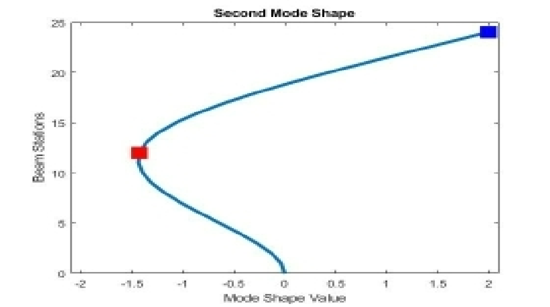
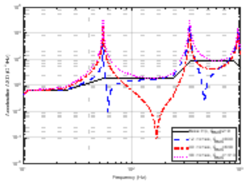
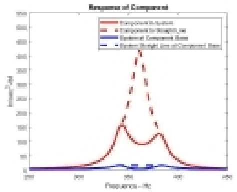
SAND2021-9588C

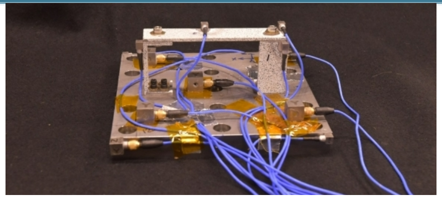
Sandia National Laboratories

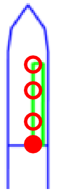
How Modal Information Can Improve Confidence in Vibration Qualification











Troy Skousen – Sandia National Laboratories
Randy Mayes – Consultant

Tutorial for the 91st Shock and Vibration Symposium
September 19-23, 2021 in Orlando, Florida

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

Outline

Motivation

Analytical Example

Traditional Methods

Basics of Modal Analysis

Modal Approach to Environments Testing

Laboratory Simulation of Component Responses

Real World Examples

Accounting for Unit-to-Unit Variability

Motivation

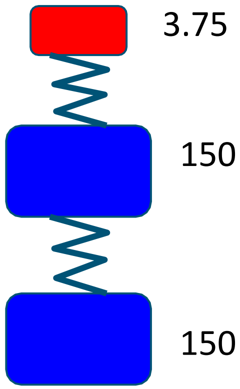
Motivation for Understanding Modal Response for Vibration Testing

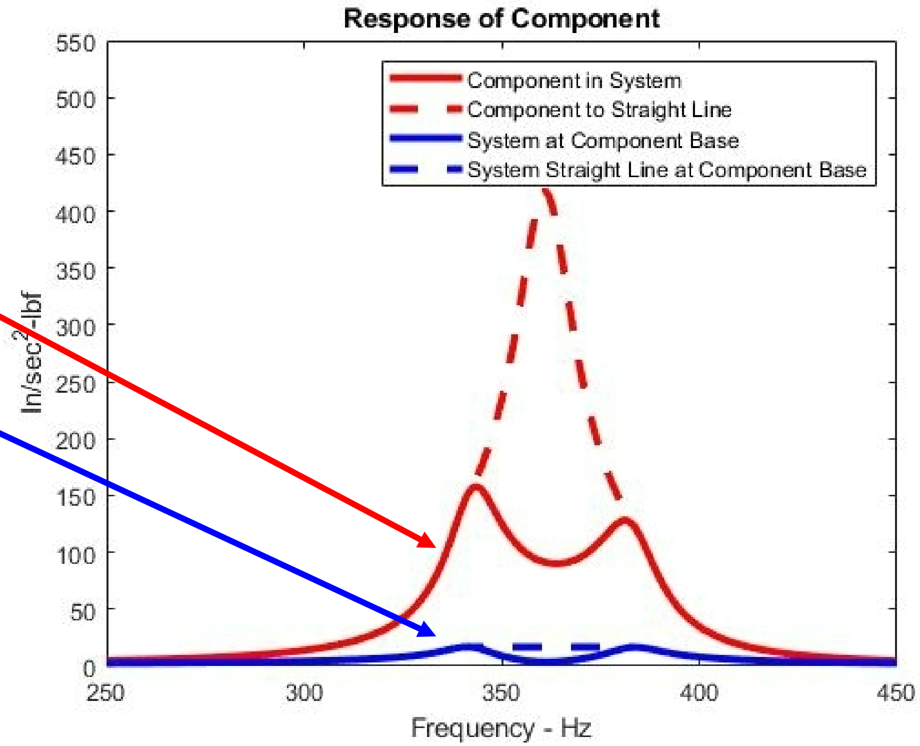
- Vibration qualification testing is designed to make sure the component (or payload) will survive the field environment.
- Even if there is a field triax near the base of the component, large uncertainties exist because:
  1. Rotations of at the component base are NEGLECTED and can ADD OR SUBTRACT large response to the component
  2. Enveloping of measurements destroys the required control notching needed at laboratory resonances
  3. Environmental specs should be modified to account for the transformation from field to lab boundary conditions
- We unnecessarily break parts and force re-design due to these large uncertainties
- Component responses in laboratory testing are usually not quantified.
  - In this course we demonstrate that a few laboratory modal quantities provide the key to mapping complex known field response to quantified laboratory component response

Enveloping component base response destroys required control notching – a 3 DOF Demonstration

- A 1 DOF base mounted component (red) with the same natural frequency as a massive 2 DOF system (blue) is mounted to the system
  - This is the definition of a vibration absorber
- A straight line envelope test spec is made for the base of the component from system response between resonances
- Compare the lab response to the field response of the component

- Large uncertainties exist
- Excessiveness of response is unquantified
- Excessive response break parts and force re-design





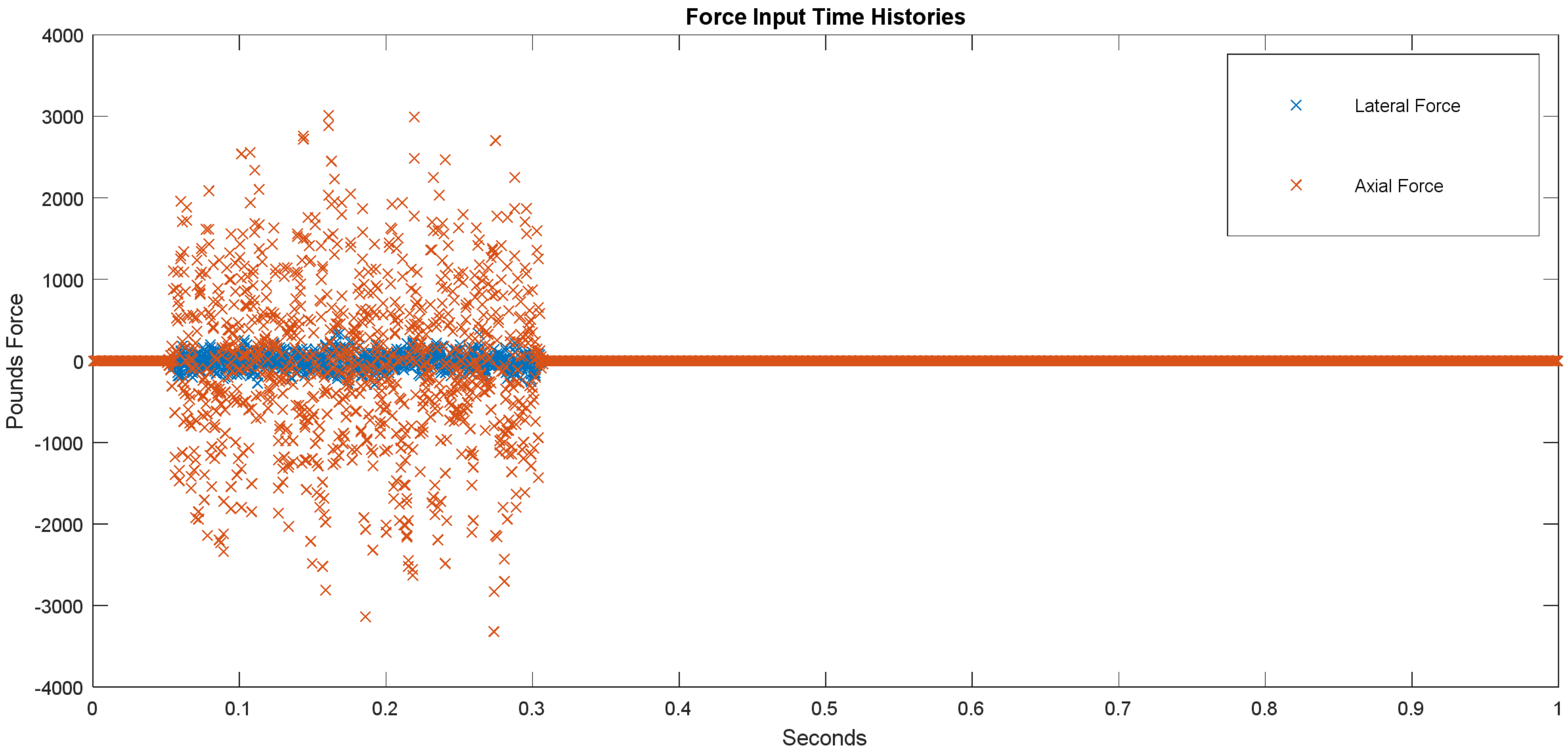
Analytical Example

Analytical Example

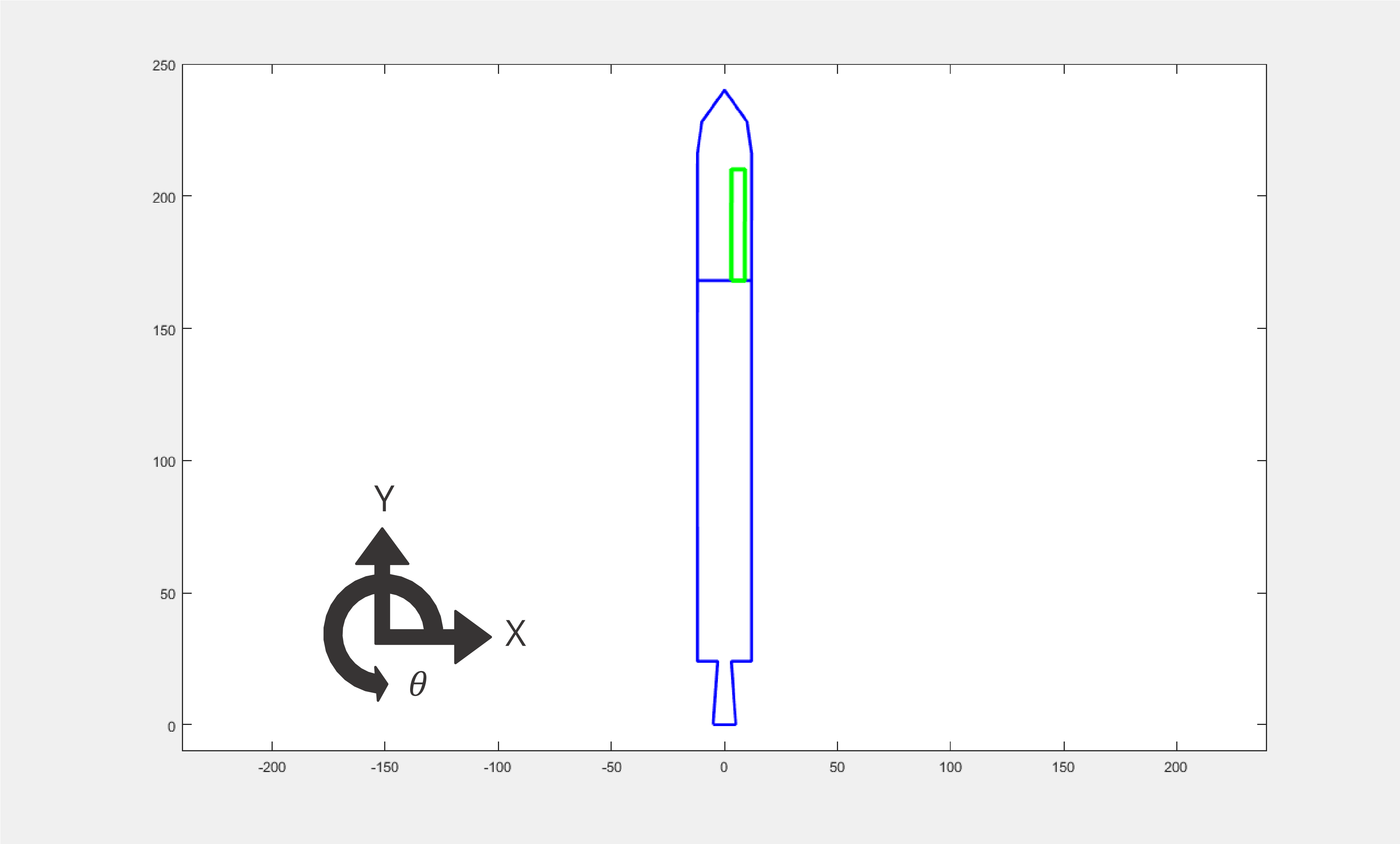
- We utilize a model of a 20-foot-long rocket with a base mounted component.
- The nozzle of the rocket is forced with 1000 lbf rms random input in the axial and 100 lbf rms in the lateral direction up to about 1000 Hz.
- FE beam models are utilized in the 2-dimensional response.
- Three rigid body modes
- First bending mode is 21 Hz
- We animate the acceleration response to 2000 Hz.

Force input

- Time Histories



Random acceleration response to 2000 Hz due to nozzle force



Traditional Methods

Traditional Single DOF Test Specification and Methods

Traditional 1DOF shaker test specifications

- Manually enter test specification breakpoints so, limited number of points
- Broad plateaus to allow for test article modes to shift due to unit to unit variability.
- Basis is typically a few or 1 component input locations in the assembly. Will use 1 in these examples

Types of test specifications discussed here

- Base input
- Response limited base input
- Least squares base input to match responses

Source Data

Acceleration response time history data along the left side of the component

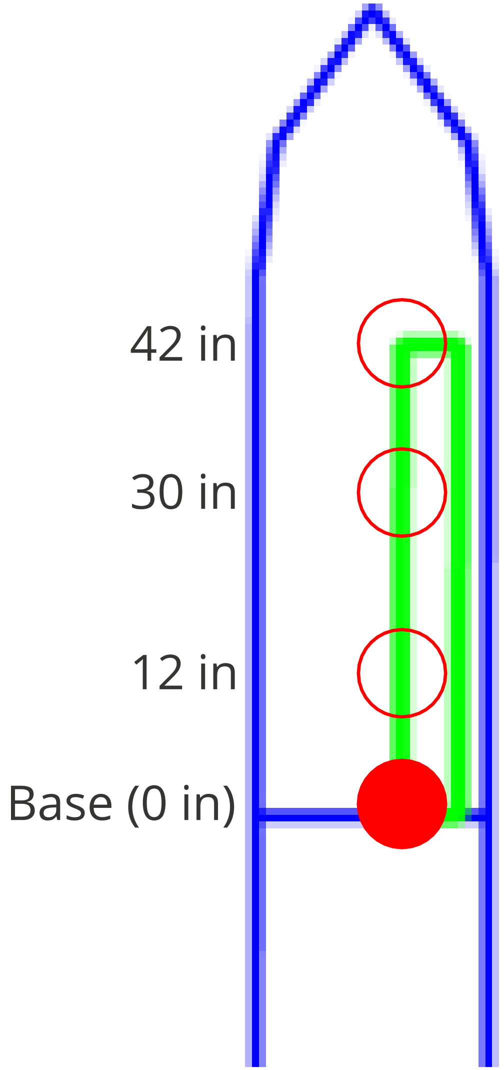
Acceleration auto-spectral densities calculated

Considered from 10 to 1000 Hz

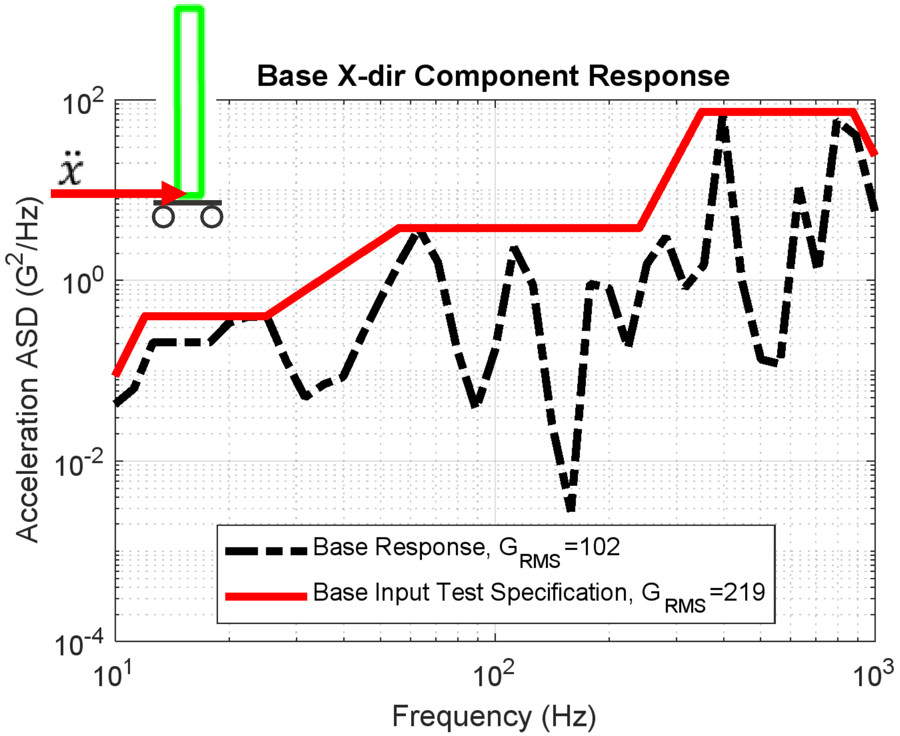
- The x-direction data drops off above 1000 Hz

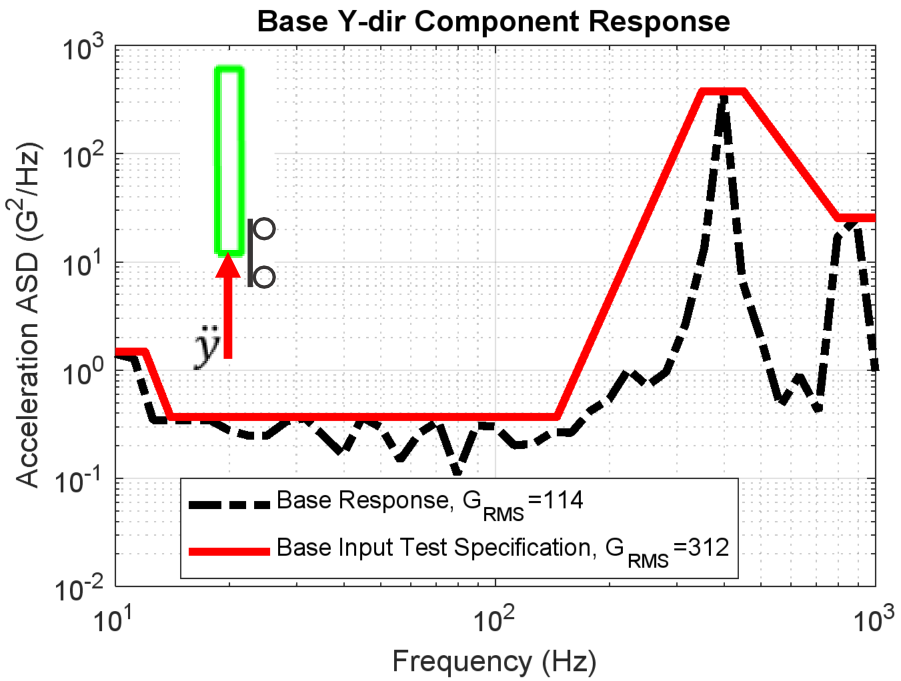
Base location is taken as the source data for the test specification

Transmissibilities in the test fixture



Traditional Single Axis Test Specifications





Acceleration time histories provided at 0, 12, 30, & 42” up the side of the component

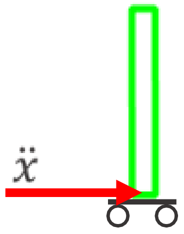
1/6th octave ASDs generated from the data

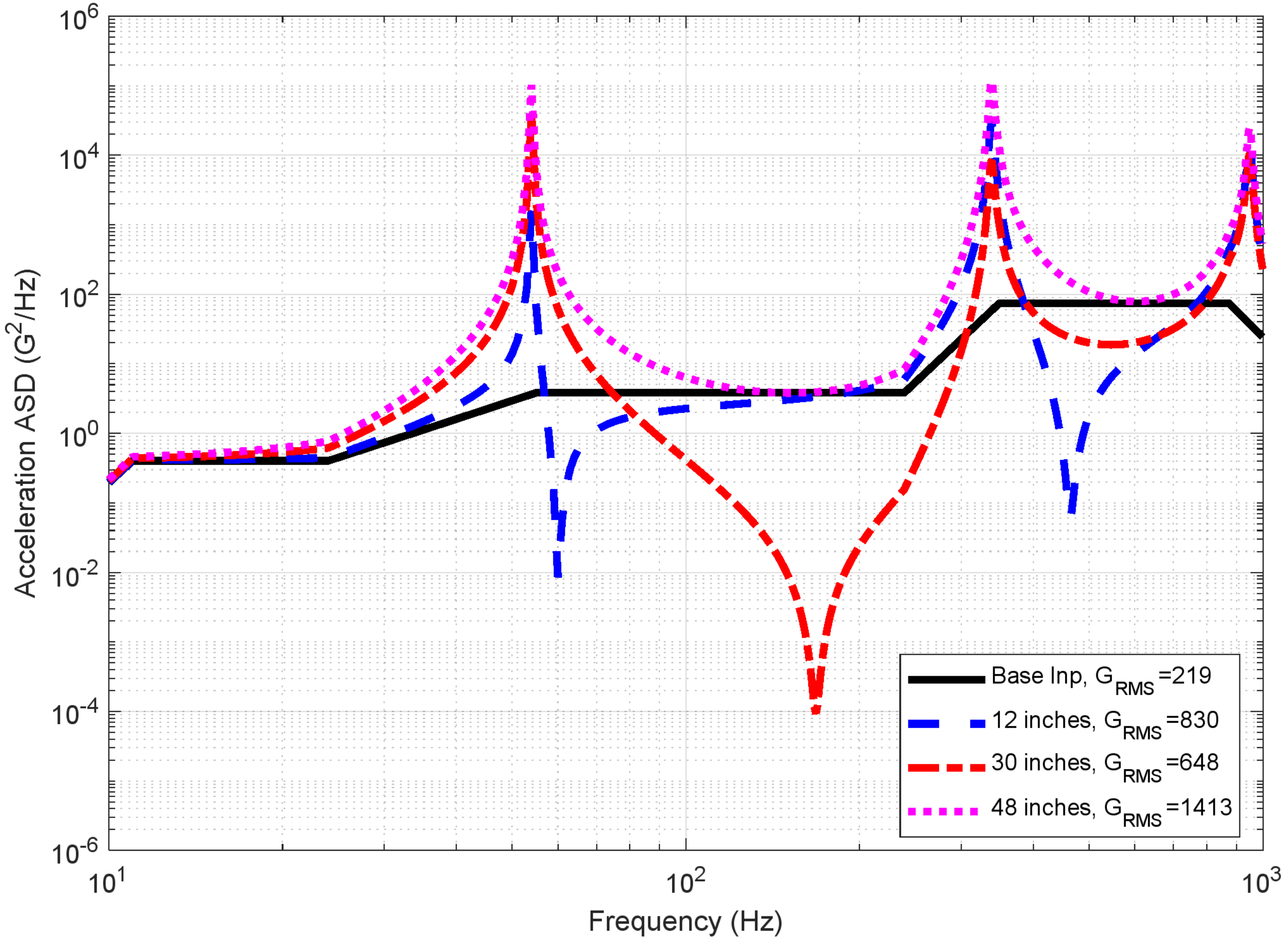
Use base (0”) in the X- and Y-directions. Ignore rotations. Ignore correlation between DOFs

Draw straight-line test specification over 1/6th octave data

Apply to base of component as the input, in turn

Responses to X-Direction Input

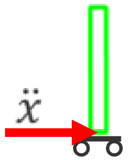


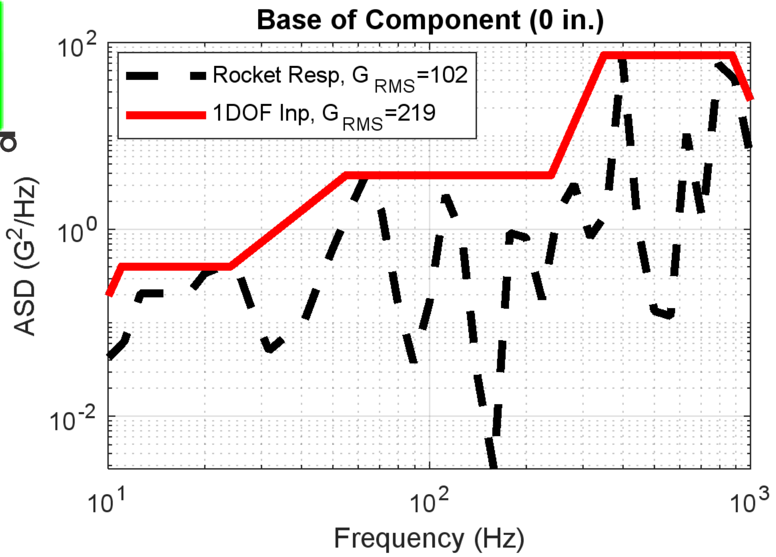


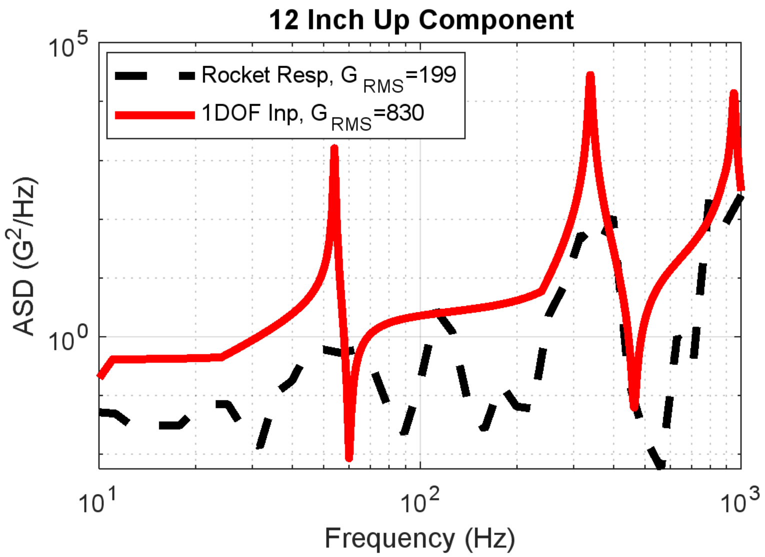
High responses at 56, 330, & 950 Hz

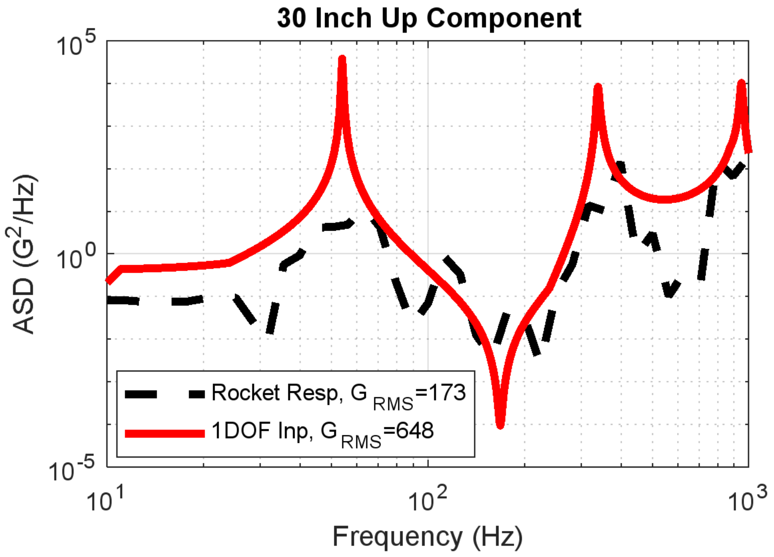
Nothing interesting going on in the y-direction, so we will focus on the x-direction

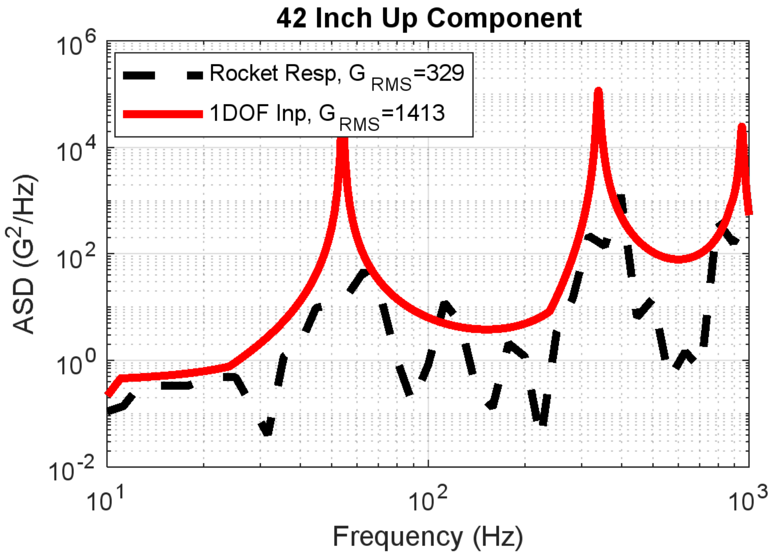
X-Direction 1DOF Input Test Response Compared to Rocket Response











Note the differences in the RMS

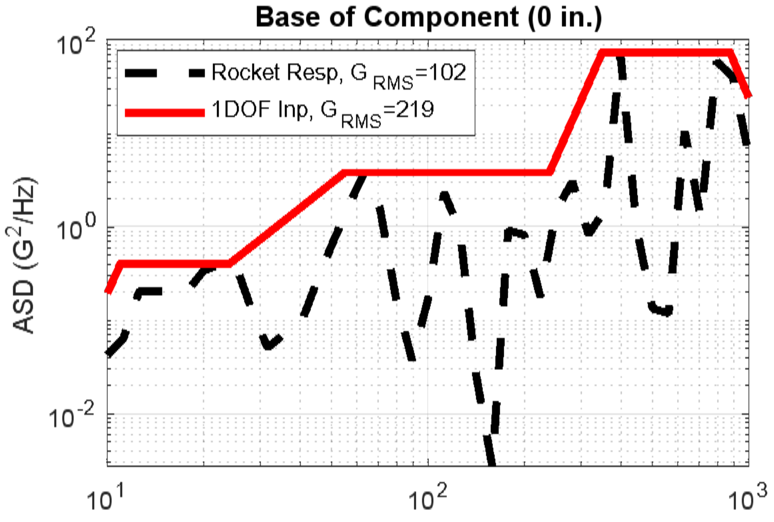
Notched

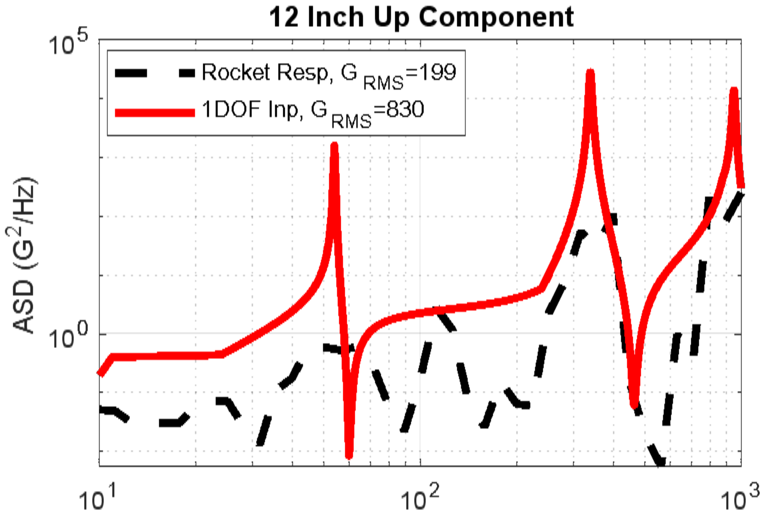
Least Squares

Fixed Base Modes

Take Away from Single DOF Testing

- Test specification is generally a conservative, coarse straight line envelope of the reference data
  - Filling in valleys in input the data
  - Wider peaks than in the data
- Very high responses relative to desired responses
  - Might be more response than the design can and should be subjected to

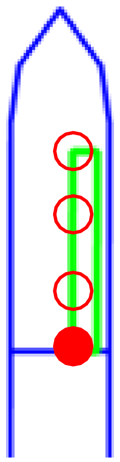




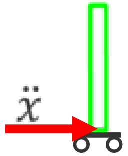
Response Limited Single DOF Testing

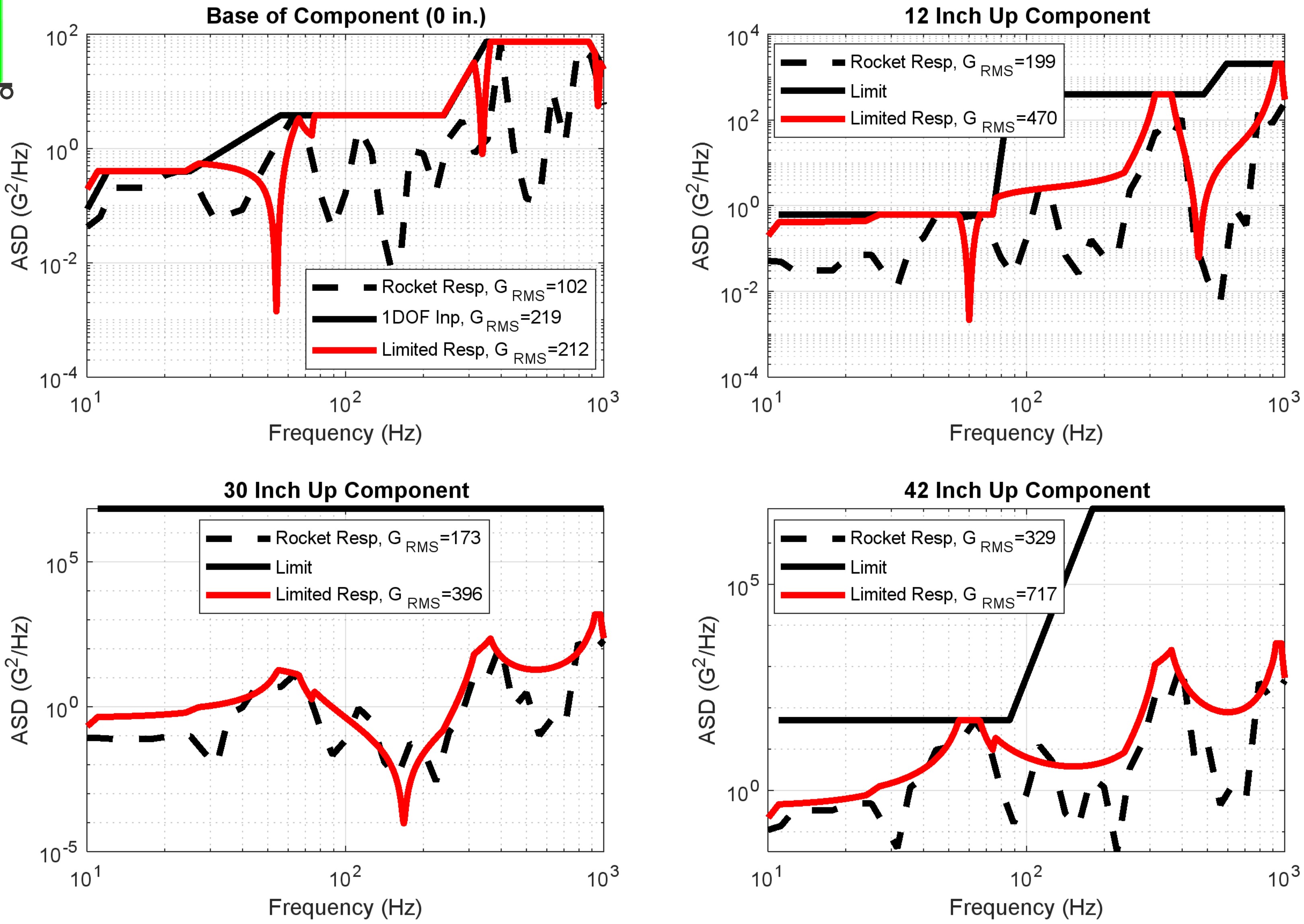
How does it work

- Compare the test response to measured or analysis based response in the next assembly for the same loading condition
- Response profile is determined as do not exceed responses.
- Shaker controls system only engages at frequency values where the test inputs cause the response to exceed the established limit
- If a response limit is exceeded, the control system reduces the input until the response matches the limit.



X-Dir Notched Inputs with Response Limits





Note the differences in the RMS

[No Notch](#)

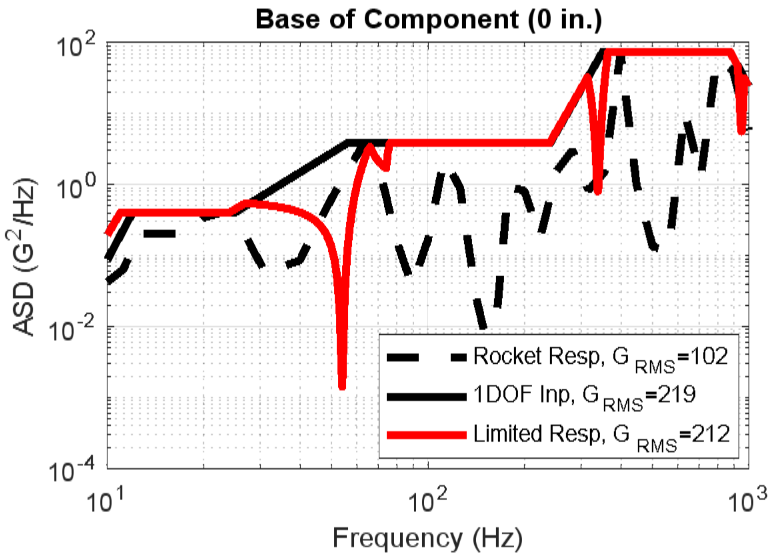
Take Away from Response Limited Single DOF Testing

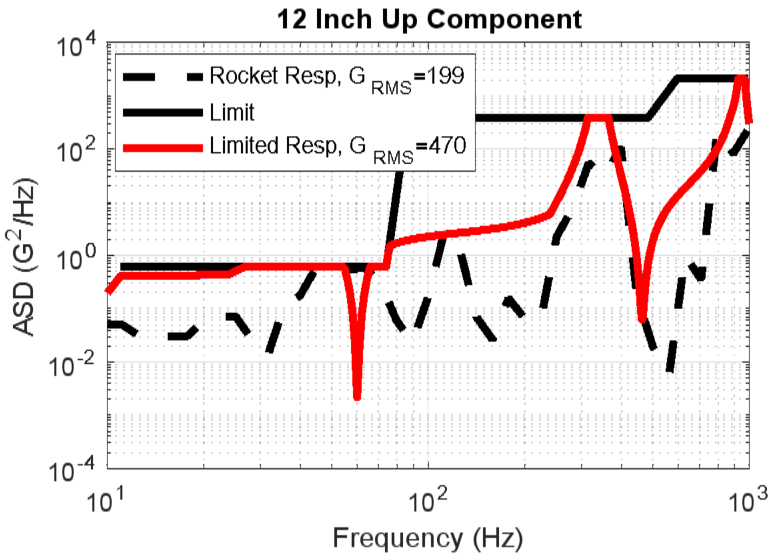
Can prevent over testing condition

Only works if you can measure the response at the location of interest in the test

Need to know the appropriate levels to limit the response to

Responses still don't match very well





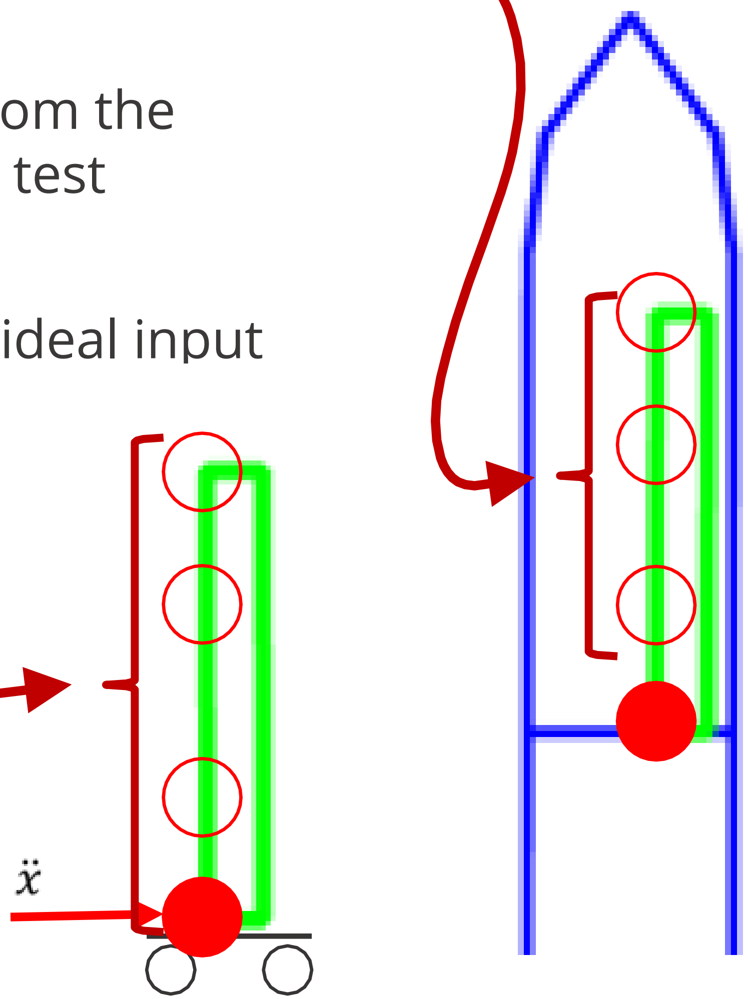
Least Squares Input Spectrum from the measured response locations

Use the responses at several locations as the basis for developing the input

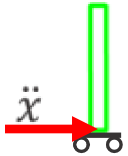
Measure the spectral density ratios from the responses to the input location in the test configuration

Use the ratios to determine what the ideal input spectral density would be to obtain each response

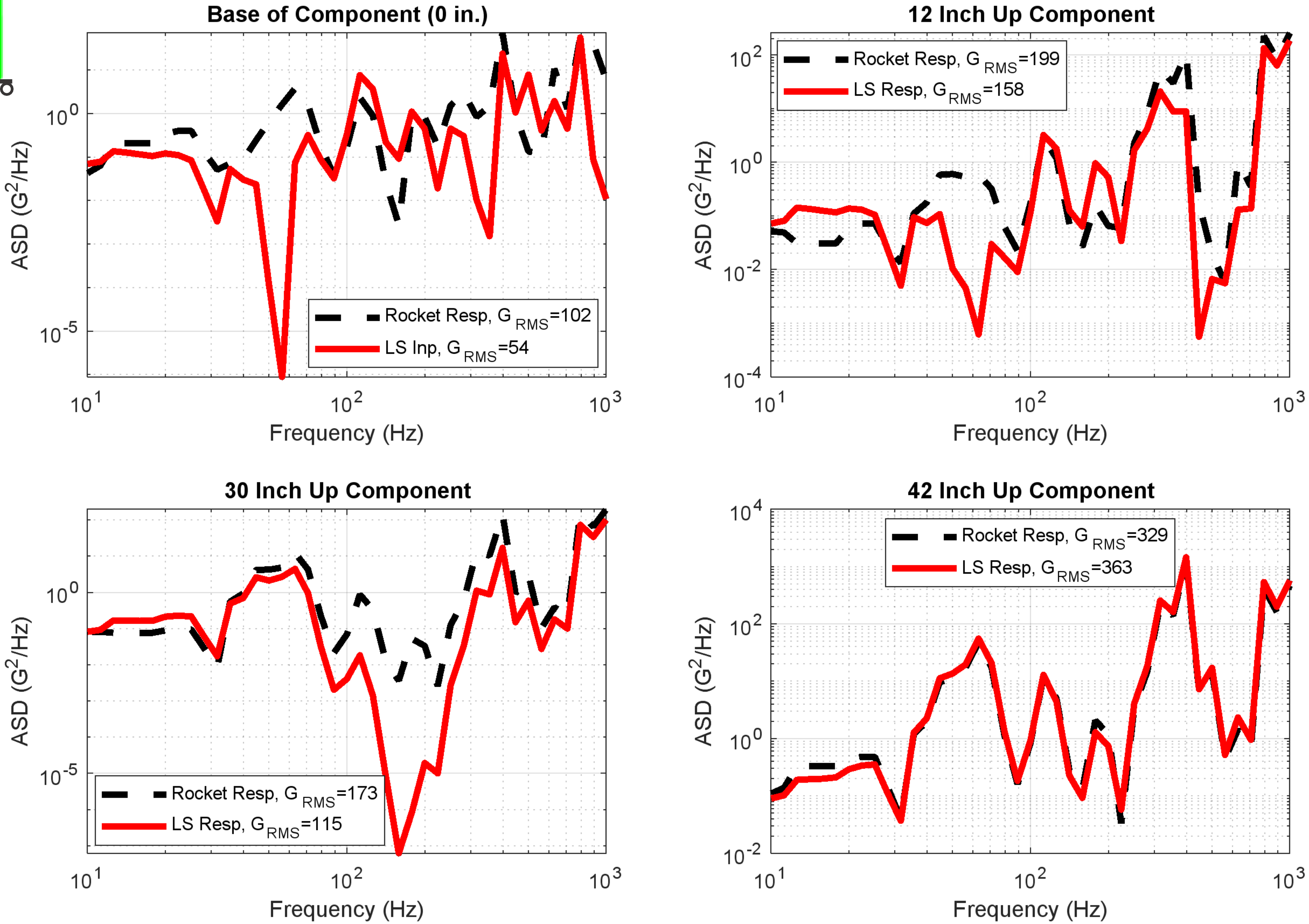
Use the least square of the multiple inputs to determine the final input



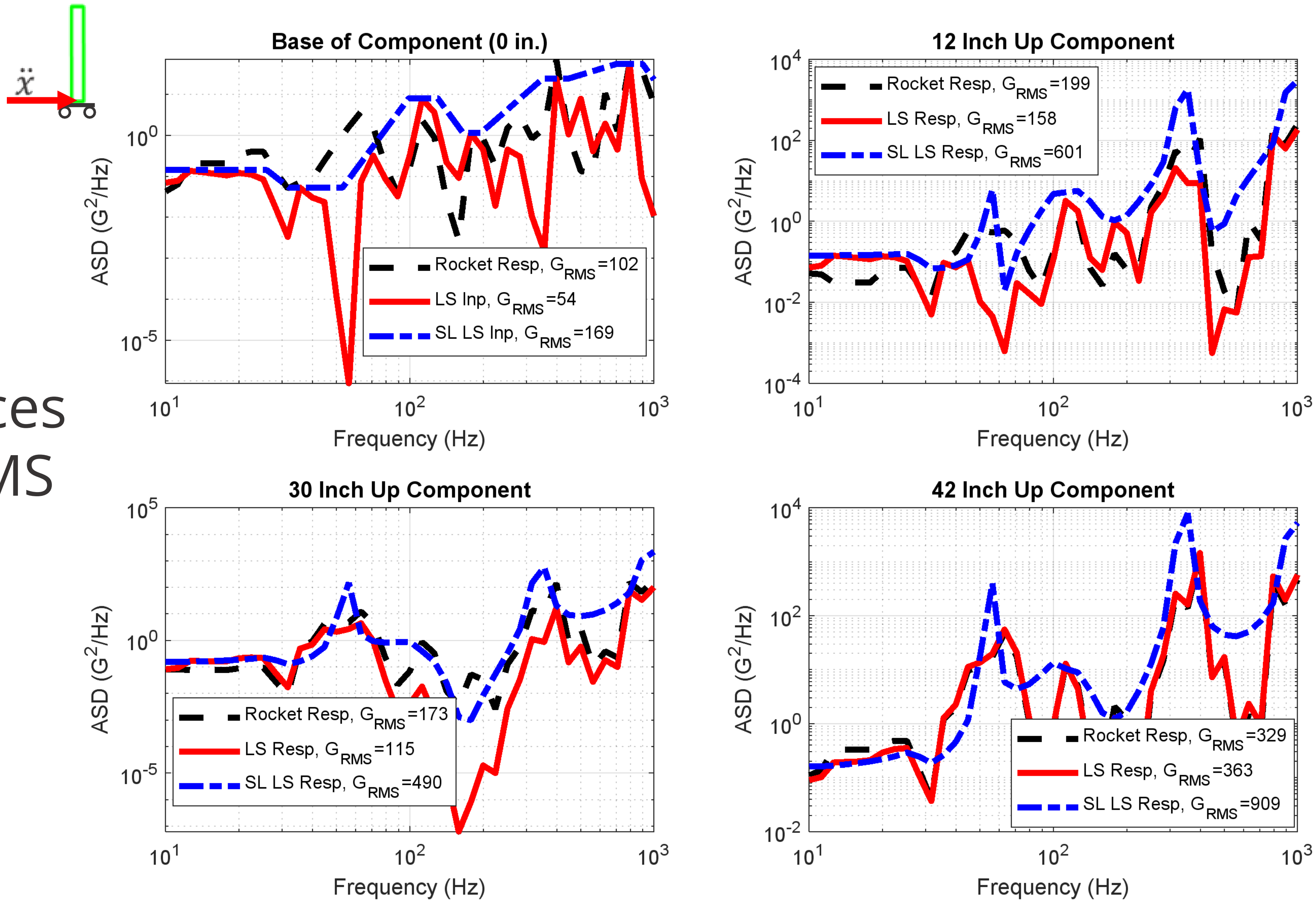
X-Dir Least Squares Input



Note the differences in the RMS



X-Dir Least Squares Straight Line Test Specification Responses



Note the differences in the RMS

No Notch

Take Away from Input Determined from Least Squares Fit to Match Test Article Responses

Can help match responses better

Input likely will not match reference data at the input location as well

Developing straight line specification can take away some of the ability to match responses

Can’t get all of the responses exactly right.

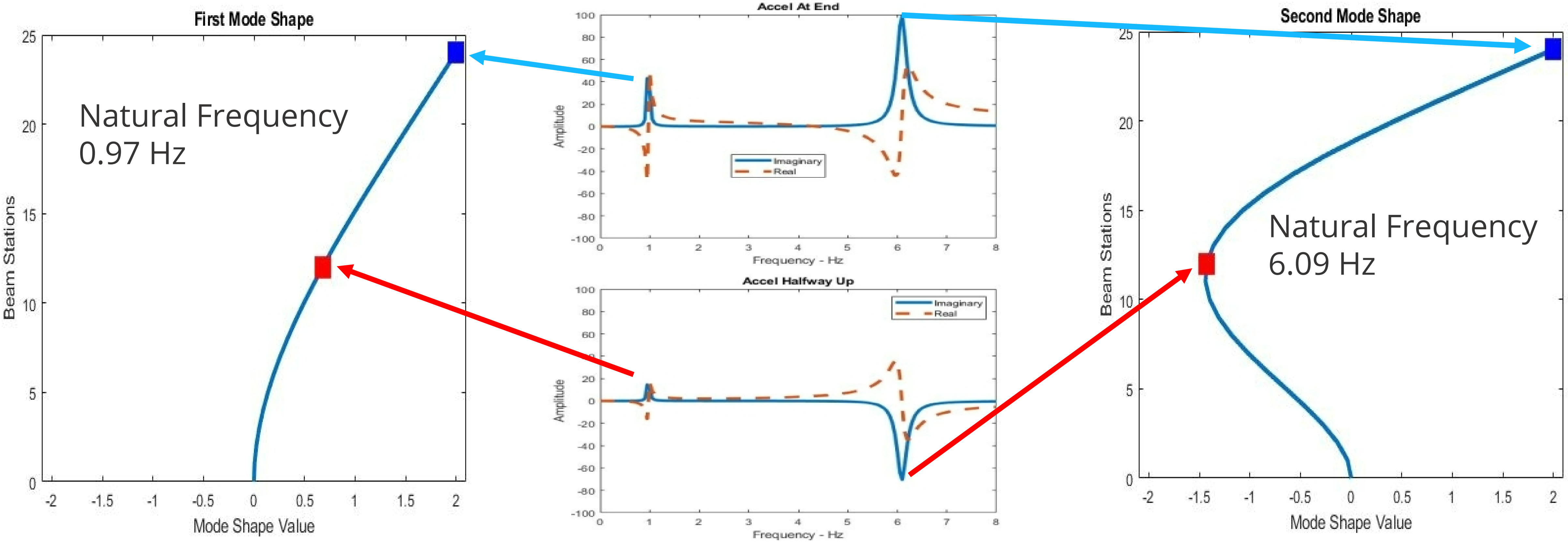
Our Perceived Shortcomings of Traditional Laboratory Testing

- Boundary Condition Discrepancies
  - Single axis testing constrains 5 of the 6 DOF introducing large forces
  - Differences in impedance shifts natural frequencies between in-service and laboratory
- Input Specifications
  - Straight line envelopes of input field measurements remove naturally occurring input notches at the resonant modes of the test article due to vibration absorber phenomenon. At these frequencies, small inputs cause huge responses
  - Single axis testing ignores rotational DOF which is a part of 75% of the transmissibilities from the in-service rigid body inputs to test article responses.
- Test Responses
  - Not easy to match responses and very easy to generate responses that are much too high
  - Methods like response limiting or least square inputs can help get the responses closer to desired levels, but still doesn’t provide an input that matches the sought after global response
  - Unquantifiable assumption of margin on the outputs (sometimes we know the input margins)
  - No indication of how well the appropriate damage mechanism is being engaged

Basics of Modal Analysis

Basics of Modal Analysis

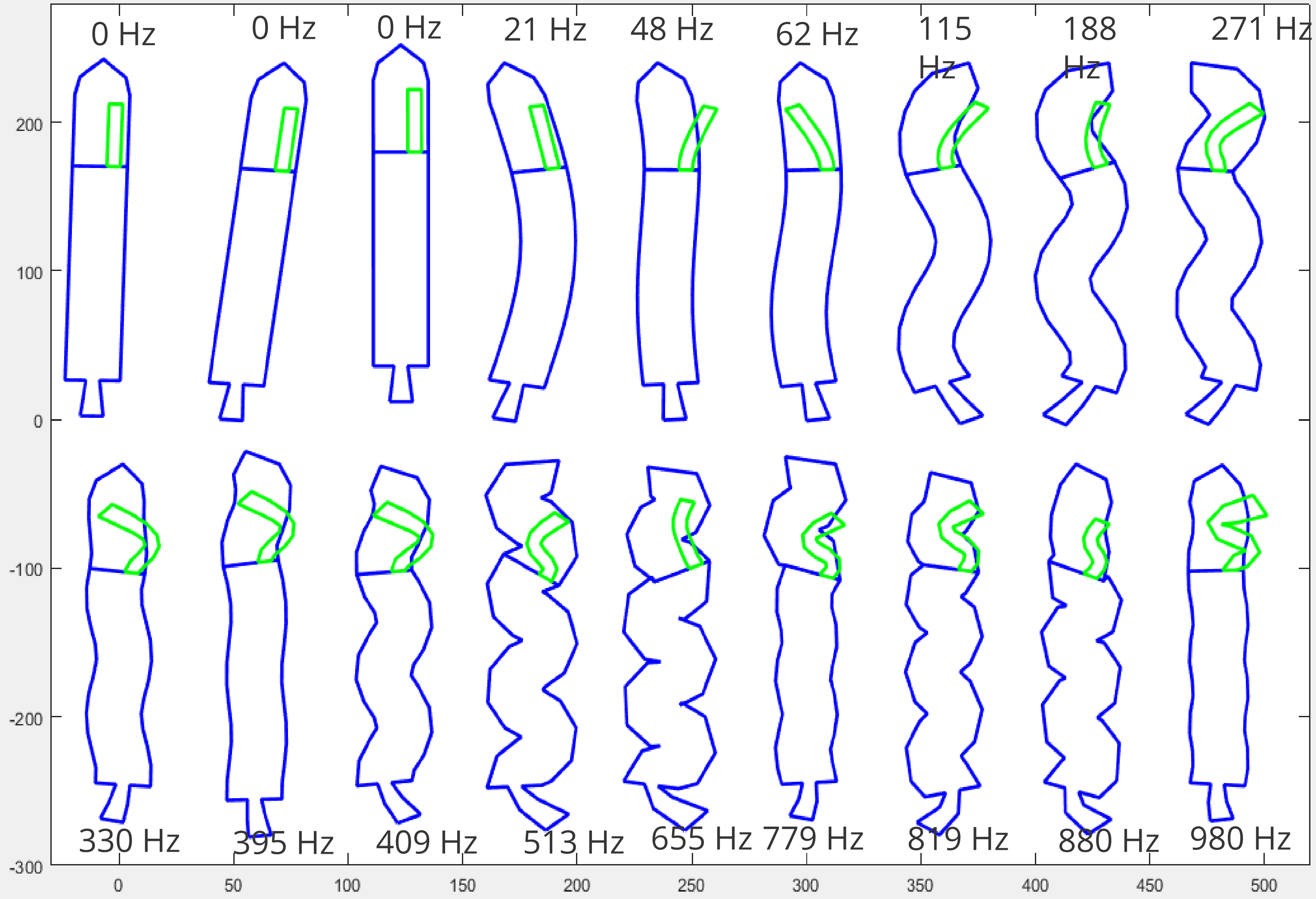
- Live beam demo – natural frequency, flexible and rigid shapes, and damping modal parameters
- Fixed base beam modes shapes related to acceleration FRFs
- The FRF is defined as the acceleration FFT/ the force input FFT



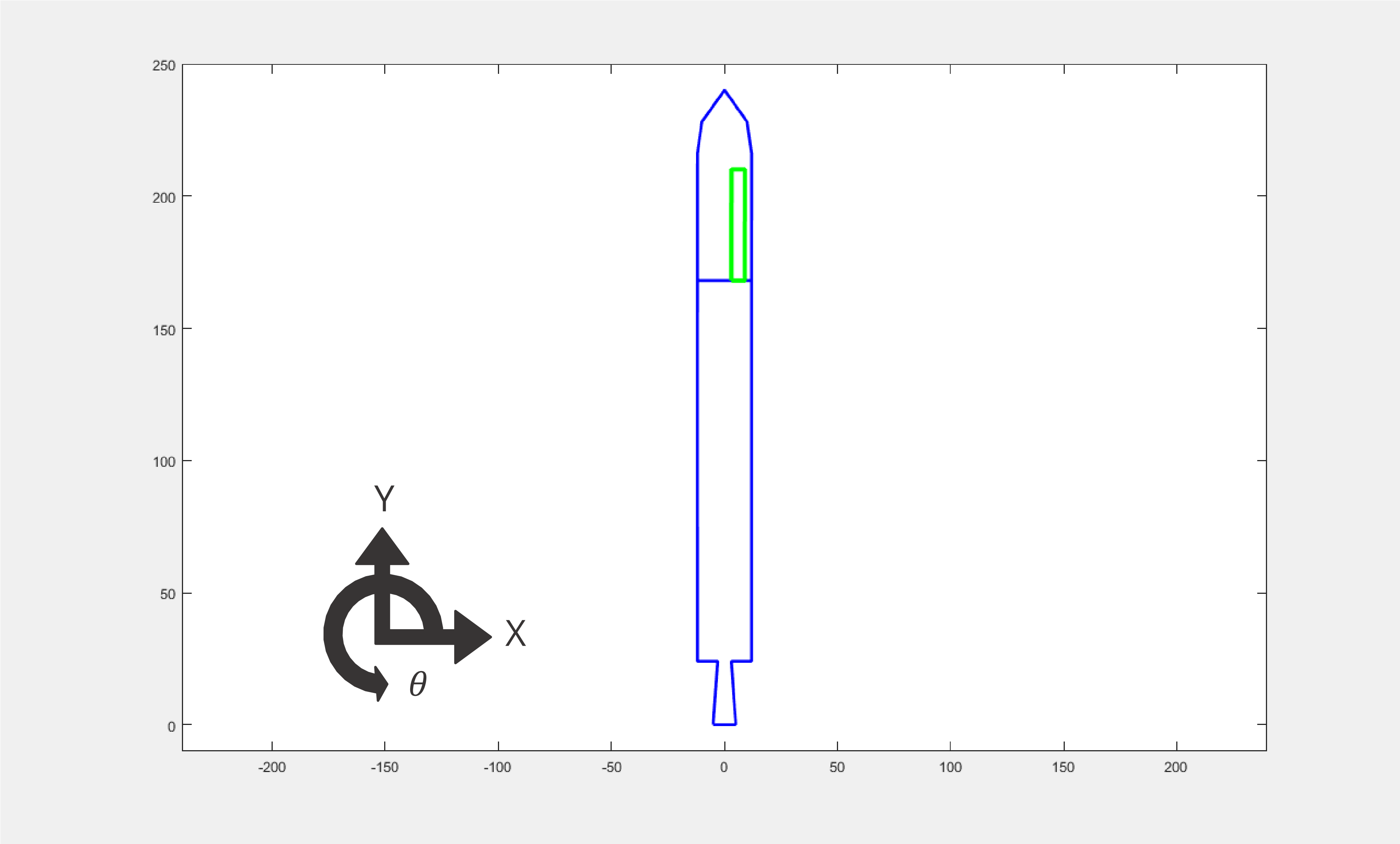
Frequency Response Functions

Modal Approach to Environments Testing

Rocket Modes up to 1000 Hz



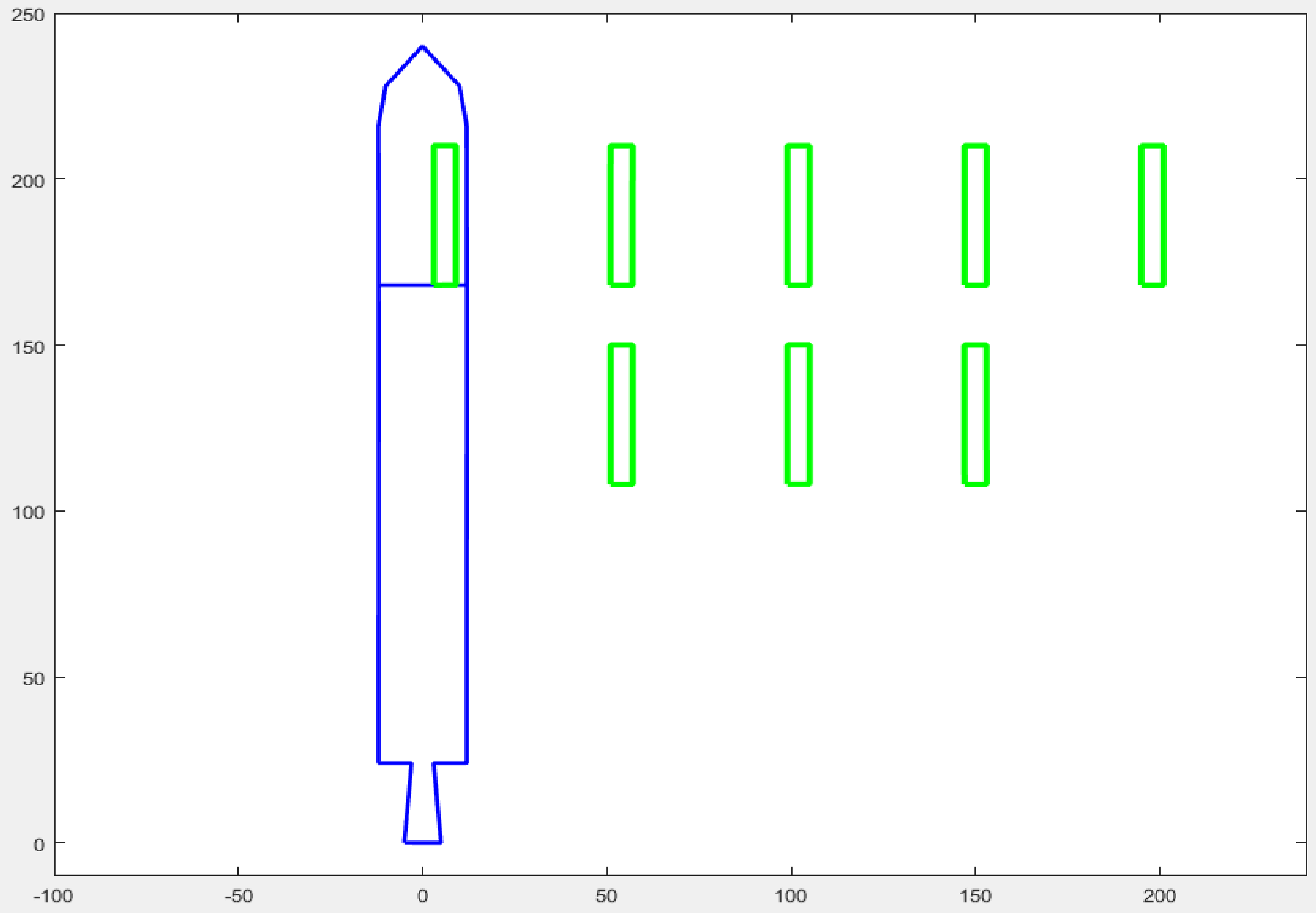
Random acceleration response to 2000 Hz due to nozzle force



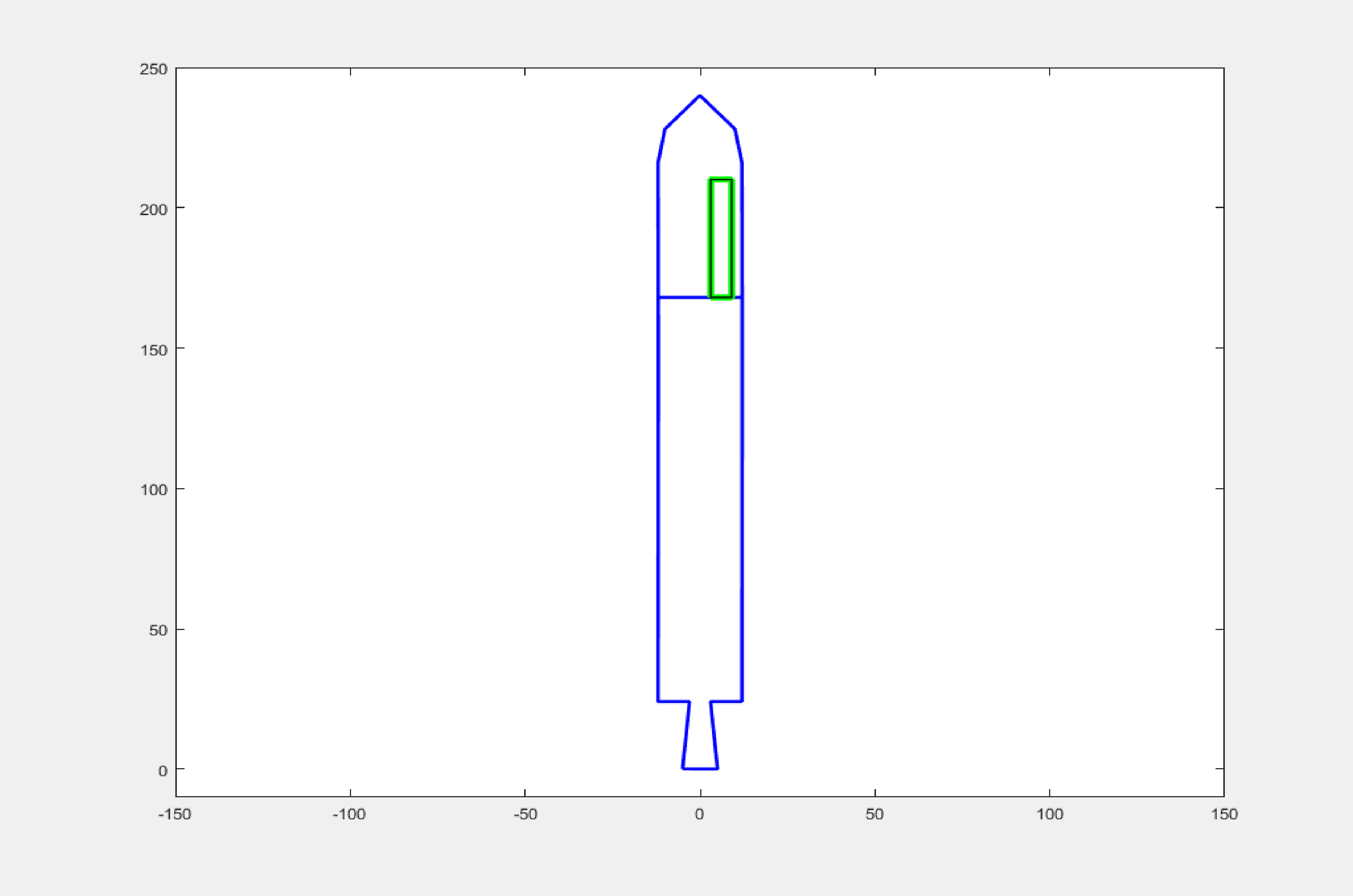
Every Dynamic Structural Response to a Load is a Linear Combination of Modal Responses

$$\{\ddot{x}\} = [\Phi]\{\ddot{q}\}$$

Rocket Field Response and rigid body/fixed base modes isolated

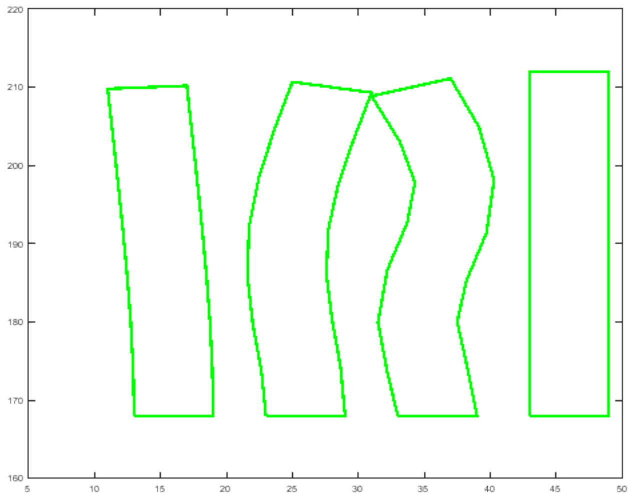


Rocket Field Response and rigid/fixed base modes fit (black line)



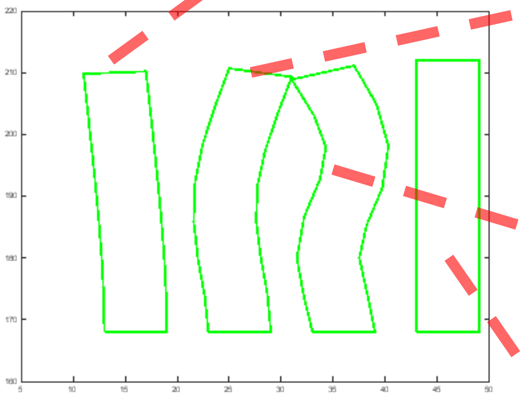
What have we learned

- We can get a pretty good simulation of component field response with just the rigid body modes and 4 fixed base modes that would be active on a 3 DoF shaker
- Insight into the component motion is quite strong
  - Damaging elastic strain response is captured with just 4 fixed base modes
  - 3 DoF table drive motion required to match field response is contained in the rigid body mode response referenced to the vibration table fixture x,y,theta coordinate system
- A significant portion of the response was driven by rigid body pitch (which is generally ignored in laboratory tests)

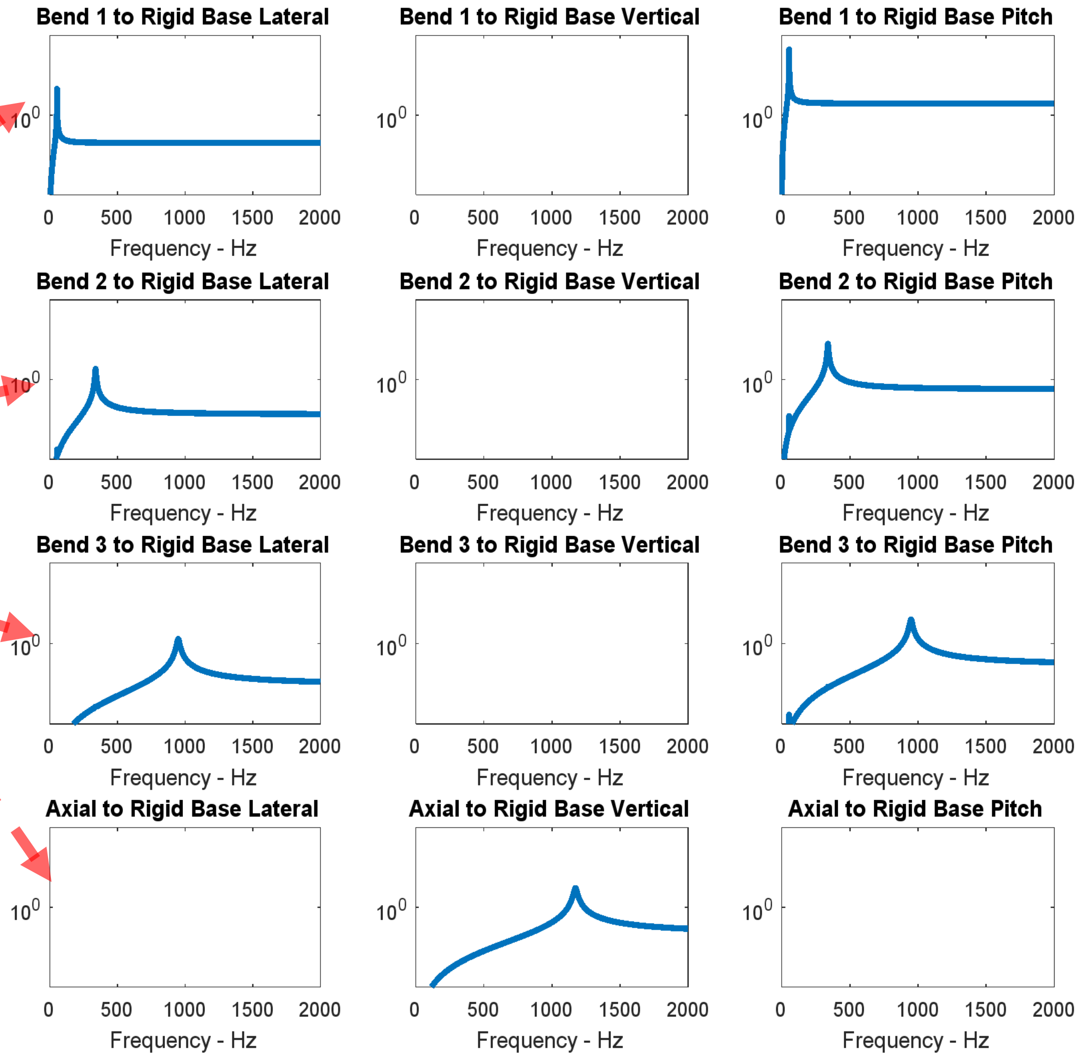


Now let's look at squiggly lines since we have a basic physical understanding of the 3 rigid body and 4 fixed base mode shape

- From a FE model, a free modal model of the component on fixture, or an uncorrelated buzz test on our 3 DoF shaker we can extract the transmissibility matrix between rigid body inputs and fixed base mode outputs
- Scale is .001 to 1000
- Bend 1 – 54 Hz
- Bend 2 – 339 Hz
- Bend 3 – 938 Hz
- Axial – 1172 Hz
- Blank plots are many orders of magnitude down

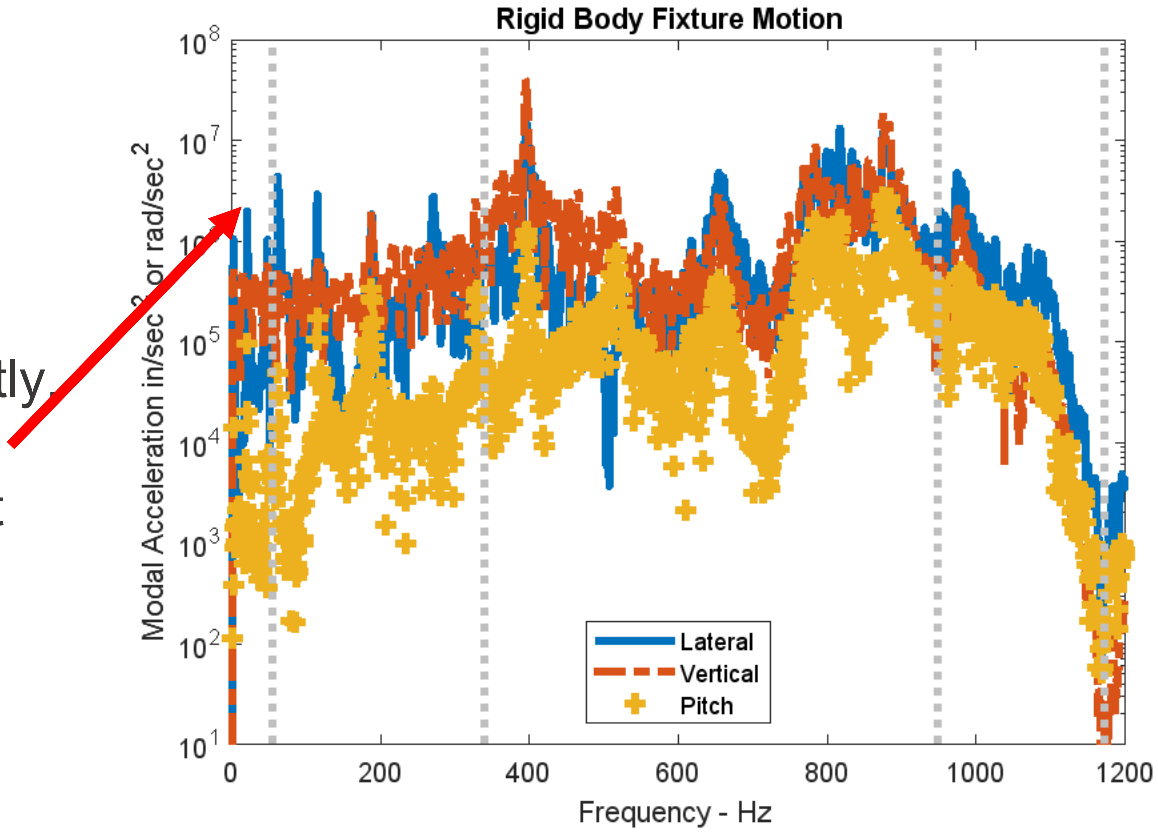


$$\{\ddot{x}\} = [\Phi]\{\ddot{p}\}$$



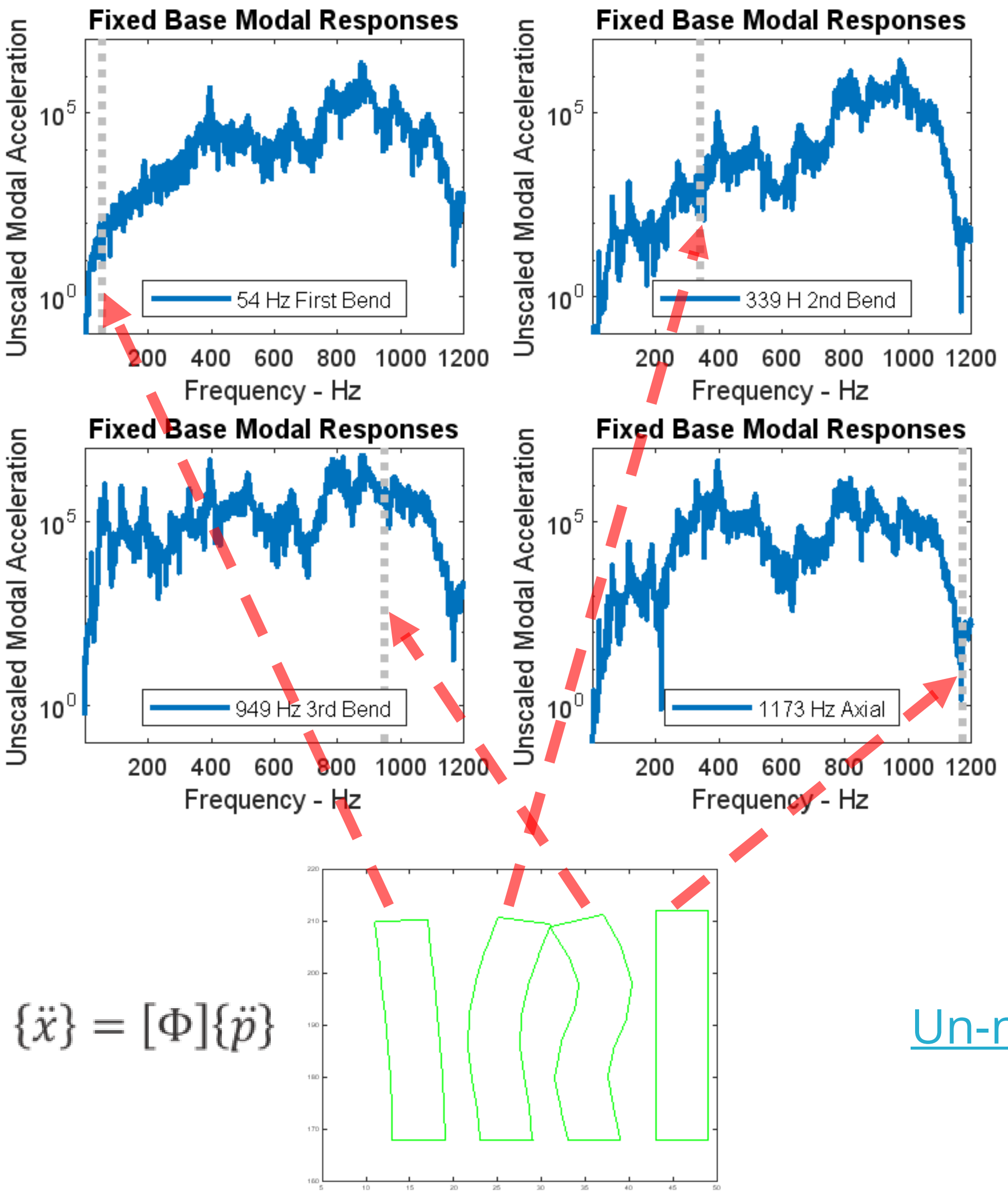
Modal Input for our 3DoF shaker - FFTs

- Note that ALL input DoF have significant response throughout the 1000 Hz excitation band
- By driving the base input significantly, we can simulate field modal responses, e.g. 21 Hz is first rocket modal frequency
- No peaks appear at the fixed base frequencies



Modal Fixed Base Responses - FFTs

- Note that ALL modal DoF have significant response throughout the 1000 Hz excitation band
- These fixed base modal responses contain the strain in the component
- By driving the base input significantly, we can simulate field modal responses to forces, e.g. 21 Hz is first rocket modal frequency
- No peaks appear at the fixed base frequencies, which act as vibration absorber in the system
- The acceleration response at each frequency line is the sum of the FFT for each mode multiplied by its mode shape
- The area under these curves is related to the strain in each of these fixed base mode shapes
- Recall the frequencies where response limiting was necessary. They are at the frequencies of the fixed base modes

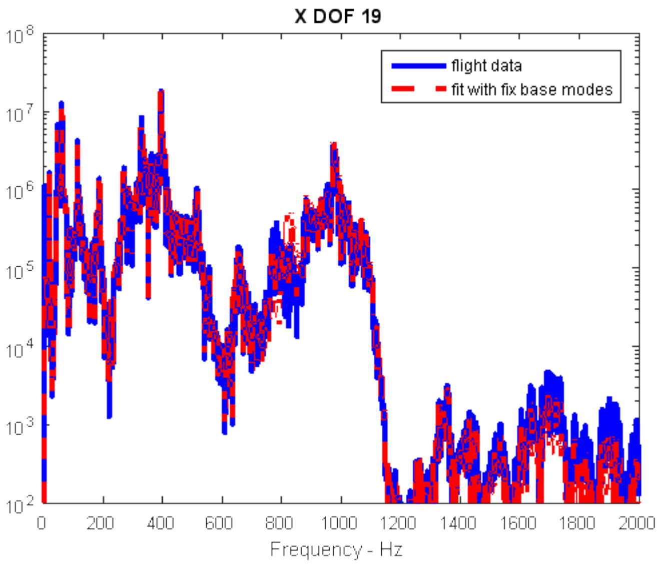


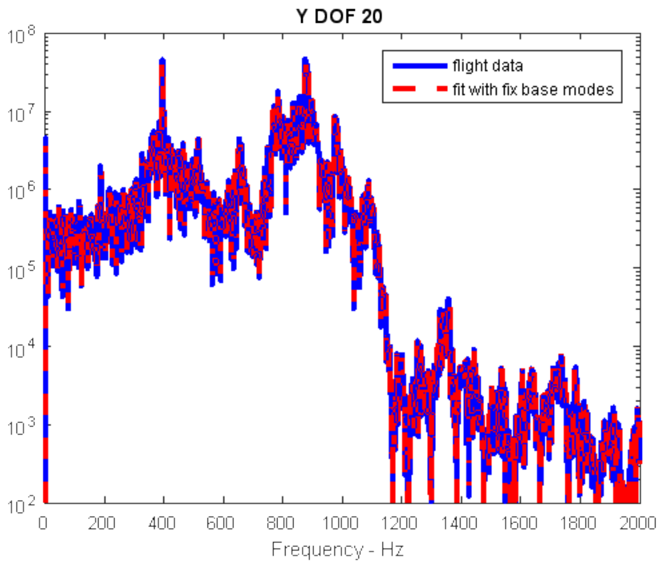
$$\{\ddot{x}\} = [\Phi]\{\ddot{p}\}$$

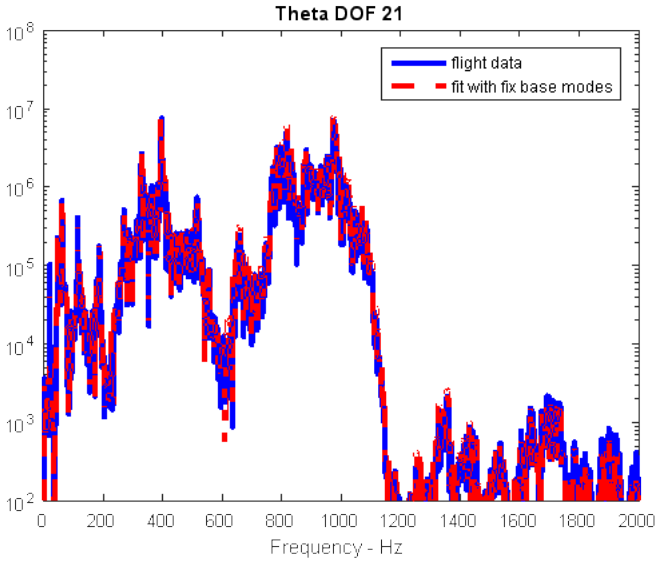
Un-notched SDOF

Lab 3 dof match to component flight response

Component beam centerline acceleration response 6” from top of component







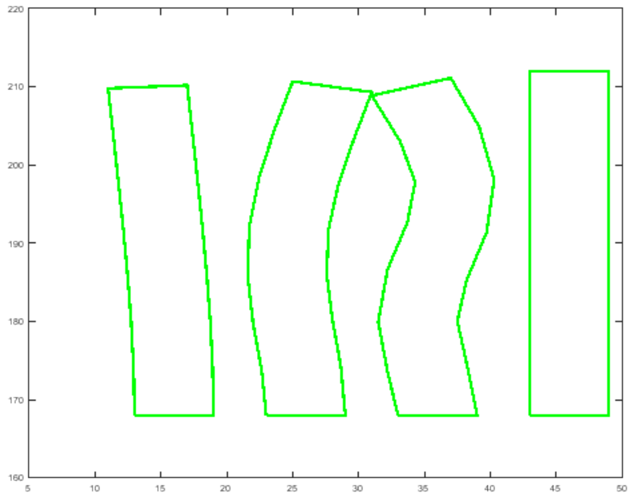
Laboratory Simulation of Component Responses

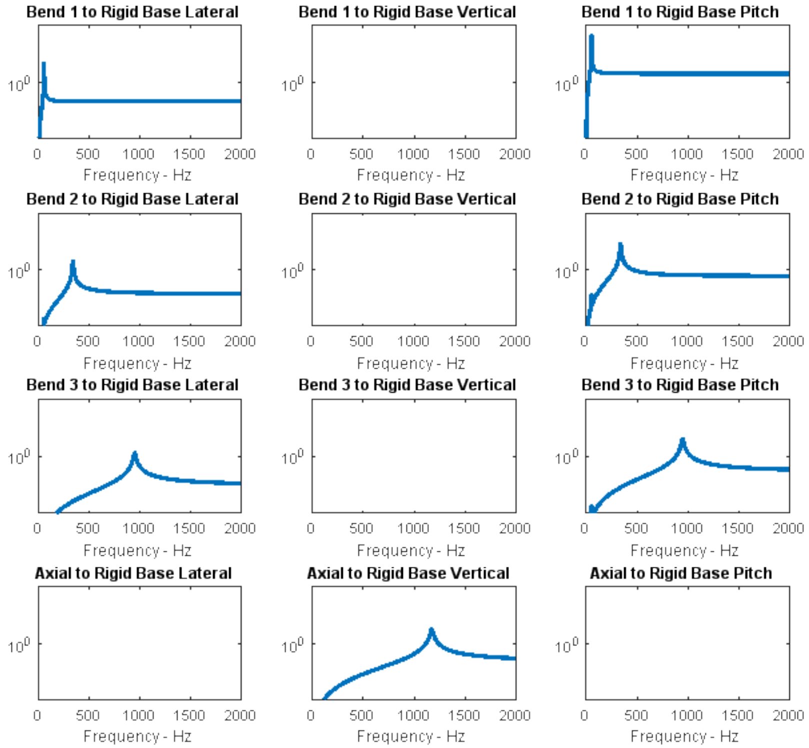
If we know the fixed base modal response, we can back out the base input needed to best match it with our transmissibility matrix

- Once we know the fixed base responses we desire, we can back out the base input required to best fit the fixed base modal response

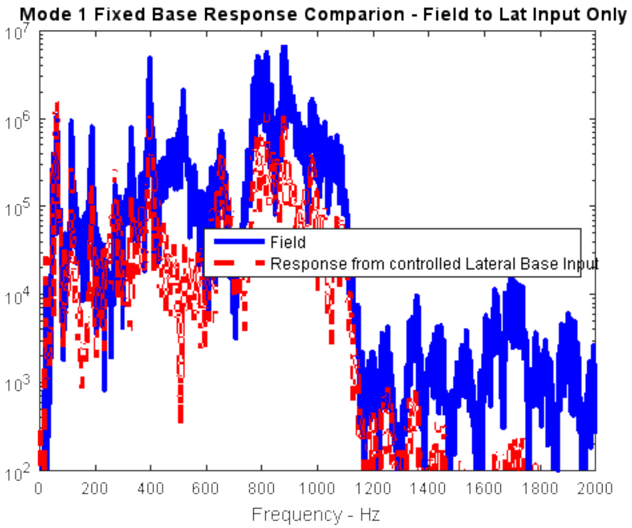
- Conversely, if the field response put in both *x* and θ to obtain a response and we only measured the *x* input, by inputting ONLY *x* we will get a very UNCERTAIN simulation

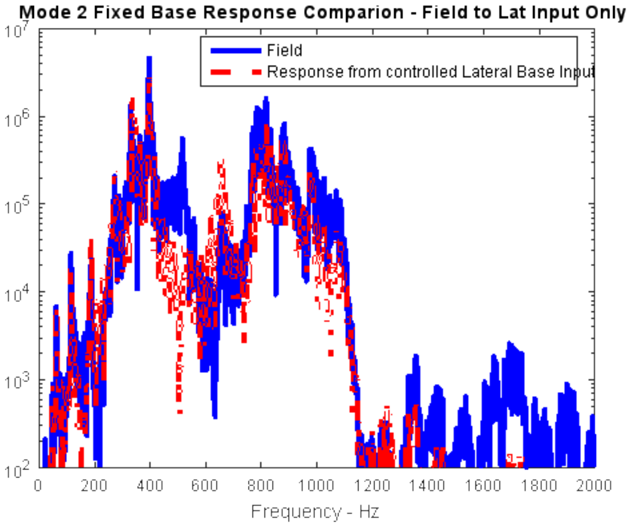
$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \\ H_{41} & H_{42} & H_{43} \end{bmatrix} \begin{Bmatrix} x \\ y \\ \theta \end{Bmatrix}$$

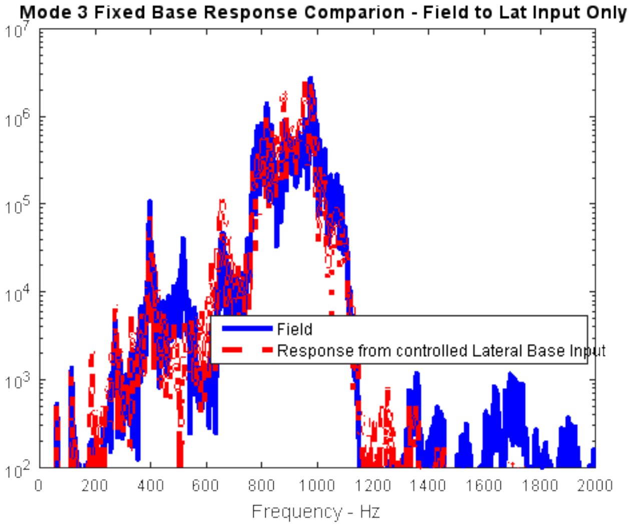


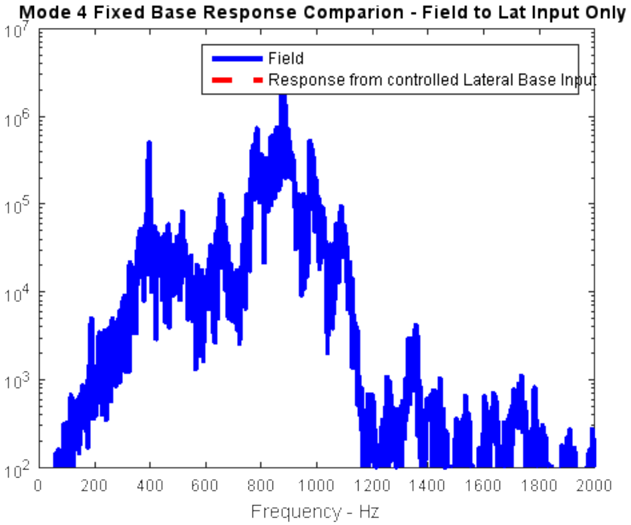


Fixed base modal response FFT to perfectly controlled x input only (no theta or vertical input)

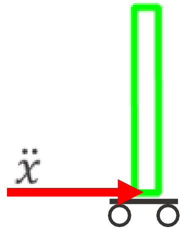


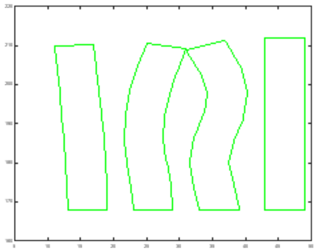






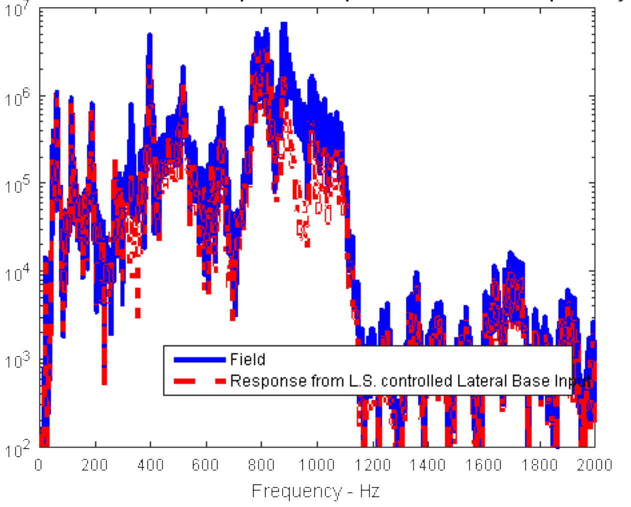
- Uncertain Simulation

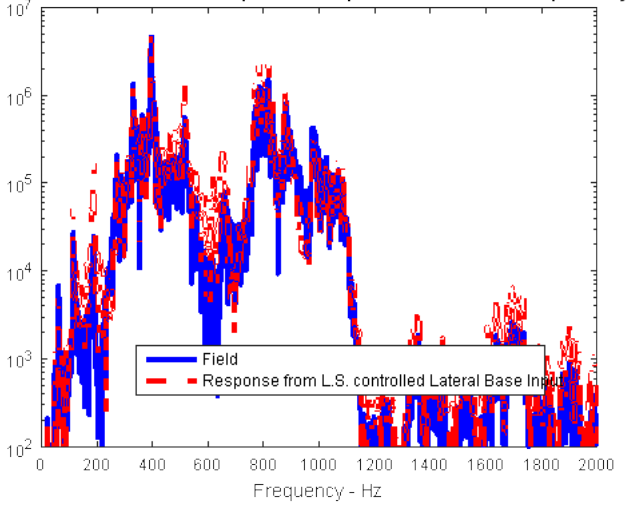


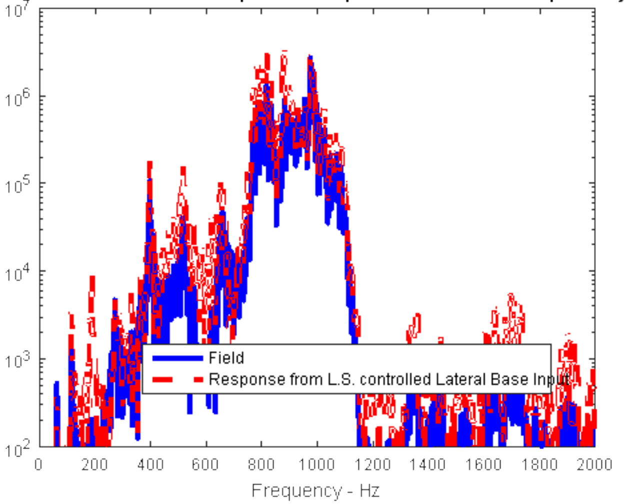


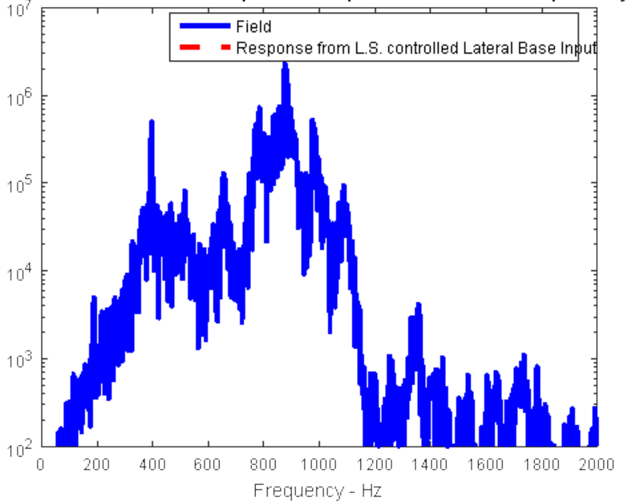
$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \\ H_{41} & H_{42} & H_{43} \end{bmatrix} \begin{Bmatrix} x \\ \cancel{y} \\ \cancel{\theta} \end{Bmatrix}$$

Fixed base modal response FFT to controlled x input least squares fit to fixed base modes 1-3

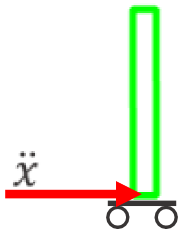


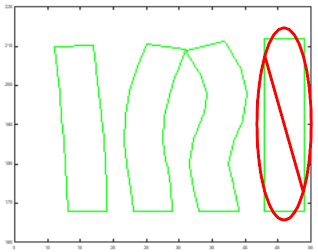






- If we only have 1 DOF input we can achieve a better simulation that the measured field input

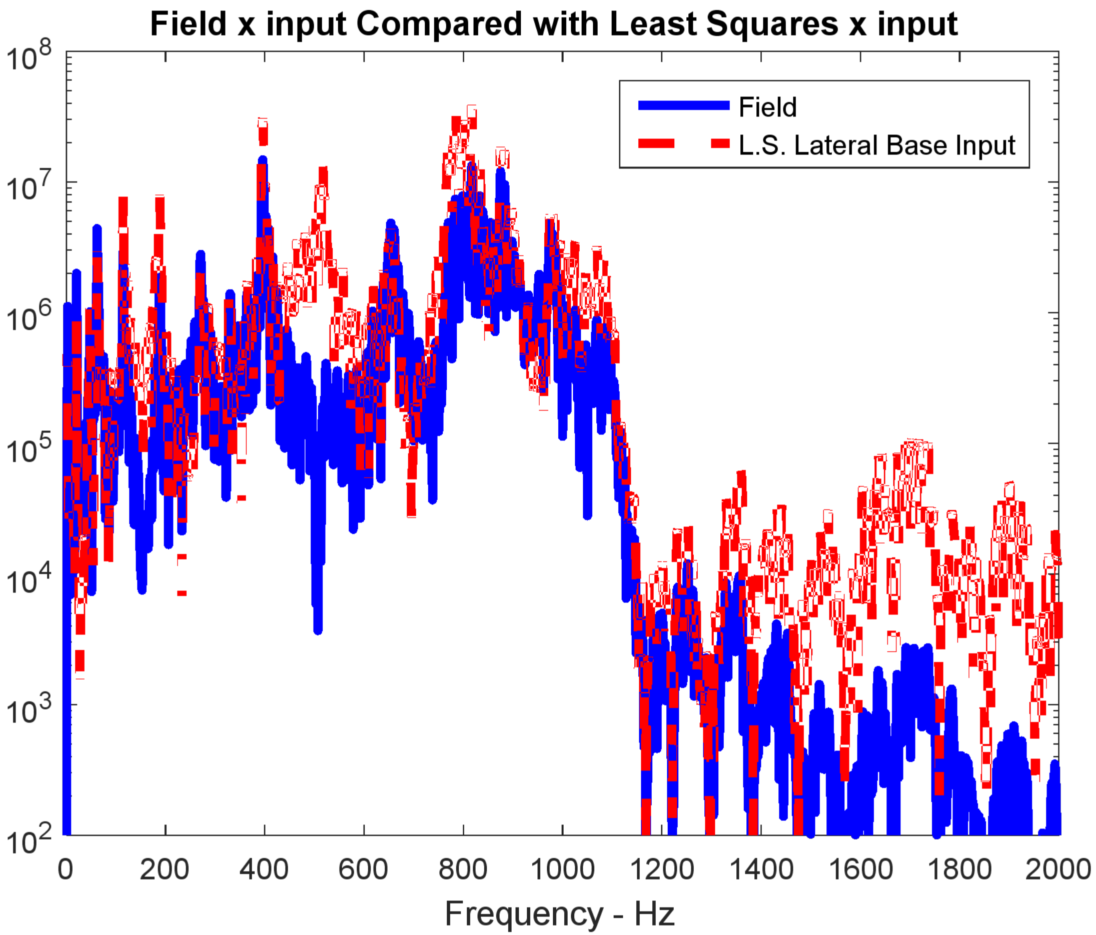


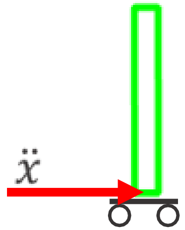


$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ \cancel{p_4} \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \\ H_{41} & H_{42} & H_{43} \end{bmatrix} \begin{Bmatrix} x \\ \cancel{y} \\ \cancel{\theta} \end{Bmatrix}$$

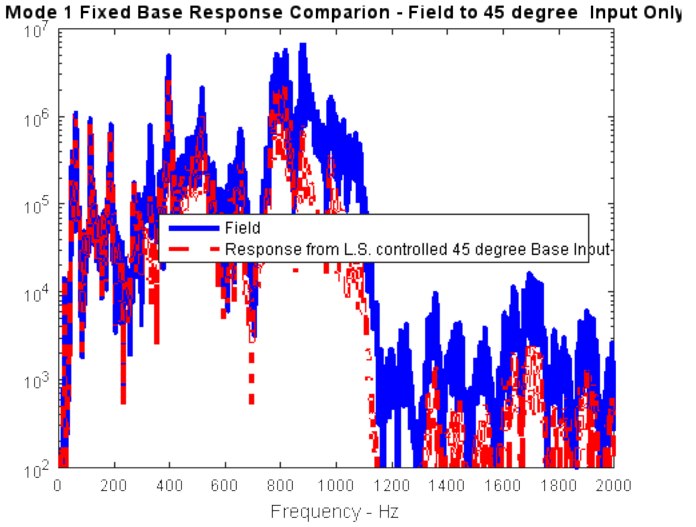
Compare field x FFT input to least squares x FFT input

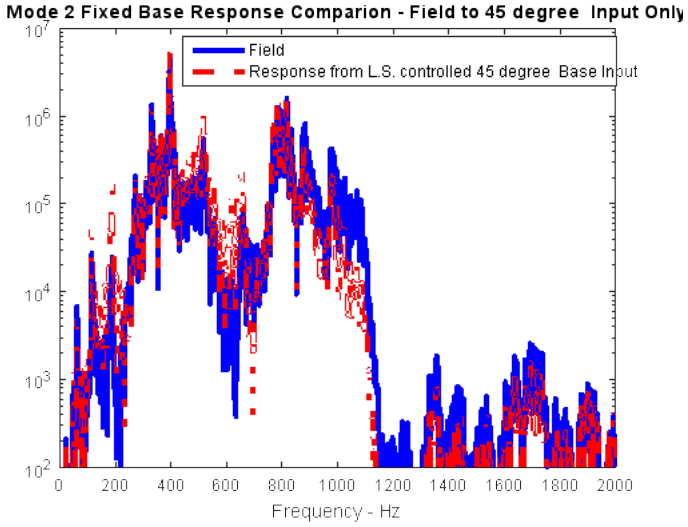
- Least squares physical x input has to compensate for lack of theta input

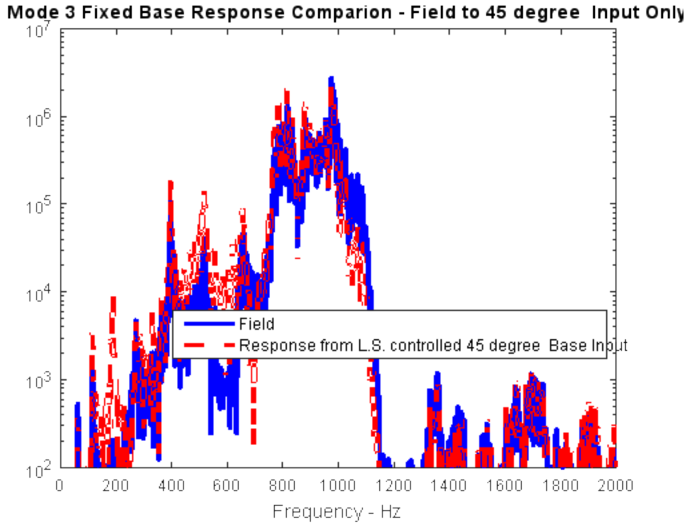


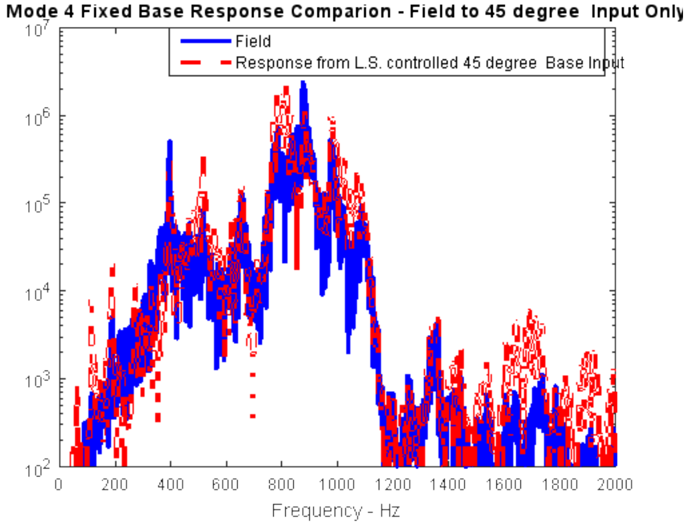


Fixed base modal response FFT to Least Squares 45 degree input only (part lateral part axial)

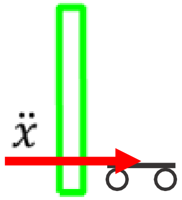


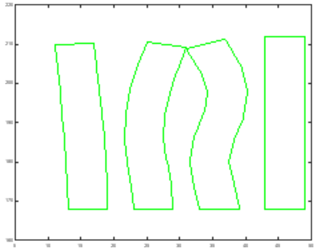






- With more 1 DOF input optimization we can do even better
  - Example: 45 degree least squares input both lateral and axial



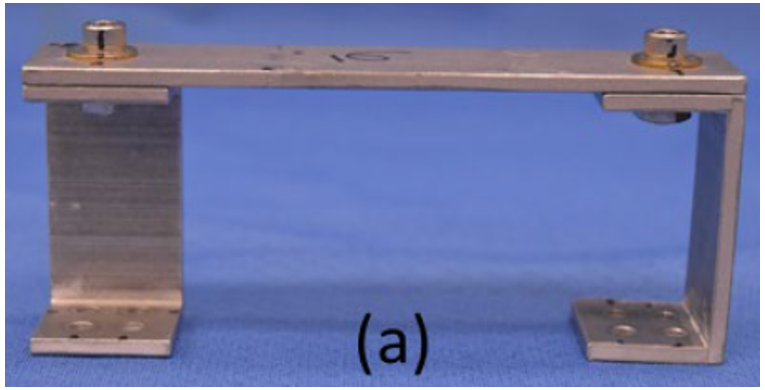


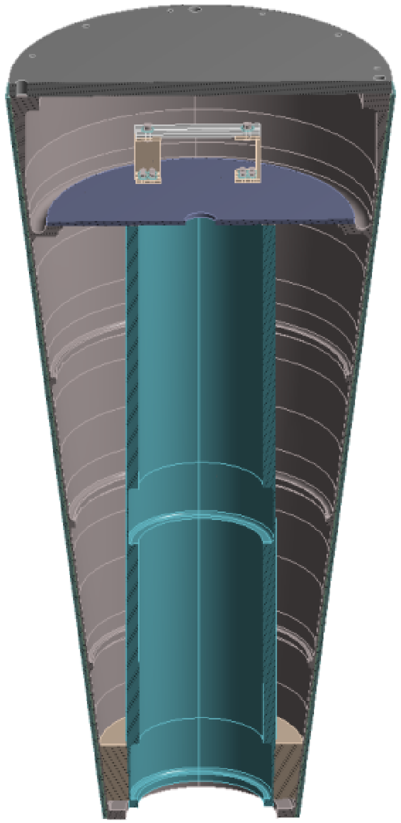
$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \\ H_{41} & H_{42} & H_{43} \end{bmatrix} \begin{Bmatrix} x \\ y \\ \cancel{\theta} \end{Bmatrix}$$

Real World Examples

Proof of Concept Field Hardware

- System was Modal Analysis Test Vehicle (MATV)
- Hardware was developed by the Atomic Weapons Establishment, AWE, UK

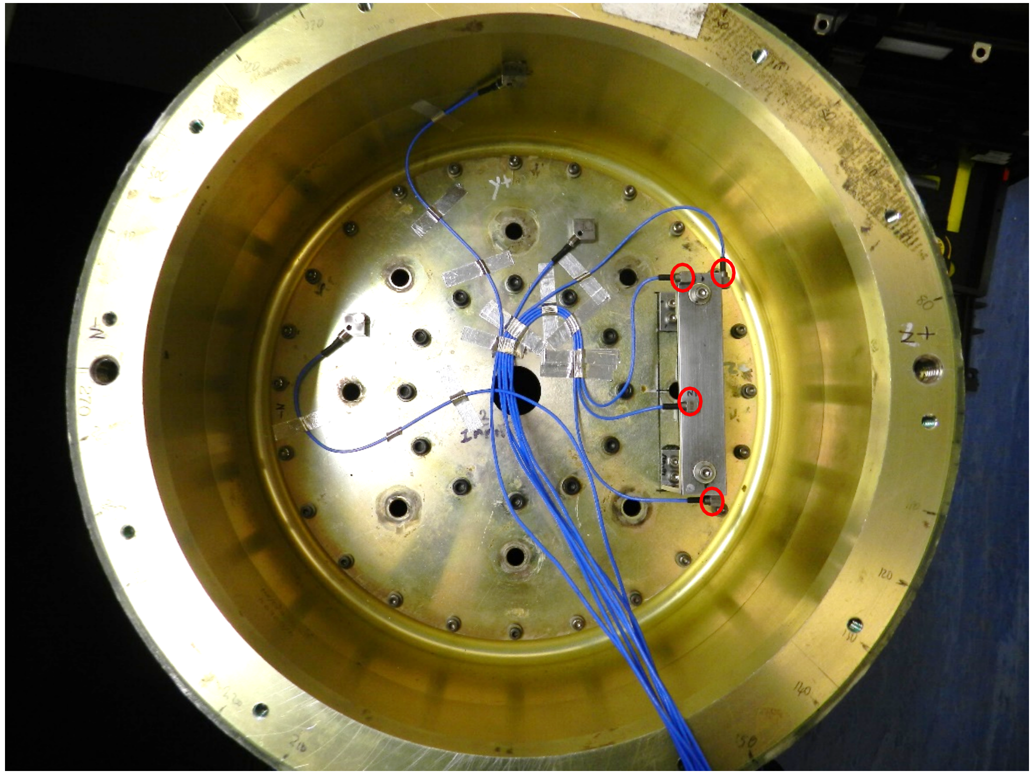




- Component is the removable component (RC), a round robin test article developed for the dynamic environments community ESTECH/SAVE/IMAC

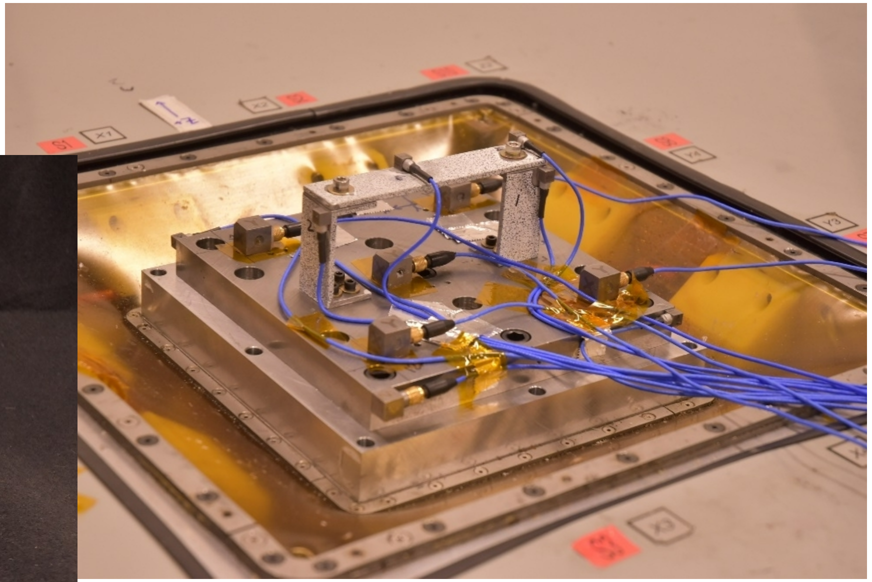
Acoustic Field Test and Instruments

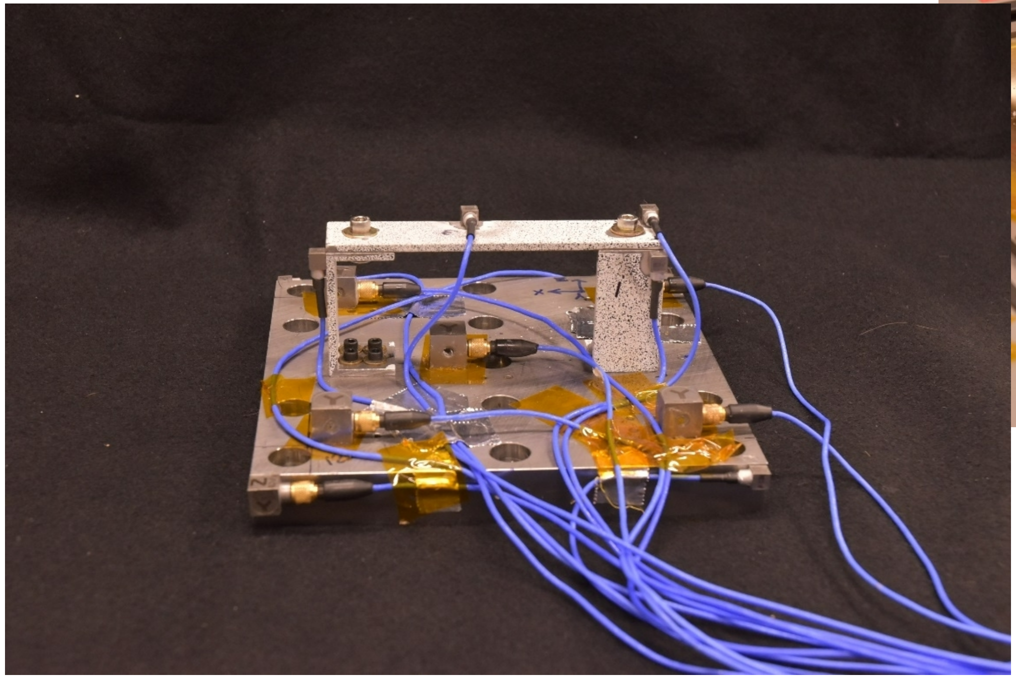
- The acoustic test was performed to 147 dB
- Data were gathered on 4 triax accelerometers on the RC
- One uniaxial accelerometer on the outside of the MATV on the plane of the component plate.



Laboratory Hardware

- Another RC was mounted on a steel plate and instrumented with 4 triax accelerometers in the same locations as the field test as well as 4 triax accelerometers on the corners of the plate.





Transform to Rigid Body Modes and Fixed Base Modes

Modal Craig-Bampton procedure transforms free-free modes to a set of fixed-base (p) modes + rigid-body (s) modes

Free-free modal params.

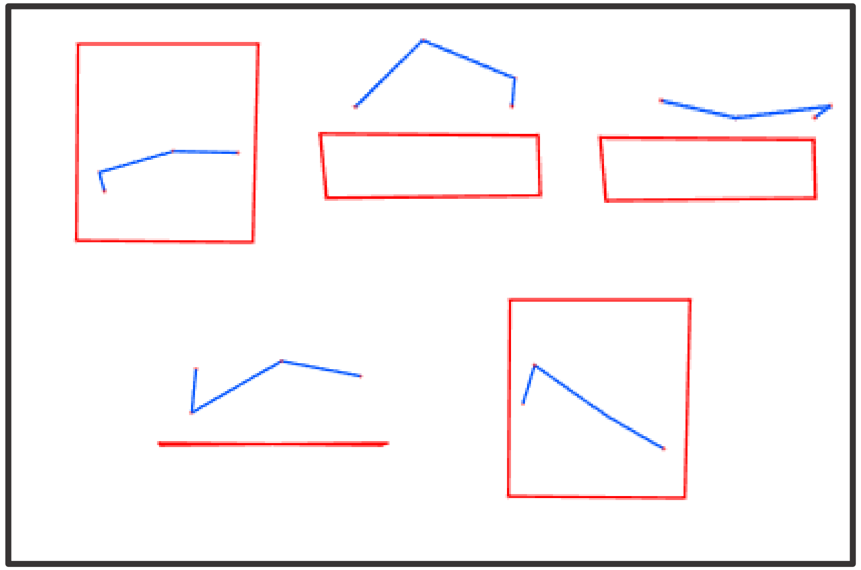
$$\mathbf{x} = \mathbf{\Phi q}$$

$$(\omega_{\text{free},r}^2 - \omega^2 + i2\zeta_{\text{free},r}\,\omega_{\text{free},r}\omega)q_r = 0$$

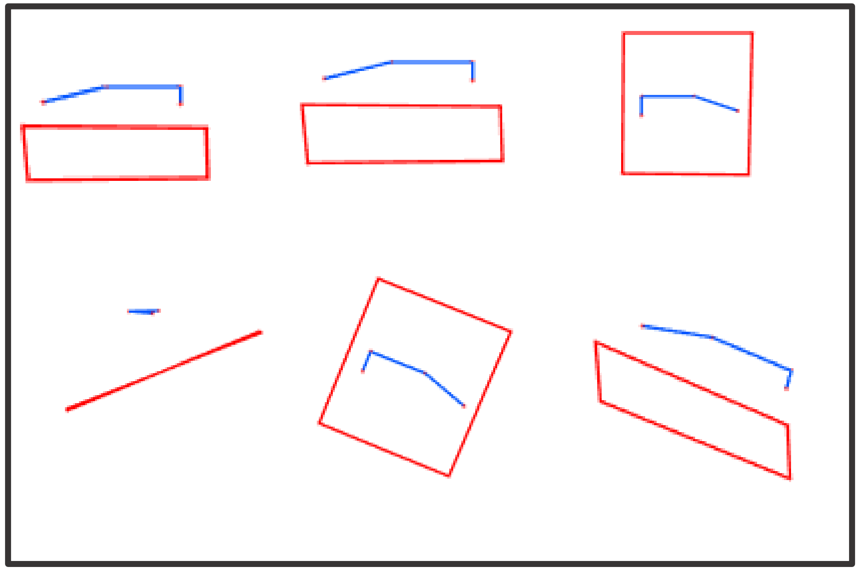
Transformation

$$\mathbf{q} = [\mathbf{T}_p \quad \mathbf{T}_s]\begin{Bmatrix}\mathbf{p}\\ \mathbf{s}\end{Bmatrix}$$

Fixed-base shapes:



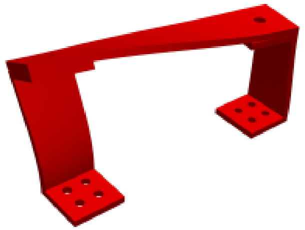
Rigid-body shapes:



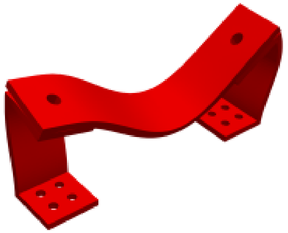
Mayes, Randall L., "A Modal Craig Bampton Substructure for Experiments, Analysis, Control and Specifications", *Proceedings of the 33rd International Modal Analysis Conference*, Orlando, FL, February 2015, paper number 353.

Better Visualization of RC Fixed Base Modes from FE Model

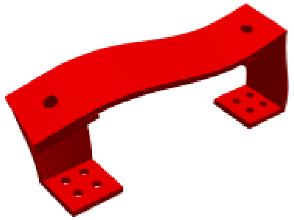
Here are the five fixed base mode shapes active up to 2000 Hz



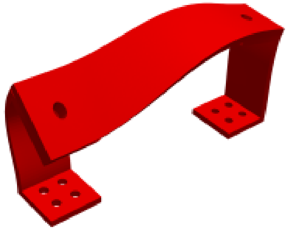
383 Hz



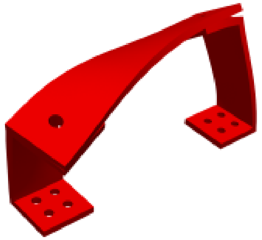
1026 Hz



1125 Hz



1651 Hz



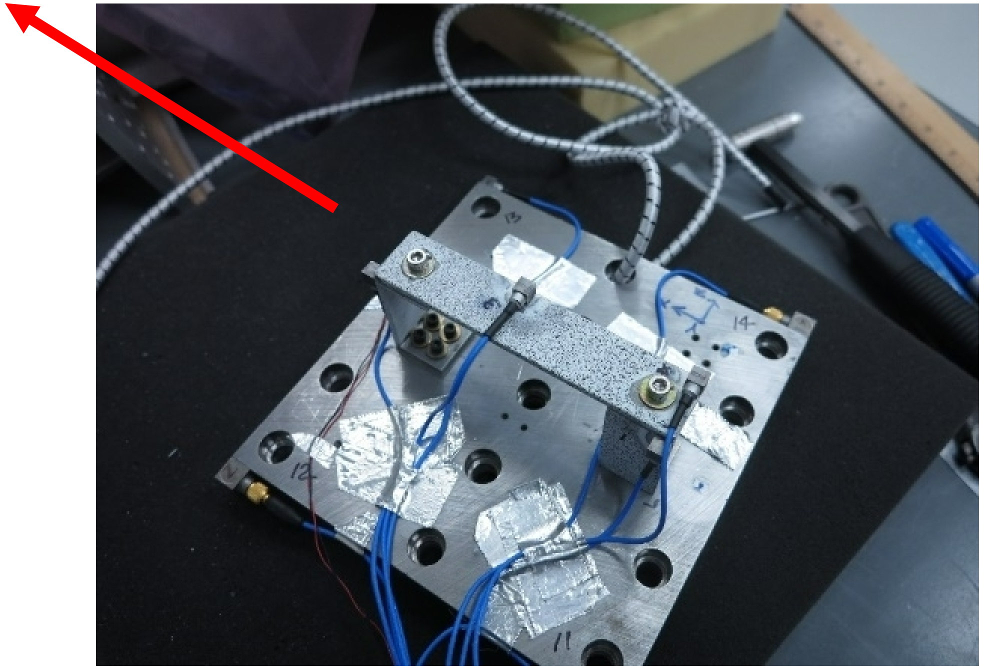
1883 Hz

Results from 1 DOF X axis input extended to physical ASDs

Transmissibilities were calculated with buzz test from the RC accelerometers to the rigid base DOF inputs (X,Y,Z,RX,RY,RZ)

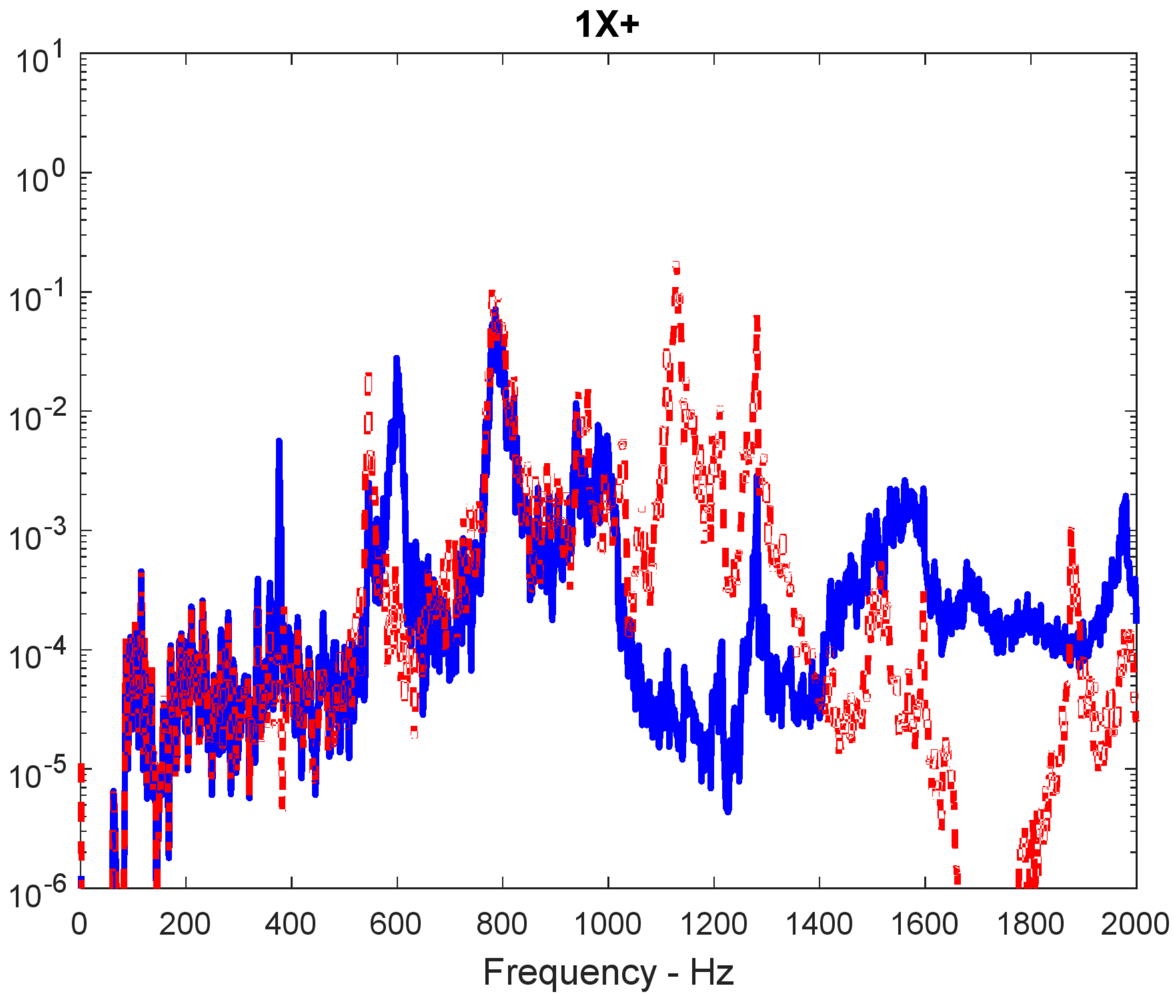
Acceleration from MATV in X direction about 3 inches away from RC was used as input in X direction only

X



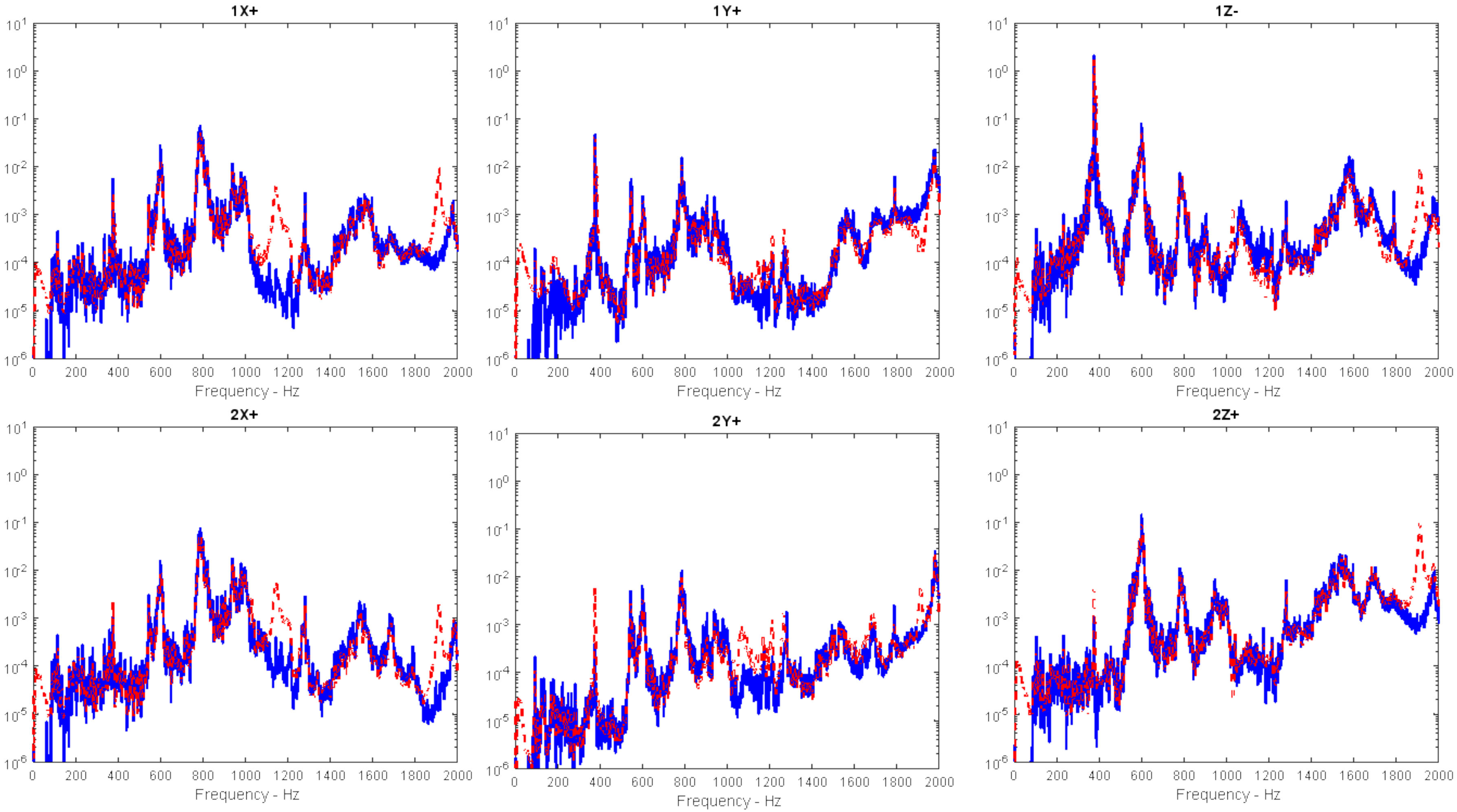
Results from 1 DOF X input analytically extended – ASD for 1X

ASDs from Acoustic Test-blue; 1 DOF shaker-red



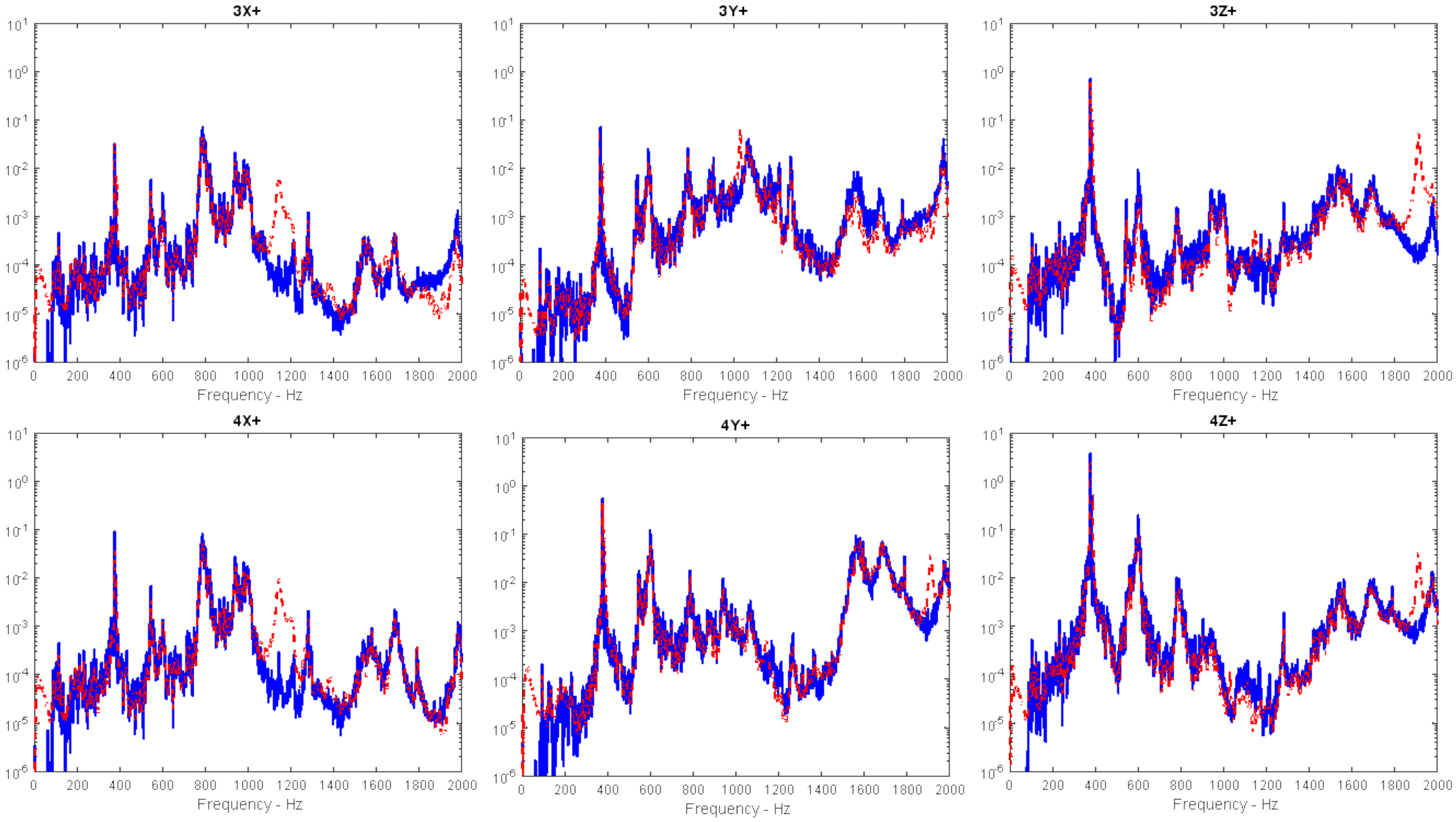
Results from 6 DOF test to control (50-2000 Hz) to 12 x 12 acoustic test cross spectral matrix (Paripovic/Nelson/Schultz)

- 6 ASDs from Acoustic Test – blue; 6 DOF shaker - red



Results from 6 DOF test to control to 12 x 12 acoustic test cross spectral matrix – page 2

- 6 ASDs from Acoustic Test – blue; 6 DOF shaker - red



Accounting for Unit-to-Unit Variability

Analysis: Develop One Specification Accounting for Unit-to-Unit Variability

Generate 20 “units”

- Perturb each fixed base modal frequency of the RC by as much as 5%

Determine the 6 DOF rigid input for each that generates responses that match MATV acoustic test

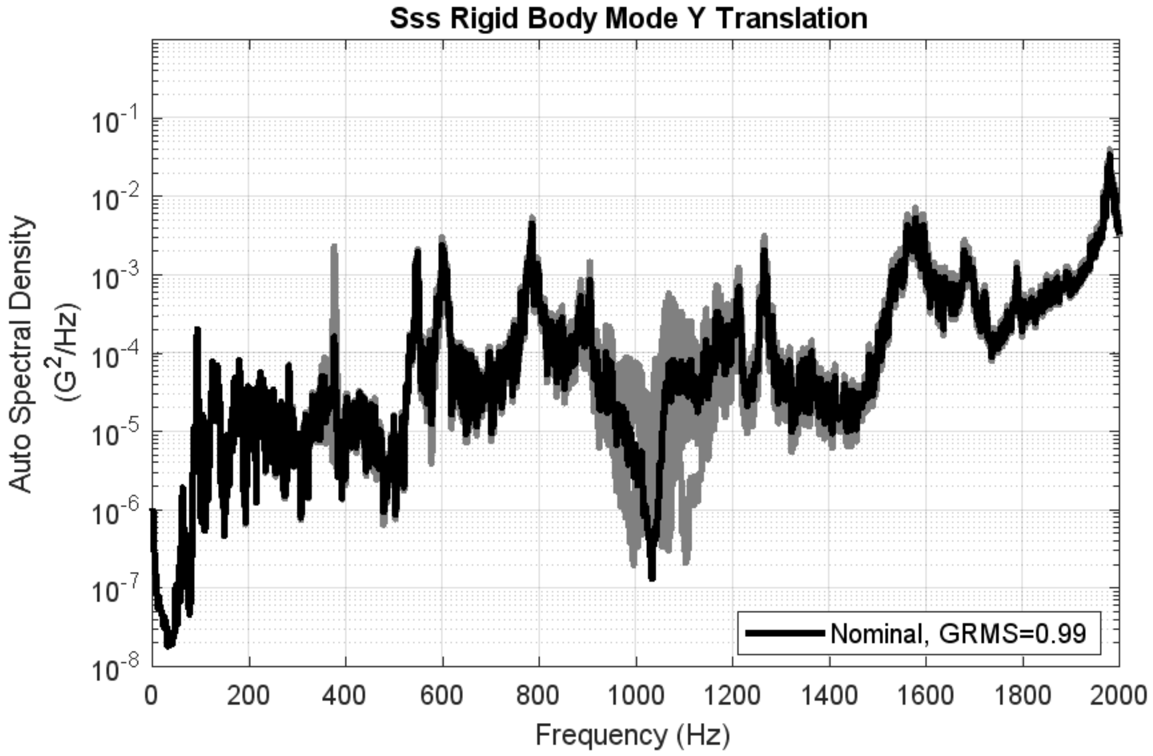
Envelope the auto-spectra for all 20 units

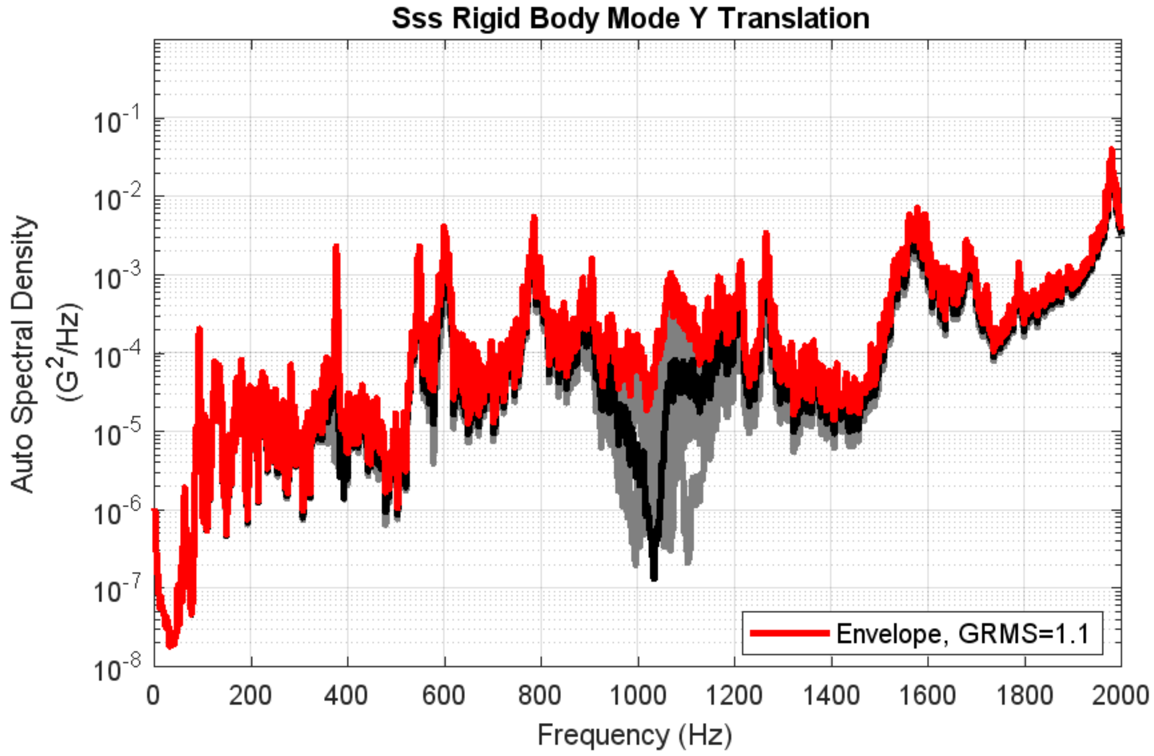
Apply the auto-spectra to a test unit

- Use the phase and coherence from that unit to fill in the 6 DOF cross spectra

Evaluate Responses

Develop One Specification Accounting for Unit-to-Unit Variability





Example Auto Spectra Inputs and Envelope

Develop One Specification Accounting for Unit ... y

Vertical lines drawn at fixed base elastic modes of the te...

Note the high responses are all connected to exci...

Enveloping Inputs

Neglects the Necessary Notches

Deemed Undesirable

Exa... ...hysical Responses on RC

Develop Independent Test Specifications for Unit-to-Unit Variability

Decided to investigate independently tailored 6 DOF rigid body inputs for each test article

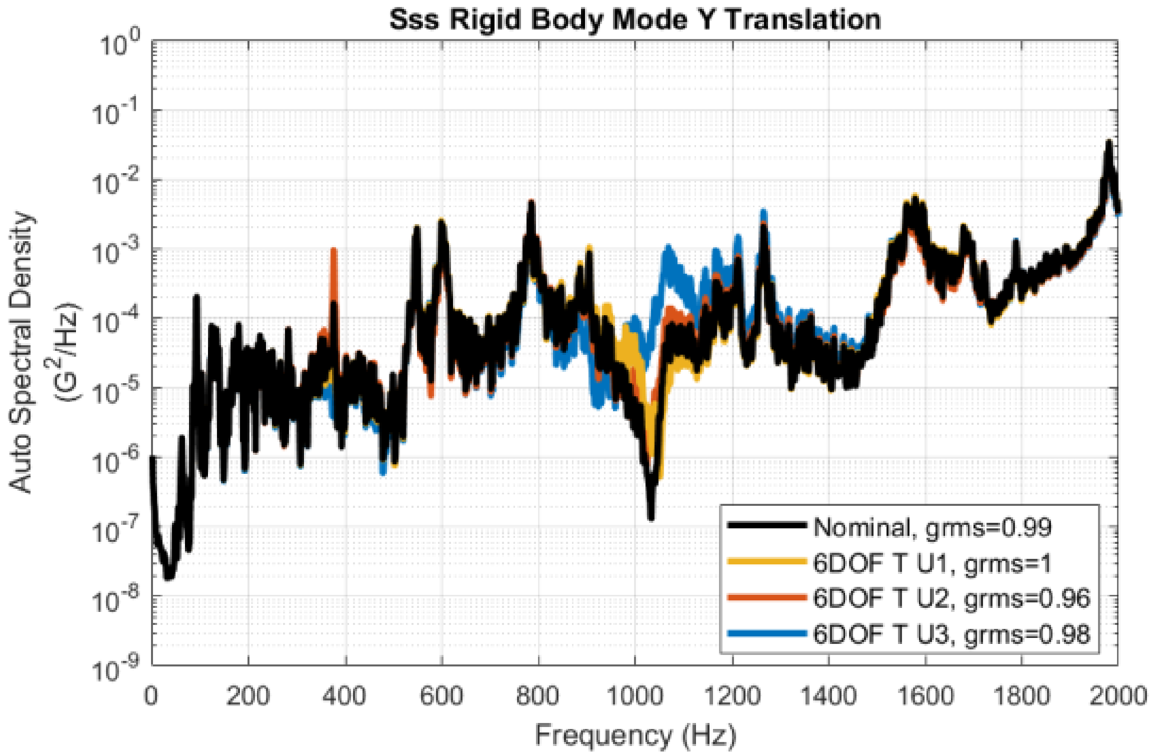
Install test article on the 6DOF shaker with the same response sensors from MATV test

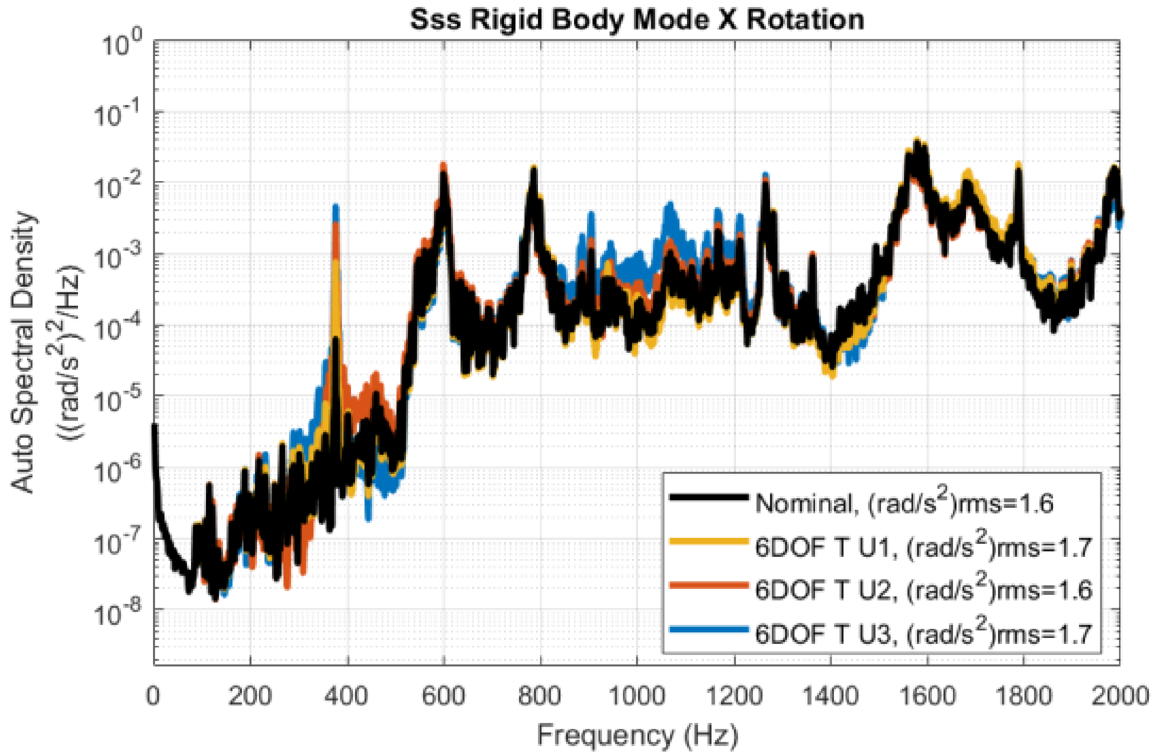
Perform a standard low-level control loop (buzz) test to determine dynamics of the test article

Generate 6DOF shaker inputs to drive the test article responses to match MATV responses

Apply inputs to the test article

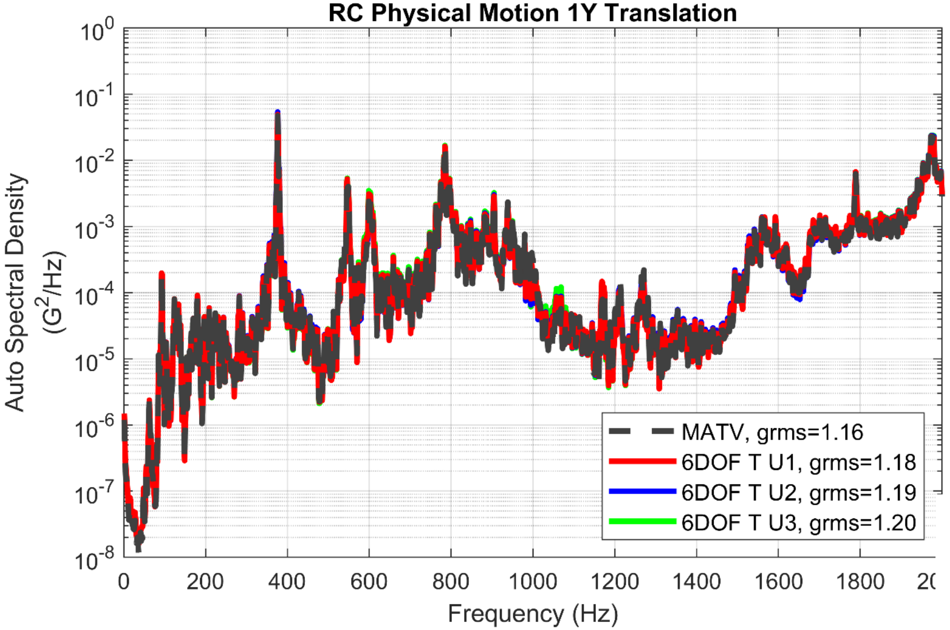
Develop Independent Test Specifications for Unit-to-Unit Variability

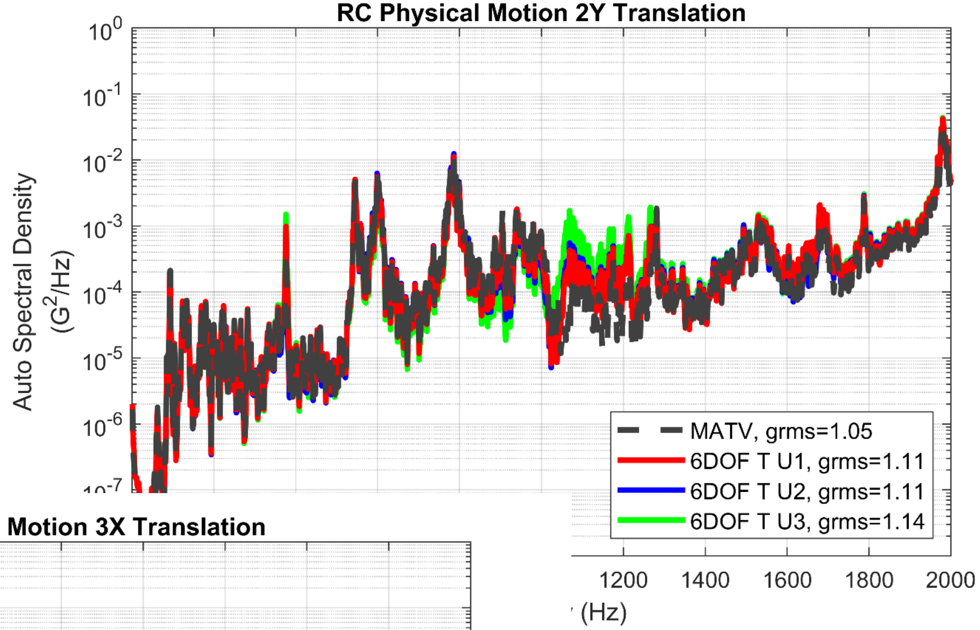


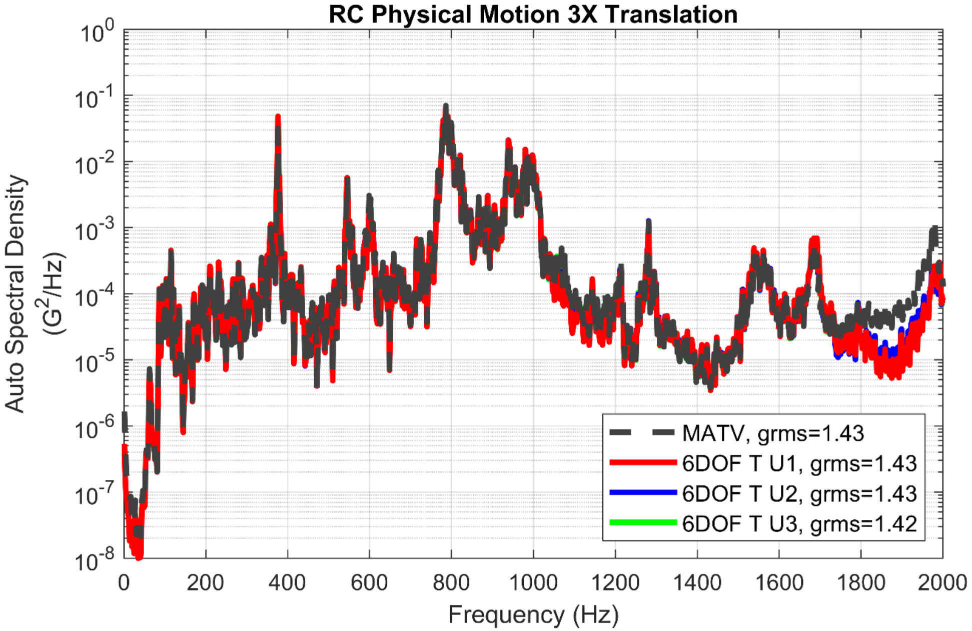


Example Auto Spectra Inputs

Develop Independent Test Specifications for Unit-to-Unit Variability







Example Physical Responses

Review Key Points

Wrap-up for base mounted component modal response

Laboratory fixed base and rigid body modes can simulate component field responses where traditional SDOF inputs cannot (even when additional traditional acceleration limiting methods are used)

Laboratory fixed base and rigid body modes allow appropriate notching removing overtests whereas traditional SDOF inputs do not

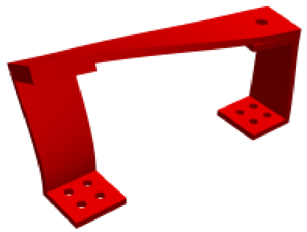
Laboratory fixed base and rigid body modes allow quantification of uncertainty whereas traditional SDOF approaches are highly uncertain in some frequency bands

Laboratory fixed base and rigid body modes can account for the difference in field and laboratory boundary conditions whereas traditional SDOF approaches do not

Laboratory fixed base and rigid body modes allow for quantification of output response acceleration margin whereas traditional SDOF approaches may quantify margin on only one base translation input (ignoring the effects of 2 other translation and 3 rotation base inputs)

Laboratory fixed base modal DOF capture intuitive strain response (and damage potential) in a few mode shapes

Transmissibilities to laboratory fixed base modes allow proper tailoring of MDOF or SDOF base inputs, greatly reducing uncertainty on responses



Wrap-up for base mounted component modal response

Laboratory fixed base modes excited by the correct rigid body base inputs can approximate field response.

This modal approach removes uncertainties associated with the differences between laboratory and field boundary conditions commonly seen in traditional testing.