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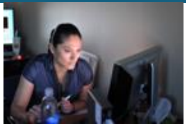
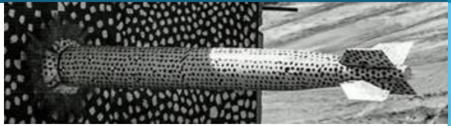
SAND2020-13296C

A Five-Moment Multifluid Model for Partially Ionized Plasmas With Arbitrarily Many Species





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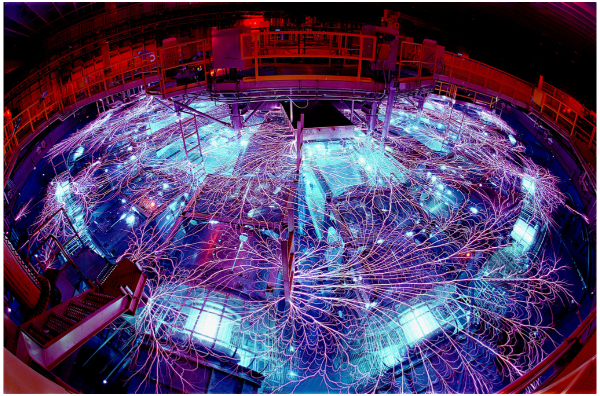
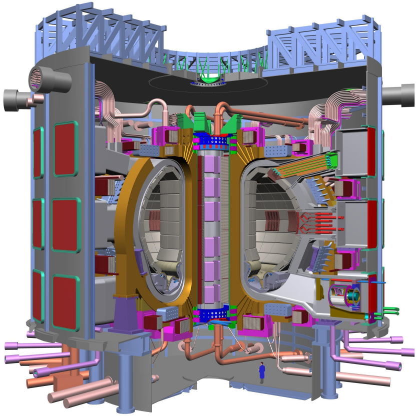
Motivation

- High-energy density plasmas.
- Higher Z elements: He, Ne, Ar.
- Wide range of density/temperature/etc.



Z-Pinch (Z Machine)
eg., Ar Gas Puff



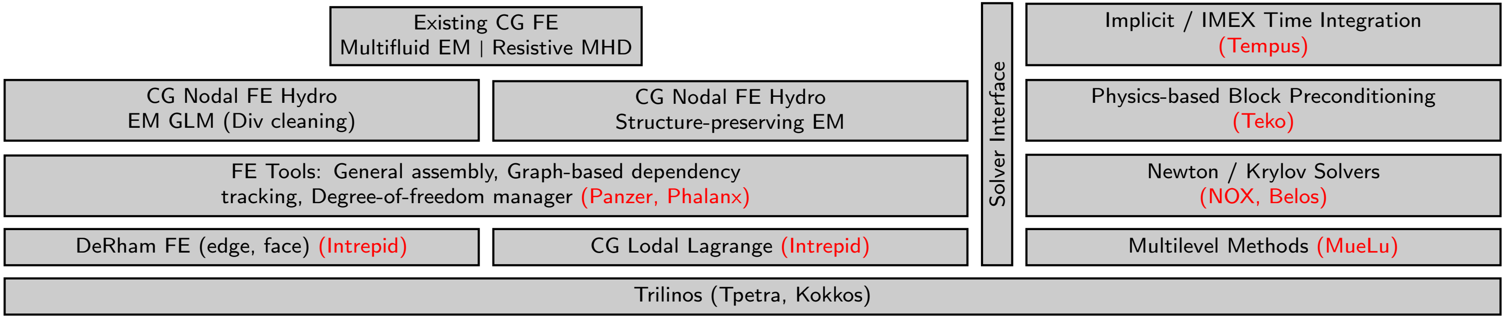
Magnetic Confinement Fusion (ITER)
eg., MGI for disruption mitigation

Context

Drekar: Resistive MHD / Multifluid with Coupled Multiplysics

- Arbitrarily many equations describing physics (continuity, momentum, energy, electromagnetics).
- ERK, DIRK, IMEX time integration (Tempus).
- 2D & 3D unstructured finite element (Intrepid):
  - Stabilized Q1, Q2 elements (high-order in process).
  - Physics compatible discretizations (node, edge, face).
  - High-resolution positivity-preserving methods.

- Advanced software capabilities:
  - MPI+X (Kokkos).
  - Linear/non-linear solvers (NOX, Belos) with robust, scalable preconditioning (Teko, MueLu).
  - Exact Jacobians through automatic differentiation (Sacado).
  - Asynchronous dependency manages multiphysics complexity (Phalanx).



Overview of Models and Equations

Kinetic equations:

$$\partial_t f_s + \mathbf{v} \cdot \nabla_{\mathbf{x}} f_s + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f_s = \mathcal{C}_s [f_s] + \mathcal{S}_s, \qquad (s \in \Lambda),$$

with

$$\mathcal{C}_s = \mathcal{C}_s^{\mathrm{sc}} + \mathcal{C}_s^{\mathrm{ion}} + \mathcal{C}_s^{\mathrm{rec}} + \mathcal{C}_s^{\mathrm{cx}} + \mathcal{C}_s^{\mathrm{rad}},$$

Fluid equations:

$$\partial_t \rho_s + \nabla \cdot (\rho_s \mathbf{u}_s) = \mathcal{C}_s^{[0]} + \mathcal{S}_s^{[0]},$$

$$\partial_t (\rho_s \mathbf{u}_s) + \nabla \cdot (\rho_s \mathbf{u}_s \otimes \mathbf{u}_s + p_s \underline{\mathbf{I}} + \underline{\mathbf{\Pi}}_s) = q_s n_s (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) + \boldsymbol{\mathcal{C}}_s^{[1]} + \boldsymbol{\mathcal{S}}_s^{[1]},$$

$$\partial_t \mathcal{E}_s + \nabla \cdot [(\mathcal{E}_s + p_s) \mathbf{u}_s + \mathbf{u}_s \cdot \underline{\mathbf{\Pi}}_s + \mathbf{h}_s] = q_s n_s \mathbf{u}_s \cdot \mathbf{E} + \mathcal{C}_s^{[2]} + \mathcal{S}_s^{[2]}.$$

$\mathcal{C}_s^{\mathrm{sc}}$: elastic scattering

$\mathcal{C}_s^{\mathrm{ion}}$: ionization reactions

$\mathcal{C}_s^{\mathrm{rec}}$: recombination reactions

$\mathcal{C}_s^{\mathrm{cx}}$: charge exchange

$\mathcal{C}_s^{\mathrm{rad}}$: radiative loss

Here:

$$s \in \Lambda = \{(\alpha, k) : \alpha = 1, \ldots, N_A; k = 0, \ldots, z_\alpha\} \cup \{e\}$$

Multifluid EM Plasma Model

Density	$\partial_t \rho_s + \nabla \cdot (\rho_s \mathbf{u}_s) = -\rho_s n_e (I_s + R_s) + m_s n_e (n_{s-1} I_{s-1} + n_{s+1} R_{s+1})$
Momentum	$\partial_t (\rho_s \mathbf{u}_s) + \nabla \cdot (\rho_s \mathbf{u}_s \otimes \mathbf{u}_s + p_s \underline{\mathbf{I}} + \underline{\mathbf{\Pi}}_s) = q_s n_s (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) + \sum_{t \neq s} \alpha_{s;t} \rho_s \rho_t (\mathbf{u}_t - \mathbf{u}_s)$ $-\rho_s \mathbf{u}_s n_e (I_s + R_s) + \frac{m_s}{m_{s-1}} n_e \rho_{s-1} \mathbf{u}_{s-1} I_{s-1} + (n_e \rho_{s+1} \mathbf{u}_{s+1} + n_{s+1} \rho_e \mathbf{u}_e) R_{s+1}$
Energy	$\partial_t \mathcal{E}_s + \nabla \cdot [(\mathcal{E}_s + p_s) \mathbf{u}_s + \mathbf{u}_s \cdot \underline{\mathbf{\Pi}}_s + \mathbf{h}_s] = q_s n_s \mathbf{u}_s \cdot \mathbf{E} + \sum_{t \neq s} \frac{\alpha_{s;t} \rho_s \rho_t}{m_s + m_t} \left[A_{s;t} k_{\mathrm{B}} (T_t - T_s) + m_t (\mathbf{u}_t - \mathbf{u}_s)^2 \right]$ $-\mathcal{E}_s n_e (I_s + R_s) + \frac{m_s}{m_{s-1}} n_e \mathcal{E}_{s-1} I_{s-1} + (n_e \mathcal{E}_{s+1} + n_{s+1} \mathcal{E}_e) R_{s+1}$
Charge & Current	$q = \sum_s q_s n_s$ $\mathbf{J} = \sum_s q_s n_s \mathbf{u}_s$
Maxwell's Equations	$\partial_t \mathbf{E} - c^2 \nabla \times \mathbf{B} + \frac{1}{\epsilon_0} \mathbf{J} = \mathbf{0}$ $\partial_t \mathbf{B} + \nabla \times \mathbf{E} = \mathbf{0}$ $\nabla \cdot \mathbf{E} = \frac{q}{\epsilon_0}$ $\nabla \cdot \mathbf{B} = 0$

1. Flexible and extensible multiphysics PDE solver framework (**Drekar**) can accommodate arbitrary numbers of species.
2. Nodal FE or structure-preserving FE discretizations (edge, face, etc.).
3. Stable and accurate AFC local bounds preserving CG FE methods.
4. IMEX capabilities for handling disparate time scales (eg., large plasma/cyclotron freq., EM CFL, etc.).
5. Robust and scalable linear and nonlinear solvers for CG/structure preserving methods
   - ⇒ Newton-Krylov with physics-based block decomposition preconditioning; enabling optimal H(curl) and H(grad) multilevel subsystem solvers.

Elastic Scattering

$$\mathbf{R}_{s;t} = \alpha_{s;t}\rho_s\rho_t\left(\mathbf{u}_t - \mathbf{u}_s\right)\Phi_{s;t}$$

$$Q_{s;t} = \frac{\alpha_{s;t}\rho_s\rho_t}{m_s + m_t}\left[A_{s;t}k_{\mathrm{B}}\left(T_t - T_s\right)\Psi_{s;t} + m_t\left(\mathbf{u}_t - \mathbf{u}_s\right)^2\Phi_{s;t}\right],$$

- Charge-charge (Coulomb):

$$\alpha_{s;t} = \frac{Z_s^2 Z_t^2 |q_e|^4 \ln\Lambda_{s;t}}{6\pi\sqrt{2\pi}\epsilon_0^2 m_s m_t m_{s;t}\left(k_{\mathrm{B}}T_s/m_s + k_{\mathrm{B}}T_t/m_t\right)^{3/2}},$$

- Charge-neutral/Neutral-neutral:

$$\alpha_{s;t} = \frac{1}{m_s + m_t}\frac{4}{3}\left[\frac{8}{\pi}\left(\frac{k_{\mathrm{B}}T_s}{m_s} + \frac{k_{\mathrm{B}}T_t}{m_t}\right)\right]^{1/2}\sigma_{s;t}.$$

⇒ Using constant cross-sections for now (most computed using hard-sphere approximation, some from QM calculations).

Ionization Rates $\Gamma^{\text{ion}}_{(\alpha,k)} = n_e n_{(\alpha,k)} I_{(\alpha,k)}$

Voronov:[a]

$$I_{(\alpha,k)} = A_{(\alpha,k)} \frac{1 + P_{(\alpha,k)}\sqrt{U_{(\alpha,k)}}}{X_{(\alpha,k)} + U_{(\alpha,k)}} \left(U_{(\alpha,k)}\right)^{K_{(\alpha,k)}} \exp\left(-U_{(\alpha,k)}\right), \qquad U_{(\alpha,k)} = \frac{\phi^{\text{ion}}_{(\alpha,k)}}{T_e}.$$

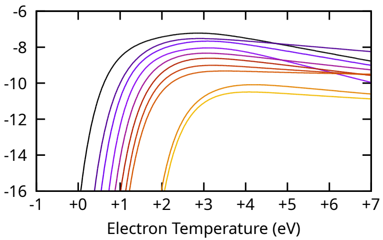
- Available for H to Ni^{27+}.
- Accurate to within 10% for T_e between 1 eV and 20 KeV.

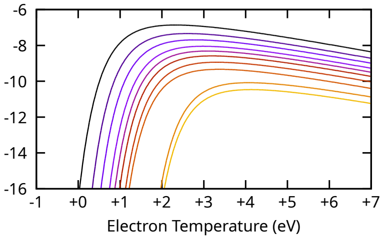
Lotz:[b,c]

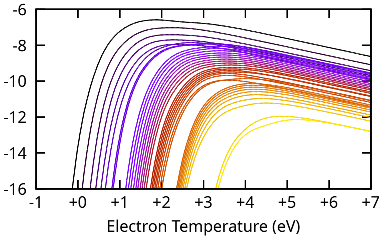
$$I_{(\alpha,k)} = (2.97\text{E}{-6}) \frac{\xi_{(\alpha,k)}}{\phi^{\text{ion}}_{(\alpha,k)}\sqrt{T_e}} E_1\left(U_{(\alpha,k)}\right),$$

- $\xi_{(\alpha,k)}$ is the number of outer electrons in the ionizing atom.
- General analytic model for any atomic species.
- Compatible with ionization potential depression models.

Others: Other sources for specific higher Z elements; eg., Mattioli, et al.[d] for Kr.







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[b] W. Lotz, *Electron-impact ionization cross-sections and ionization rate coefficients for atoms and ions from hydrogen to calcium*, Zeitschrift für Physik, 216 (1968), pp. 241–247.

[c] W. Lotz, *Electron-impact ionization cross-sections and ionization rate coefficients for atoms and ions from scandium to zinc*, Zeitschrift für Physik, 220 (1969), pp. 466–472.

[d] M. Mattioli, G. Mazzitelli, K. B. Fournier, M. Finkenthal, and L. Carraro, *Updating of atomic data needed for ionization balance evaluations of krypton and molybdenum*, Journal of Physics B: Atomic, Molecular and Optical Physics, 39 (2006), pp. 4457–4489.

Radiative Recombination Rates $\Gamma^{\mathrm{rec}}_{(\alpha,k)} = n_e n_{(\alpha,k)} \left(R^{\mathrm{rad}}_{(\alpha,k)} + R^{\mathrm{die}}_{(\alpha,k)} \right)$

Badnell, et al.:[a]

$$R^{\mathrm{rad}}_{(\alpha,k)} = A_{(\alpha,k)} \left[\sqrt{T_e / T_0^{(\alpha,k)}} \left(1 + \sqrt{T_e / T_0^{(\alpha,k)}} \right)^{1-D_{(\alpha,k)}} \left(1 + \sqrt{T_e / T_1^{(\alpha,k)}} \right)^{1+D_{(\alpha,k)}} \right]^{-1},$$

$$D_{(\alpha,k)} = B_{(\alpha,k)} + C_{(\alpha,k)} \exp\left(-T_2^{(\alpha,k)} / T_e \right)$$

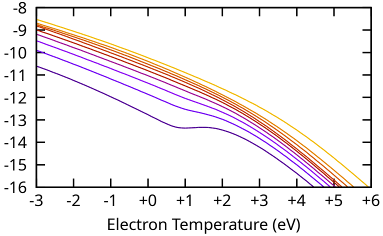
- Available for H through Zn, plus Kr, Mo, Xe.
- Fits of calculated data from AUTOSTRUCTURE code.

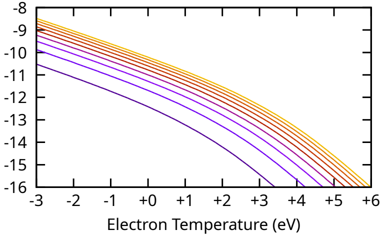
Kotelnikov, et al.:[b]

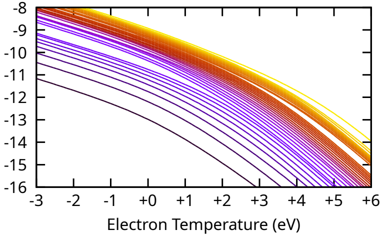
$$R^{\mathrm{rad}}_{(\alpha,k)} = \frac{8.414 k \alpha^4 c a_0^2 \left[\ln(1+\lambda) + 3.499 \right]}{(1/\lambda)^{1/2} + 0.6517\,(1/\lambda) + 0.2138\,(1/\lambda)^{3/2}}, \qquad \lambda = \frac{h R_\infty c k^2}{k_{\mathrm{B}} T_e}$$

- Generic hydrogenic approximation.
- Valid in both high- and low-temperature limits.

Others: Other sources for specific higher Z elements; eg., Mattioli, et al.[c] for Kr.







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[b] I. A. Kotelnikov and A. I. Milstein, *Electron radiative recombination with a hydrogen-like ion*, Physica Scripta, 94 (2019), p. 055403.

[c] M. Mattioli, G. Mazzitelli, K. B. Fournier, M. Finkenthal, and L. Carraro, *Updating of atomic data needed for ionization balance evaluations of krypton and molybdenum*, Journal of Physics B: Atomic, Molecular and Optical Physics, 39 (2006), pp. 4457–4489.

Dielectronic Recombination Rates $\Gamma^{\mathrm{rec}}_{(\alpha,k)} = n_e n_{(\alpha,k)} \left(R^{\mathrm{rad}}_{(\alpha,k)} + R^{\mathrm{die}}_{(\alpha,k)} \right)$

Badnell, et al.:[a]

$$R^{\mathrm{die}}_{(\alpha,k)} = T_e^{-3/2} \sum_{i=1}^{N_{(\alpha,k)}} c_i^{(\alpha,k)} \exp\left(-E_i^{(\alpha,k)} / T_e\right)$$

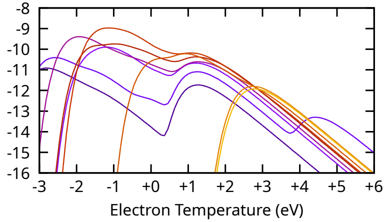
- Available by isoelectronic sequence, through Si sequence.
- Fits of calculated data from AUTOSTRUCTURE code.

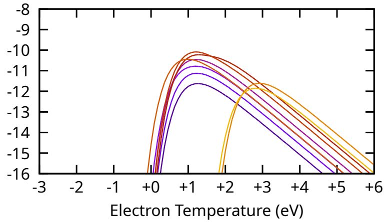
Landini, et al.:[b]

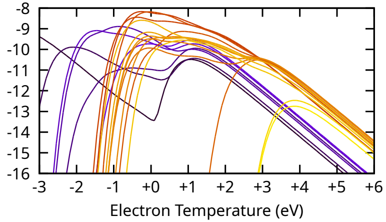
$$R^{\mathrm{die}}_{(\alpha,k)} = A_{(\alpha,k)} T_e^{-3/2} \exp\left(-T_0^{(\alpha,k)} / T_e\right) \left(1 + B_{(\alpha,k)} \exp\left(-T_1^{(\alpha,k)} / T_e\right)\right)$$

- Less resolved, but data available for a wider range of species (eg., low charge states of Ar).

Others: Other sources for specific higher Z elements; eg., Mattioli, et al.[c] or Sterling[d] for Kr.







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[a] N. R. Badnell, M. G. O'Mullane, H. P. Summers, Z. Altun, M. A. Bautista, J. Colgan, T. W. Gorczyca, D. M. Mitnik, M. S. Pindzola, and O. Zatsarinny, *Dielectronic recombination data for dynamic finite-density plasmas: I. Goals and methodology*, Astronomy & Astrophysics, 406 (2003), pp. 1151–1165.

[b] M. Landini and B. C. Monsignori Fossi, *The X-UV spectrum of thin plasmas*, Astronomy & Astrophysics Supplement Series, 82 (1990), pp. 229–260.

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[d] N. C. Sterling, *Atomic data for neutron-capture elements II. photoionization and recombination properties of low-charge krypton ions*, Astronomy & Astrophysics, 533 (2011), p. A62.

Verification: Two-Species Damped Oscillation (Constant coefficients)

Two-fluid system with no spatial gradients:

$$\partial_t \left(\rho_s \mathbf{u}_s\right) = q_s n_s \left(\mathbf{E} + \mathbf{u}_s \times \mathbf{B}\right) + \alpha_{s;t} \rho_s \rho_t \left(\mathbf{u}_t - \mathbf{u}_s\right)$$

$$\partial_t \mathcal{E}_s = q_s n_s \mathbf{u}_s \cdot \mathbf{E} + \frac{\alpha_{s;t} \rho_s \rho_t}{m_s + m_t} \left[A_{s;t} k_{\mathrm{B}} \left(T_t - T_s\right) + m_t \left(\mathbf{u}_t - \mathbf{u}_s\right)^2\right]$$

$$\partial_t \mathbf{E} = -\frac{1}{\varepsilon_0} \mathbf{J}$$

Analytic solution can be derived if $\alpha_{s;t}$ is constant:

$$\mathbf{u}_e(t) - \mathbf{u}_i(t) = \left(\mathbf{u}_e(t_0) - \mathbf{u}_i(t_0)\right) \exp\left(-\frac{\nu_{e;i} t}{2}\right) \frac{\cos\left(\eta_{e;i} t - \phi_{e;i}\right)}{\cos \phi_{e;i}}$$

$$\mathbf{E}(t) = -\left(\mathbf{u}_e(t_0) - \mathbf{u}_i(t_0)\right) \frac{q_e n_i}{\epsilon_0} \frac{\exp\left(-\nu_{e;i} t/2\right)}{\omega_{e;i}^2 \cos \phi_{e;i}} \left[\eta_{e;i} \sin\left(\eta_{e;i} t - \phi_{e;i}\right) - \frac{\nu_{e;i}}{2} \cos\left(\eta_{e;i} t - \phi_{e;i}\right) + \exp\left(\nu_{e;i} t/2\right) \left(\frac{\nu_{e;i}}{2} \cos \phi_{e;i} + \eta_{e;i} \sin \phi_{e;i}\right)\right]$$

$$T_e(t) - T_i(t) = \exp\left(-A_{e;i} t\right) C_{e;i} + \frac{D_{e;i} \exp\left(-\nu_{e;i} t\right)}{\cos^2 \phi_{e;i}} \left[4\eta_{e;i}^2 + 2G_{e;i}^2 \cos^2\left(\eta_{e;i} t - \phi_{e;i}\right) + 4\eta_{e;i} G_{e;i} \sin\left(\eta_{e;i} t - \phi_{e;i}\right) \cos\left(\eta_{e;i} t - \phi_{e;i}\right)\right]$$

Equilibrium velocity and temperature can be computed by conservation of momentum and energy (ideal gas):

$$\mathbf{u}_* = \frac{\sum_s \rho_s \mathbf{u}_s}{\sum_s \rho_s}, \qquad T_* = \frac{\sum_s \mathcal{E}_s - \frac{1}{2} \mathbf{u}_*^2 \sum_s \rho_s}{\frac{1}{\gamma - 1} k_{\mathrm{B}} \sum_s n_s}.$$

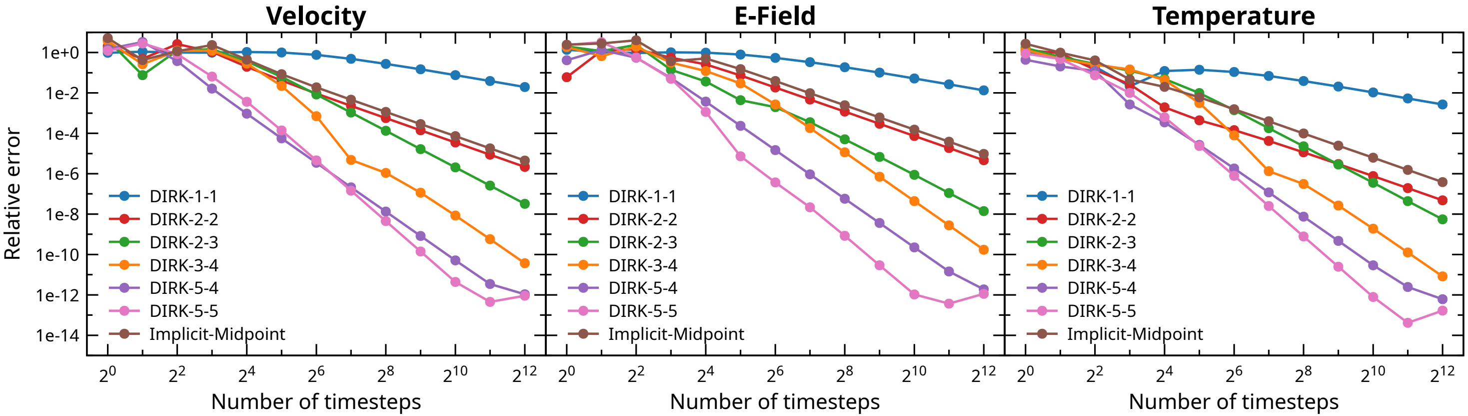
⇒ Compare Drekar and ODE solver (Python odeint) with analytic solution and equilibrium values.

Verification: Two-Species Damped Oscillation (Constant coefficients)

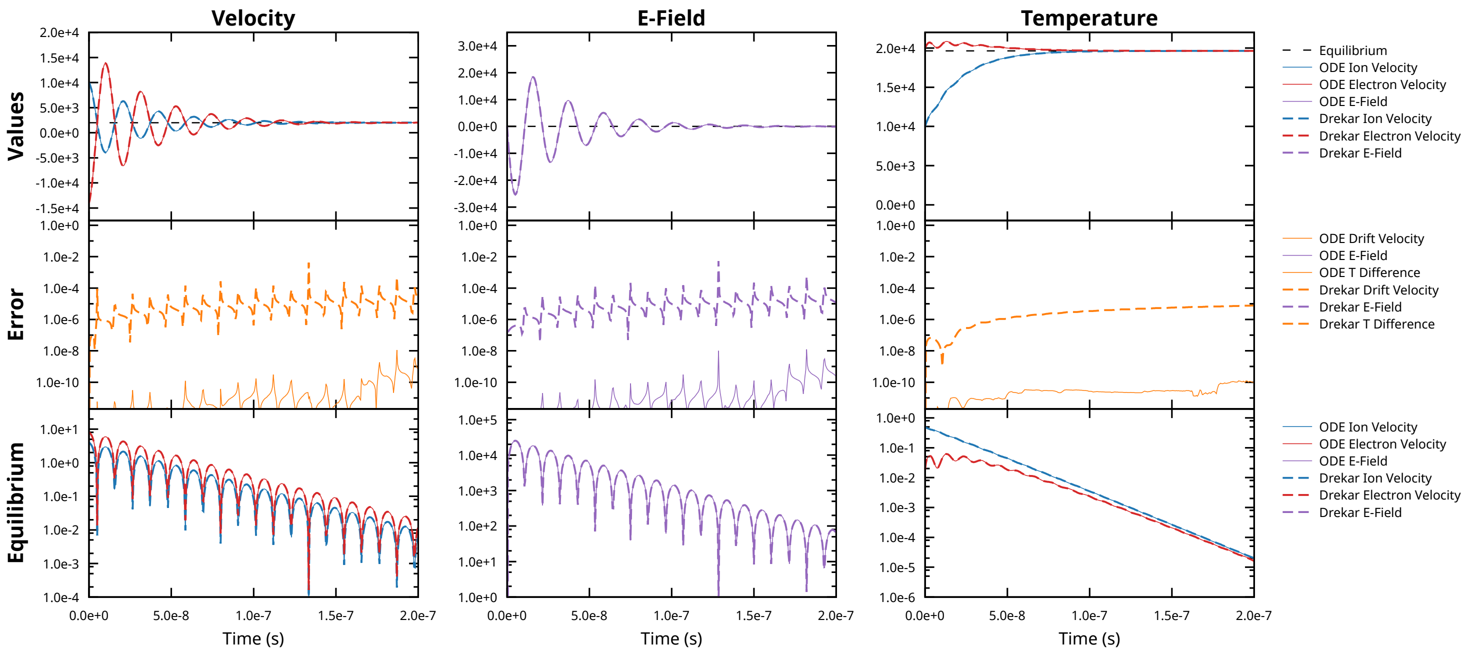
$$\alpha_{e;i} \equiv 1.0\text{E}{+}18$$

	Ion (i)	Electron (e)
m_s	2.0E-27	1.0E-27
T_s	1.0E+4	2.0E+4
$\mathbf{u}_s$	$\begin{bmatrix} 1.0\text{E}{+}4 \\ 0.0 \\ 0.0 \end{bmatrix}$	$\begin{bmatrix} -1.4\text{E}{+}4 \\ 0.0 \\ 0.0 \end{bmatrix}$
n_s	2.0E+16	2.0E+16

	Ion (i)	Electron (e)
ρ_s	4.0E-11	2.0E-11
$\rho_s \mathbf{u}_s$	$\begin{bmatrix} 4.0\text{E}{-}7 \\ 0.0 \\ 0.0 \end{bmatrix}$	$\begin{bmatrix} -2.8\text{E}{-}7 \\ 0.0 \\ 0.0 \end{bmatrix}$
$\rho_s e_s$	4.1430E-3	8.2860E-3
$\mathcal{E}_s$	6.1430E-3	1.0246E-2



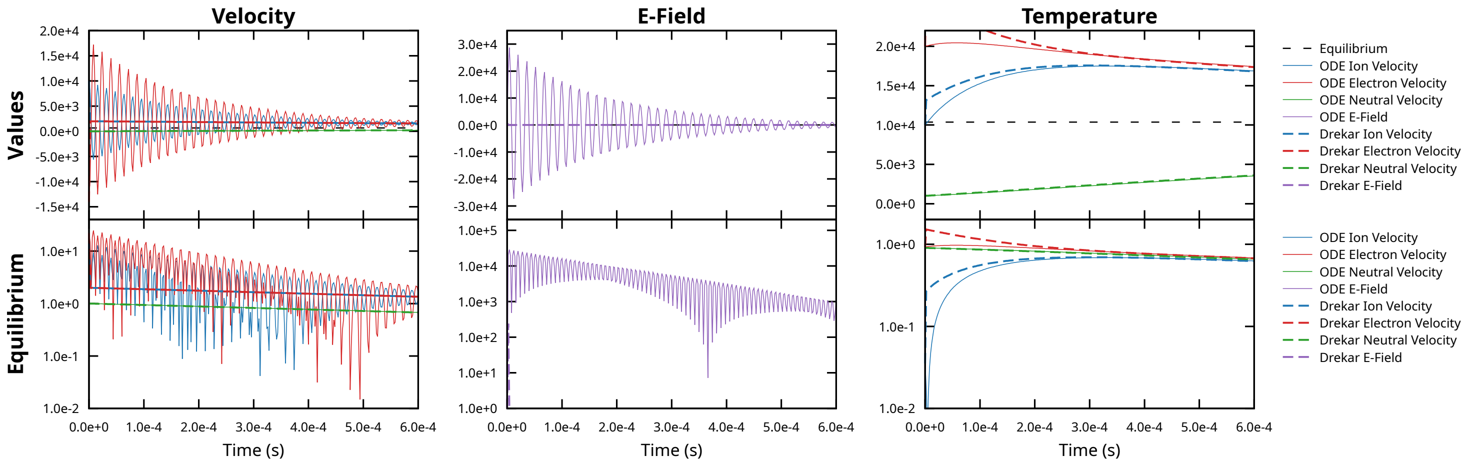
Verification: Two-Species Damped Oscillation (Constant coefficients)



Verification: Three-Species Damped Oscillation (Physical coefficients)

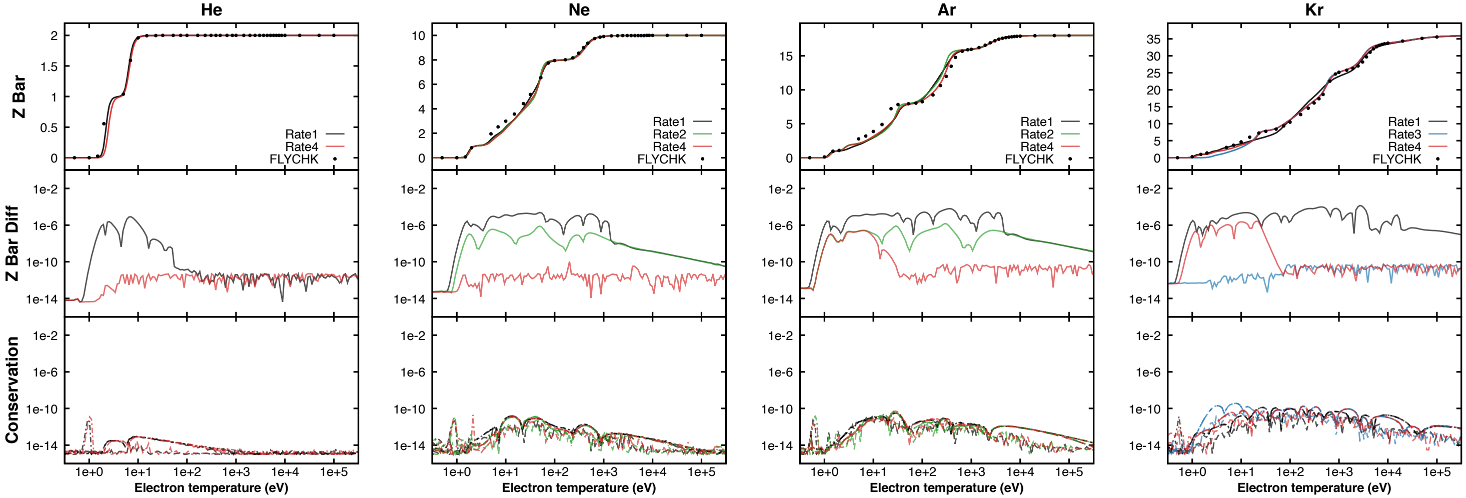
	Neutral (n)	Ion (i)	Electron (e)
m_s	3.0E-27	2.0E-27	1.0E-27
T_s	1.0E+3	1.0E+4	2.0E+4
$\mathbf{u}_s$	$\begin{bmatrix}0.0\\0.0\\0.0\end{bmatrix}$	$\begin{bmatrix}1.0\text{E}{+}4\\0.0\\0.0\end{bmatrix}$	$\begin{bmatrix}-1.4\text{E}{+}4\\0.0\\0.0\end{bmatrix}$
n_s	4.0E+16	2.0E+16	2.0E+16

	Neutral (n)	Ion (i)	Electron (e)
ρ_s	1.2E-10	4.0E-11	2.0E-11
$\rho_s \mathbf{u}_s$	$\begin{bmatrix}0.0\\0.0\\0.0\end{bmatrix}$	$\begin{bmatrix}4.0\text{E}{-}7\\0.0\\0.0\end{bmatrix}$	$\begin{bmatrix}-2.8\text{E}{-}7\\0.0\\0.0\end{bmatrix}$
$\rho_s e_s$	8.2860E-4	4.1430E-3	8.2860E-3
$\mathcal{E}_s$	8.2860E-4	6.1430E-3	1.0246E-2



Verification: Ionization/Recombination

- Equilibrium ion fractions computed by linear solve.
- Compare long-time integration (with non-equilibrium initial condition) to equilibrium $\bar{z}$.
- Long-time solution recovers equilibrium values.
- Conservation of total mass and momentum is also good.
- Rate coefficients give equilibrium values comparable to FLYCHK library.



CG Finite Element Discretization

Hyperbolic system for each fluid:

$$\partial_t \mathbf{U}_s + \nabla \cdot \mathbf{F}_s(\mathbf{U}_s) = \mathbf{S}_s(\mathbf{U}, \mathbf{E}, \mathbf{B}), \qquad \mathbf{U}_s = \left(\rho_s, \rho_s \mathbf{u}_s, \mathcal{E}_s\right)^T.$$

Semi-discrete scheme:

$$\mathbf{M}_{\mathrm{C}} \partial_t \hat{\mathbf{U}}_s + \mathbf{K}_s(\hat{\mathbf{U}}_s) + \mathbf{B}_s(\hat{\mathbf{U}}_s) + \mathbf{S}_s(\hat{\mathbf{U}}, \hat{\mathbf{E}}, \hat{\mathbf{B}}) = \mathbf{0},$$

where

$$\mathbf{M}_{\mathrm{C}} = \left\{M_{k,\ell} = m_{k,\ell} I_{N \times N}\right\}_{k,\ell} \qquad m_{k,\ell} = \int_\Omega \phi_k \phi_\ell \, d\mathbf{x}$$

$$\mathbf{K}_s(\hat{\mathbf{U}}_s) = \left\{\mathbf{k}_{s,k}\right\}_k \qquad \mathbf{k}_{s,k} = -\int_\Omega \nabla \phi_k \cdot \mathbf{F}_s(\mathbf{U}_s) \, d\mathbf{x}$$

$$\mathbf{B}_s(\hat{\mathbf{U}}_s) = \left\{\mathbf{b}_{s,k}\right\}_k \qquad \mathbf{b}_{s,k} = \int_\Gamma \phi_k \mathbf{F}_s(\mathbf{U}_s) \cdot \mathbf{n} \, d\Gamma$$

$$\mathbf{S}_s(\hat{\mathbf{U}}, \hat{\mathbf{E}}, \hat{\mathbf{B}}) = \left\{\mathbf{S}_{s,k}\right\}_k \qquad \mathbf{S}_{s,k} = -\int_\Omega \phi_k \mathbf{S}_s(\mathbf{U}, \mathbf{E}, \mathbf{B}) \, d\mathbf{x}$$

AFC Stabilization

Stabilized system:

$$\mathbf{M}_\mathrm{L}\partial_t\hat{\mathbf{U}}_s + \mathbf{K}_s(\hat{\mathbf{U}}_s) + \mathbf{B}_s(\hat{\mathbf{U}}_s) + \mathbf{S}_s(\hat{\mathbf{U}},\hat{\mathbf{E}},\hat{\mathbf{B}}) + \mathbf{D}_s(\hat{\mathbf{U}}_s)\hat{\mathbf{U}}_s + \alpha_s\mathcal{F}_s = \mathbf{0},$$

where

$$\mathcal{F}_s = -\mathbf{D}_s(\hat{\mathbf{U}}_s)\hat{\mathbf{U}}_s + (\mathbf{M}_\mathrm{C} - \mathbf{M}_\mathrm{L})\,\partial_t\hat{\mathbf{U}}_s \qquad \text{and} \qquad \alpha_s \in [0,1].$$

Question: How to choose $\mathbf{D}_s$, α_s, $\mathcal{F}_s$ for each species?

First try: Stabilize each fluid separately.

- $\mathbf{D}_s$ is constructed based on the sound speed of species s.
- α_s is chosen based on the behavior of species s.

Alternative: Synchronize stabilization between fluids.

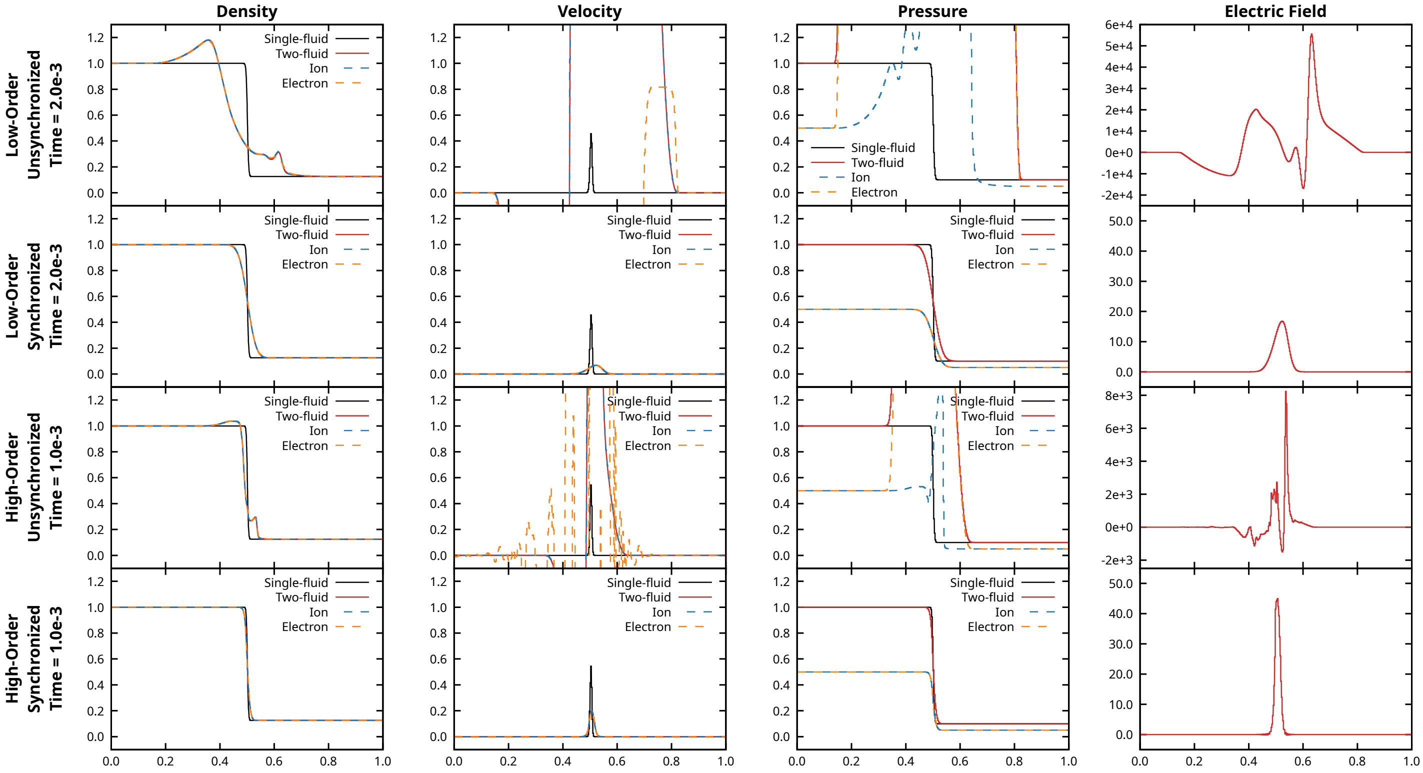
- $\mathbf{D}_s$ is constructed based on the maximum sound speed of *all* species (all $\mathbf{D}_s$ equal).
- α_s is chosen based on the behavior of *all* species (all α_s equal).

Two-Fluid Sod Problem

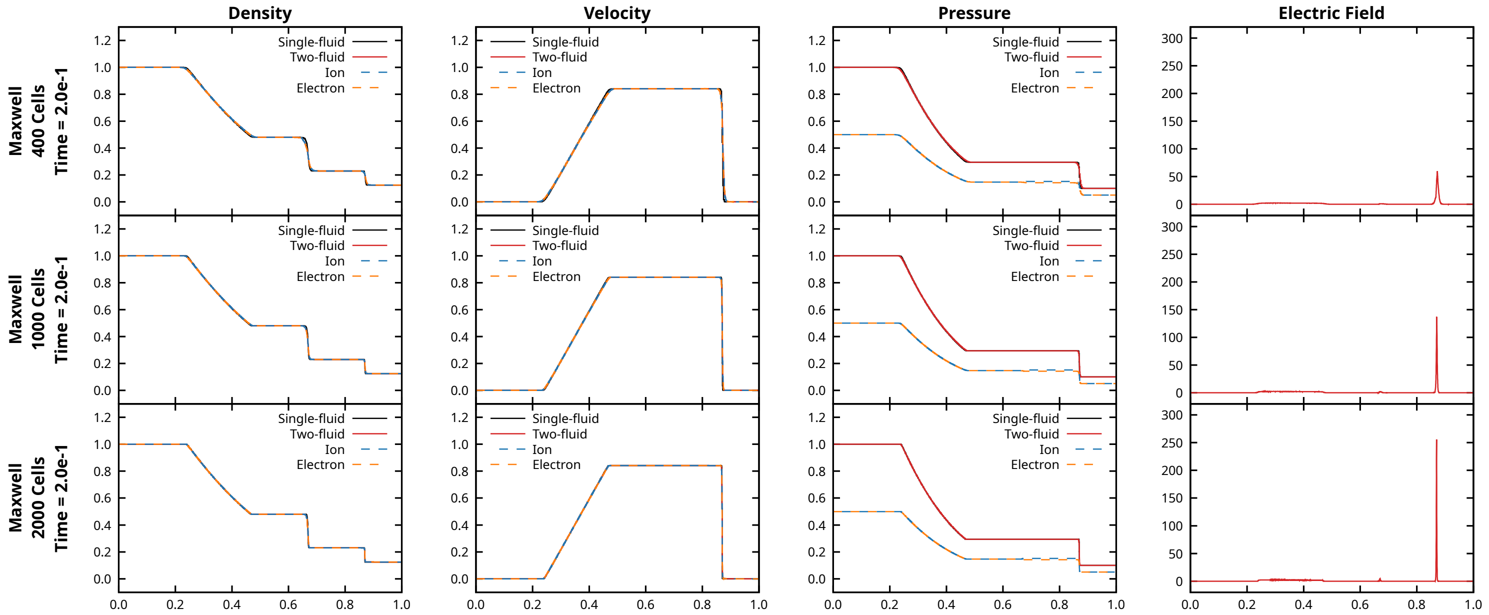
- Position $x \in [0,1]$
- Time $t \in [0, 0.2]$
- Density $\rho = \begin{cases} 1.0, & x < 0.5 \\ 0.125, & x > 0.5 \end{cases}$
- Pressure $p = \begin{cases} 1.0, & x < 0.5 \\ 0.1, & x > 0.5 \end{cases}$
- $\varepsilon =$ Particle mass ratio.
- $\epsilon_0 = \varepsilon^2$
- Two-fluid solution recovers single-fluid behavior as $\varepsilon \to 0$.
- Use $\varepsilon = 1\text{E}{-4}$

Quantity	Electron	Ion
Mass (m_s)	$\frac{\varepsilon}{1+\varepsilon}$	$\frac{1}{1+\varepsilon}$
Density (ρ_s)	$\frac{\varepsilon}{1+\varepsilon}\rho$	$\frac{1}{1+\varepsilon}\rho$
Pressure (p_s)	$p/2$	$p/2$

Two-Fluid Sod Problem: Stabilization (Electrostatic)



Two-Fluid Sod Problem: Electrostatic Mesh Refinement



Divergence Cleaning for Maxwell's Equations

Divergence constraints *must* be adequately satisfied!

$$\nabla \cdot \mathbf{B} = 0, \qquad \epsilon_0 \nabla \cdot \mathbf{E} = q.$$

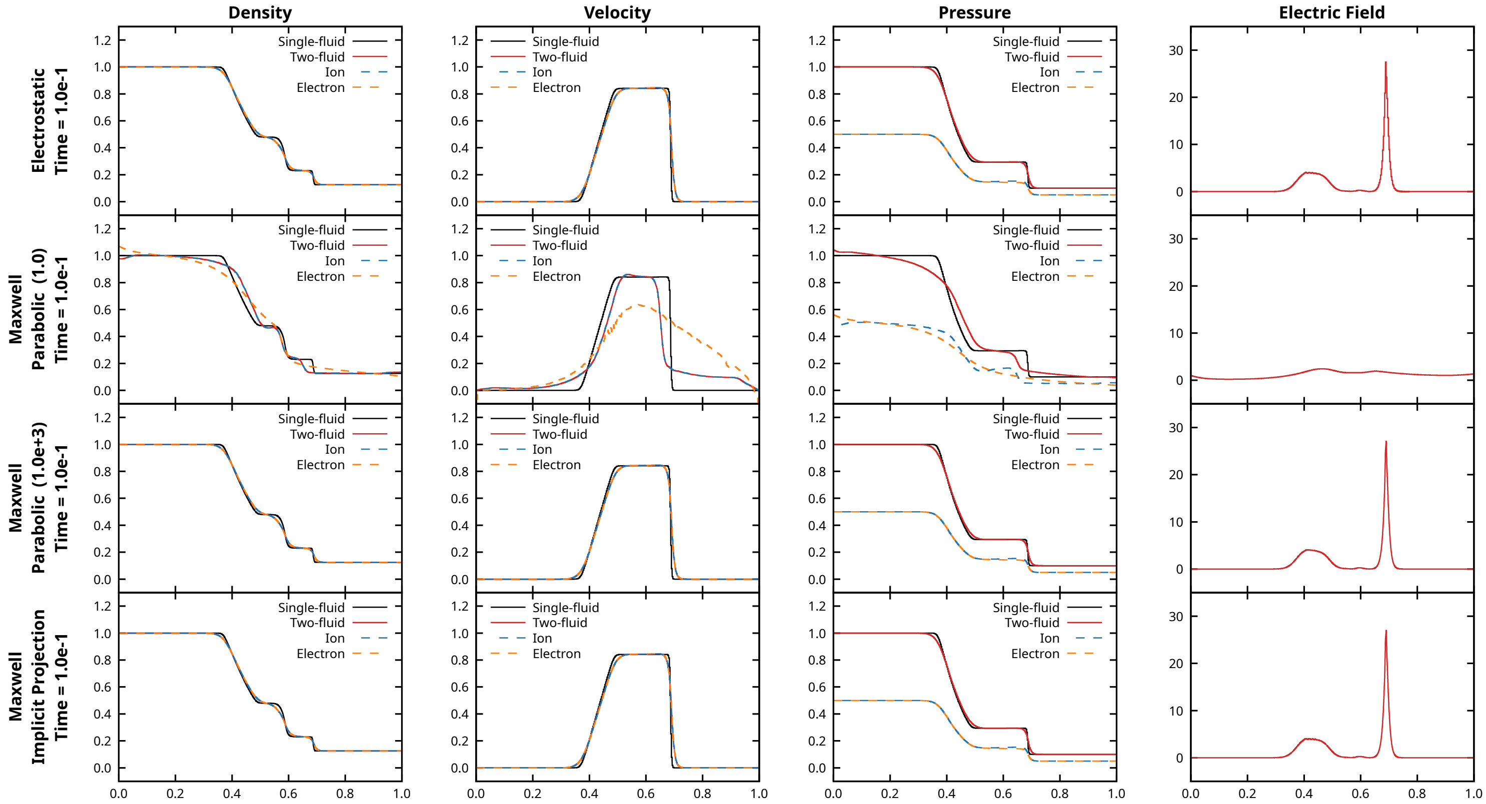
Eliminated Parabolic Cleaning: Add penalty term to Ampere's Law: $\nabla \cdot \left[c_p^2 \left(\epsilon_0 \nabla \cdot \mathbf{E} - q \right) I \right]$

- Requires c_p parameter to be specified.
- c_p must be "large enough".

Implicit Projection: Set $\hat{\mathbf{E}} = \mathbf{E} + \nabla \phi$, where $\epsilon_0 \Delta \phi + \epsilon_0 \nabla \cdot \mathbf{E} - q = 0$.

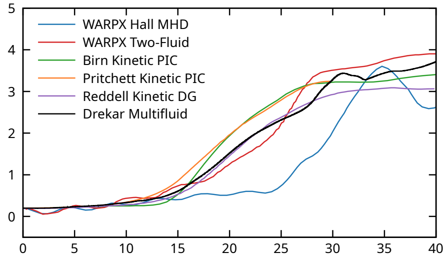
- Requires elliptic solve.
- Controls divergence error to machine/solver precision.
- Boundary conditions can be non-trivial.

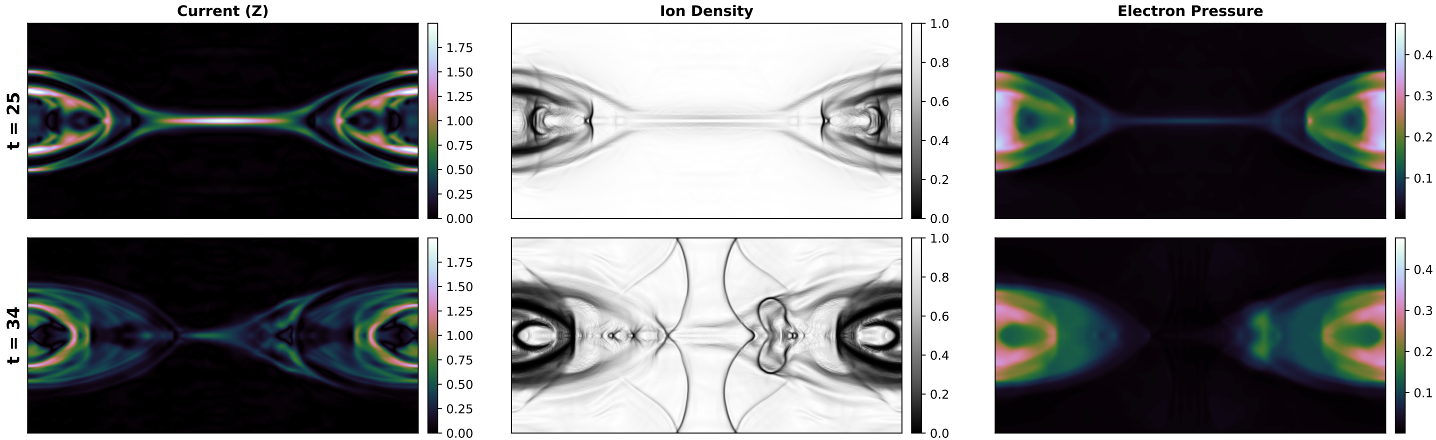
Two-Fluid Sod Problem: Maxwell Divergence Cleaning



GEM Challenge Problem

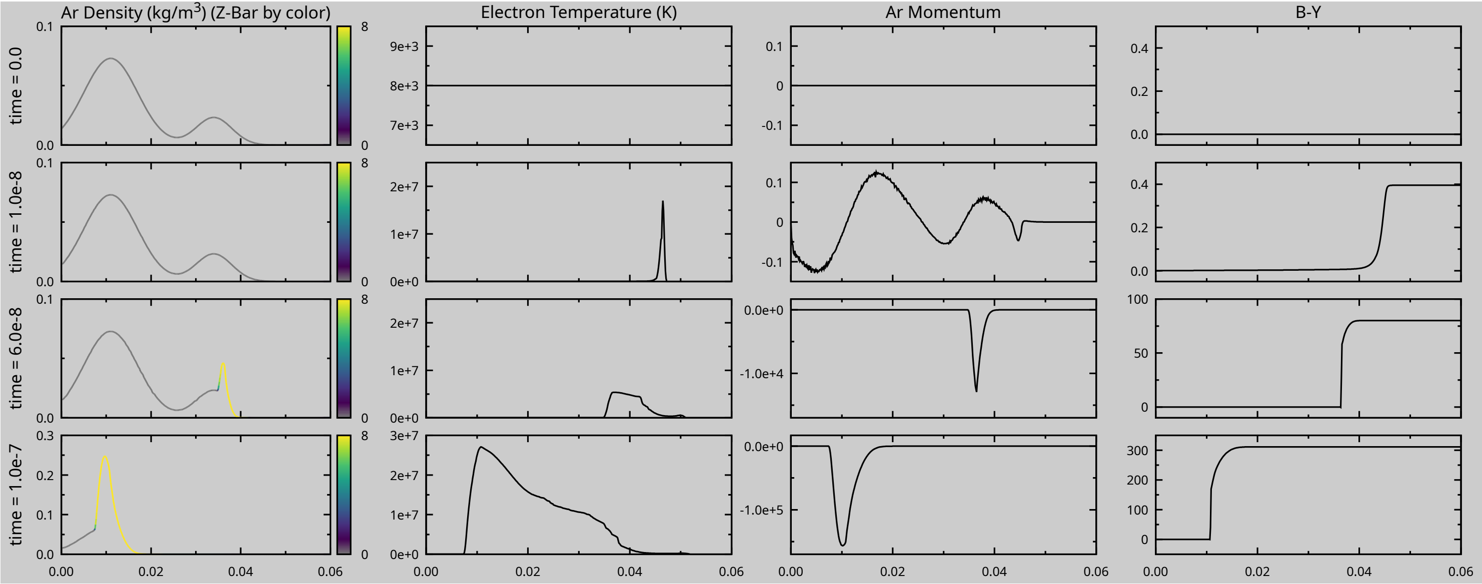
- Test magnetic reconnection of a perturbed current sheet equilibrium.
- Electron/Hall effects are crucial (resistive MHD behaves differently).
- Standard comparison is based on integral of magnetic field.
- Our results compare favorably with kinetic and other two-fluid codes.





1D Argon Gas Puff (Proof of Concept)

- Argon gas ($z = 0$ to 8^+) plus electrons.
- Potential form of Maxwell's equations.
- Driven by EM field applied at the boundary.
- Ionization + collisions yields resistive heating.
- Implicit time integration follows ion fluid CFL.
- Overall behavior is qualitatively reasonable.



Thank you