

THEORY OF SHEATHS NEAR POSITIVELY BIASED ELECTRODES

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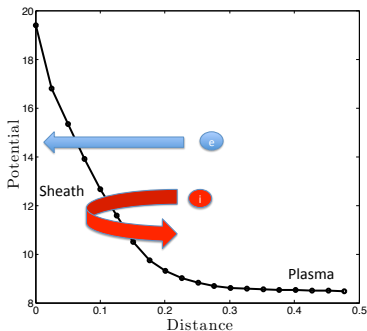
ICOPS 2016, Banff, Alberta
June 22, 2016



Electron Sheaths - Sheaths that are Electron Rich

- Surround boundaries in a plasma biased above the plasma potential
- Electron-rich
- Global current balance requires that for a monotonic electron sheath

$$A_E/A_w < \sqrt{2.3m_e/M_i} \quad [1]$$



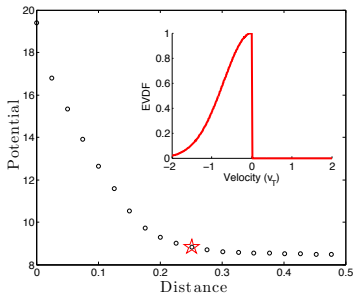
[1] S. D. Baalrud, N. Hershkowitz, and B. Longmier Physics of Plasmas 14, 042109 (2007)

Electron Sheaths are Accompanied by Fast Electron Flows

- Electron sheaths are important for
 - Langmuir probes collecting the electron saturation current
 - tethered space probes and plasma contactors
 - near electrodes used to induce circulation in dusty plasmas
 - sheath prior to the formation of anode spots
- In this work we find that
 - Electron sheaths are accompanied by a presheath
 - The presheath flow velocities approach v_{T_e}
 - Velocity distributions for electrons have a flow shift along with a loss-cone like truncation

Conventional Picture: Electron Sheaths Collect a Random Flux of Electrons

- Flux collected by an electron sheath is the random flux [2,3]
- The electron velocity distribution function (EVDF) is a truncated Maxwellian at the sheath edge [4]
- The electron sheath equivalent of the Bohm criterion is trivially satisfied \implies no need for presheath [5,6]



[2] H. M. Mott-Smith and I. Langmuir, Physical Review 28, 727 (1926)

[3] N. Hershkowitz, Physics of Plasmas 12, 055502 (2005)

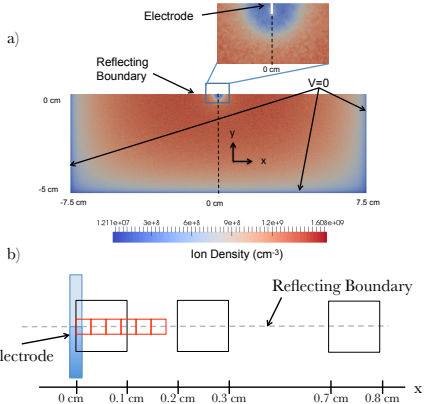
[4] G. Medicus, Journal of Applied Physics 32, 2512 (1961)

[5] K.-U. Riemann, Journal of Physics D Applied Physics 24, 493(1991)

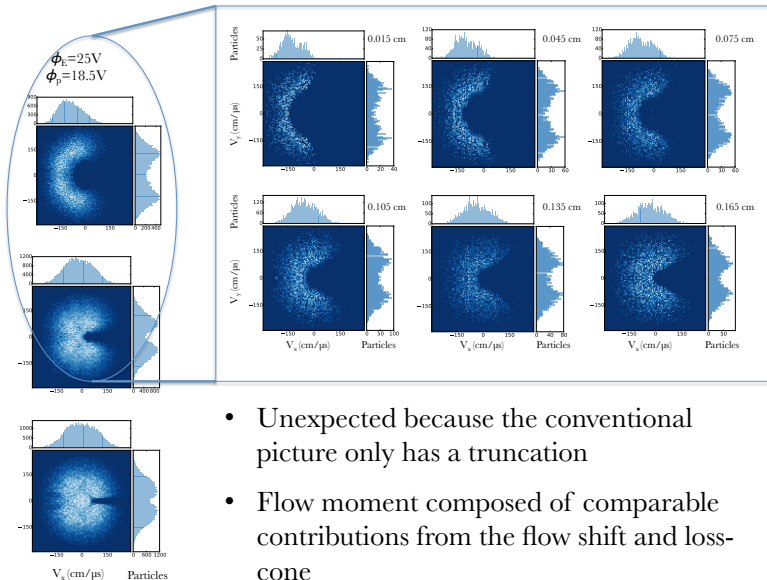
[6] F. F. Chen, Plasma Sources Science Technology 15, 773 (2006)

Particle-In-Cell Simulations

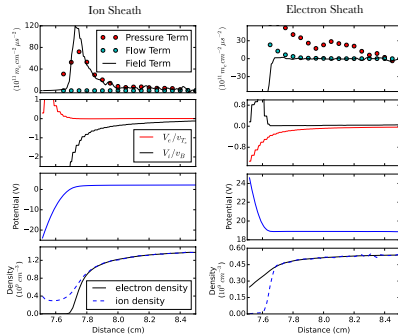
- Electrostatic PIC code Aleph
- Helium plasma was generated at $10^8 \text{ cm}^{-3} \mu\text{s}^{-1}$ within the volume
- $\Delta t = 0.5 \text{ ns}$, 0.02 cm mesh scale, $\lambda_{De} = 0.059 \text{ cm}$, $\omega_{pe}^{-1} = 0.63 \text{ ns}$
- No collisions with neutrals were included
- Simulations ran for $40 \mu\text{s}$ of physical time
- Will study EVDFs at different locations in front of the electrode



Electron Velocity Distribution Functions Have a Flow Shift and Truncation



Pressure Tensor Gradients Dominates the Presheath Electron Flow



$$\underbrace{m_e n_e \mathbf{V}_e \cdot \nabla \mathbf{V}_e}_{\text{flow term}} = \underbrace{-n e \mathbf{E}}_{\text{field term}} \underbrace{- \nabla \cdot \mathcal{P}_e}_{\text{pressure term}} - \underbrace{\mathbf{R}_e}_{\text{friction}}$$

Why Does the Electron Flow Approach the Electron Thermal Speed?

- **Sheath Criterion:** At the sheath edge $\rho \approx 0$ and $|d\rho/d\phi|_{\phi=\phi_0}| > 0$ [7]
- This can be rewritten as

$$\sum_s q_s \frac{dn_s}{dz} \leq 0$$

- Using the continuity equation (no source)

$$\sum_s q_s \frac{n_s}{V_s} \frac{dV_s}{dz} \leq 0$$

- The electron presheath has a minimum velocity determined by the momentum equation, approximating this as

$$m_e n_e \mathbf{V}_e \cdot \nabla \mathbf{V}_e = -n e \mathbf{E} - \nabla p_e \hat{z}$$

the flow velocity is

$$V_e \geq \sqrt{\frac{T_e + T_i}{m_e}} \approx v_{Te}$$

- Stress moments due to the loss cone will slightly modify this value

Pressure Gradient Plays a Larger Role than the Electric Field

- Assume a Boltzmann ion density profile, then quasineutrality implies

$$\frac{dn_e}{dz} = \frac{dn_i}{dz} = -\frac{en_i E}{T_i}$$

- Using $\mathcal{P}_e = p_e \mathcal{I} + \Pi_e$, with $p_e = n_e T_e$ the z component of the momentum equation shows the pressure term is T_e/T_i larger than the electric field term

$$V_e \frac{dV_e}{dz} = -\frac{e}{m_e} E - \underbrace{\frac{T_e}{m_e n_e} \left(-\frac{en_i E}{T_i} \right)}_{\frac{1}{m_e n_e} \nabla p_e} - (\text{stress, friction, other terms})$$

In the electron presheath the electric field causes a density (pressure) gradient. The acceleration of the flow velocity is dominated by the pressure gradient.

Significantly different from ion presheaths

Flow Moment Means Thicker Electron Sheath

This compares well with previous simulations [8]

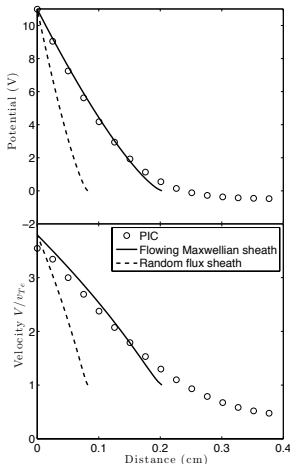
- Combining the momentum and continuity equations, ignoring stress and friction terms, and integrating

$$\left(\frac{V_e}{v_{Te}}\right)^2 - 2 \log \left(\frac{V_e}{v_{Te}}\right) = \frac{2e\phi}{T_e} + 1$$

- Integrating Poisson's equation twice within the sheath:

$$\frac{z}{\lambda_{De}} = 0.79 \left(\frac{e\Delta\phi}{T_e}\right)^{3/4}$$

- The numerical constant is typically assumed to be 0.32 based on the random flux assumption.



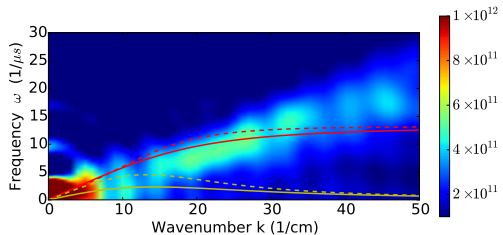
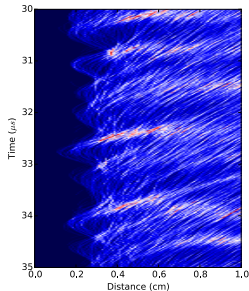
[8] B. Scheiner, S. D. Baalrud, B. T. Yee, M. M. Hopkins, and E. V. Barnat Physics of Plasmas 22, 123520 (2015)

Instabilities

- The dielectric response for a plasma where the electrons are Maxwellian with flow V_e and stationary Maxwellian ions is

$$\epsilon(\mathbf{k}, \omega) = 1 - \frac{\omega_{pe}^2}{k^2 v_{Te}^2} Z'(\xi_e) - \frac{\omega_{pi}^2}{k^2 v_{Ti}^2} Z'(\xi_i) \quad (1)$$

where $\xi_e = \frac{\omega - \mathbf{k} \cdot \mathbf{V}_e}{k v_{Te}}$ and $\xi_i = \frac{\omega}{k v_{Ti}}$



[8] B. Scheiner, S. D. Baalrud, B. T. Yee, M. M. Hopkins, and E. V. Barnat Physics of Plasmas 22, 123520 (2015)

Summary: Electron Flow in the Electron Presheath is Caused by a Loss-Cone Type Truncation and a Flow Shift

- Electrons flow with a velocity comparable with v_{Te}
- The flow velocity is composed of two kinetic effects, 1) a loss-cone type truncation and 2) a shift in the maximum value of the EVDF
- The loss-cone truncation is a geometric effect
- This flow is accelerated in a presheath characterized by pressure gradients
- Sheath models with a flow are in better agreement with simulations than the conventional picture