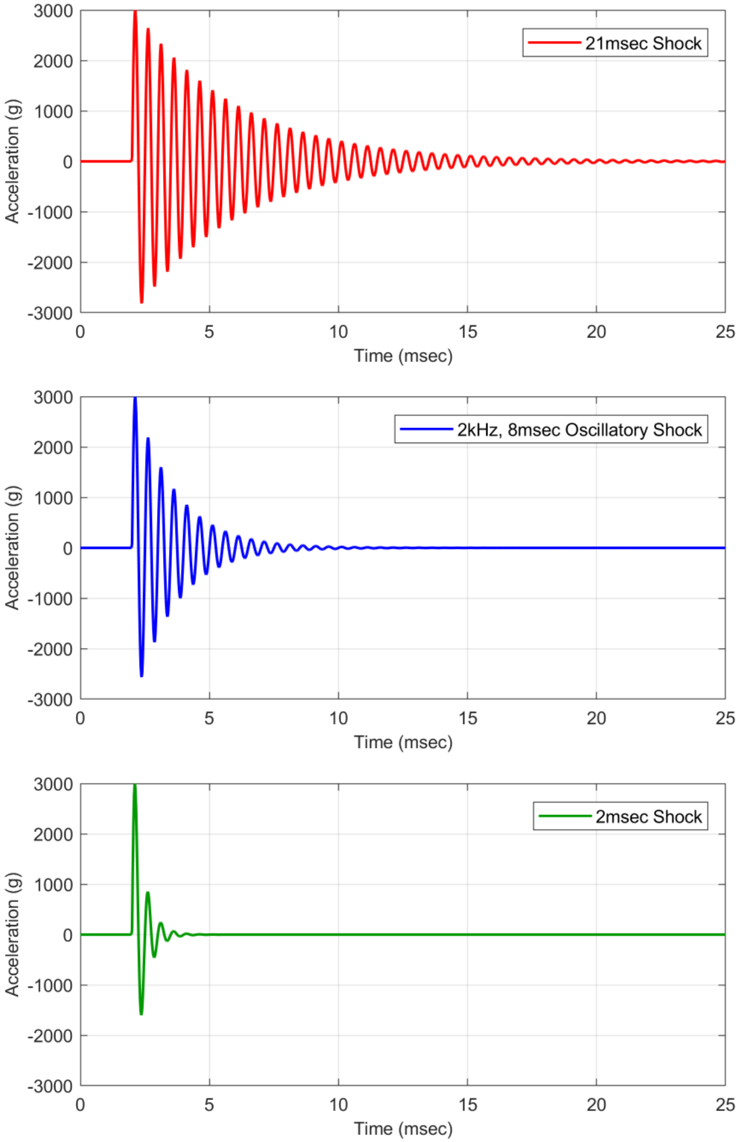
In contrast to the classical shock, there is also significantly less energy in the lead-up to the primary frequency

Clearly seen in the FFT plot and the pseudo-velocity SRS plot





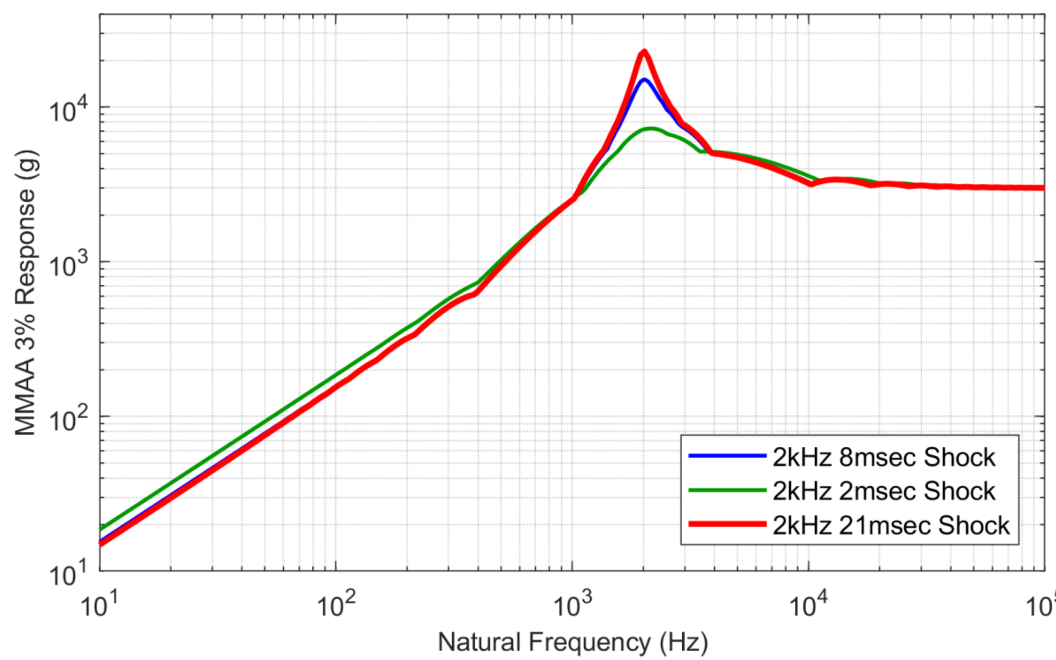
Oscillatory Shocks – Influence of Decay Rate on the SRS



The longer the shock rings, the higher the SRS peak

This is an artifact of the SDOF oscillator resonating

If the test article does not amplify like the SDOF oscillator then these levels will not be reached

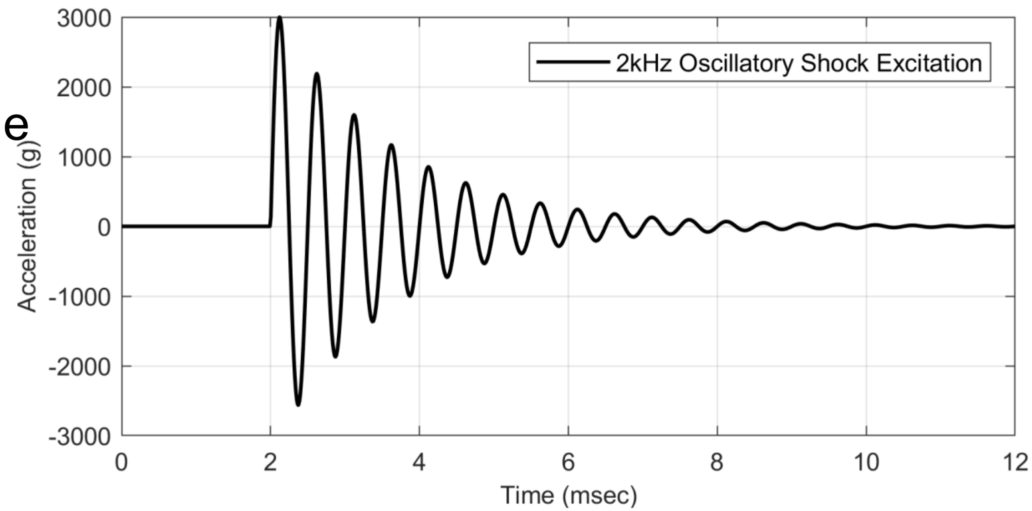


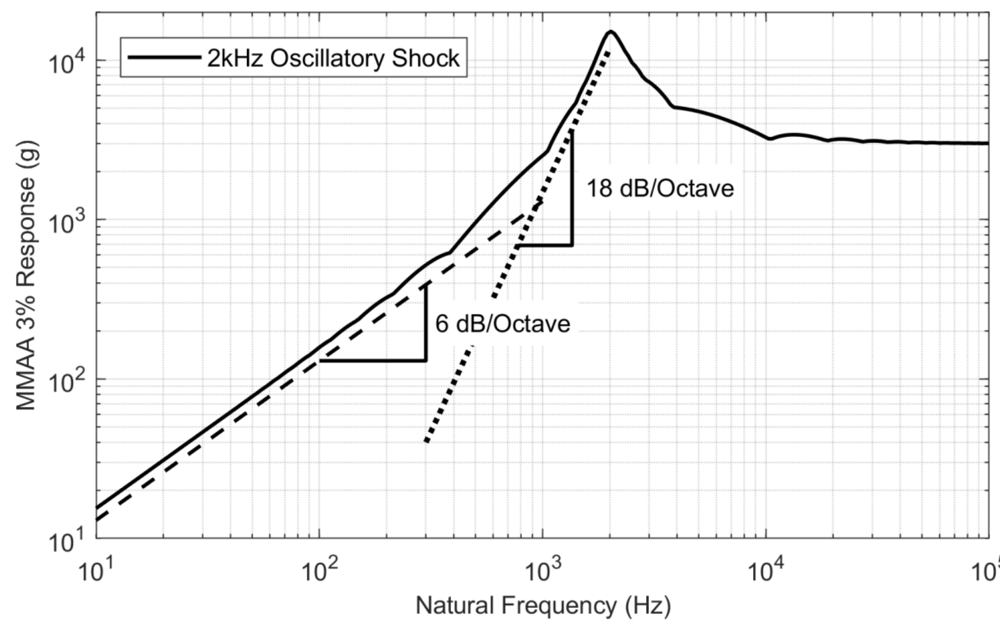
Oscillatory Shocks – What Are Those Inflection Points?

SRS of a pure decaying sine tone has inflection points at:

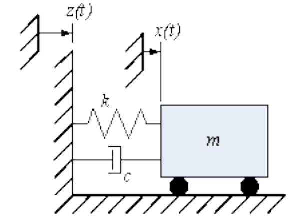
- 1/2 and 1/5 of the primary frequency
- 2 and 5 times the primary frequency

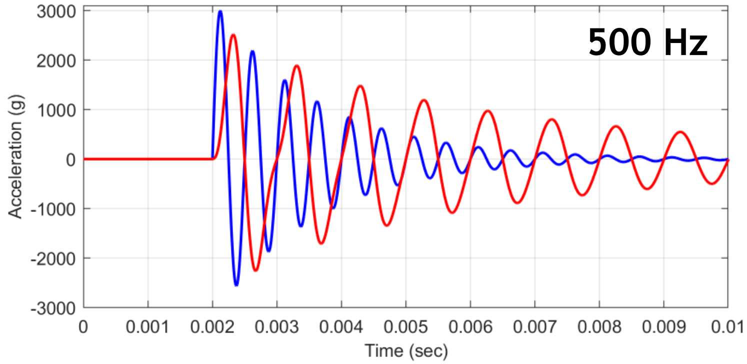
What causes this?

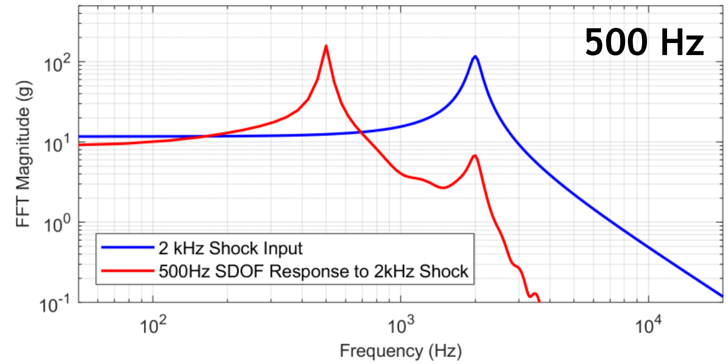


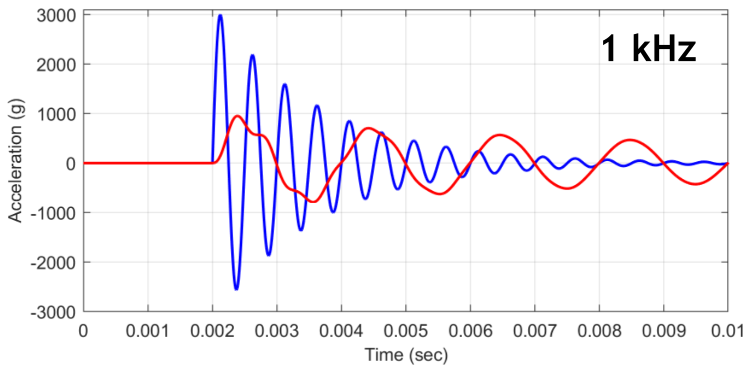


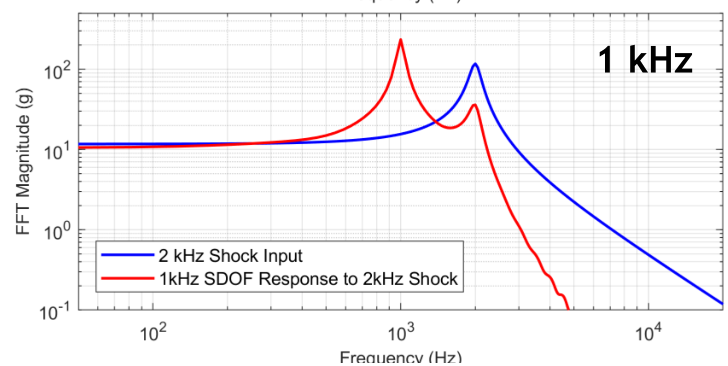
Oscillatory Shocks – Origin of the Inflection Points

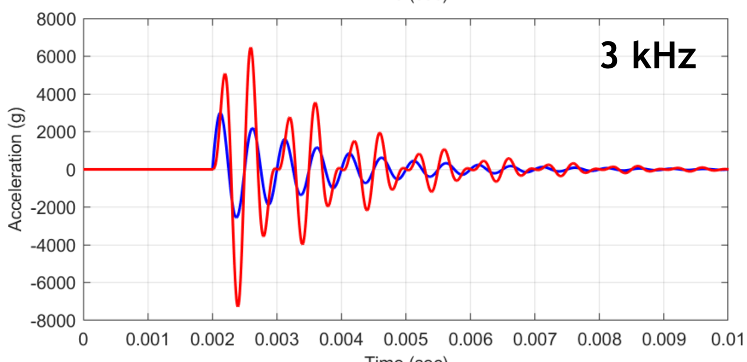


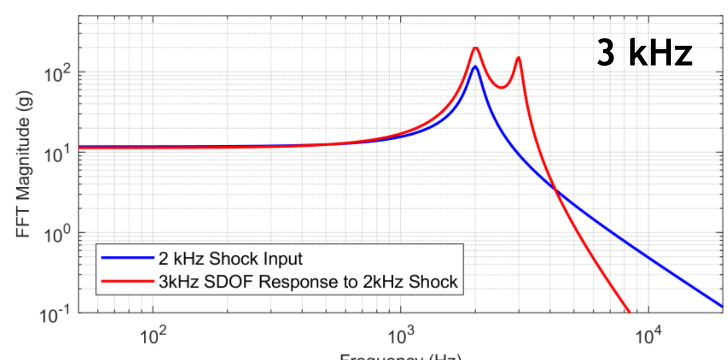












Pure Two-Sides Shock Pulse

Not actually an oscillatory shock but demonstrates similar characteristics

True zero velocity change shock

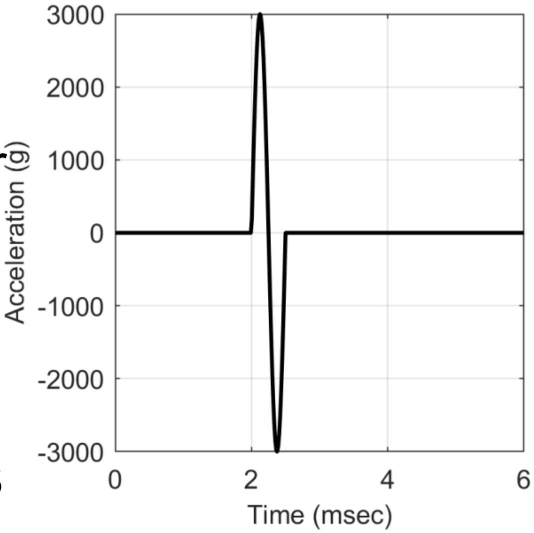
Straight 12 dB/octave low-frequency slope in MMAA SRS

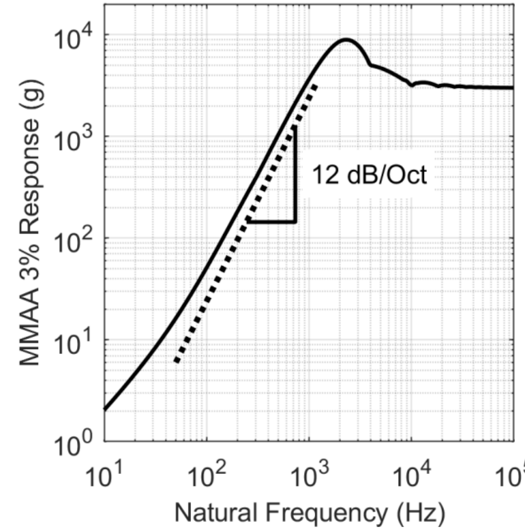
- Some rolling over at very low frequencies

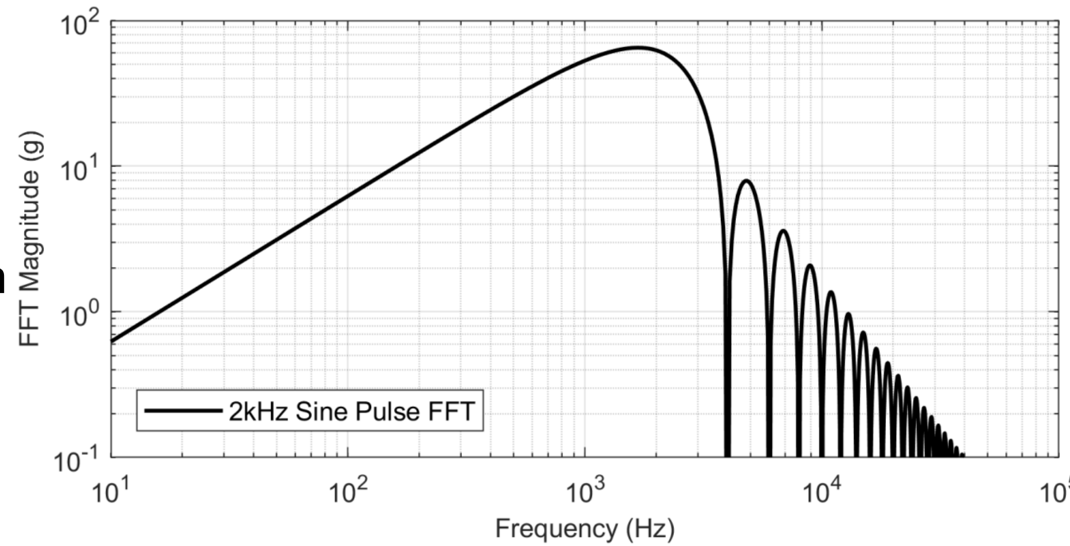
Inflection points exist above the pulse frequency but not below

Shock bandwidth is higher than for a pure oscillatory shock

Not particularly practical for laboratory shock work







Complex Shocks

Complex Shocks

Complex shocks are shocks that can be described as a linear summation of two or more basic shocks

- Classical shock and an oscillatory shock
- Two or more classical shocks
- Two or more oscillatory shocks

SRS is generally additive

- An SRS calculated from the sum of two shocks is essentially equal to the sum of the two underlying SRS curves
- Sometimes the sum will generate its own additional frequency content but typically relatively minor

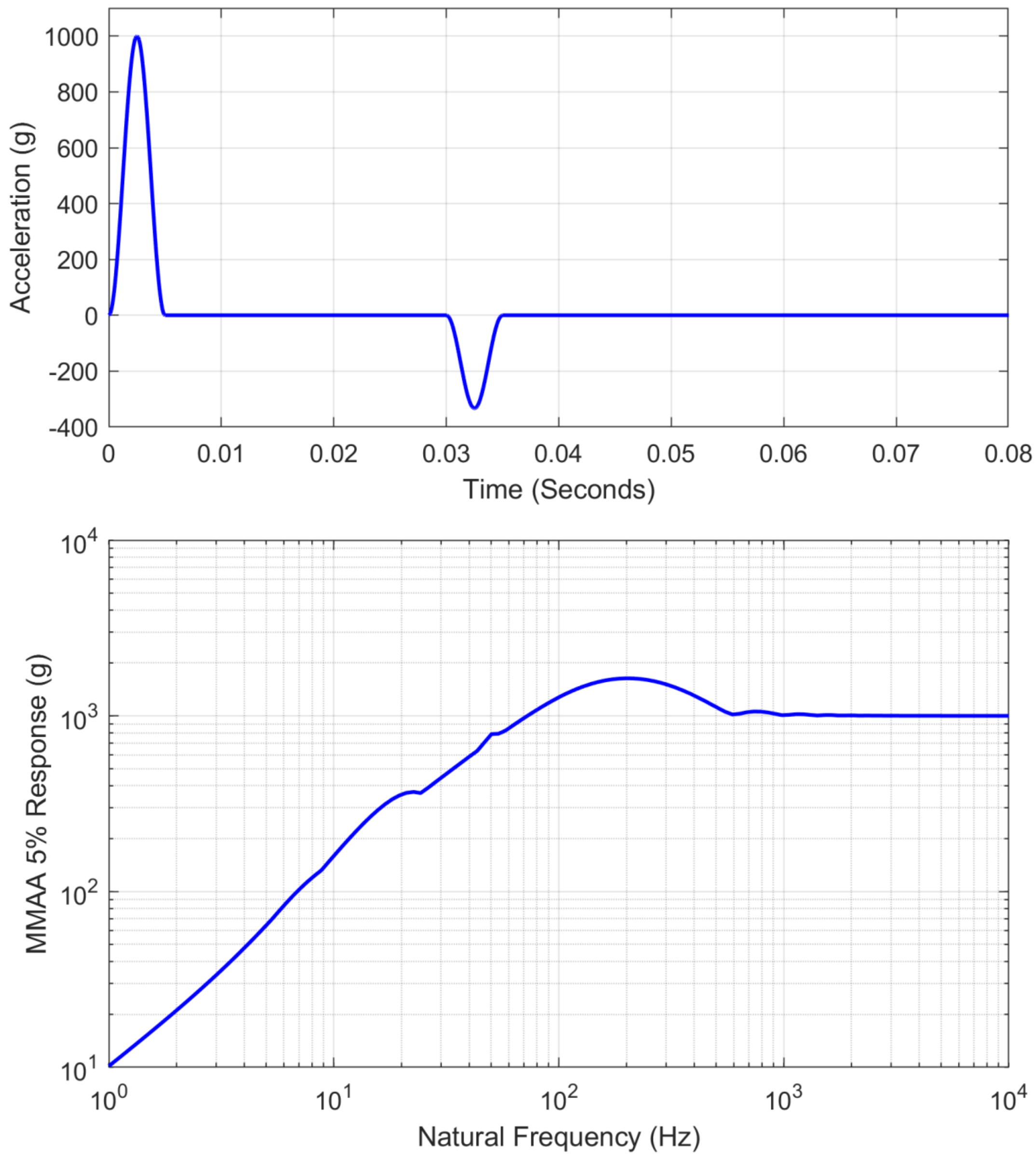
Complex Shocks

How does a double hit alter the original SRS?

In this example with the low-level return shock, an extra hump or two is added to the low-frequency portion of the MMAA SRS

Location will move with separation time between two shock pulses

How severe is this for the component?



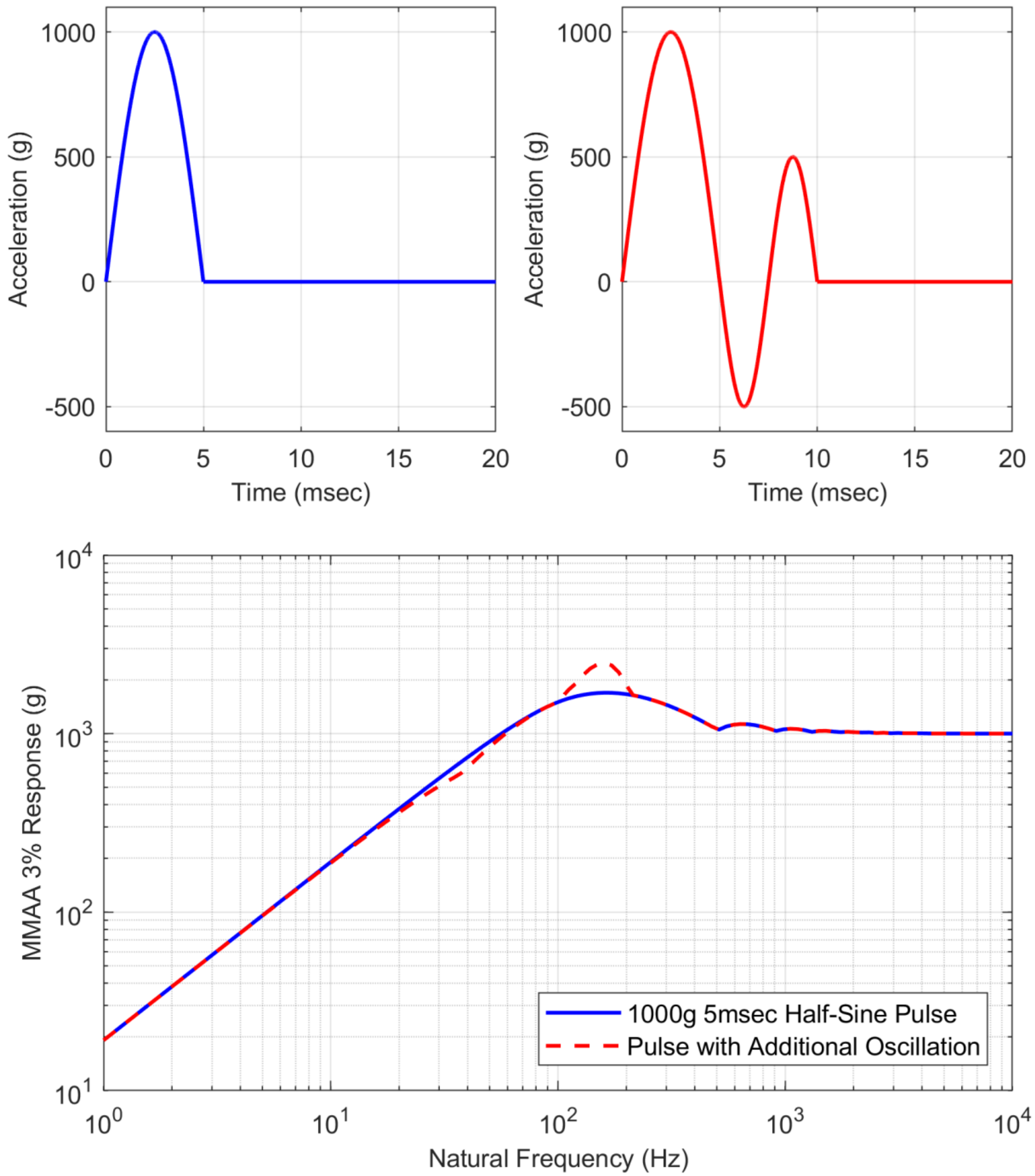
Complex Shocks

How does an extra cycle influence the SRS?

SRS has the low-frequency response of the classical shock with a higher peak similar to an oscillatory shock

How severe is this for the component?

- Depends on the failure mode



Complex Shocks

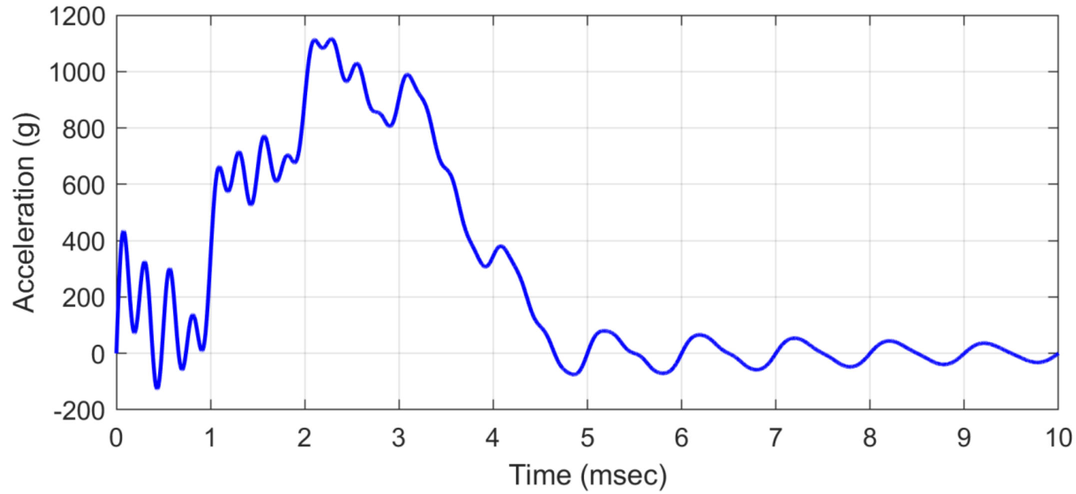
What kind of shock is this?

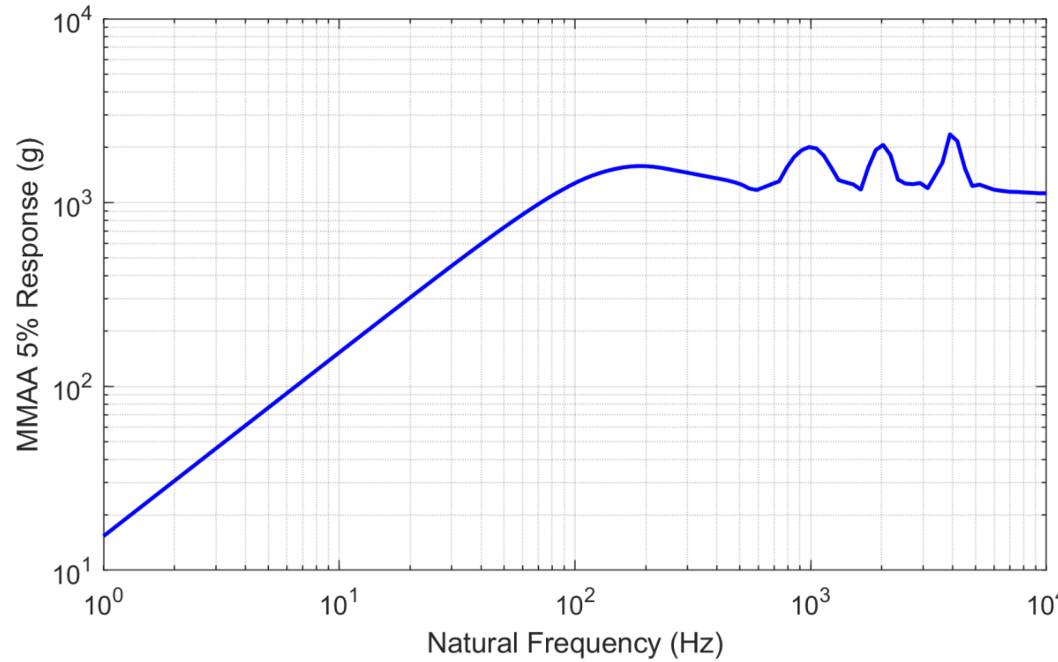
Looks like a drop shock but appears to have extra frequency content

How severe is the high-frequency content?

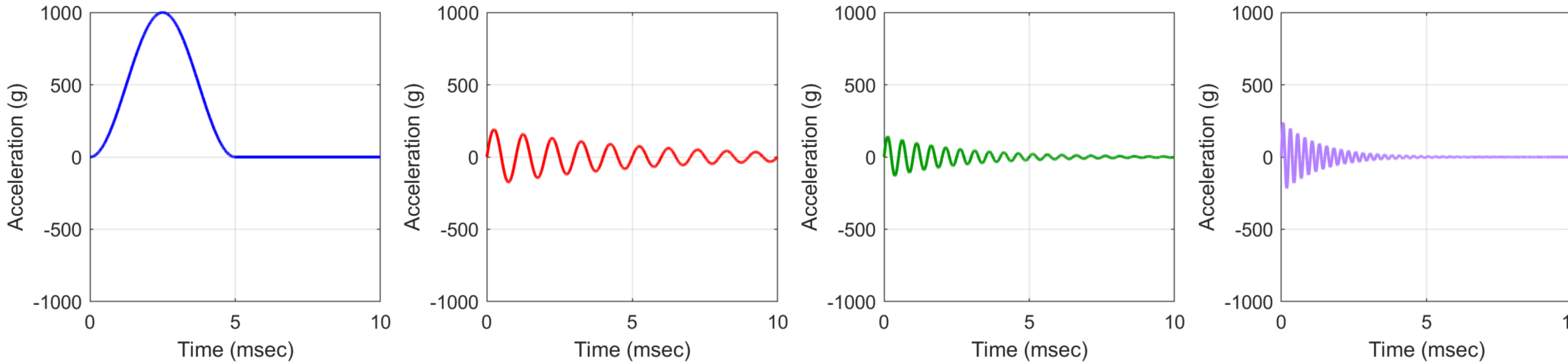
Appears that it could be severe in the SRS calculation

What does the high-frequency content represent?





Complex Shocks

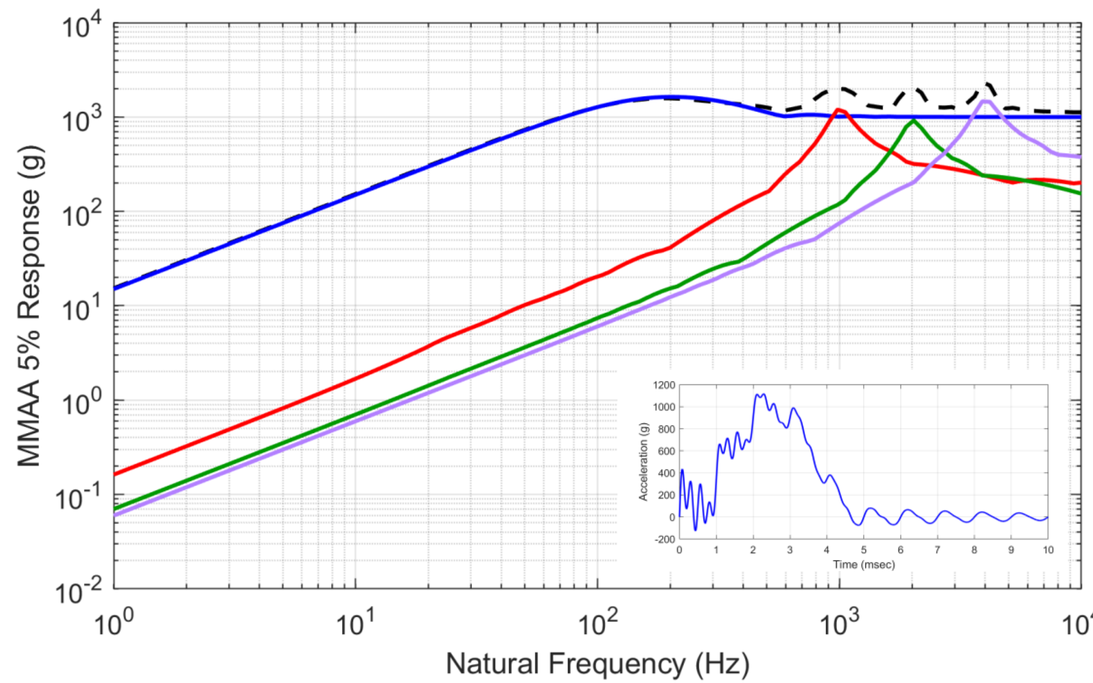


The previous shock was made of four curves added together

Shock levels for the last three curves appear higher in the combined than separated

May not be important overall

Will not be excited by a drop shock test due to limited shock bandwidth

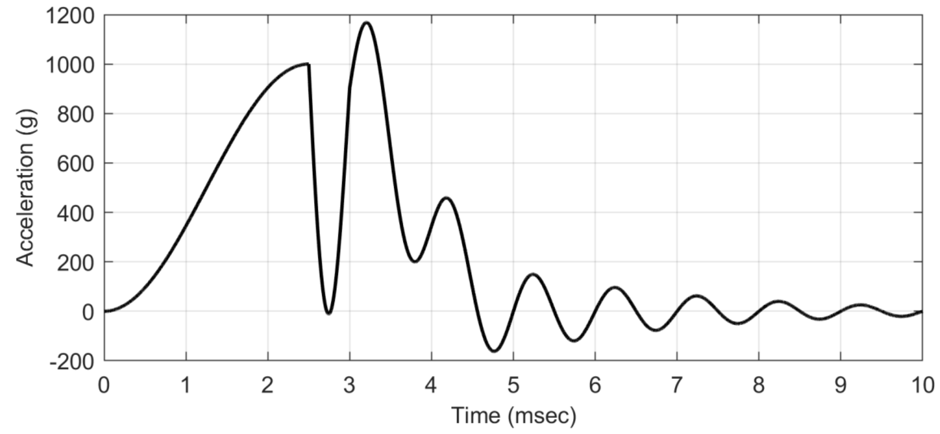


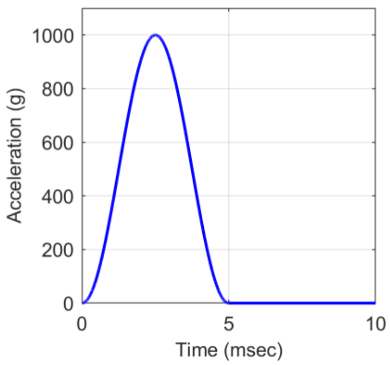
Complex Shocks

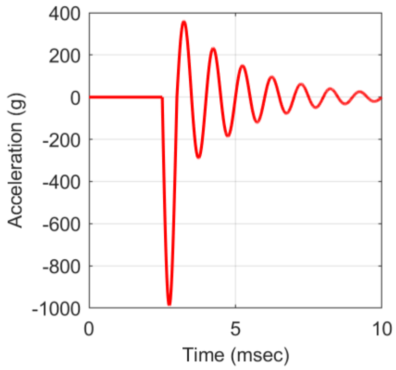
Haversine shock combined with a delayed oscillating shock

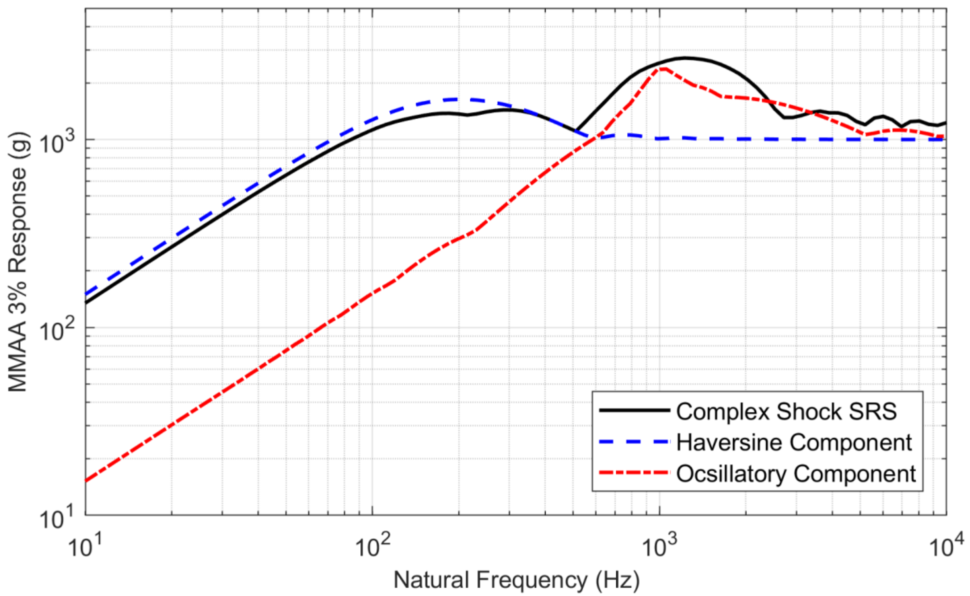
Two distinct frequencies visible in SRS plot from two distinct shock events

What does this mean?









Sisemore, SRS Primer, 91st Shock & Vibration Symposium, 19 – 23 Sept 2021

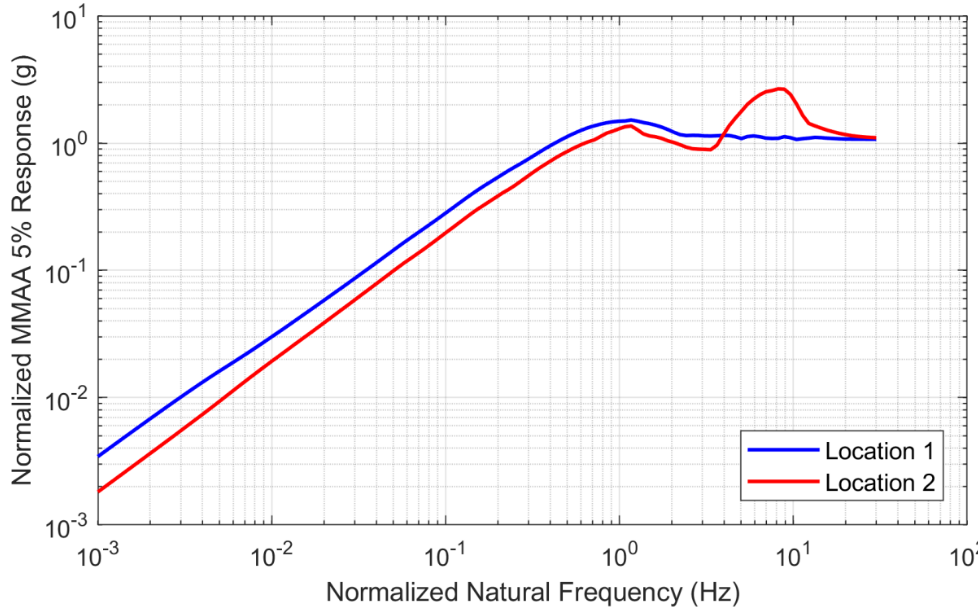
Complex Shocks

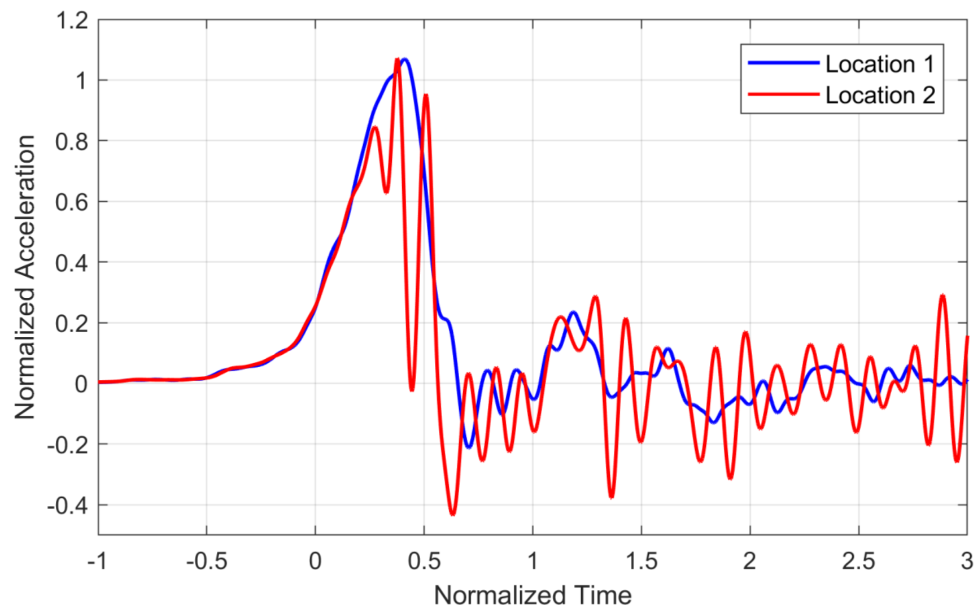
A set of sample test data

Two accelerometer locations were relatively close together

Significantly different MMAA SRS
- Different velocity change
- Different high-frequency content

What happened here?
- What caused the different frequency content?
- What caused the different SRS velocity change?
- Is it real?





Uniqueness & Non-Uniqueness of the SRS Transformation

Non-Uniqueness of the SRS

The SRS transformation substantially reduces the quantity of shock data

- Can reduce a million time history data points to a few spectral points

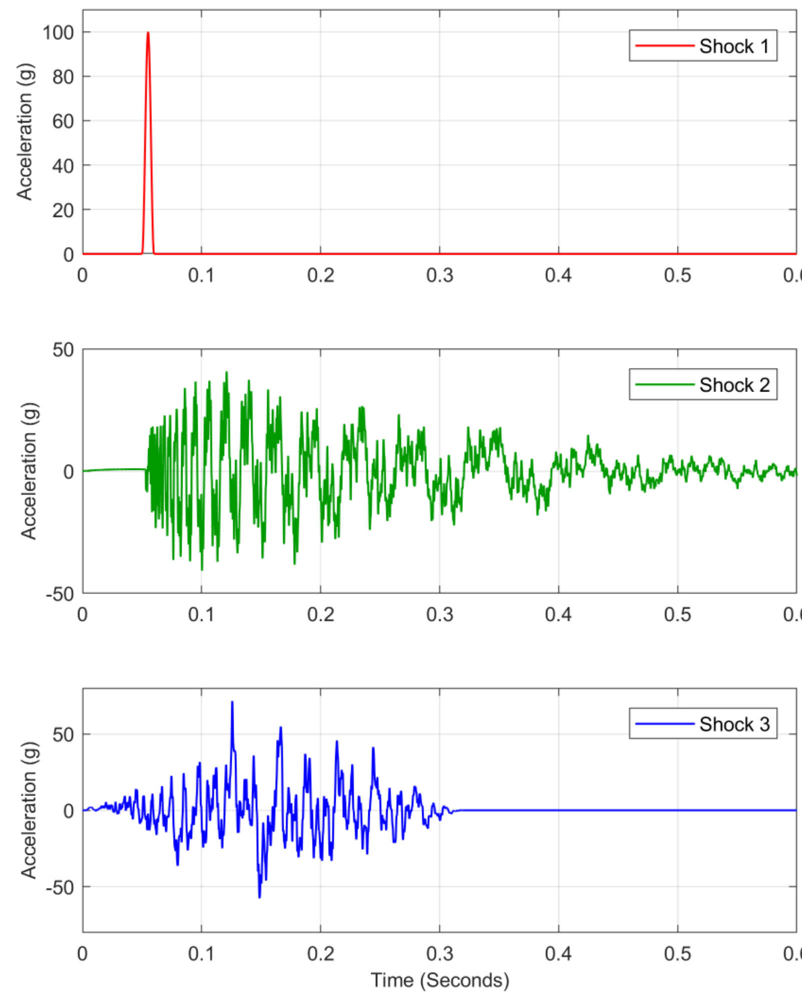
Since the SRS is an incomplete transform it is also non-unique

- Multiple shock time histories can yield the same nominal SRS

★ A fundamental assumption of the SRS is that all shocks with the same SRS are equally damaging

All three of these time histories have nominally the same SRS but they are obviously not equal

- Are they equally damaging?



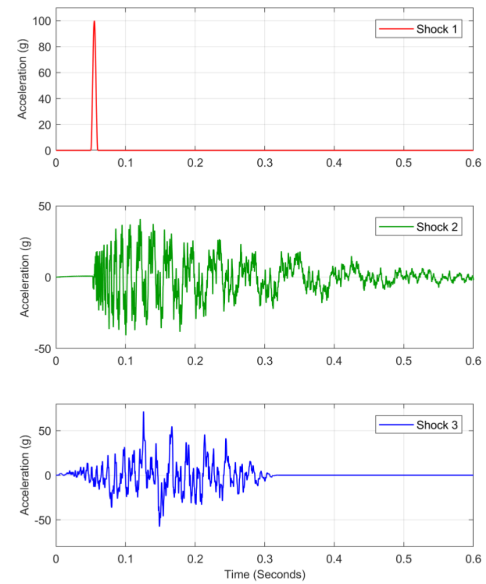
Non-Uniqueness of the SRS

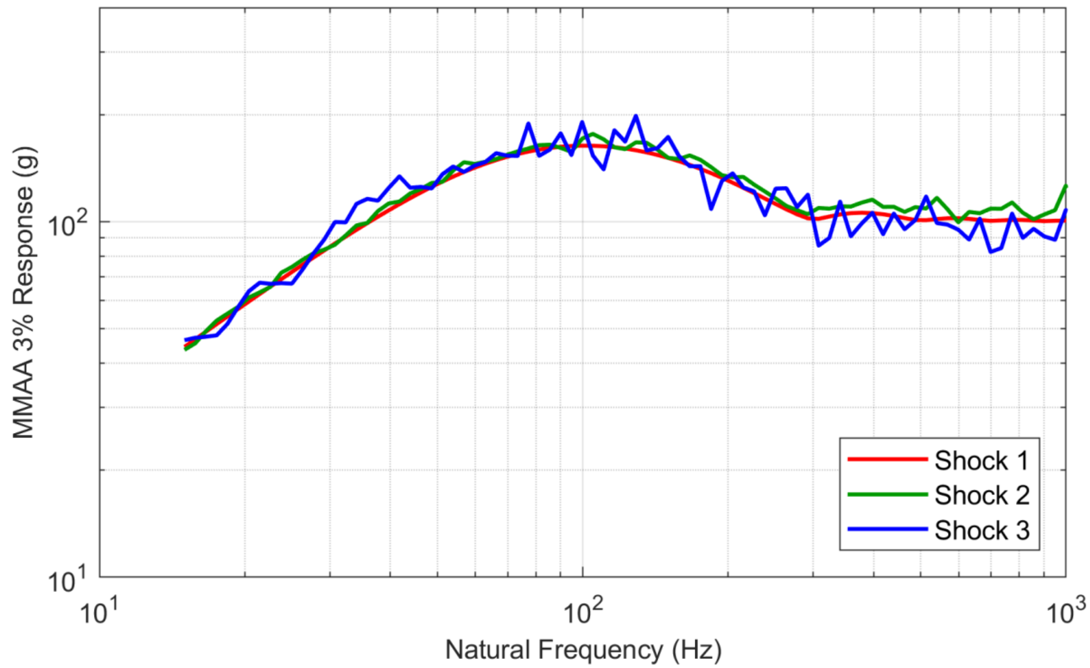
Substantially different shock amplitudes

Substantially different number of oscillations for each shock event

Substantially different shock duration

Nominally the same SRS





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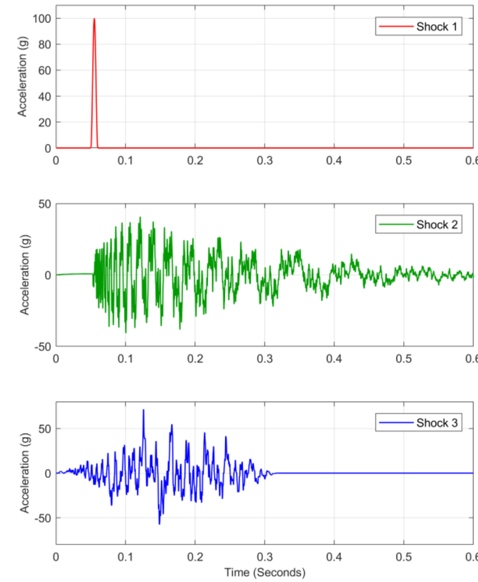
Non-Uniqueness of the SRS?



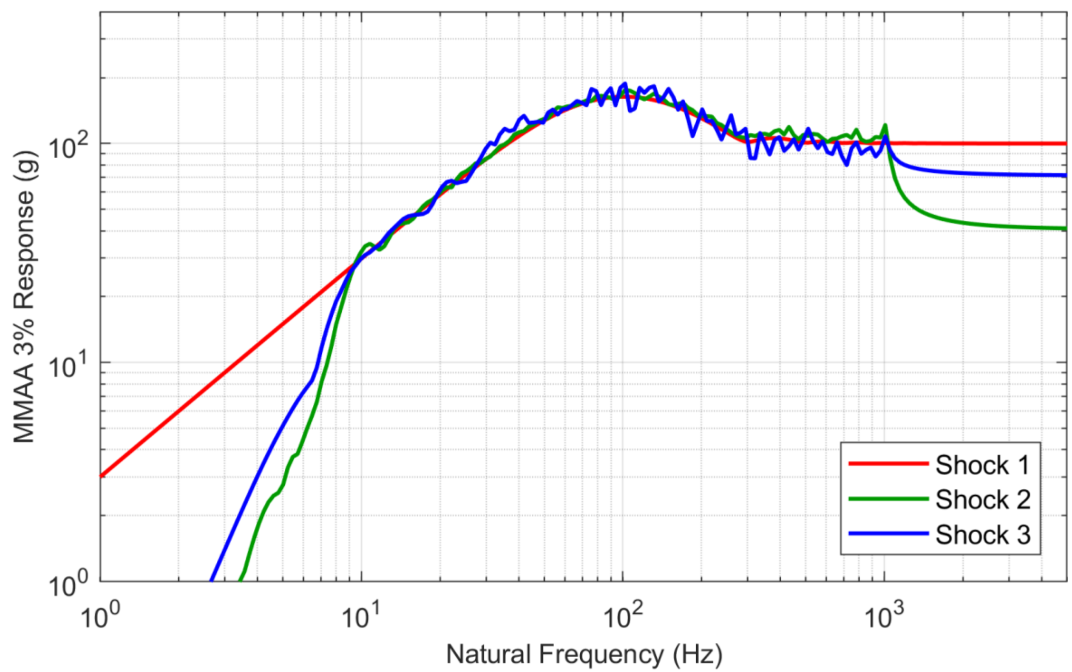
In reality these shocks are not the same

Only the same over a defined frequency range

- Frequency range could be expanded to better match SRS

Usually this type of equivalence is done so that a shock test can be performed on a shaker table

- Maybe it is alright
- Maybe not
- Where is your part susceptible to damage?



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Judging Shock Severity

Stress is Proportional to Velocity

There is a well-known relationship between stress and velocity

- Well, maybe not well-known exactly but it has been proven numerous times
- Gaberson, H. A. and Chalmers, R. H., “Modal Velocity as a Criterion of Shock Severity,” *Shock and Vibration Bulletin*, No. 40, Part 2, SVIC, U.S. Naval Research Lab, Washington, D.C., December 1969
- Gaberson, H. A., “The Pseudo Velocity Stress Analysis Stress Velocity Fountation,” *Proceedings of the 30th international Modal Analysis Conference*, Jacksonville, Florida, 2012

Derivation is based on the theory of one-dimensional stress waves

Stress is Proportional to Velocity

Solution to the long rod problem is the one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 u}{\partial x^2}$$

Solution is a complex harmonic solution of the form:

$$u(t,x) = Ae^{i(\omega t - \lambda x)} + Be^{i(\omega t + \lambda x)}$$

The full solution is quite exciting and will not be repeated here

However, the summary is that the stress is related to velocity, displacement, and acceleration by the three equations

$$\sigma_{max} = V_0\sqrt{E\rho}$$

$$\sigma_{max} = x_{max}\omega\sqrt{E\rho}$$

$$\sigma_{max} = \frac{\ddot{x}_{max}}{\omega}\sqrt{E\rho}$$

Stress is Proportional to Velocity

What does this mean?

Velocity can be compared directly since stress is proportional to velocity

$$\sigma_{max} = V_0\sqrt{E\rho}$$

Acceleration can only be compared when the frequencies are the same

If one shock has substantially more acceleration than another, you have to know the frequency content to decide which is more severe

$$\sigma_{max} = \frac{\ddot{x}_{max}}{\omega}\sqrt{E\rho}$$

Same for displacement—must have knowledge of displacement and frequency content

$$\sigma_{max} = x_{max}\omega\sqrt{E\rho}$$

How Do We Judge Shock Severity

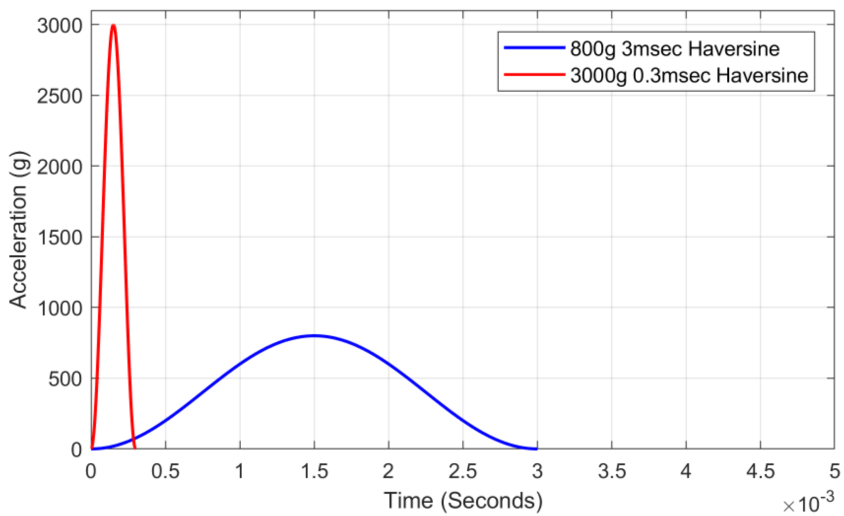
There is not always a clear-cut answer to this question

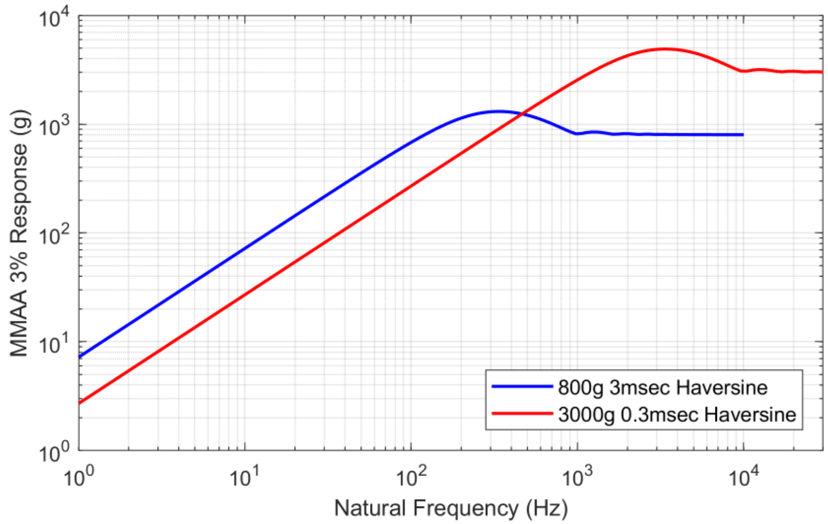
Rather there are some guidelines to consider when evaluating shock severity

Do we look at acceleration, velocity, or the time history?

What are the potential failure modes?

- Structural failure from overstress
- Modal failure from a system resonance
- Functional failure





How Do We Judge Shock Severity

Another set of sample test data

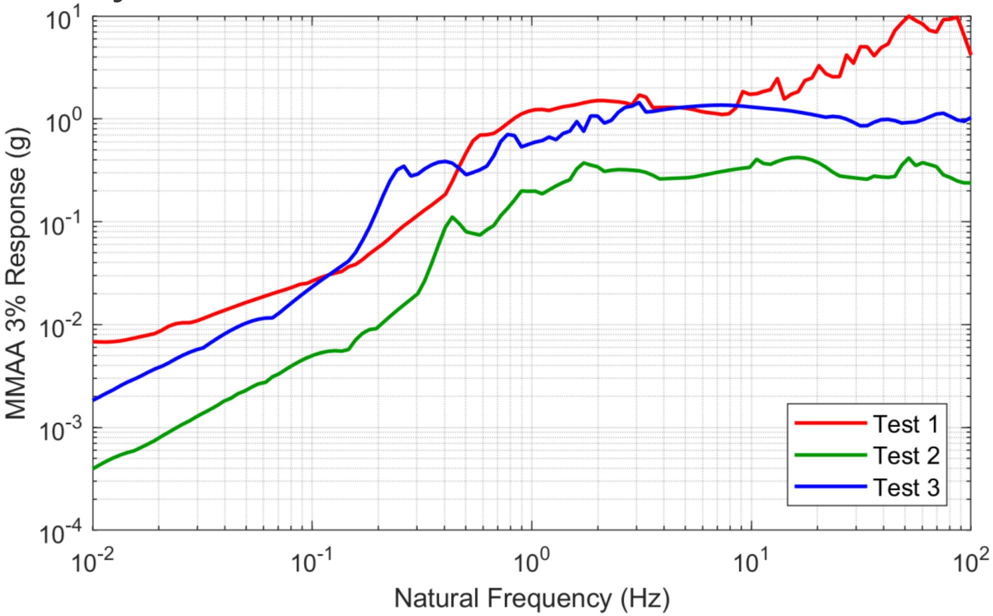
Which shock is more severe?

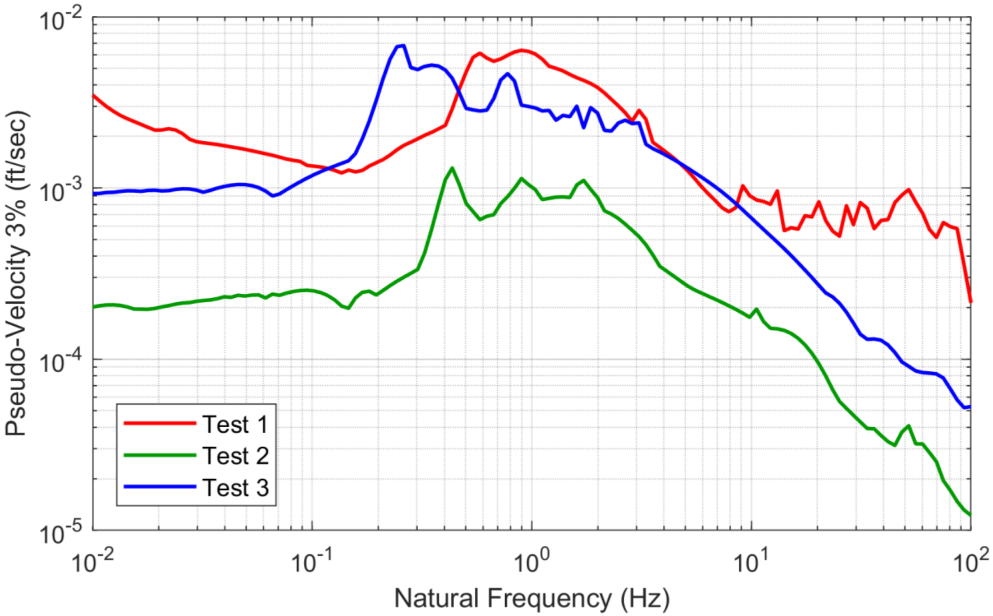
Obviously not Test 2

Is the choice the same if you look at MMAA SRS or Pseudo-velocity SRS?

- They are the same data, just in different formats

Depends on where your part is sensitive





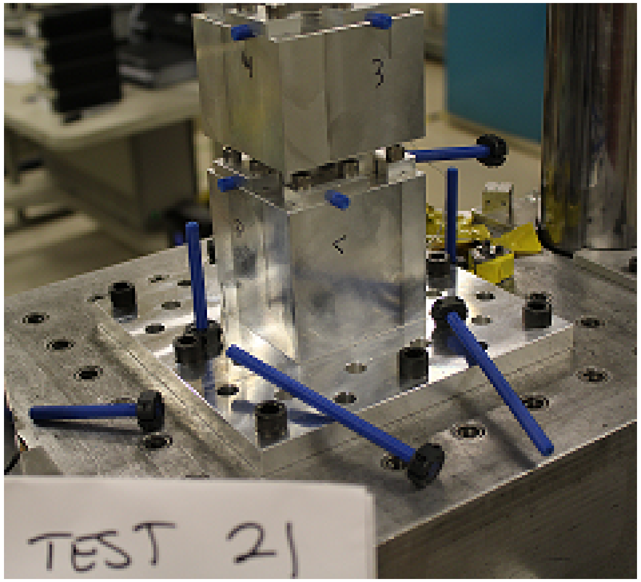
How Do We Judge Shock Severity

Which shock has the highest velocity change?

Which shock will excite the resonant modes of interest for your part?

Which shock excites the unique failure modes of your component?

Is your part subject to failure by a change in momentum?



Questions?