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A reduced space simultaneous approach for optimization of nonlinear index-1 differential algebraic equations

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A reduced space simultaneous approach for optimization of nonlinear index-1 differential algebraic equations

Intern: Robert Parker, Carnegie Mellon University, **Virtual at:** Pittsburgh, PA
Mentor: Bethany Nicholson, 1464 Discrete Math and Optimization

Abstract Power generation equipment such as boilers, reactors, and adsorbers may be described by systems of nonlinear differential and algebraic equations (DAEs). A common approach for optimization of these systems is to include the fully discretized equations in a nonlinear program (NLP) as equality constraints. These problems can be difficult to converge if algebraic equations are poorly scaled. We propose an NLP formulation for index-1 DAEs in which algebraic equations are removed from the optimization problem and replaced with equivalent implicit functions. These implicit functions admit exact first and second derivatives via the implicit function theorem, allowing the NLP algorithm to maintain its convergence properties. Furthermore, the algebraic subsystem may decompose via block triangularization, improving the speed and reliability of the implicit function evaluation.

Problem Domain

Dynamic optimization

Technical Approach

Implicit functions

Mission Application

Power generation

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Prototype implementation is completed; trials are underway

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A semi-explicit index-1 DAE has the form:

$$\begin{aligned} \dot{x} &= f(x, y, u) \\ 0 &= g(x, y, u), \ \det(\nabla_y g) \neq 0 \end{aligned} \tag{1}$$

Our implicit function implementation handles systems:

$$\begin{aligned} F(z, y) &= 0 \\ G(z, y) &= 0, \ \dim(G) = \dim(y) \end{aligned} \tag{2}$$

First and second derivatives are via the implicit function theorem:

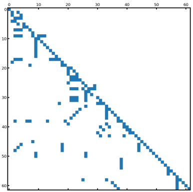
$$\nabla_z y = -\nabla_y G^{-1} \nabla_z G$$

$$\nabla^2_{zz} y = -\nabla_y G^{-1} \left(\nabla^2_{zz} G + \left(\nabla^2_{zy} G^T \nabla_z y + \nabla_z y^T \nabla^2_{zy} G \right) + \nabla_z y^T \nabla^2_{yy} G \nabla_z y \right) \tag{3}$$

Application: Chemical looping combustion (CLC) reactor
Algebraic equations are:

- Highly nonlinear
- Highly decomposable

Figure: Algebraic Jacobian, $\nabla_y g$, of CLC reactor model



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