SAND2020-2193C

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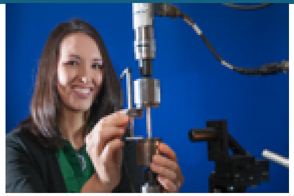
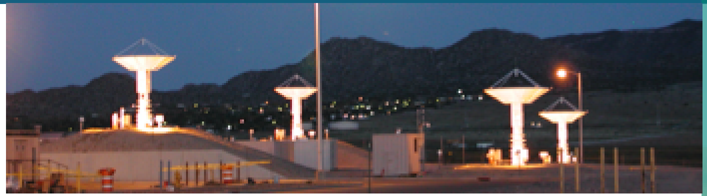
Using the Restoring Force Surface Method to Model Modal Coupling





PRESENTED BY

Ben Moldenhauer

- University of Wisconsin-Madison

In Association with Dan Roettgen & Ben Pacini

- Sandia National Laboratories

IMAC 38
February 11, 2020

U.S. DEPARTMENT OF ENERGY   NNSA

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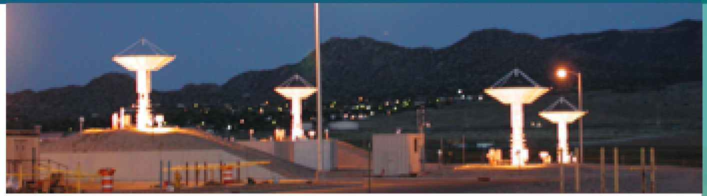
How Well Does a Model Constructed with RFS Actually Represent the Original System?



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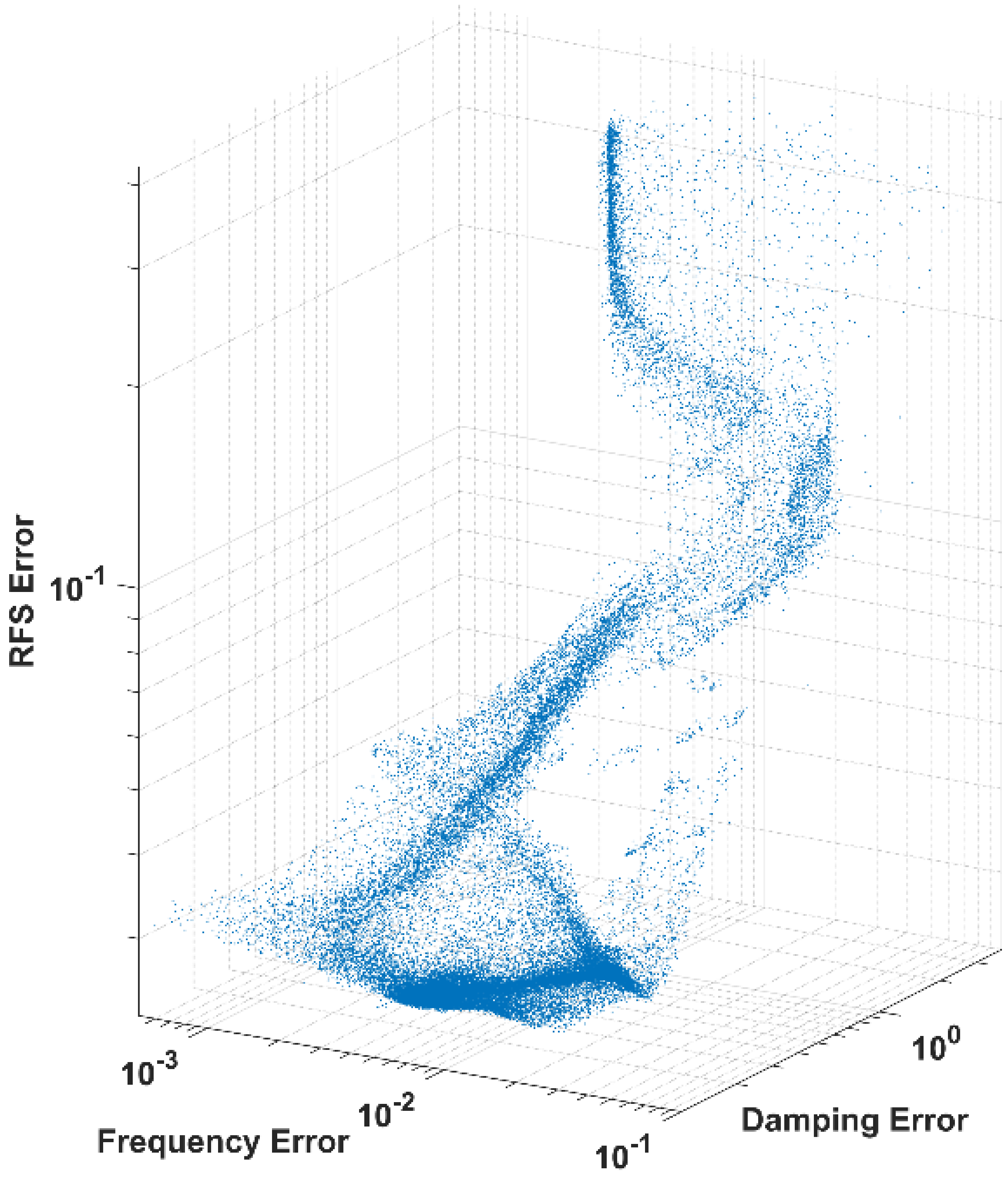
Overview

- Restoring Force Surface Method
  - The Restoring Force
  - Nonlinear Polynomials
    - Fractional Exponents
  - Model Fitting Procedure
    - Monte Carlo Simulations
    - Ideal System Demonstration
- Experimental Data
  - Cylinder-Plate-Beam Structure
  - Nonlinear Testing & Data
- Fitting Isolated Experimental Modes
  - Restoring Force Fit vs. Model Accuracy
- Conclusions & Coupling



Restoring Force Surface Method

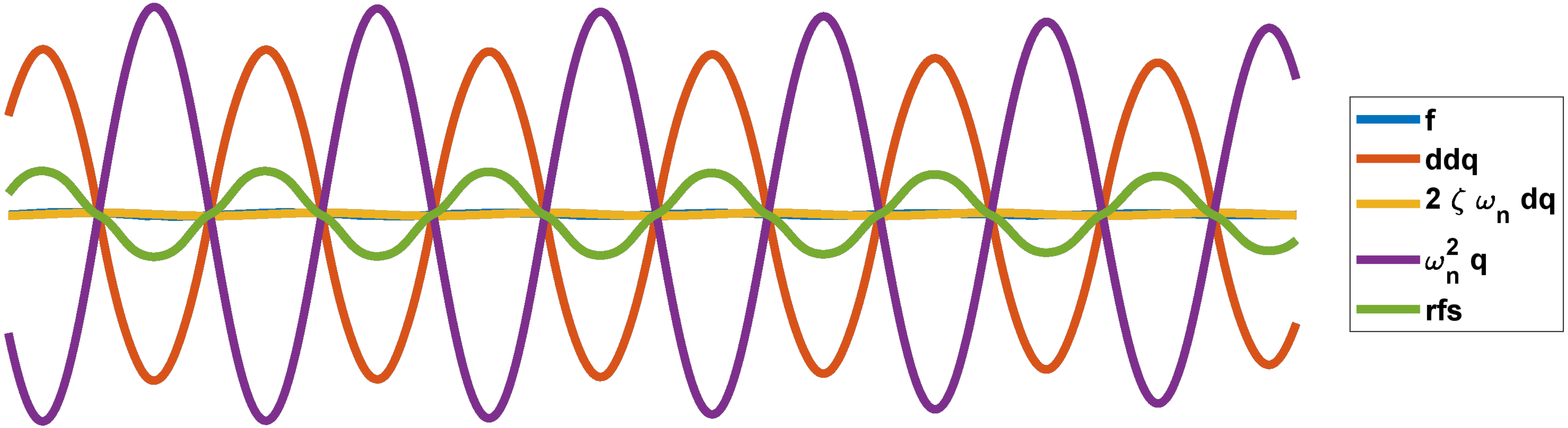
The Restoring Force

- Standard SDOF EOM with General Nonlinear Term

$$\ddot{q} + 2\zeta\omega_{\mathrm{n}}\dot{q} + \omega_{\mathrm{n}}^2 q + f_{\mathrm{NL}}(q,\dot{q}) = f_{\mathrm{ext}}$$

- Rearrange terms → Nonlinearity is residual of linear responses

$$f_{\mathrm{NL}}(q,\dot{q}) = f_{\mathrm{ext}} - \ddot{q} - 2\zeta\omega_{\mathrm{n}}\dot{q} - \omega_{\mathrm{n}}^2 q$$



Fit Nonlinear Polynomials to the Restoring Force

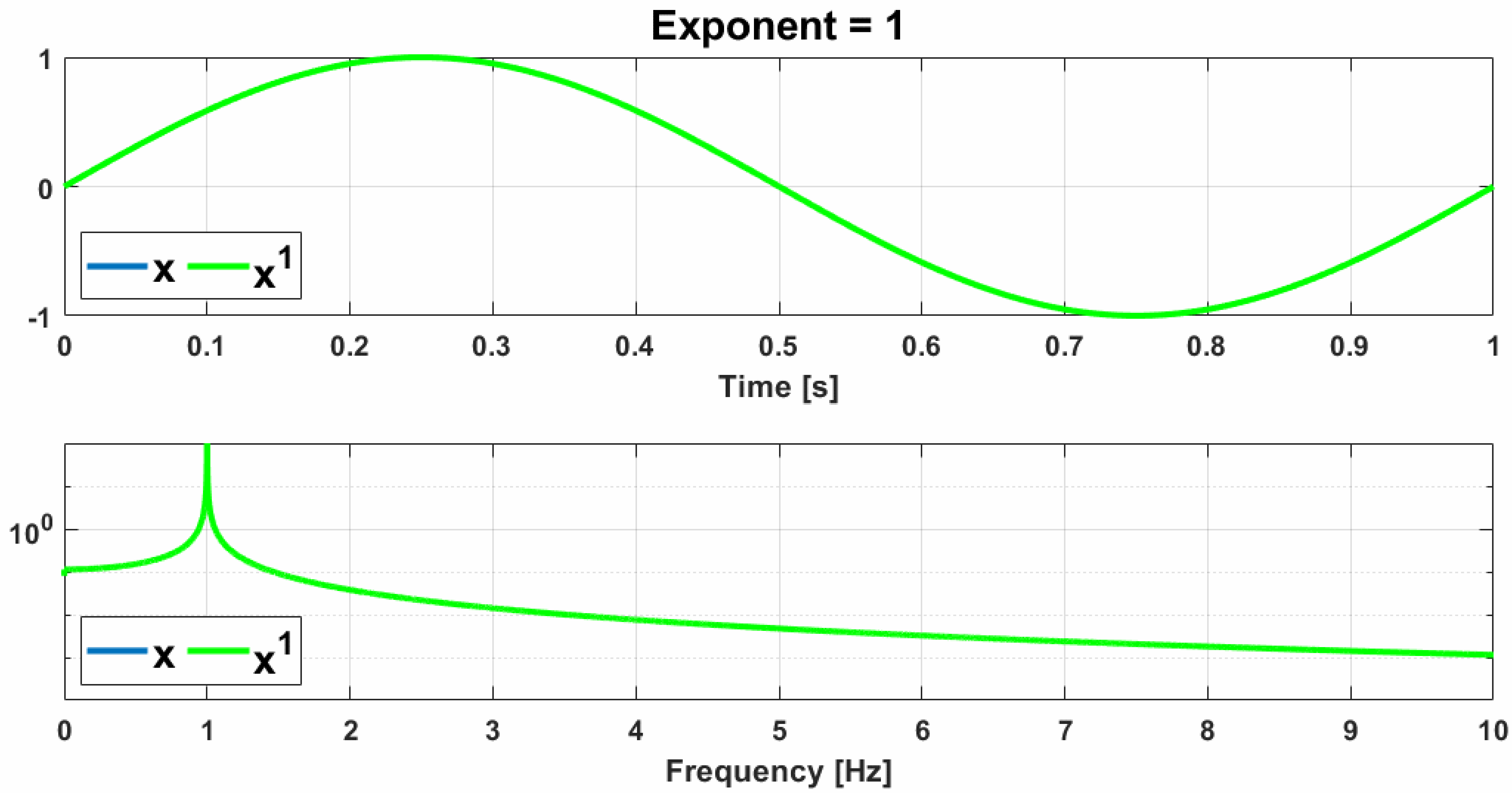
- Form nonlinear polynomials from the displacement and velocity
  - 4 types of terms used here: Odd and Even variations of q and $\dot{q}$

$$f_{\mathrm{NL}}(q,\dot{q}) = \sum k_{\mathrm{q}}|q|^i \mathrm{sgn}(q) + \sum k_{|\mathrm{q}|}|q|^j + \sum c_{\dot{\mathrm{q}}}|\dot{q}|^k \mathrm{sgn}(\dot{q}) + \sum c_{|\dot{\mathrm{q}}|}|\dot{q}|^l$$

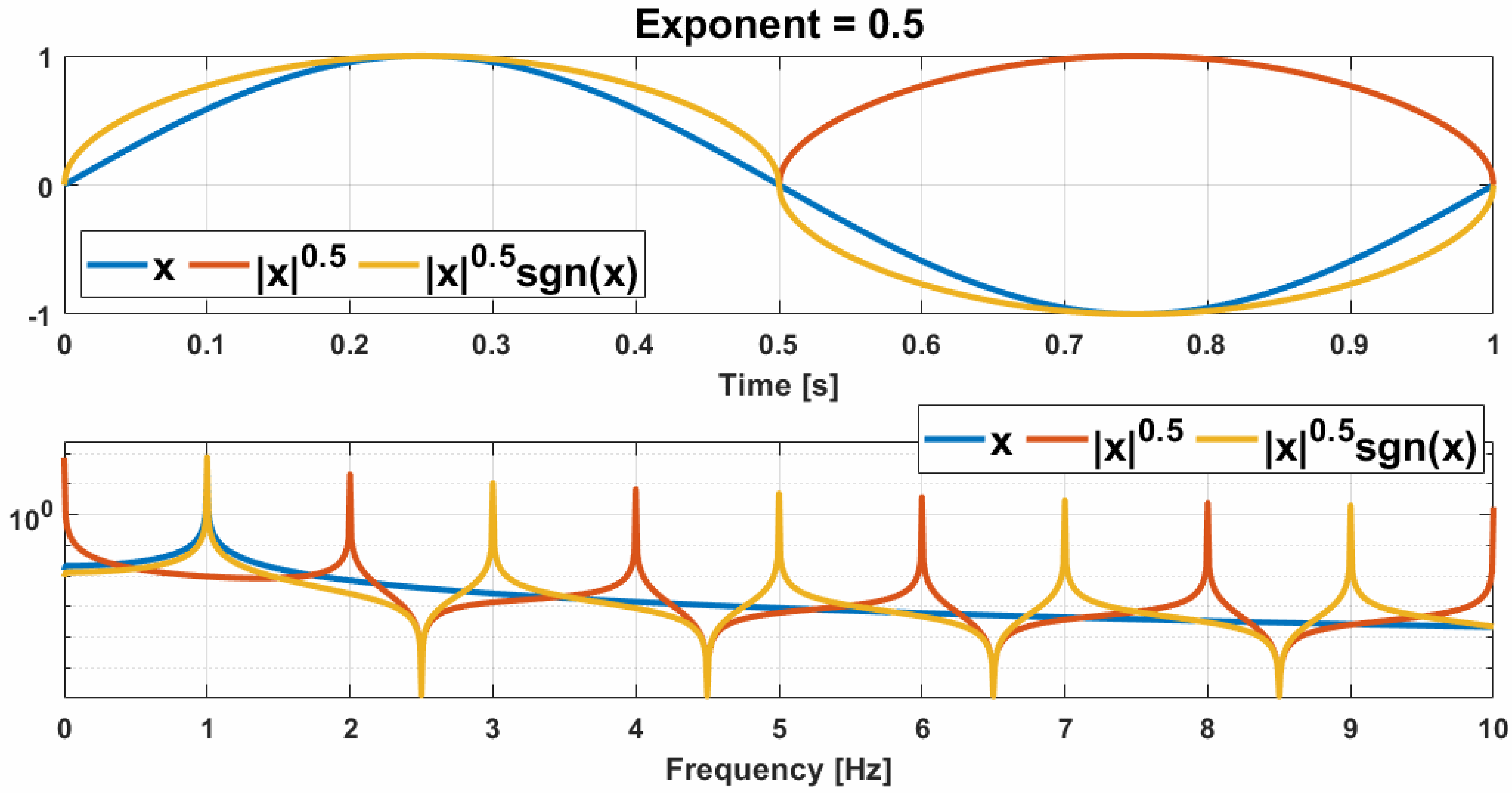
- Least-squares solution yields the best fit coefficients for the proposed nonlinear terms

$$\begin{bmatrix} k_q \\ k_{|\mathrm{q}|} \\ c_{\dot{\mathrm{q}}} \\ c_{|\dot{\mathrm{q}}|} \end{bmatrix} = \begin{bmatrix} |q|^i \,\mathrm{sgn}(q) & |q|^j & |\dot{q}|^k \,\mathrm{sgn}(\dot{q}) & |\dot{q}|^l \end{bmatrix}^{\dagger} \left[f_{\mathrm{ext}} - \ddot{q} - 2\zeta\omega_{\mathrm{n}}\dot{q} - \omega_{\mathrm{n}}^2 q \right]$$

Integer Exponents Produce Associated Harmonics

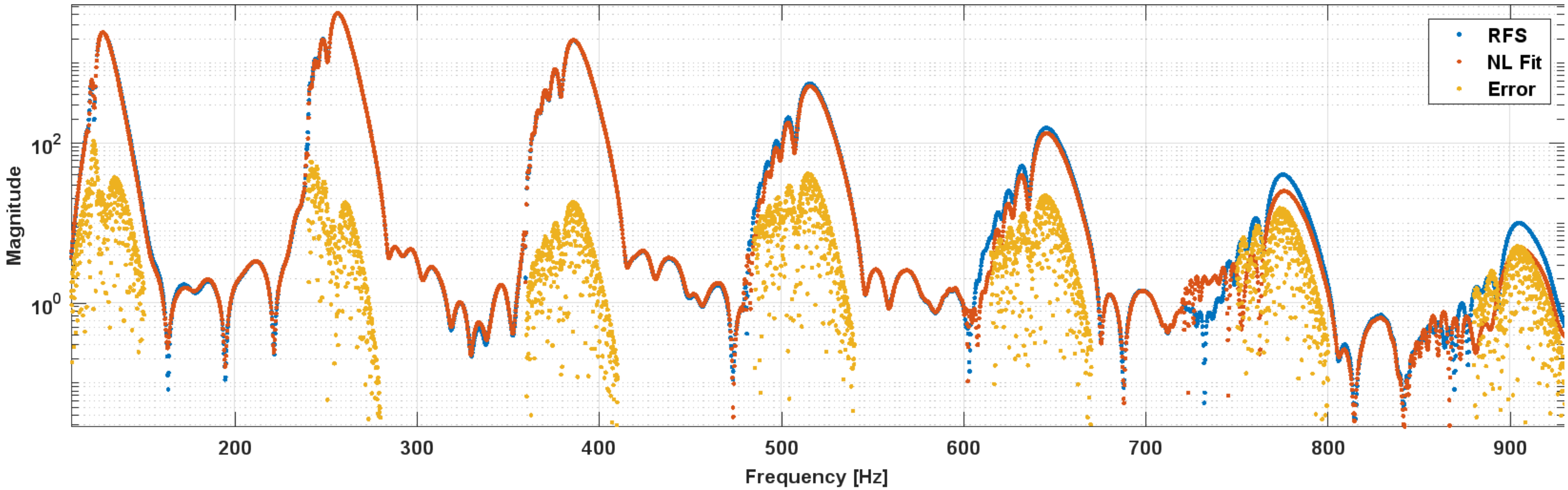


Fractional Exponents Alter Relative Harmonic Amplitudes



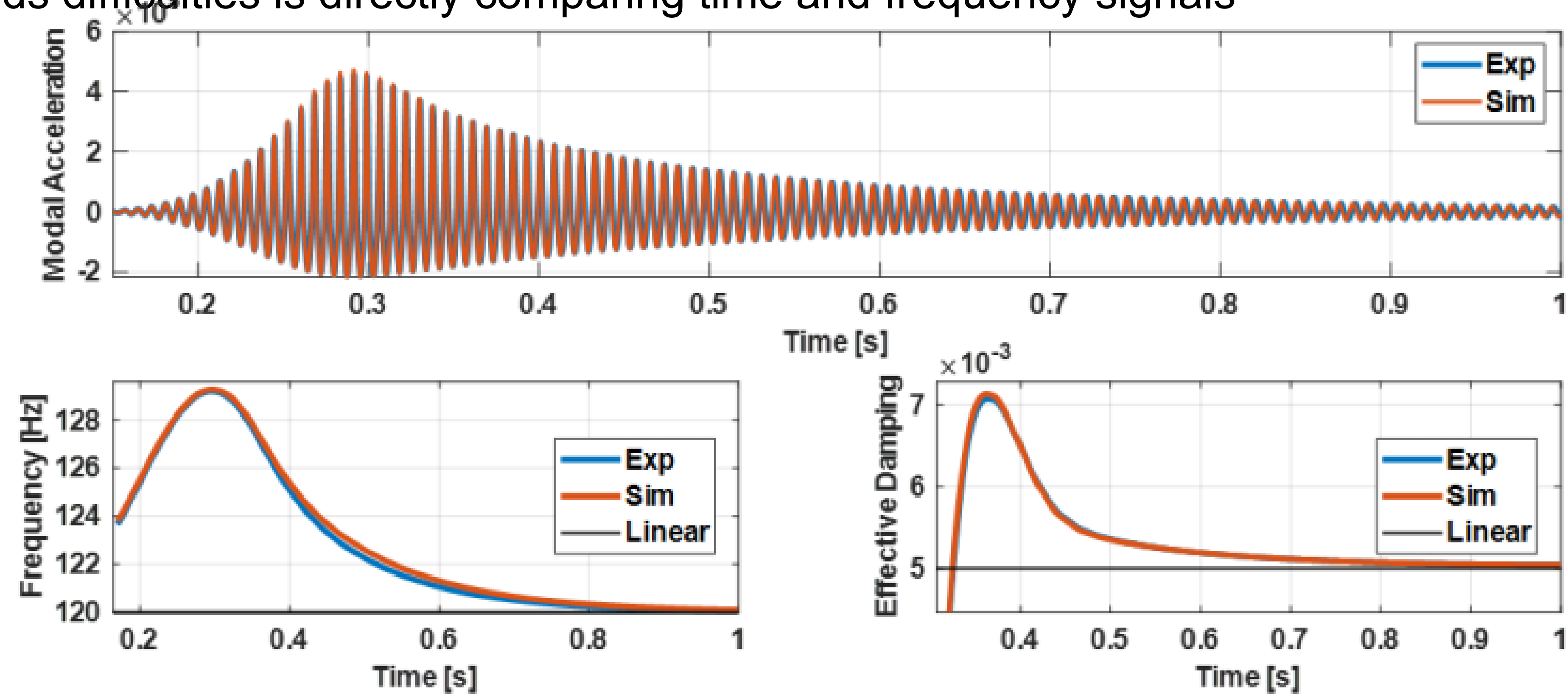
RFS Nonlinear Model Fitting Procedure

- Potential Models trialed with Monte Carlo Simulation
  - Randomly select number of terms and their exponents
  - Typically choose up to 10 terms of each type with exponents from (1.1, 1.2, 1.3, … , 7)
  - Example: $q^3 + \dot{q}^2$ being fit by $|q|^{2.9}\,\mathrm{sgn}(q) + |\dot{q}|^{2.1}$
- Least squares fit to restoring force to determine coefficients



RFS Nonlinear Model Fitting Procedure

- Simulate model response to experimental modal force
- Compute instantaneous frequency and effective damping and compare to the experimental data
  - Avoids difficulties is directly comparing time and frequency signals



RFS Nonlinear Model Fitting Procedure - Monte Carlo Simulation

- $\omega_n = 120$ [Hz] , $\zeta = 0.005$, and Nonlinear Terms $(1e6)x^3$ and $(1)\dot{x}^2$
- Numerically integrate response to obtain acceleration, velocity, displacement, and force.

Randomly select several terms and their exponents

Least Squares Fit to Restoring Force to get Coefficients

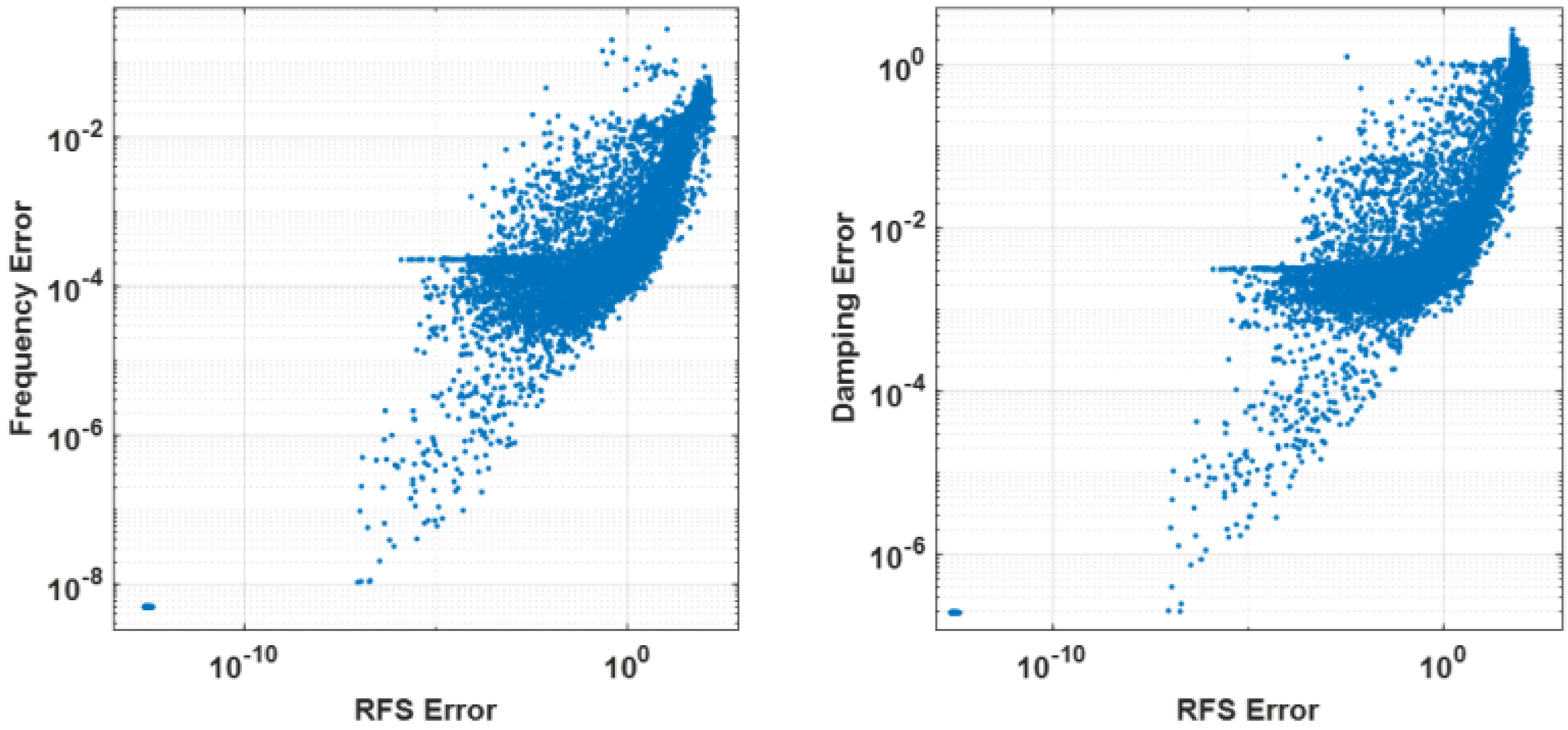
ODE Solver to simulate response of model

Compute Instantaneous Frequency and Damping

Log model and errors

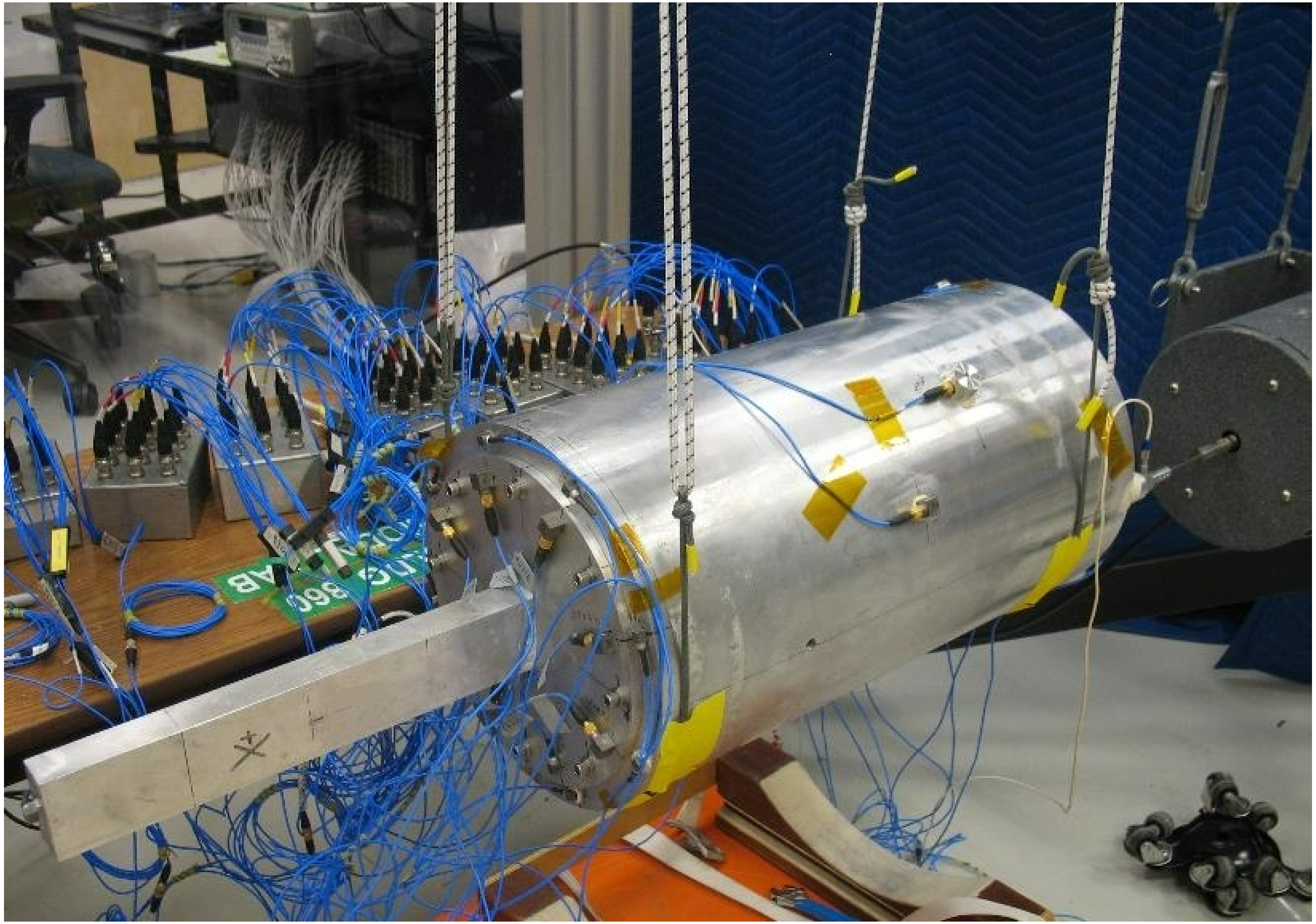
From 10,000 Iterations, a Better RFS Fit Tends to Yield a Better Model

- In this idealized scenario, a smaller RFS fit error tends to produce a model that more accurately represents the original system.



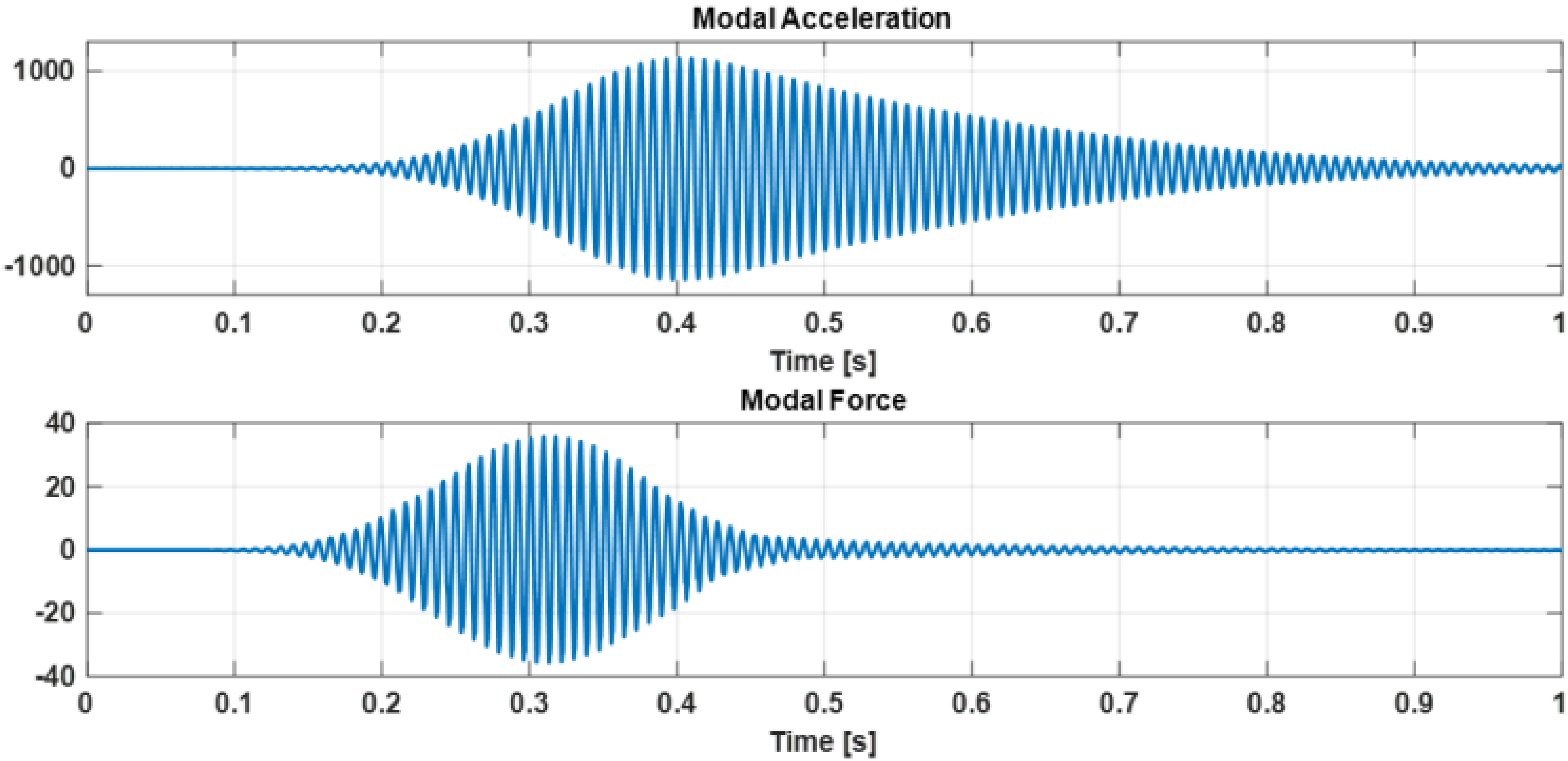
Experimental Data

Cylinder-Plate-Beam Experimental Structure



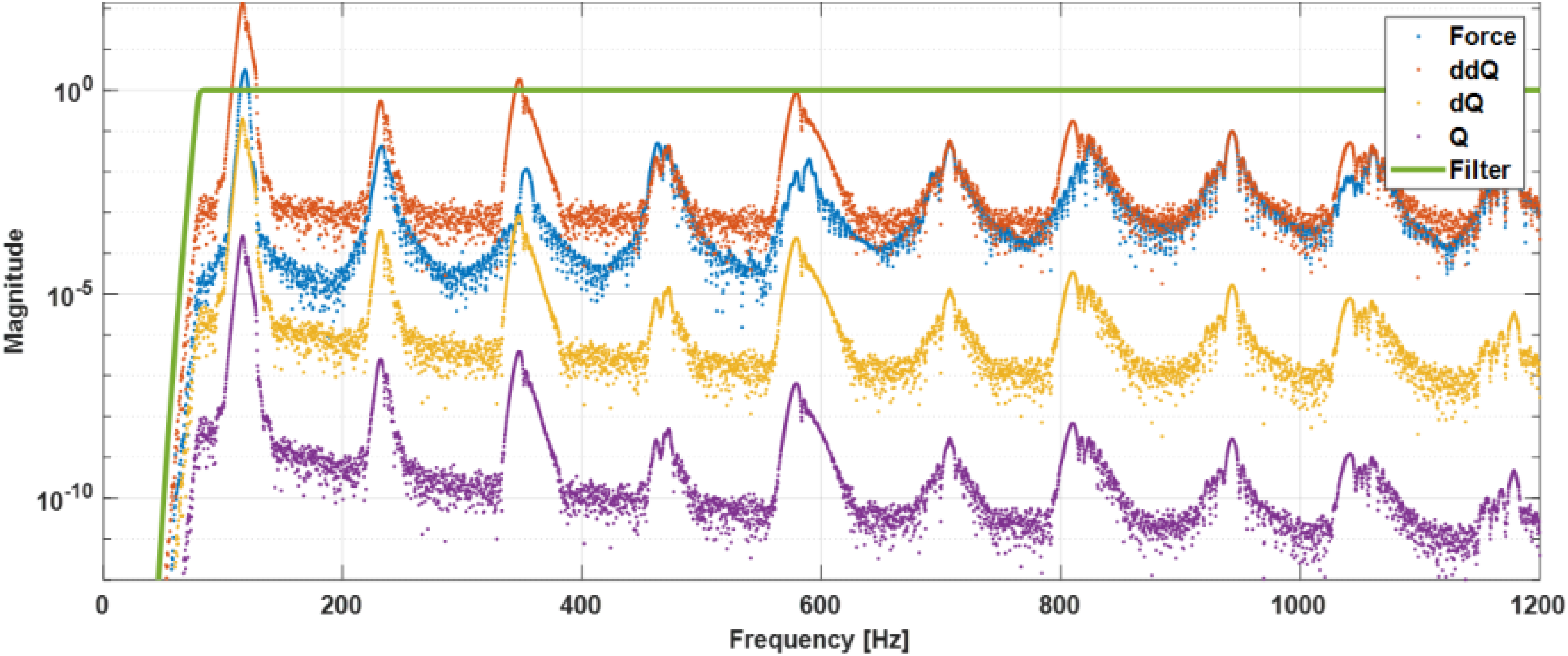
Linear & Nonlinear Data Collection

- Curve Fit low-level burst random excitation data to get linear $\omega_n, \zeta,$ and Φ
- Excite individual modes to nonlinear level with sine beats and modal filter responses with Φ



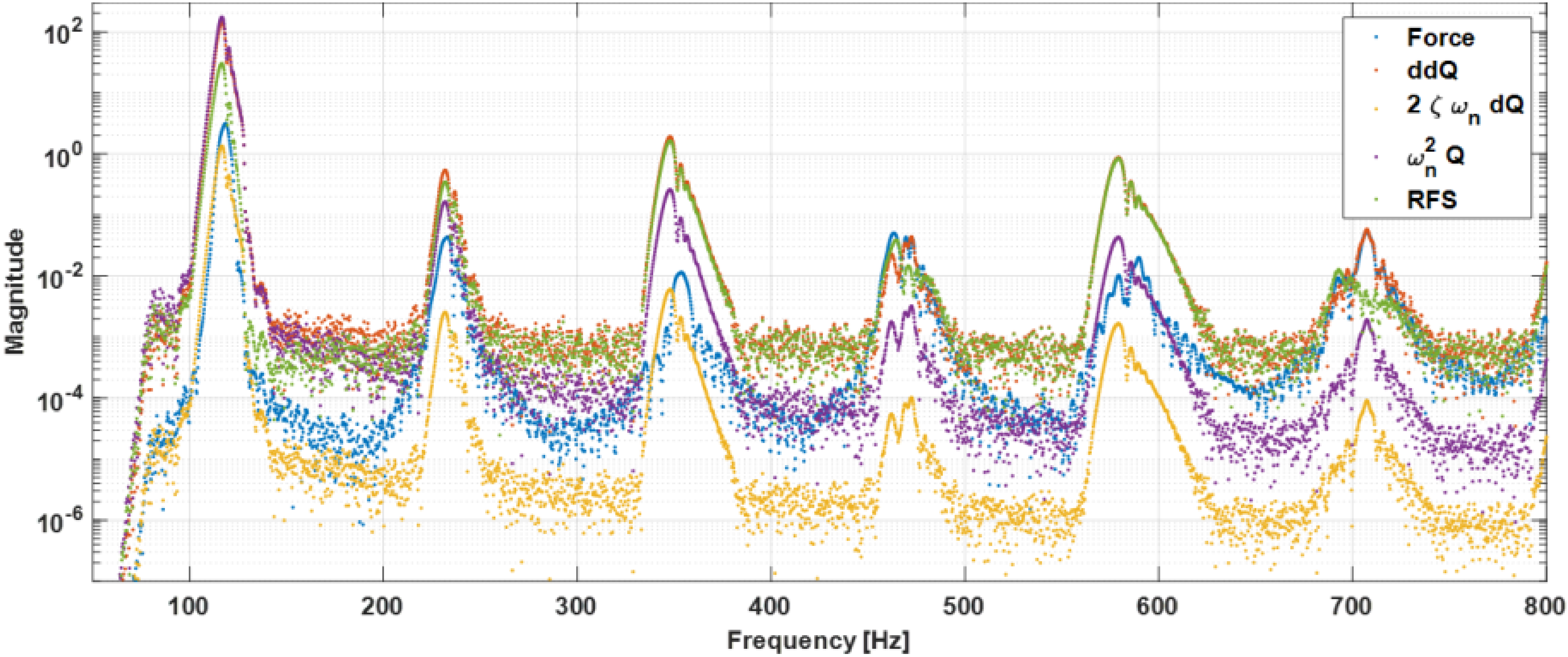
Integrate Acceleration in Frequency Domain to get Velocity and Displacement

- Nonlinearity produces many high order harmonics
- Low frequency drift from integration must be filtered out



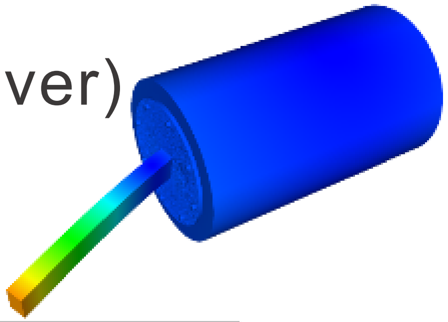
Forming the Restoring Force

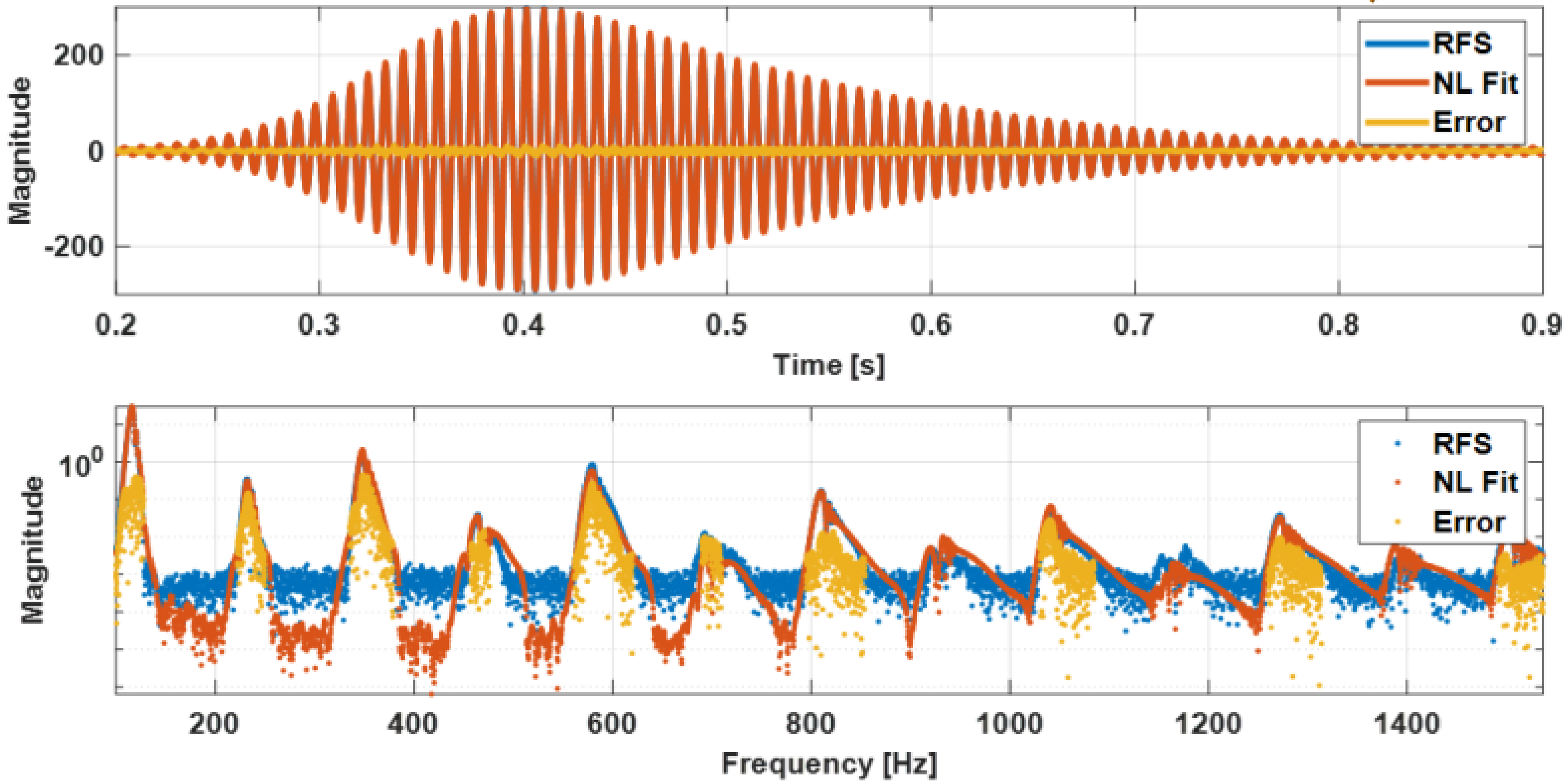
- Least squares fit is done with the Fourier Coefficients of the Restoring Force and the Nonlinear Terms



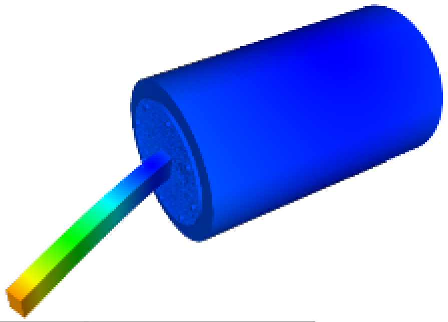
Fitting Experimental Modes

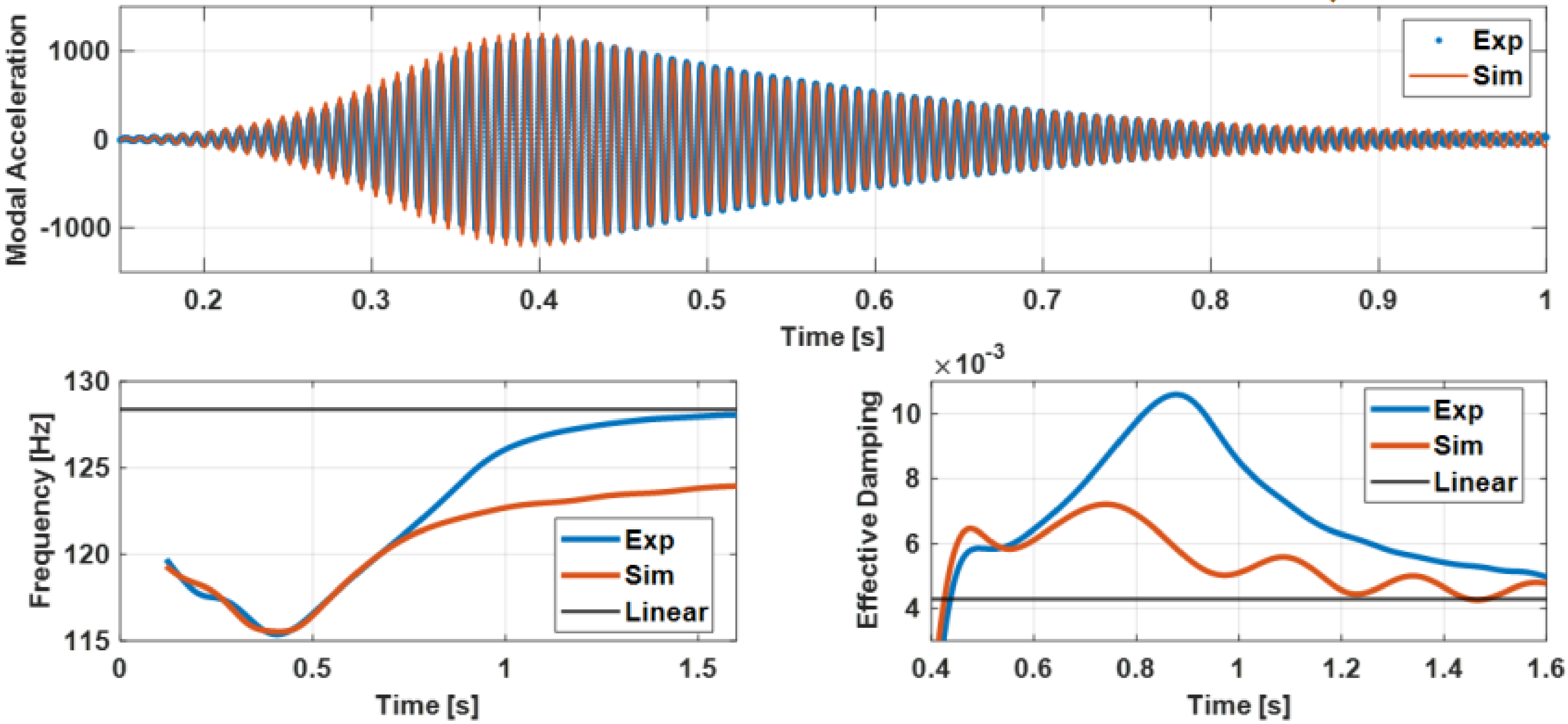
Best Restoring Force Fit for First Mode (Soft Beam Cantilever)



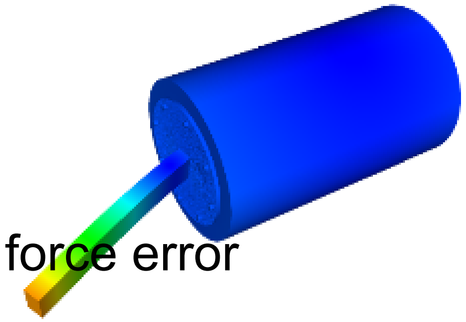


Simulated Response to Best Restoring Force Fit

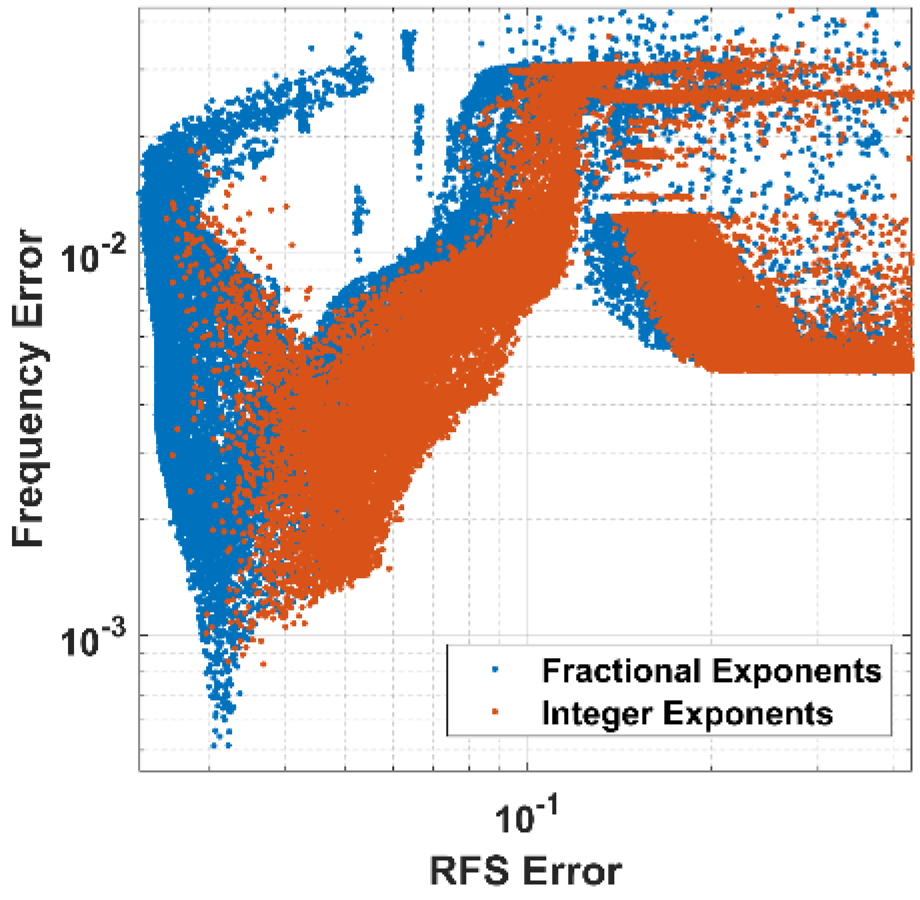


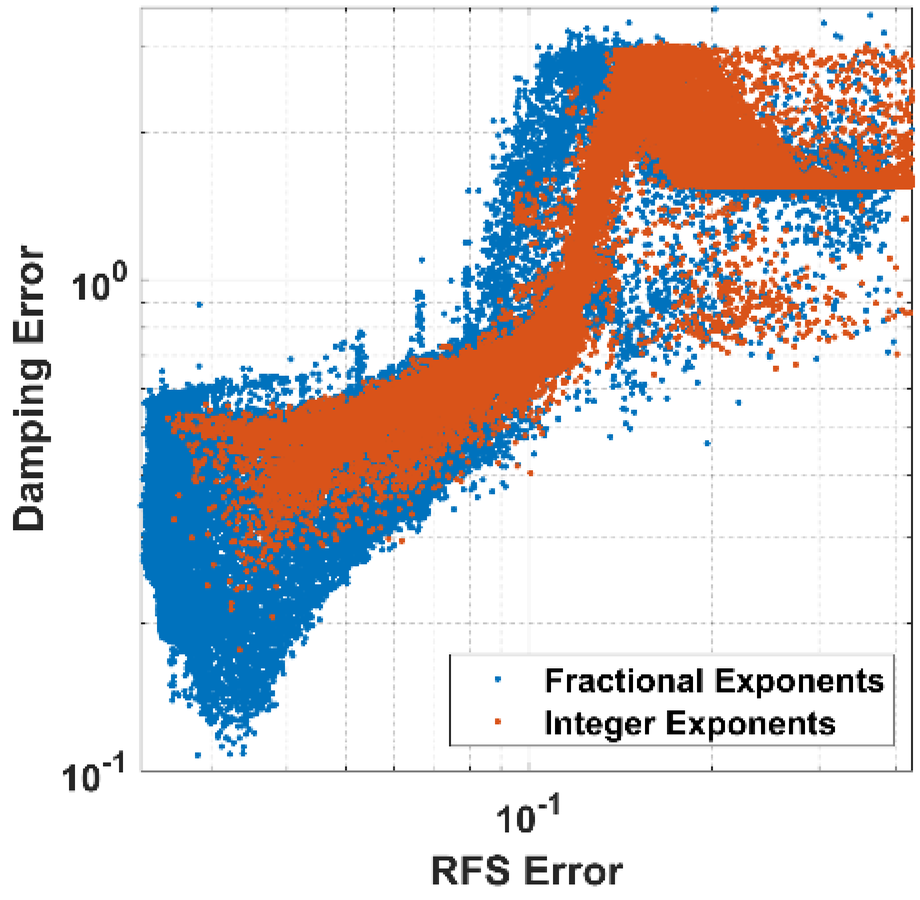


Restoring Force Fit vs Accuracy of Simulated Model

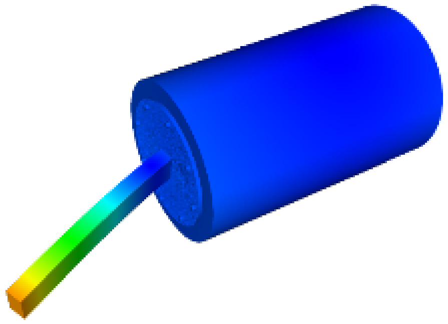


- Minimum error in simulated response occurs well before the minimum restoring force error
- Integer exponents yield inferior results to implementing fractional exponents

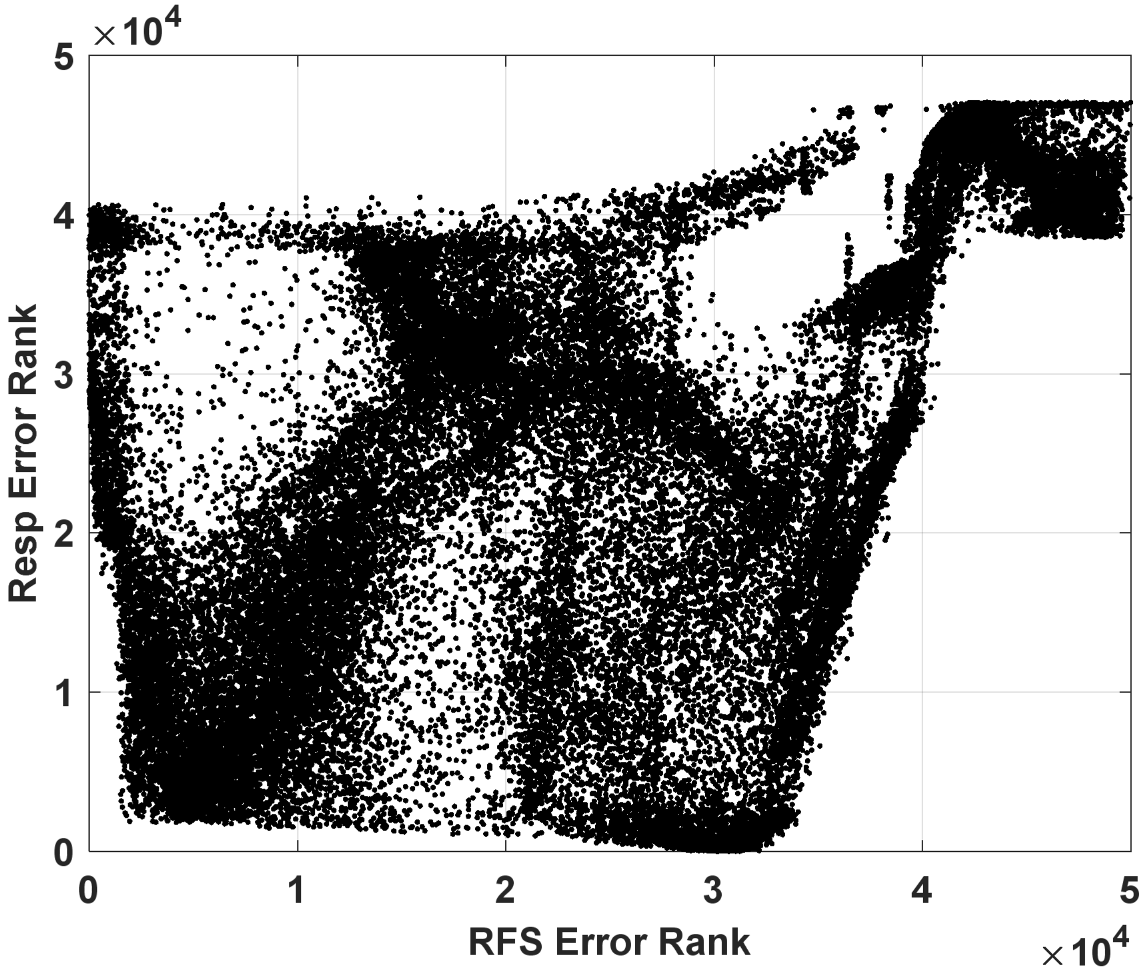


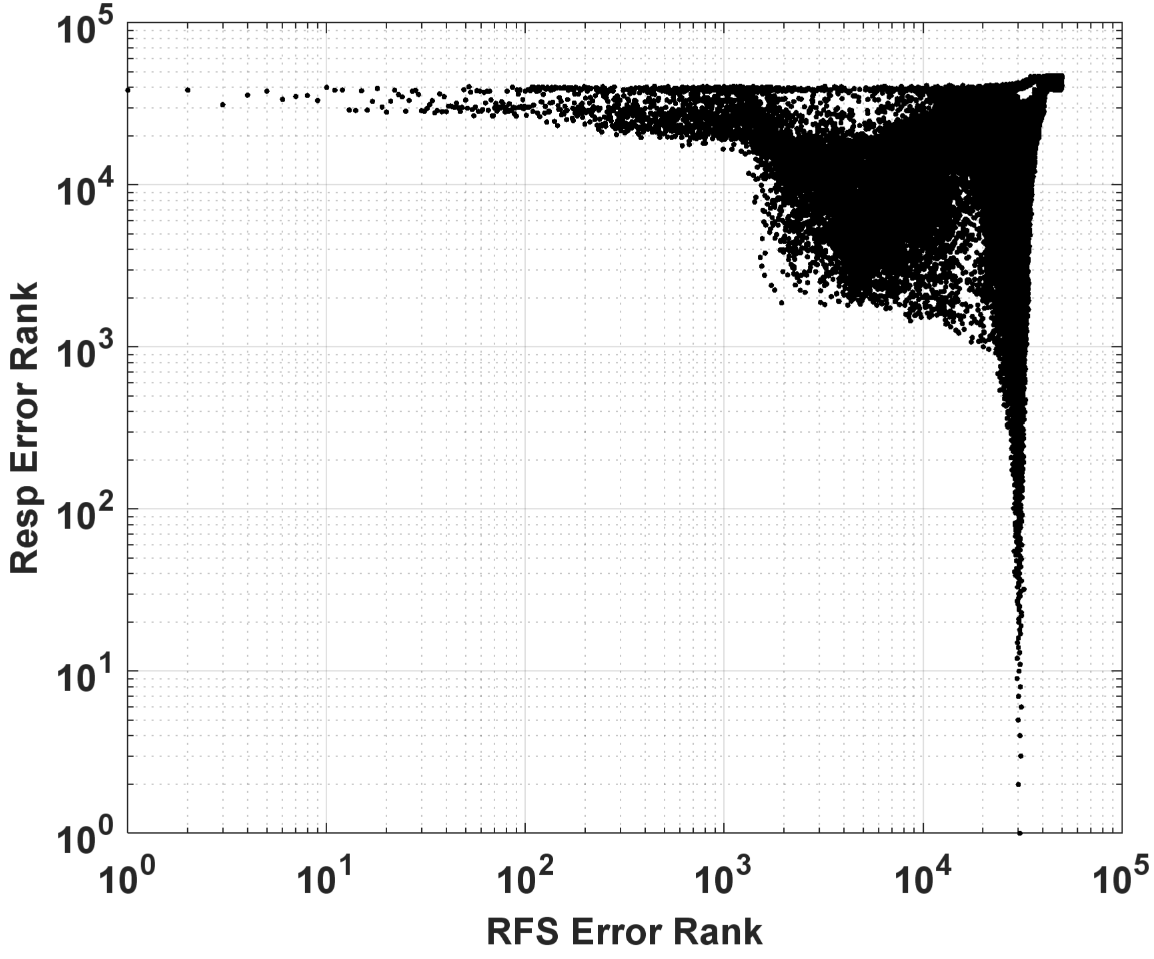


Restoring Force Fit vs Accuracy of Simulated Model

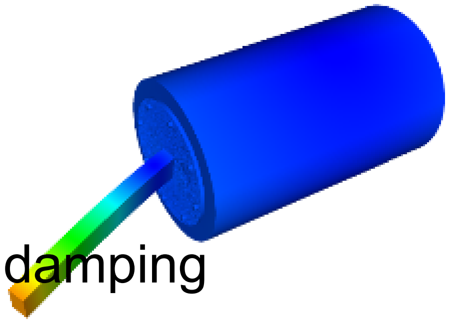


- Alternative Representation: Rank of errors. i.e minimum=1, next=2 and so o

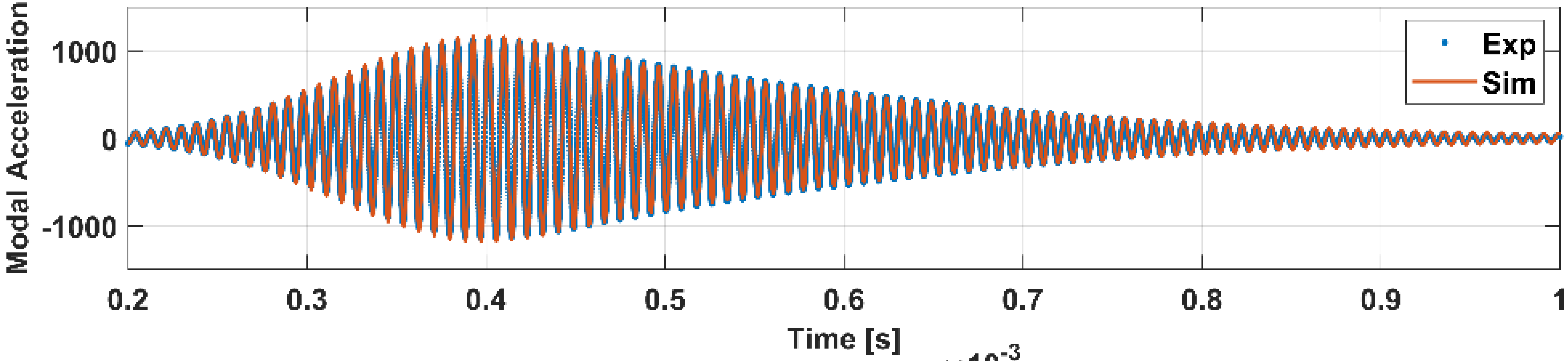


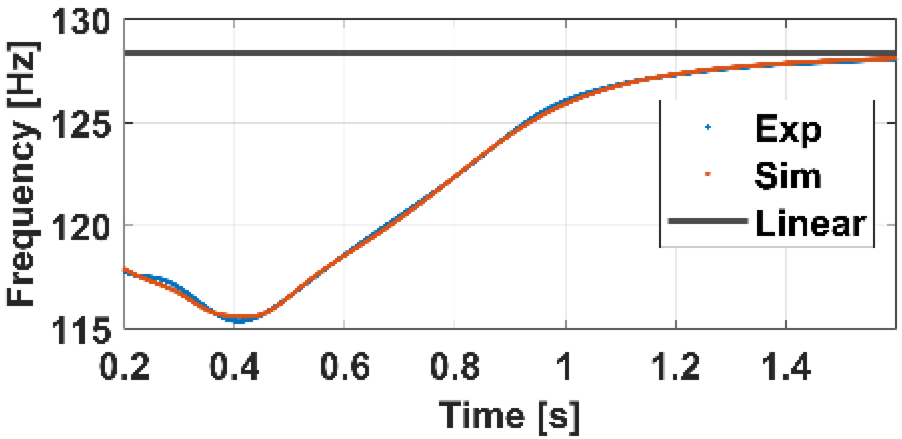


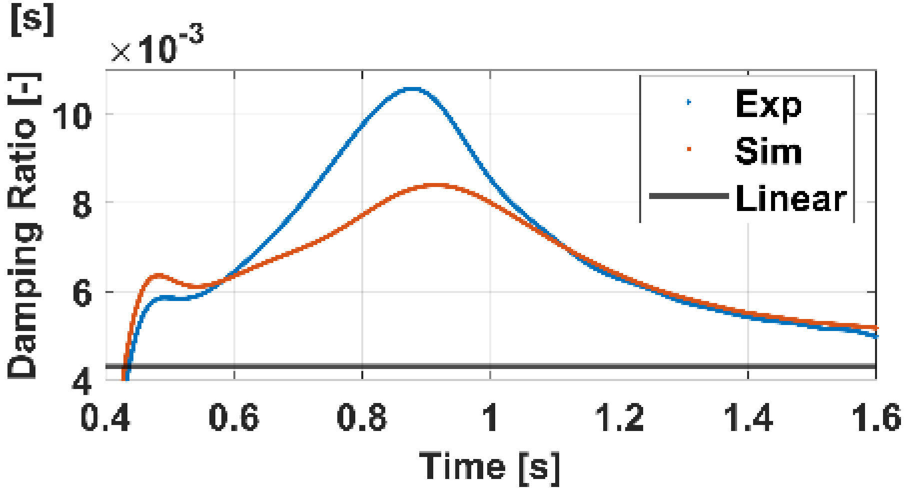
Best Simulated Response for First Mode



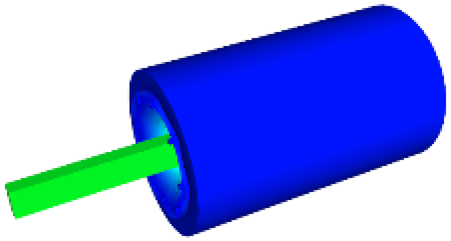
- Frequency is modeled very well, but was unable to achieve correct effective damping



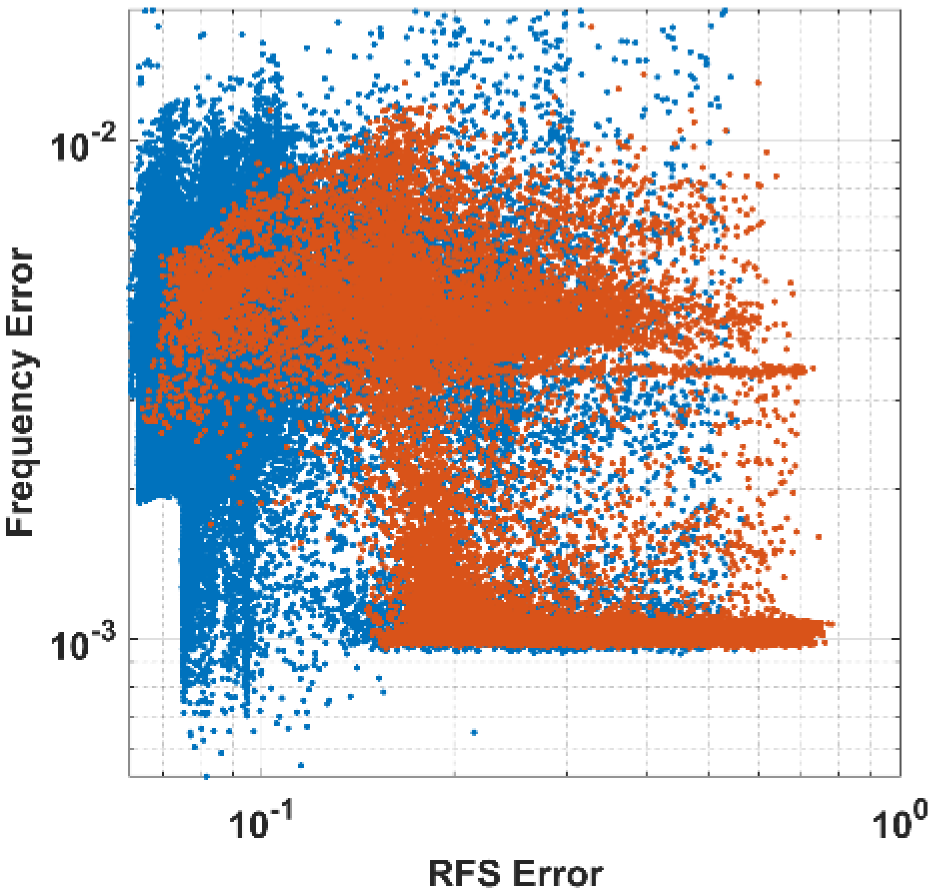


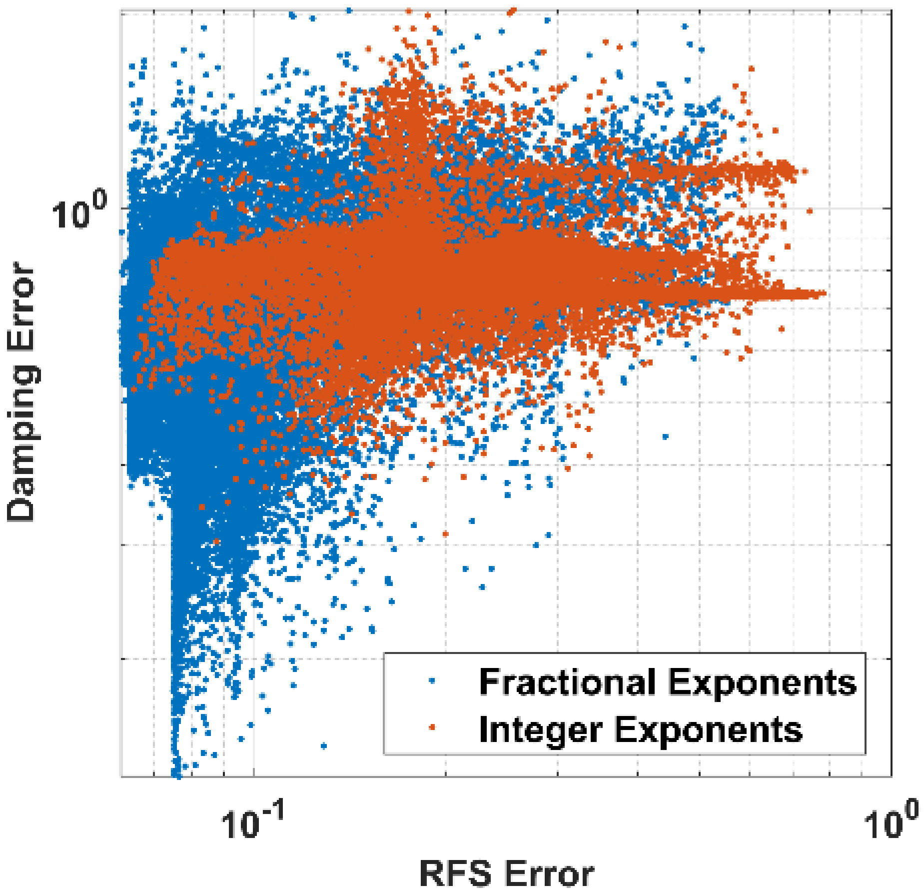


Similar result for 3rd Mode – Plate Drum

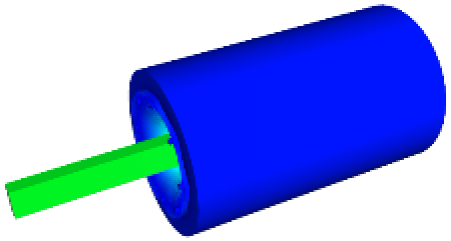


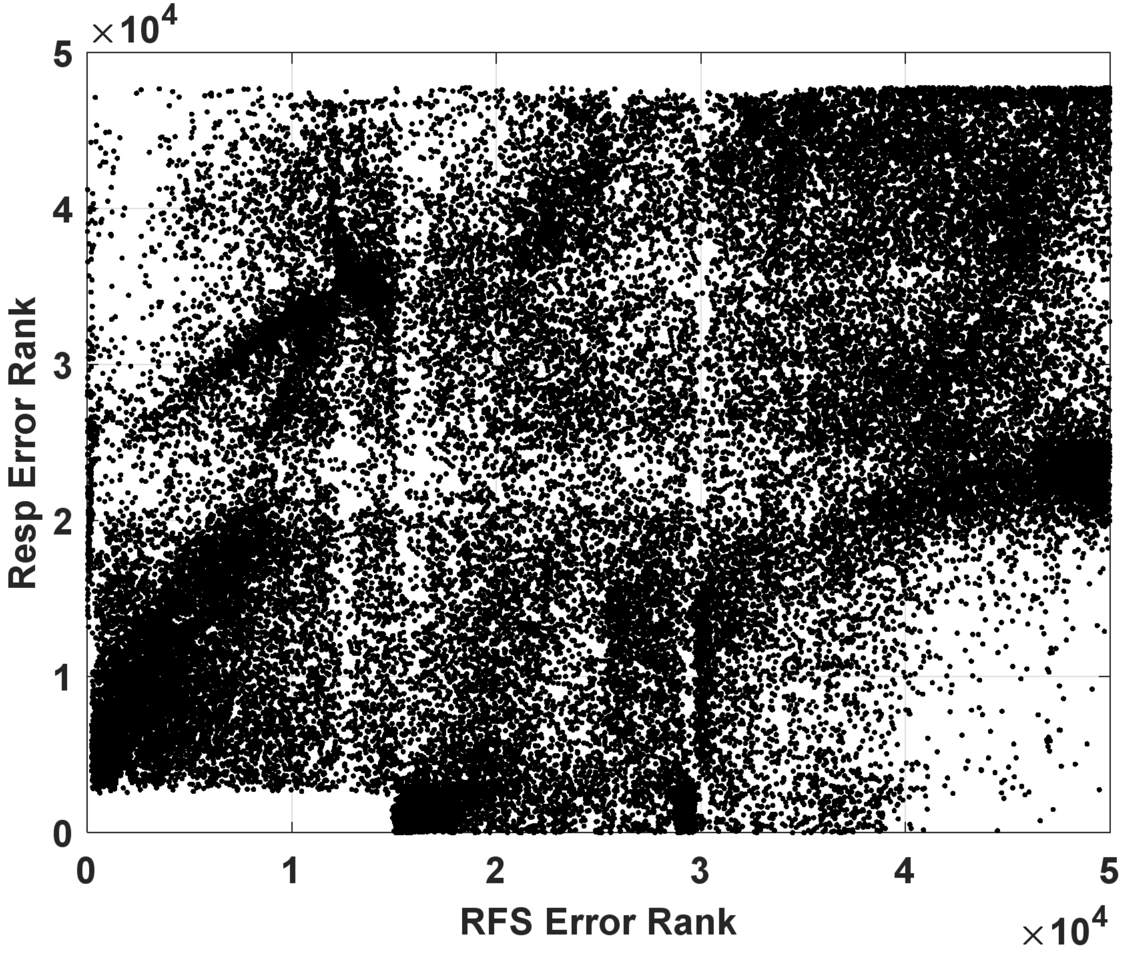
- As before, best simulated response does not occur at minimum restoring force error
- The best response has the same RFS error as the worst models

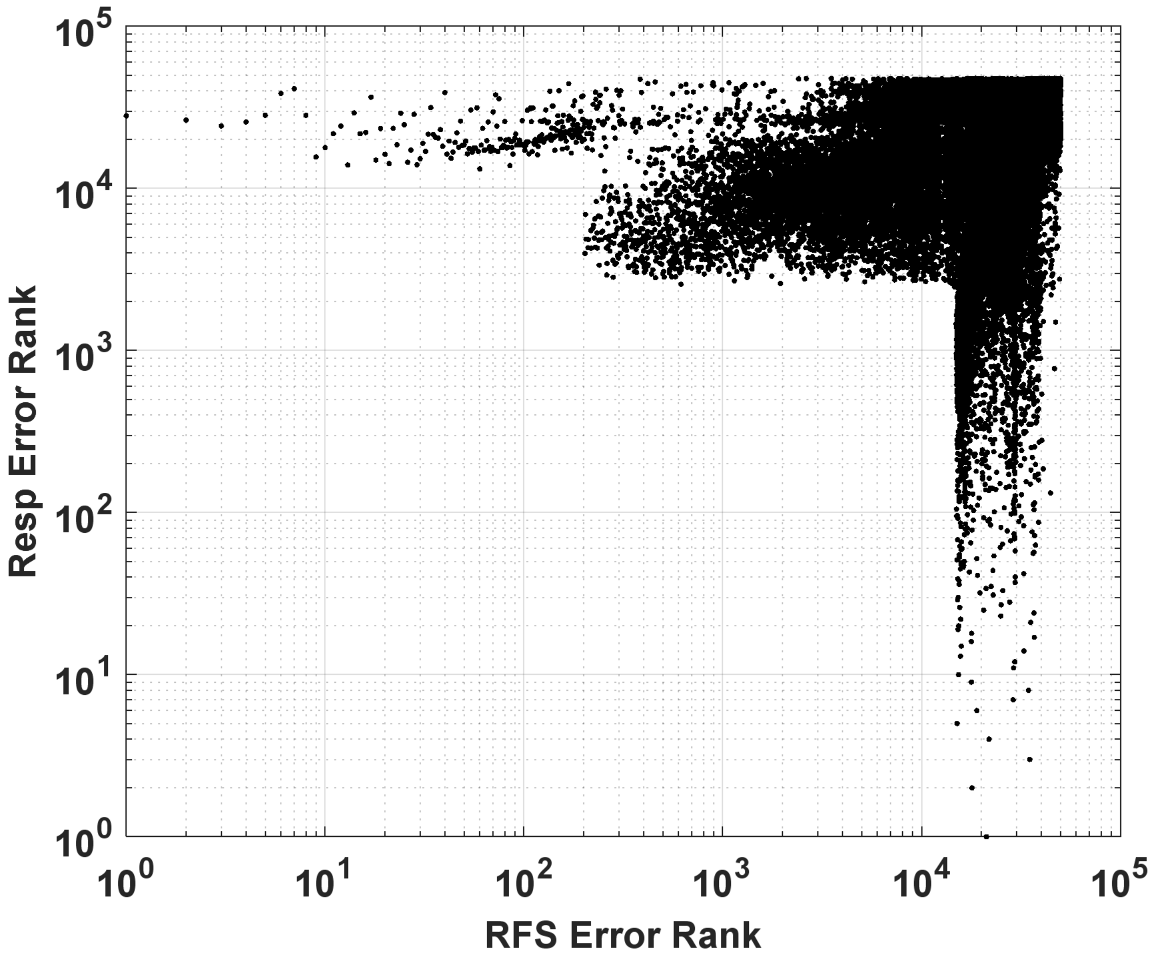




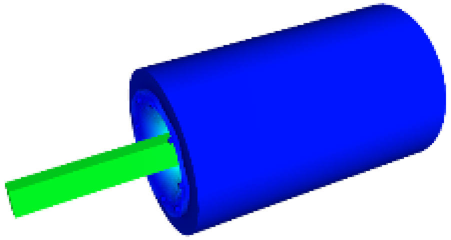
Restoring Force Fit vs Simulated Model



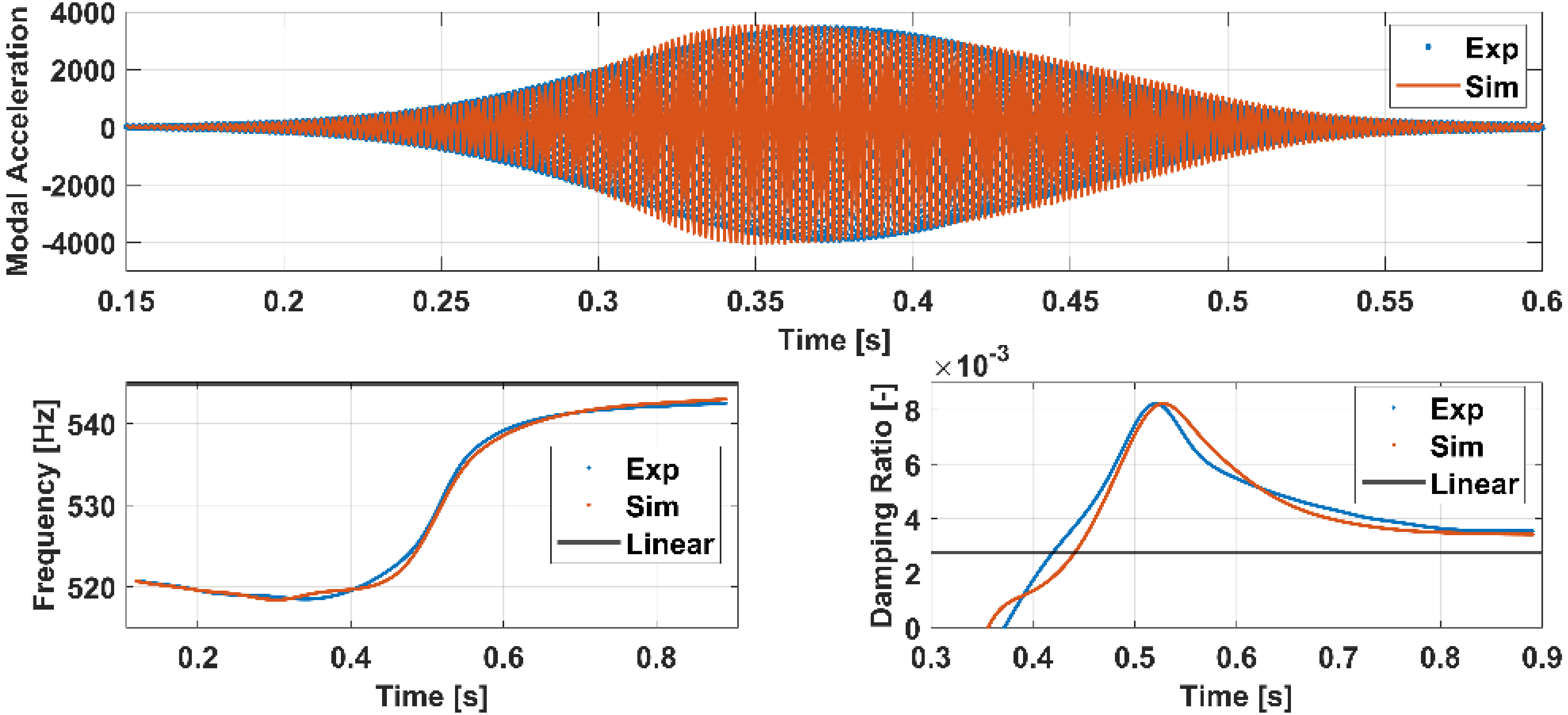




Best Simulated Model for 3rd Mode



- Vaguely matches the measured response, but oscillates around it



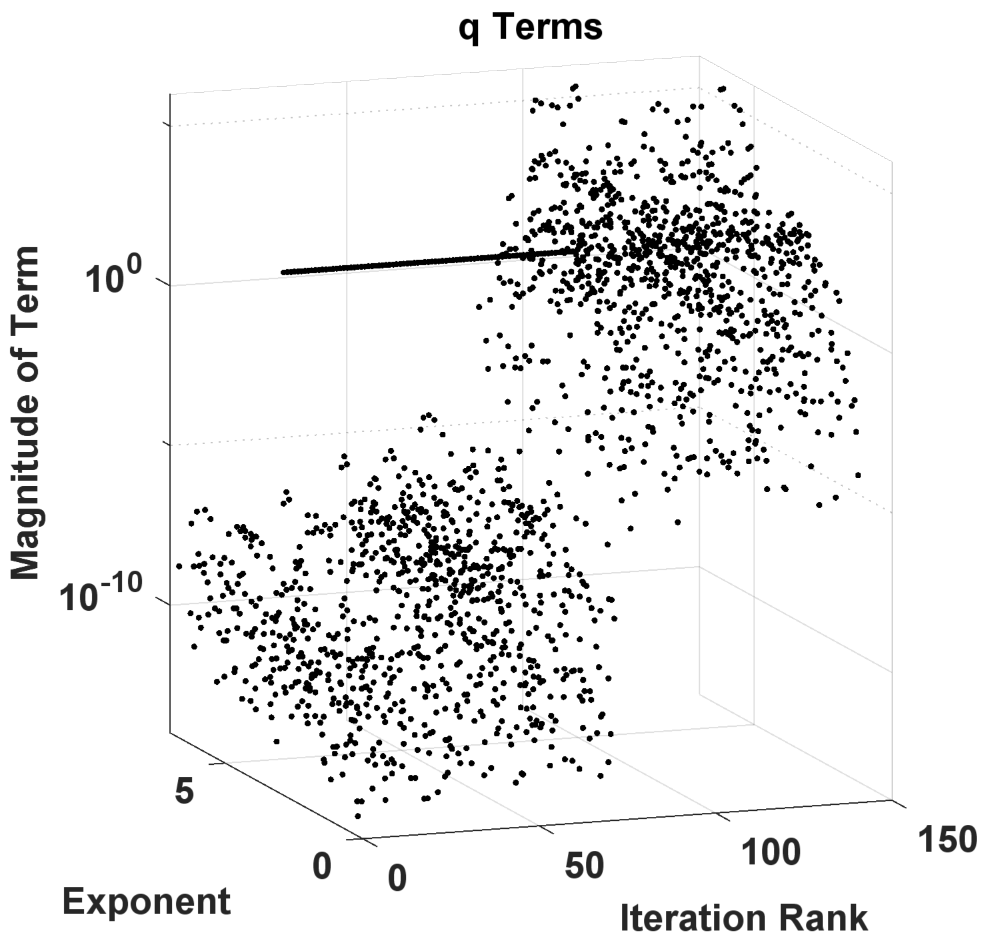
Conclusions / Where's the Modal Coupling?

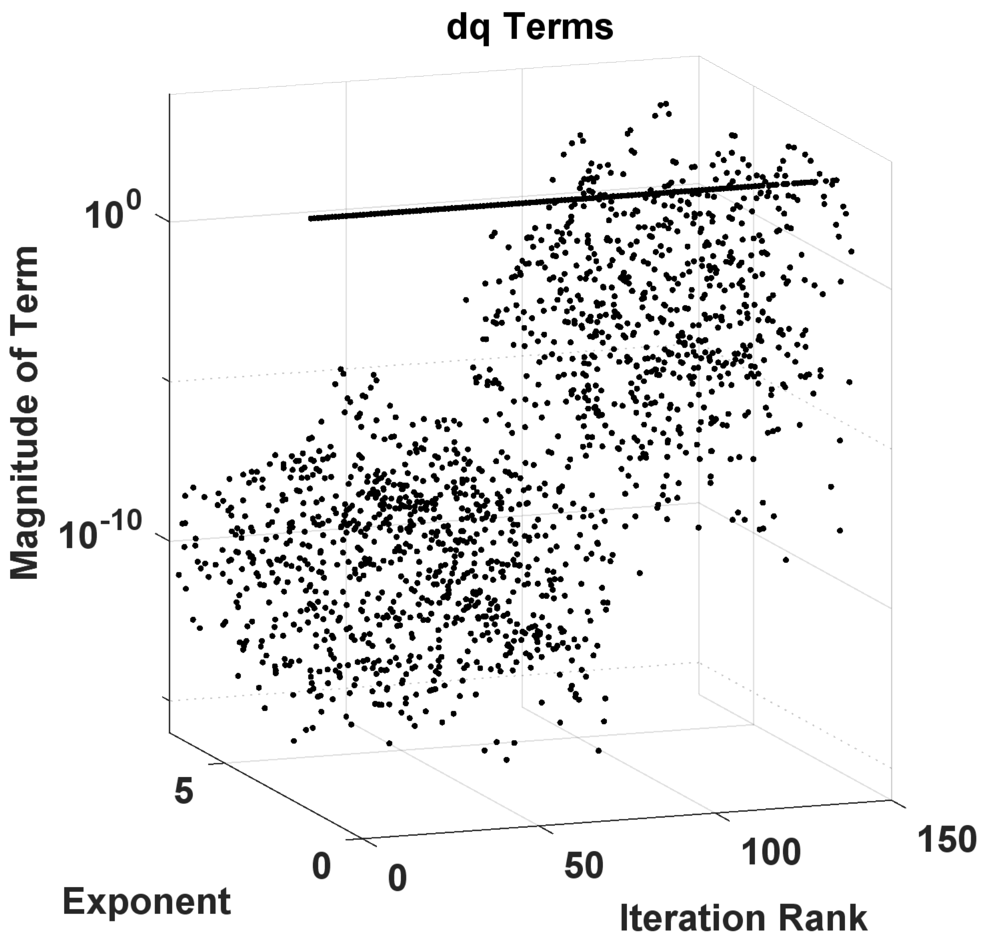
- Least-squares is straightforward but does not give any indication if the proposed nonlinear terms are 'correct'.
  - A model can be formed from basically any possible set of terms and least squares will assign them coefficients, but is the model any good?
- Simulating the response of the model shows that for this experimental data, the best fits to the restoring force do not actually represent the system as well as others.
- Two models with the same RSF error can simulate completely differently, and vice versa.
- No discernable patterns were observed so far in determining what makes a particular model accurate.
  - Some indication is needed in terms of what exponents are required to model a system
  - Running 100,000+ simulations is not a practical method to generate a usable predictive model.
- To model modal coupling, residual from the individual nonlinear mode models is assumed to be modal coupling effects, so any error from the single mode fits will be taken to be coupling.
- Bonus observation: Filtering out low frequency content can negatively impact fit.
  - Typical methods for integrating experimental (noisy) data all cause low frequency drift that is high-passed out

Extra Slides that might be used in presentation

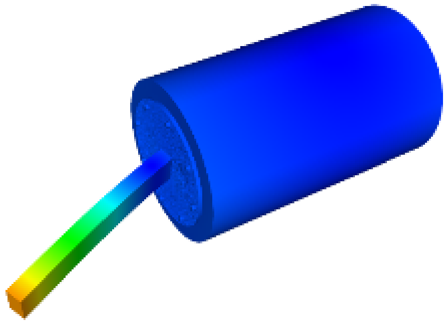
Exact Fit Zeros Extra Terms

- About 80 Monte Carlo models contained the exact terms, x^3 and $\dot{x}^2$. The extra terms are zeroed.
- The remining models consist of other terms that the least squares solution assigns significant values

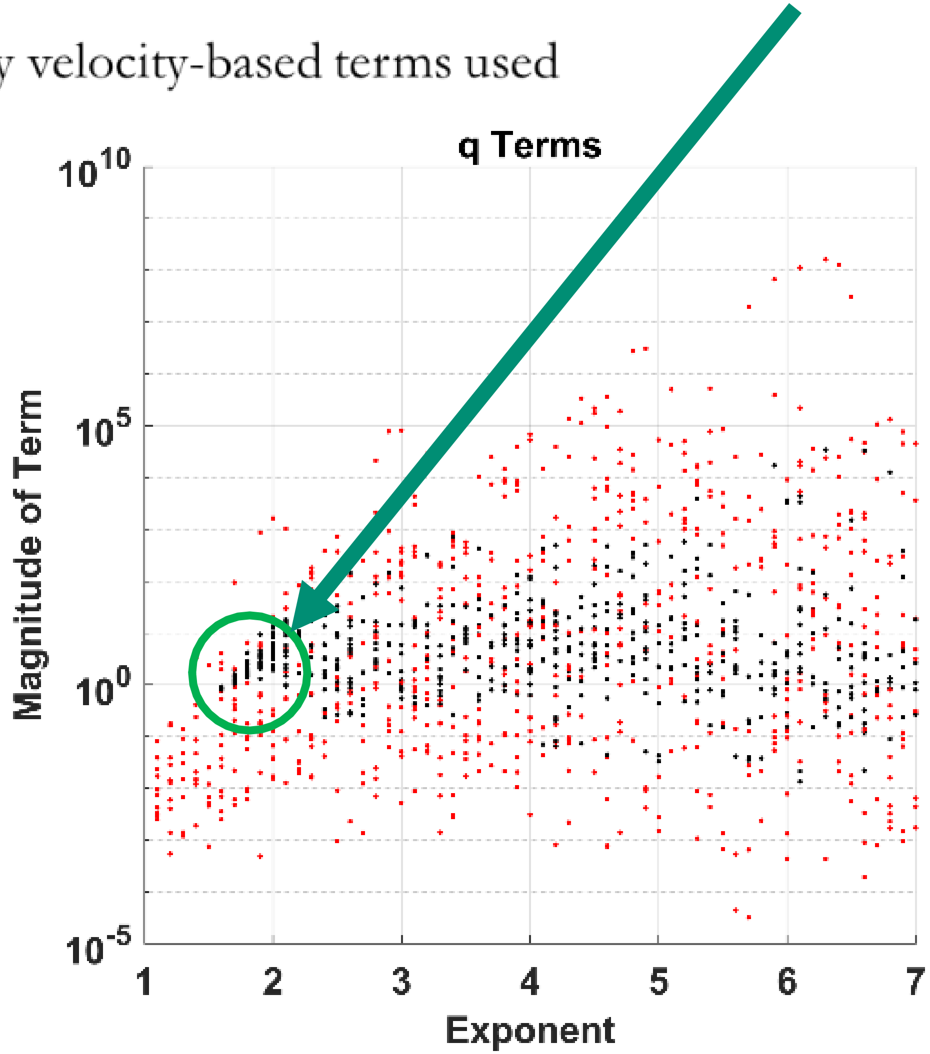


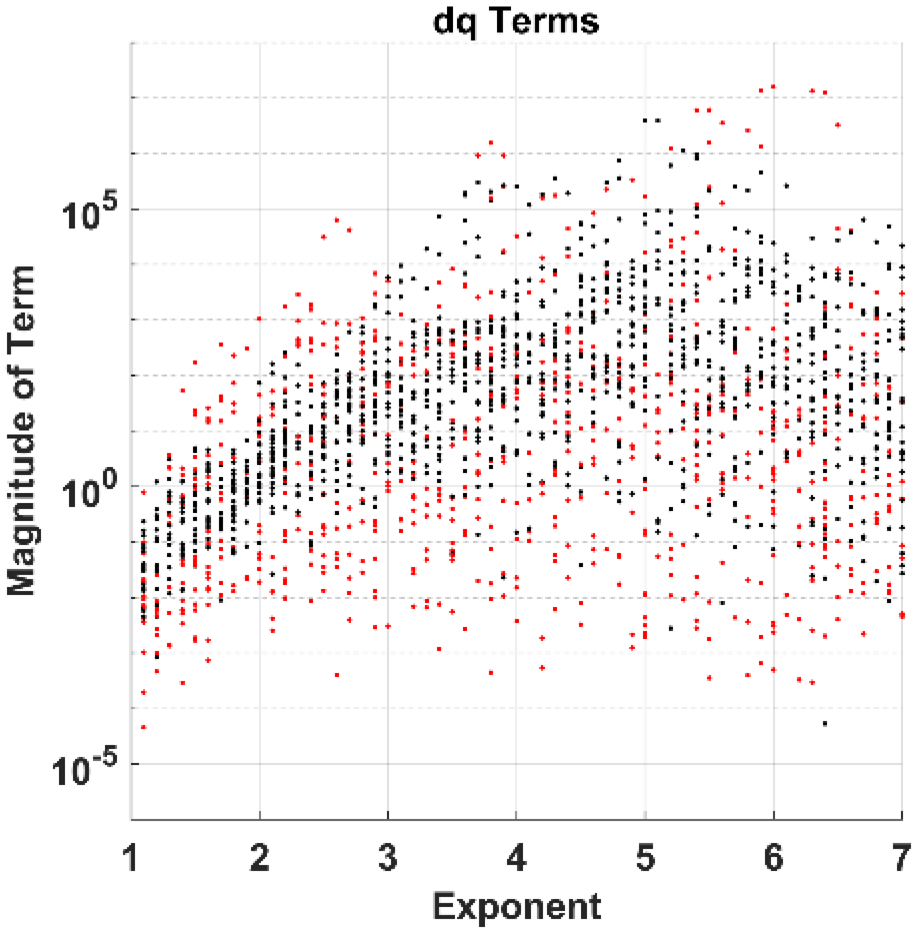


Little Agreement on Important Terms

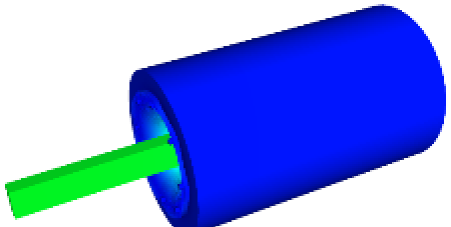


- Best 150 models – Most have $|q|^i \mathrm{sgn}(q)$ terms with exponents near 1.6
- Many velocity-based terms used

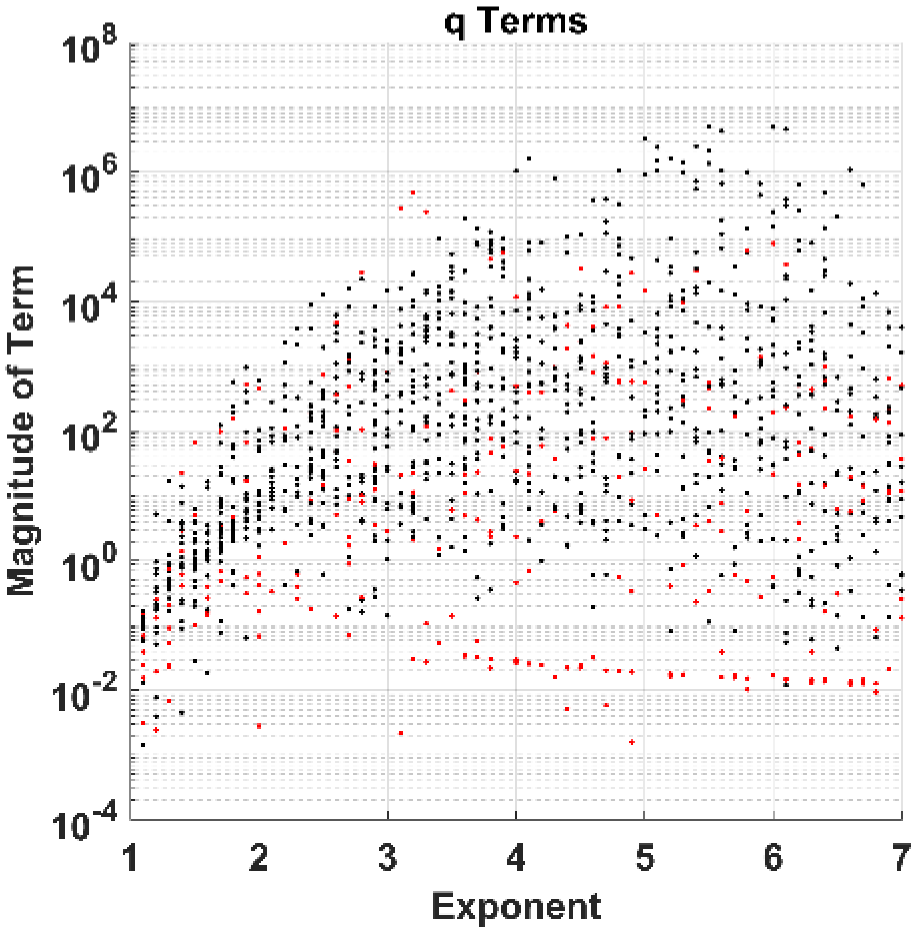


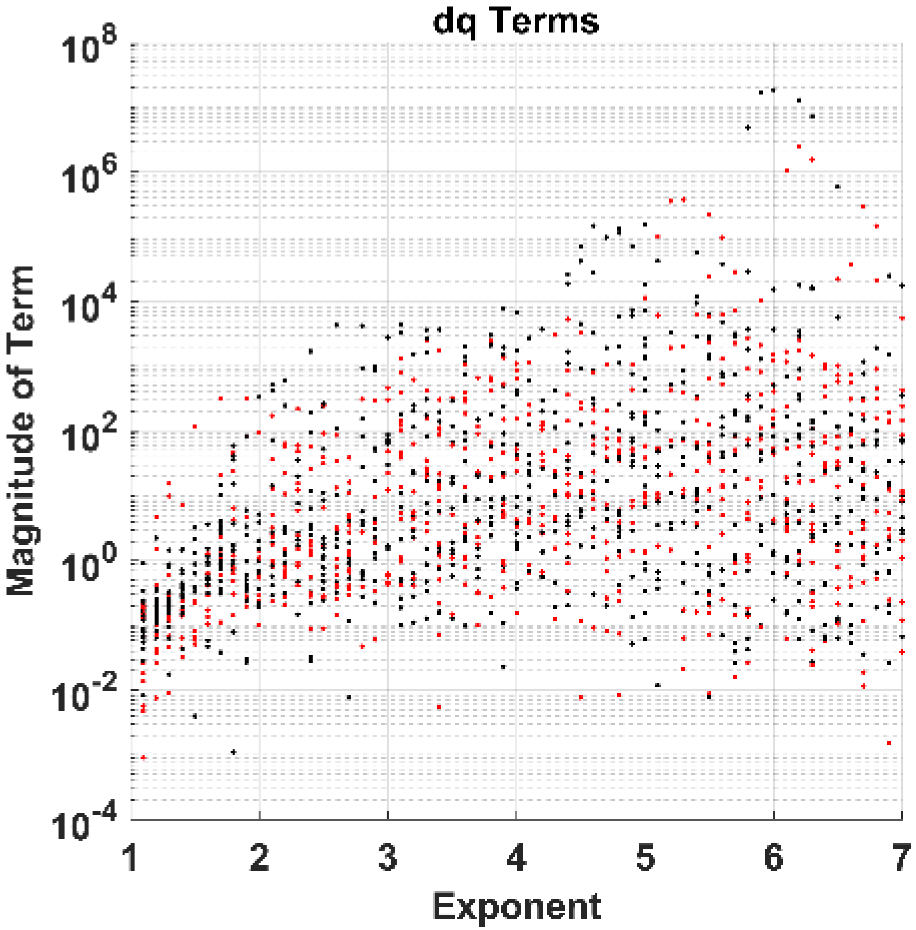


Large spread in Term Magnitude



- Only discernable pattern is that lower order terms have a lower magnitude than higher terms





Terms for Mode 1 Fit

- -177465229.050906 [1,1,1.9,1]
- 5288685276.06863 [1,1,2.4,1]
- -673302860701.045 [1,1,3.3,1]
- 3.12289514840e+17 [1,1,5.7,1]
- 178.655382989733 [1,2,2.0,1]
- -98026.3166282509 [1,2,3.5,1]
- 130764.575820313 [1,2,3.6,1]
- -244852.638032219 [1,2,4.5,1]
- 351128.759543513 [1,2,4.8,1]
- -332716.197208410 [1,2,5.4,1]
- 621958.742628561 [1,2,5.8,1]
- -442121.883231656 [1,2,5.9,1]
- 13686.7580836147 [1,2,6.3,1]
- 0.69803975921237 [1,2,1.3,0]
- -4.99609595305738 [1,2,1.7,0]
- 0.30310822778175 [1,2,5.4,0]

Terms for Mode 3 Fit

- -83366161.7478319 [1,1,1.5,1]
- 1017969766938.89 [1,1,2.8,1]
- -1.23011683368e+16 [1,1,4.2,0]
- 111.372529765949 [1,2,1.3,1]
- -133.975092372418 [1,2,1.9,1]
- 37.5706720619877 [1,2,5.8,1]
- -356.741135062190 [1,2,1.9,0]
- 64.6922246954489 [1,2,3.4,0]
- -12.7263420066765 [1,2,5.7,0]