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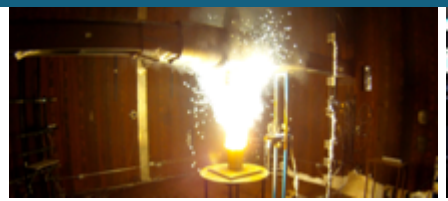
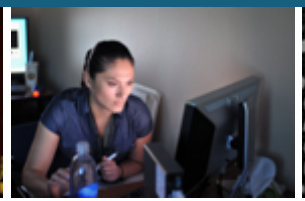
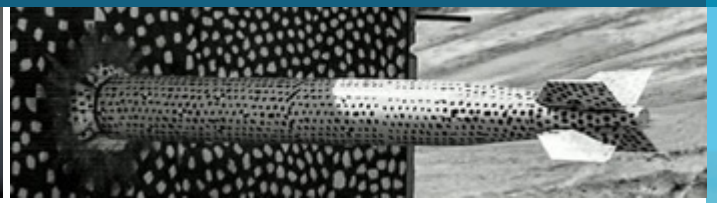
Sandia National Laboratories

Variational, Stable, and Self-Consistent Coupling of 3D Electromagnetics to 1D Transmission Lines







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U.S. DEPARTMENT OF ENERGY | NNSA National Nuclear Security Administration

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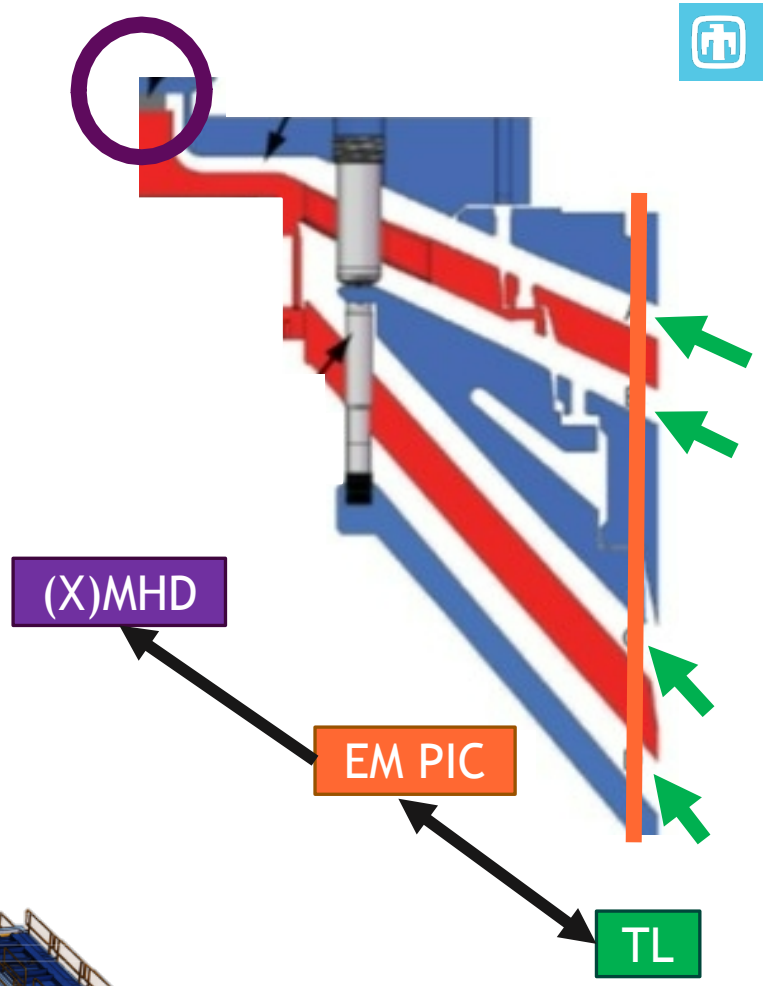
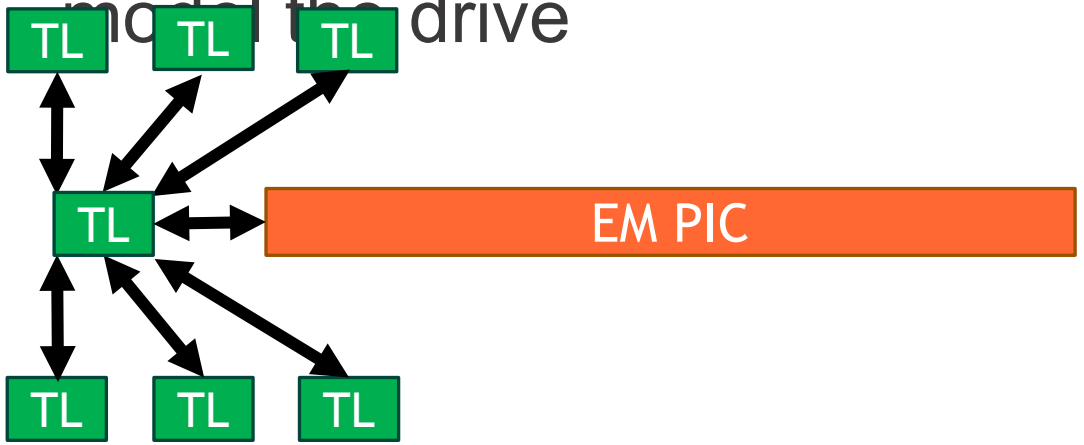
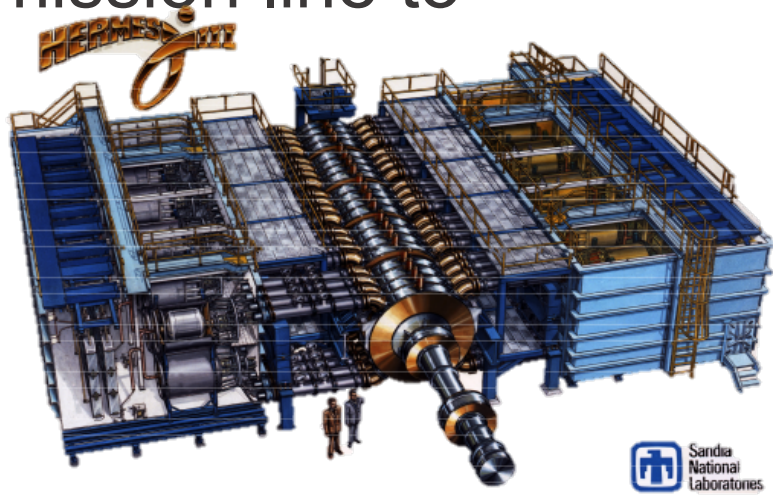
Motivation

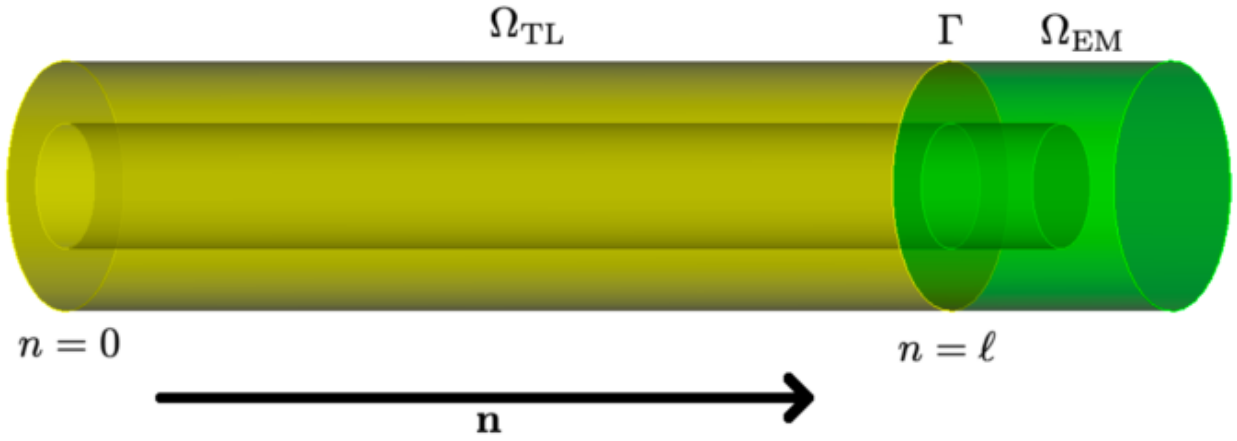
- Many plasma applications (e.g. pulsed power devices) involve kinetic physics in a complex domain driven by relatively simple electromagnetic pulses in simple geometries
- Two-way coupling between the drive and the kinetics is important (changing impedance at interface)
- We can dramatically reduce the cost of such simulations by using a 1D transmission line to model the drive

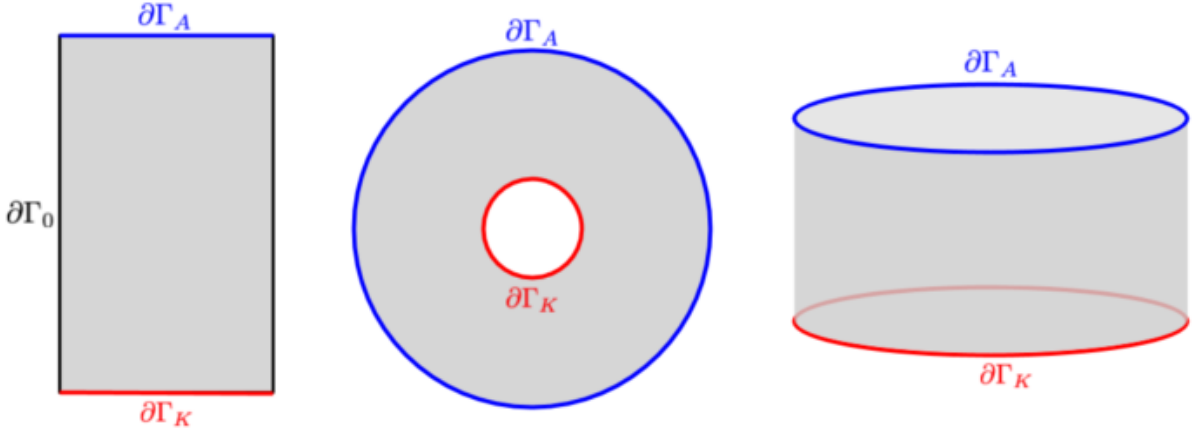






Transmission Line Model for TEM modes





- In a TL, we assume an 2D cross-section geometry is extruded in the normal direction
- TEM mode is computed by a Laplace solve on the cross-section

$$\begin{cases} \mathbf{E}_0 + \mathbf{grad}_{\boldsymbol{\tau}}\, \varphi_0 = 0, \\ \mathrm{div}_{\boldsymbol{\tau}}\, \epsilon_0 \mathbf{E}_0 = 0, \\ \varphi_0|_{\partial\Gamma_K} = 0, \\ \varphi_0|_{\partial\Gamma_A} = 1, \\ \mathbf{E}_0 \cdot \boldsymbol{\xi}|_{\partial\Gamma_0} = 0. \end{cases}$$

- Ansatz for solution in TL domain

$$\mathbf{E} = \mathbf{E}_0(\tau_1, \tau_2) V(n,t), \qquad \mathbf{H} = \mathbf{H}_0(\tau_1, \tau_2) I(n,t), \qquad \mathbf{E}_0 \perp \mathbf{H}_0 \perp \mathbf{n},$$

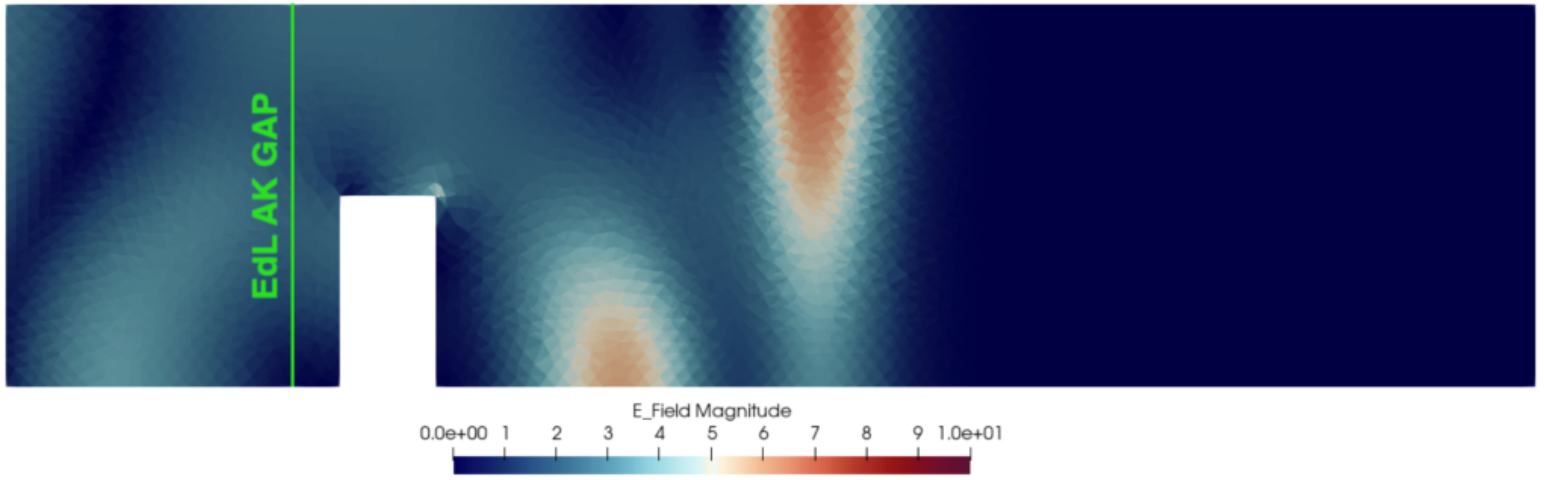
- Maxwell's equations reduce to the 1D telegrapher's equations in the TL domain

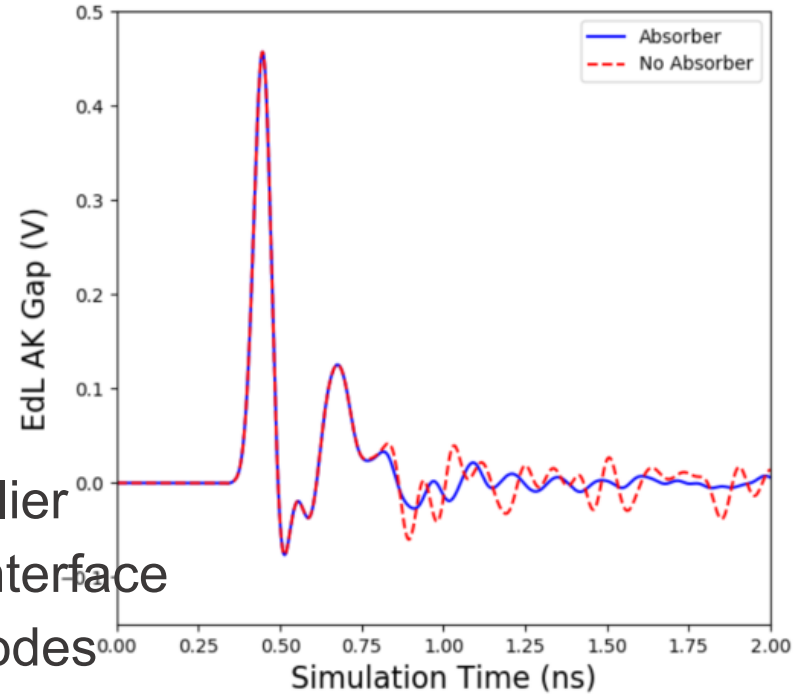
$$\begin{cases} C\dfrac{\partial V}{\partial t} + \dfrac{\partial I}{\partial n} = 0, \\ L\dfrac{\partial I}{\partial t} + \dfrac{\partial V}{\partial n} = 0, \end{cases} \qquad C = \int_\Gamma \epsilon |\mathbf{E}_0|^2 \, da, \qquad CL = \epsilon\mu.$$

- Model extends to non-uniform cross-sections by adjusting capacitance and inductance to geometry

New Features of Our Coupling Algorithm

- Variational – couple through surface integrals – this allows it to apply to arbitrary meshes
- Implicit coupling – introduces no new stability constraints – we developed an efficient linear solver to handle implicit solves
- Self-consistent – coupling is based directly on assumptions made to by the TL model – we enforce continuity of voltage and current at the interface





- We enforce voltage continuity via a constraint and a Lagrange multiplier
- Allows us to apply an additional boundary conditional at the EM/TL interface
- We apply an absorbing BC at the interface that absorbs non-TEM modes
- Reduces unphysical ringing due to reflection of non-TEM modes

Variational Formulation for the Coupled System

$$(\mathbf{E}, \mathbf{B}, \lambda, V, I) \in \mathbf{H}(\mathbf{curl}, \Omega_{\mathrm{EM}}) \times \mathbf{H}(\mathrm{div}, \Omega_{\mathrm{EM}}) \times \mathbb{R} \times H^1(0,\ell) \times L^2(0,\ell) :$$

$$\begin{cases}
\displaystyle\int_{\Omega_{\mathrm{EM}}} \left(\epsilon \frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{\Psi} + \mathbf{J} \cdot \mathbf{\Psi} - \mu^{-1} \mathbf{B} \cdot \mathbf{curl}\, \mathbf{\Psi} \right) \, dv - \int_{\Gamma} \left[(Z^{-1}(\mathrm{I} - \Pi_{\mathrm{TEM}})(\mathbf{E}) + \lambda \epsilon \mathbf{E}_0) \times \mathbf{n} \right] \cdot (\mathbf{\Psi} \times \mathbf{n}) \, da, \\
\displaystyle\int_{\Omega_{\mathrm{EM}}} \left(\frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{\Phi} + \mathbf{curl}\, \mathbf{E} \cdot \mathbf{\Phi} \right) \, dv = 0, \\
\displaystyle\int_{\Gamma} \epsilon \, (\mathbf{E} \times \mathbf{n}) \cdot (\mathbf{E}_0 \times \mathbf{n}) \, da = CV(\ell), \\
\displaystyle\int_0^{\ell} \left(C \frac{\partial V}{\partial t} \psi - I \frac{\partial \psi}{\partial n} \right) \, dn + I_{\mathrm{EM}} \psi(\ell) = \frac{V_{\mathrm{OC}} - V(0)}{R_g} \psi(0), \\
\displaystyle\int_0^{\ell} \left(L \frac{\partial I}{\partial t} \varphi + \frac{\partial V}{\partial n} \varphi \right) \, dn = 0,
\end{cases}$$

$$\forall (\mathbf{\Psi}, \mathbf{\Phi}, \psi, \varphi) \in \mathbf{H}(\mathbf{curl}, \Omega_{\mathrm{EM}}) \times \mathbf{H}(\mathrm{div}, \Omega_{\mathrm{EM}})$$

Maxwell's equations

Open circuit BC

Absorbing BC for non-TEM modes

Current continuity + energy stability (I_{EM} comes from residual of Ampere's law)

$$I_{\mathrm{EM}} = \int_{\Omega_{\mathrm{EM}}} \left(\epsilon \frac{\partial \mathbf{E}}{\partial t} \cdot \overline{\mathbf{E}}_0 + \mathbf{J} \cdot \overline{\mathbf{E}}_0 - \mu \mathbf{B} \cdot \mathbf{curl}\, \overline{\mathbf{E}}_0 \right) \, dv.$$

Linear Solver for the Coupled System

- Fully coupled linear system

$$\mathcal{A}\mathbf{x} = \begin{pmatrix} \mathcal{A}_M & f & g\Pi_{V_s} \\ f^t & 0 & -C\Pi_{V_s} \\ \Pi_{V_s}^t k^t & 0 & \mathcal{A}_{TL} \end{pmatrix} \begin{pmatrix} \mathbf{x}_M \\ \lambda \\ \mathbf{x}_{TL} \end{pmatrix}$$

- Exact block LU decomposition

$$\mathcal{A} = \begin{pmatrix} \mathcal{A}_M & 0 & 0 \\ f^t & s_\lambda & 0 \\ \Pi_{V_s}^t k^t & -\Pi_{V_s}^t k^t \mathcal{A}_M^{-1} f & \mathcal{S}_{TL} \end{pmatrix} \begin{pmatrix} \mathcal{I} & \mathcal{A}_M^{-1} f & \mathcal{A}_M^{-1} g\Pi_{V_s} \\ 0 & 1 & (-C - f^t \mathcal{A}_M^{-1} g) s_\lambda^{-1} \Pi_{V_s} \\ 0 & 0 & \mathcal{I} \end{pmatrix}$$

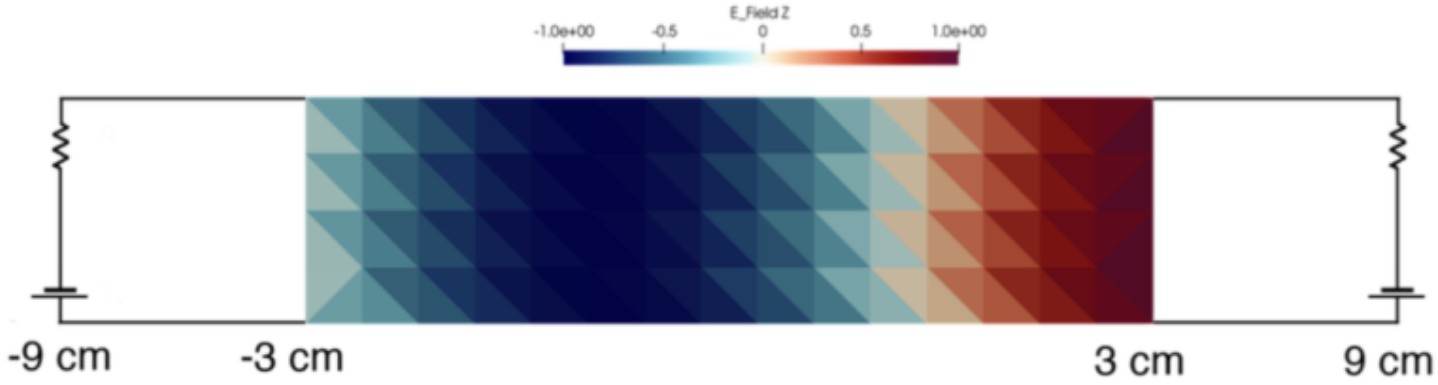
$$s_\lambda = -f^t \mathcal{A}_M^{-1} f,$$

$$\mathcal{S}_{TL} = \mathcal{A}_{TL} - [k^t \mathcal{A}_M^{-1} g + k^t \mathcal{A}_M^{-1} f (C + f^t \mathcal{A}_M^{-1} g) s_\lambda^{-1}] \Pi_{V_s}^t \Pi_{V_s}$$

- Exact inverse requires 1 EM solve plus 2 for each TL whenever the time-step changes
- An augmented TL system solve (S_{TL}) for each TL (assumed small compared to EM solve – we do a small direct solve on each MPI rank)
- For fixed time-step simulations, the cost per time-step is essentially the same as solving EM and TL decoupled

Verification: O-Wave between Parallel Plates

- Simulating a transient O-wave where the middle of the domain is 3D EM and the sides are 1D TLs



(a) CFL varies from $(4.35, 7.53)$

p.p.w.	p.p.p.	$\mathscr{E}_E$	rate	$\mathscr{E}_B$	rate
21.8	2.5	1.43	–	1.64	–
43.6	5	8.24e-1	0.79	1.10	0.69
87.2	10	1.99e-1	2.04	2.56e-1	1.98
174.5	20	4.99e-2	1.99	6.11e-2	2.06

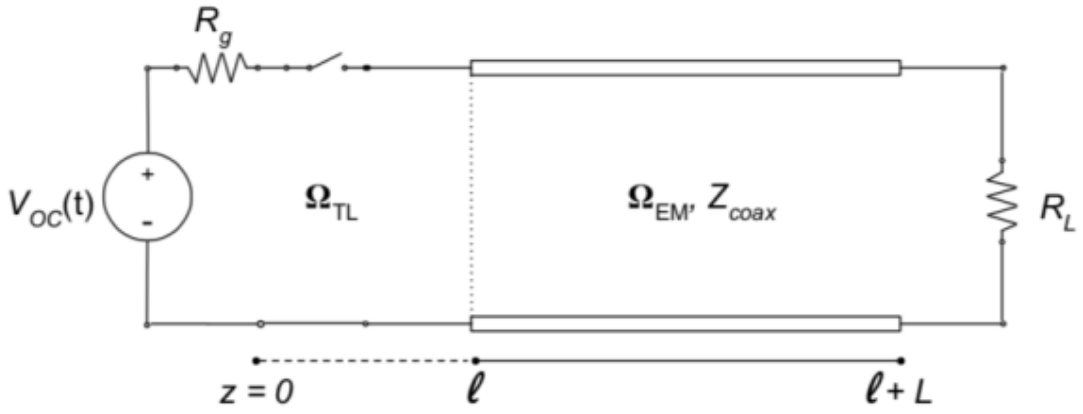
(b) CFL varies from $(0.54, 0.94)$

p.p.w.	p.p.p.	$\mathscr{E}_E$	rate	$\mathscr{E}_B$	rate
21.8	10	7.84e-2	–	7.06e-2	–
43.6	20	3.37e-2	1.21	2.41e-2	1.55
87.2	40	1.61e-2	1.07	1.03e-2	1.22
174.5	80	7.97e-3	1.01	5.16e-3	0.99

- Using implicit midpoint time-stepping (second order) and first order conforming finite elements for the EM
- Large CFL – error is dominated by time discretization – obtain expected second order convergence
- Small CFL – error is dominated by spatial discretization – obtain expected first order convergence

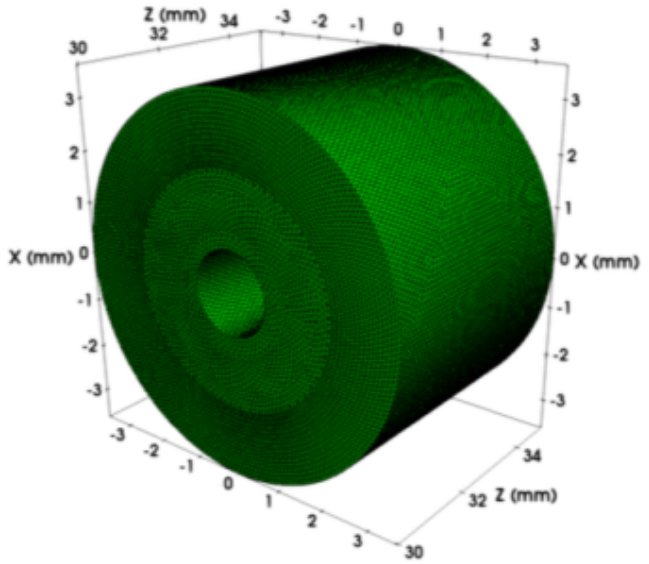
Verification: Coaxial Waveguide

- Coaxial waveguide driven by an equivalent circuit
- Obtained first order convergence to analytic steady-state solution



(a) 1D circuit representation of $\Omega_{TL} \cup \Omega_{EM}$

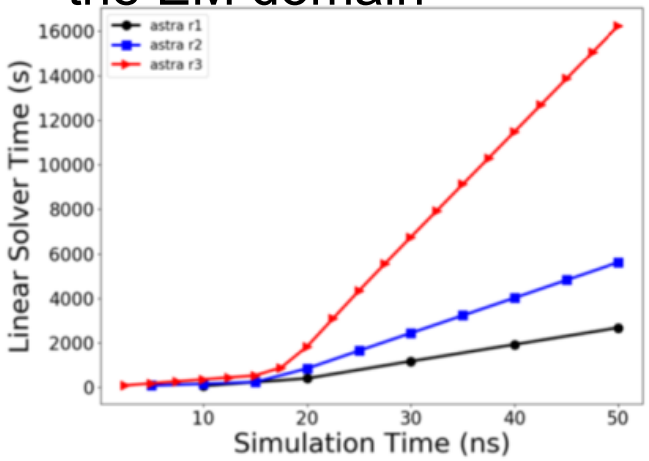
h (m)	$\frac{\text{Gap}}{h}$	$\mathscr{E}_E$	rate	$\mathscr{E}_B$	rate
$1.00e-03$	5.70	$7.89e-02$	–	$5.01e-02$	–
$5.99e-04$	9.70	$7.30e-02$	0.15	$4.70e-02$	0.12
$4.64e-04$	12.3	$6.77e-02$	0.29	$4.49e-02$	0.17
$3.59e-04$	15.9	$5.57e-02$	0.76	$3.64e-02$	0.82
$2.78e-04$	20.5	$4.21e-02$	1.09	$2.67e-02$	1.20
$2.15e-04$	26.5	$3.25e-02$	1.00	$2.03e-02$	1.07
$1.67e-04$	34.2	$2.52e-02$	1.00	$1.55e-02$	1.05
$1.29e-04$	44.3	$1.91e-02$	1.08	$1.17e-02$	1.07
$1.00e-04$	57.0	$1.45e-02$	1.06	$8.96e-03$	1.06

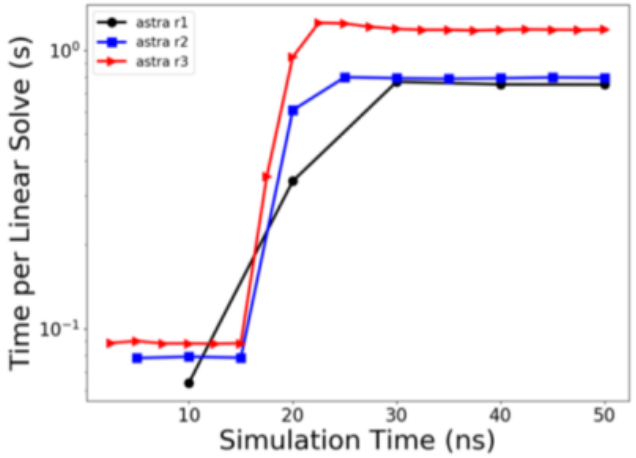


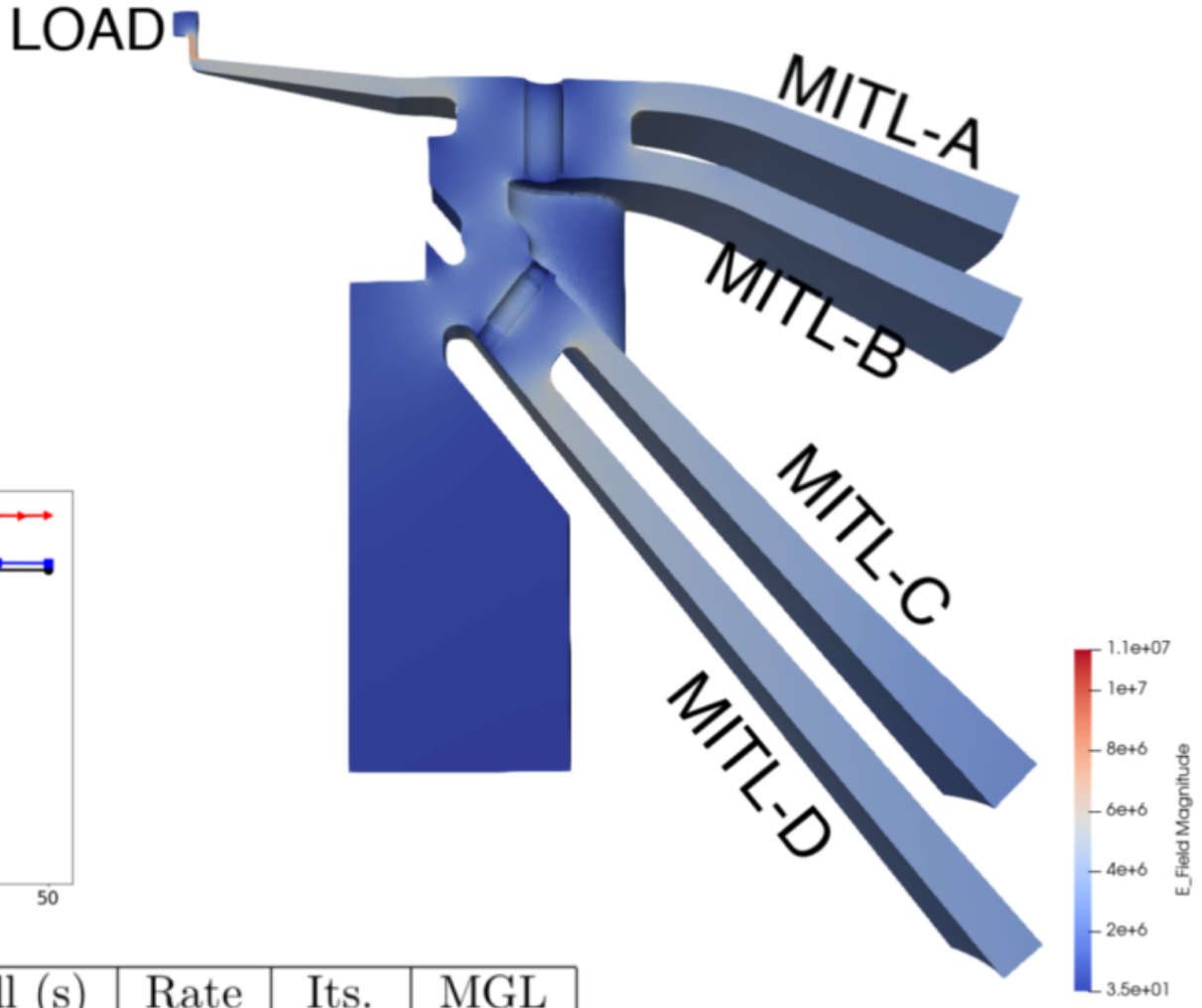
(b) 3D domain Ω_{EM}, 0.1 mm mesh

Demonstration: 18a Convolute

- EM simulation on 18a geometry driven by 4 TLs equivalent to BERTHA model
- Have demonstrated good agreement with CHICAGO for same problem elsewhere
- Coupled linear solver scales similarly to EM alone
- Early time-steps are cheap until pulses reach the EM domain







Ref.	Els.	Rate	Steps	CFL DR	Nodes	Wall (s)	Rate	Its.	MGL
1	2.14 M	–	5000	19.7	2	2667	–	23.4	2
2	19.3 M	9.01	10000	19.6	16	5614	2.10	18.4	3
3	154.7 M	8.0	20000	23.3	128	16241	2.89	27.9	4

Future Work

- Publication has been submitted: D. McGregor, E. Phillips, D. Sirajuddin, T. Pointon, “Variational, Stable, and Self-Consistent Coupling of 3D Electromagnetics to 1D Transmission Lines in the Time Domain, ” submitted to JCP

- Many extensions based on same coupling methodology:
  - Coupling through TE and TM modes in addition to TEM modes
  - Capturing plasma effects in the TL itself (1D MITL models coupled to 3D PIC)
  - Using TLs to couple different physics (power flow to target coupling)
  - General networks of transmission lines (network connected to multiple surfaces)
  - Coupling EM to arbitrary nonlinear circuits (XYCE)

- Code optimization and simplified workflow