SAND2021-9970C

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Simplified Method for Determining the Estimated Uncertainty for Digital Sampling Instruments

Sara DiGregorio, R&D S&E Systems Engineer

CO-AUTHOR: A. RODERICK MAHONEY, MAHONEY FARM & CONSULTATION

August 24, 2021

NCSL International Workshop and Symposium

U.S. DEPARTMENT OF ENERGY NNSA

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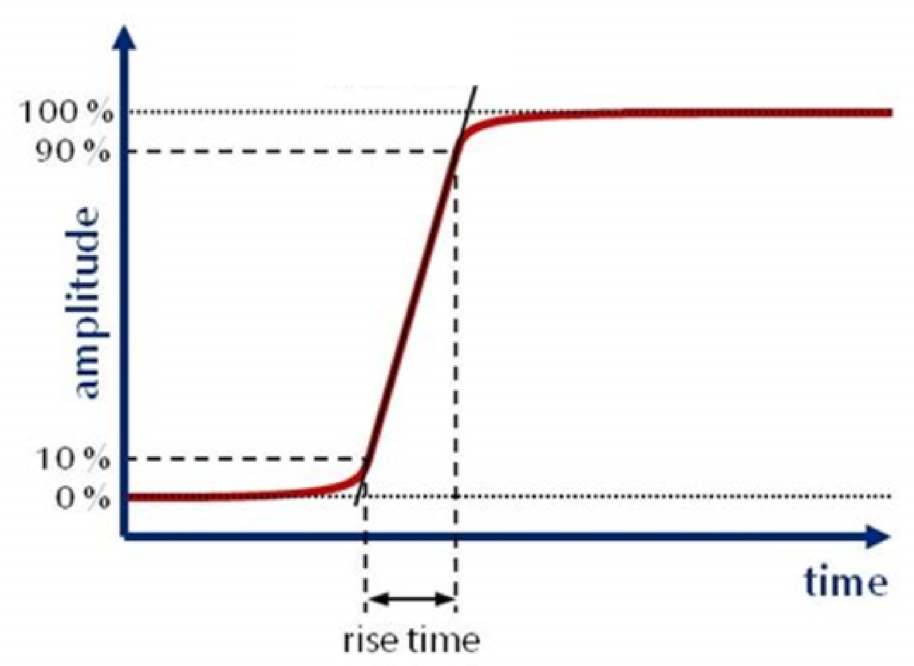
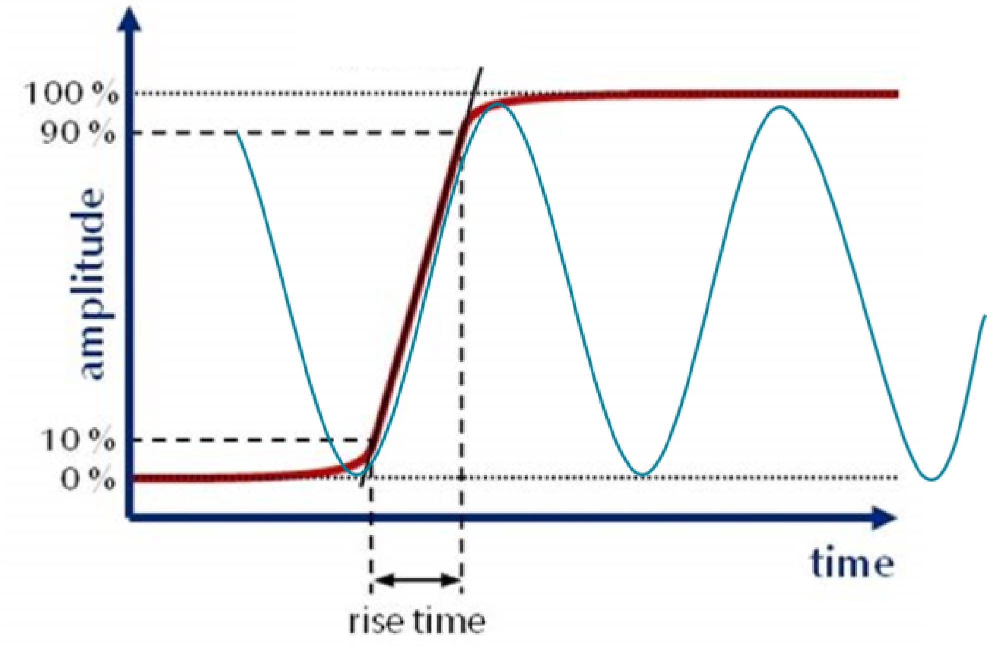
Account for measurement errors associated with capture and evaluation of time values

- In our organization the digital sampling timing error has been misunderstood and poorly documented in our Measurement Assurance process
- Reported timing errors have ONLY considered the instrument time base clock, typically 10-100 parts per million
- Reported timing errors have neglected the error contributions of periodic sampling of analog waveforms generated by digital instruments when determining the difference between two cardinal time points, such as:
  - Rise Time, Fall Time, and FWHM intervals
- Results could significantly influence reported uncertainty, challenge our ability to meet our measurement requirements, and will aid in selecting appropriate sampling technology

Estimating uncertainty in waveform rise time (t_r)

- t_r is defined as the time duration of a signal to transition from the 10% to 90% relative to the peak of the waveform
- t_r uncertainty will be estimated by a method based from a sinusoidal approximation using the trigonometric unit circle



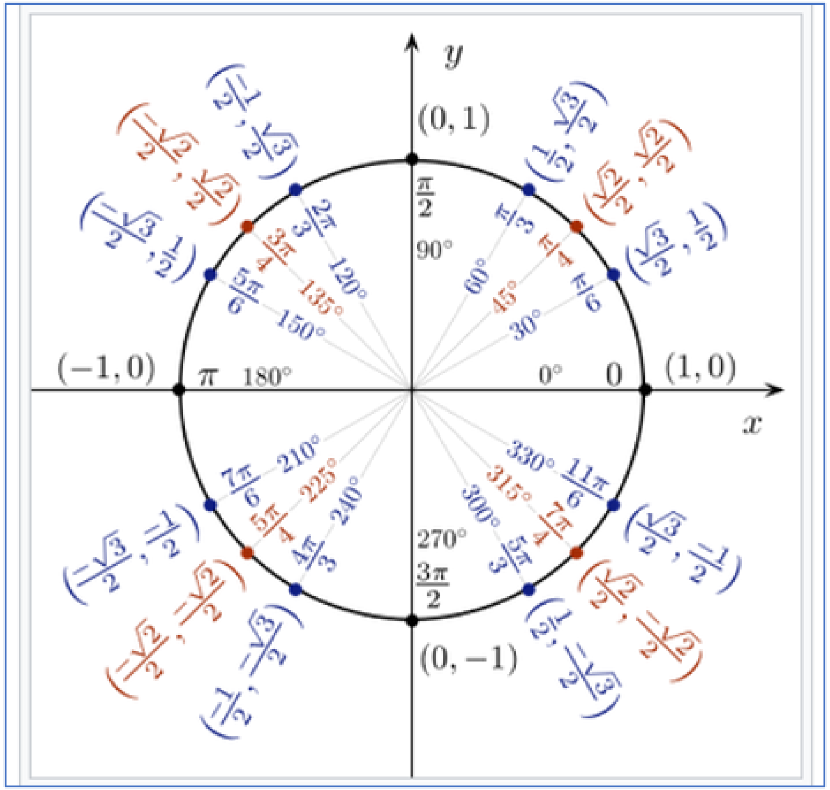


Two main error contributors to t_r

- Sampling rate of the digitizing instrument
- Overall voltage measurement error for the specific measurement channel
  - Combines the instrument voltage accuracy with the errors of additional components

Fall Time
+ Peak to Zero Baseline

Rise Time
Zero Baseline to + Peak

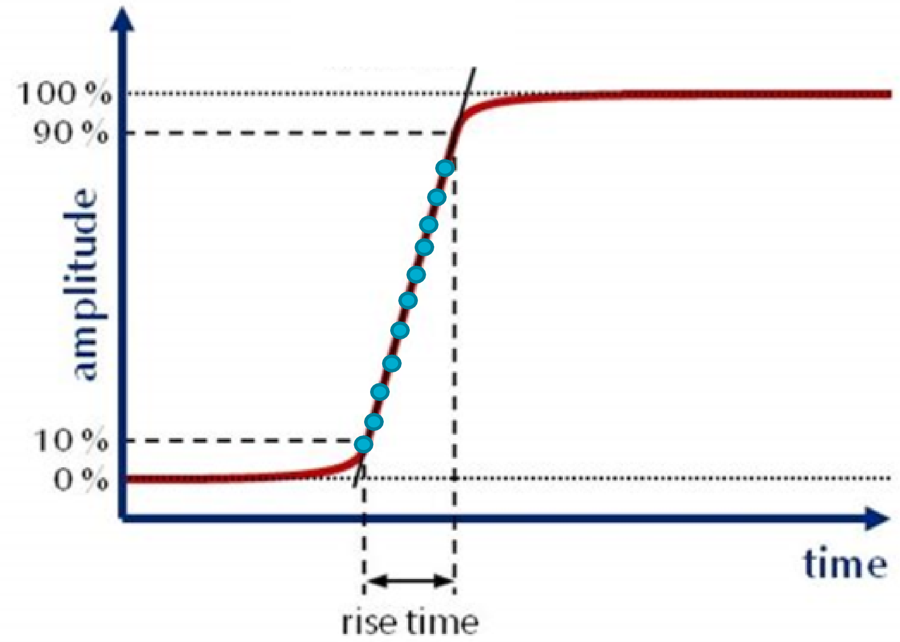


Fall Time
Zero Baseline to - Peak

Rise Time
- Peak to Zero Baseline

Determining uncertainty due to the sample rate contribution

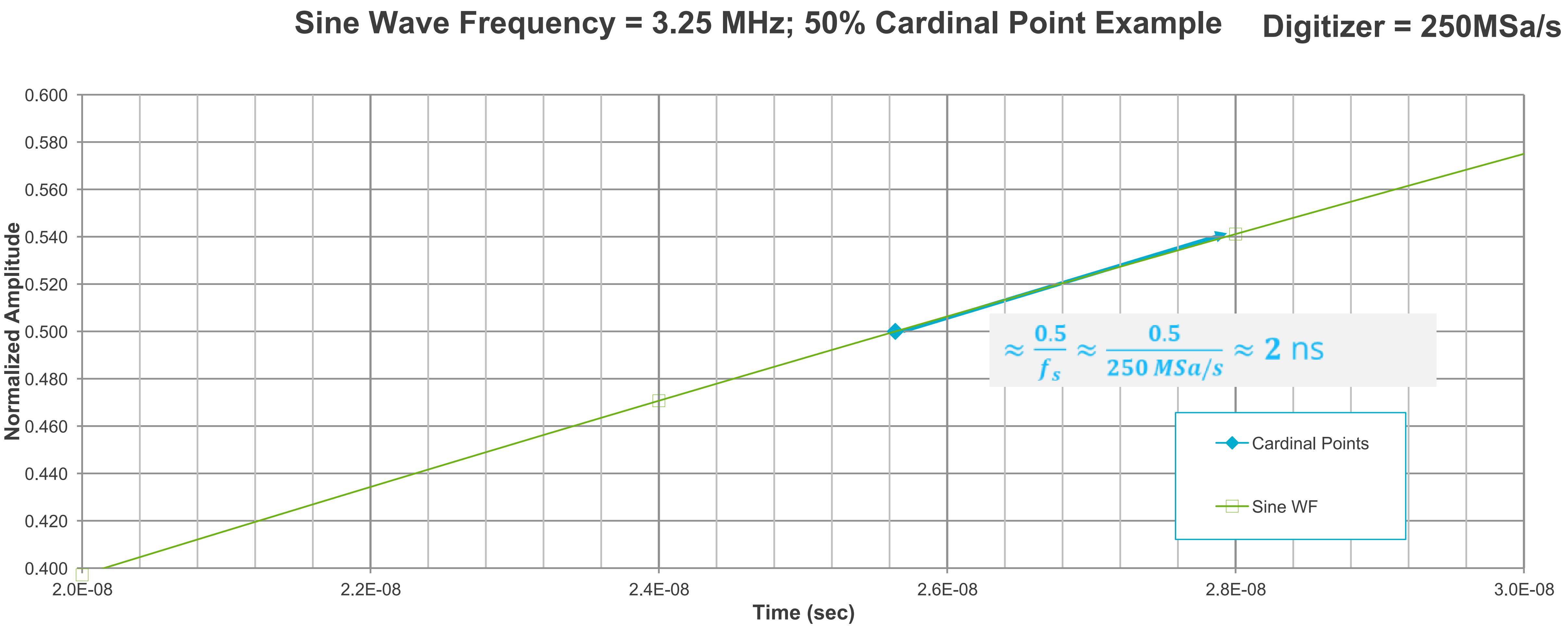
- Step 1: Determine the number of samples available within the waveform rise time interval, $Sa_{\#}$
- Use the instrument sample rate, f_s
- $\boldsymbol{Sa_{\#} = t_r[sec] * f_s}$
- $Sa_{\#}$ should be equal to or greater than 25
  - Generally satisfies the 5X Nyquist criteria
- If $Sa_{\#}$ is less than 25, a higher sampling rate instrument should be selected, where possible



Determining uncertainty due to the sample rate contribution

- Step 2: Assign a sampling rate error for any individual data point, $f_{s_{err\ single\ point}}$
- This will be the **estimated timing error associated with each single point** along the interval
- $\boldsymbol{f_{s_{err\ single\ point}}[sec] \approx \frac{0.5}{f_s}}$
- For any waveform attribute metric taking the difference between two data points, $f_{s_{err\ t_r}}$, **the interval timing error results in one sampling step (or 2X the value for a single point)**
- $\boldsymbol{f_{s_{err\ t_r}}[sec] \approx \frac{1}{f_s}}$

Determining uncertainty due to the sample rate contribution



Determining uncertainty due to the overall voltage

- The contribution to the uncertainty in t_r due to voltage is derived from the manufacturer's specification for the instrument
- Commonly expressed as "vertical accuracy" and is a function of "% of reading"
- Uncertainty to the voltage contribution can be estimated according to the following expression:
- $V_{err}[sec] \cong 2U * t_r[sec]$
- Only include this uncertainty if there is special need to include additional conservatism
- A more detailed instrument accuracy study could be performed to better determine the magnitude of this uncertainty contribution

Accuracy

Resolution	12 bits

Table 2. DC Accuracy[2], warranted

Range ($V_{pk\text{-}pk}$)	Accuracy
0.2 V and 0.4 V	±(0.65% of input + 1.8 mV)
1 V and 2 V	±(0.65% of input + 2.1 mV)
4 V, 10 V, and 20 V[3]	±(0.65% of input + 10.0 mV)

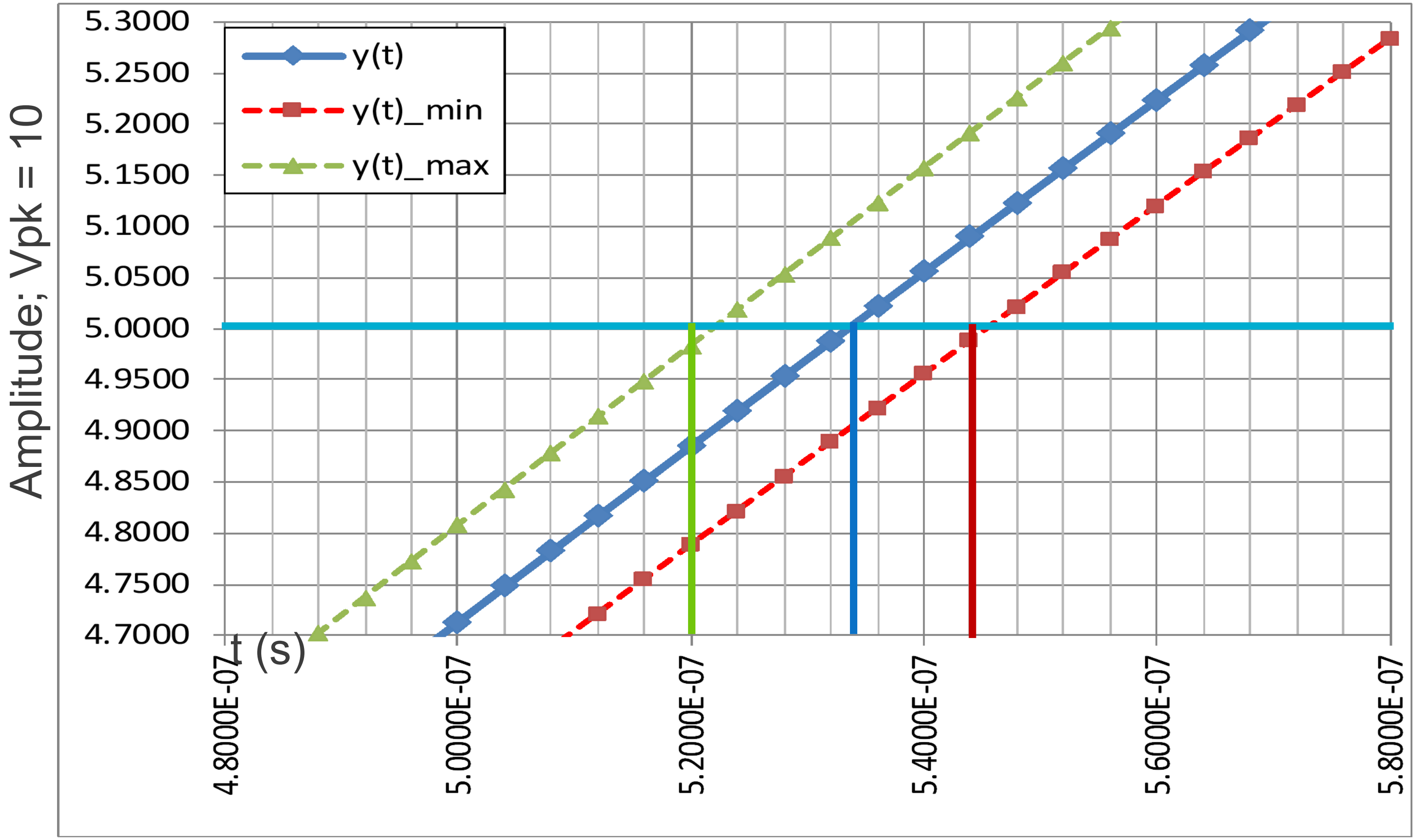
Programmable vertical offset accuracy[4]	±0.4% of offset setting, warranted

Determining uncertainty due to the overall voltage

Sample Rate
2.50E+08
Time Step
4.00E-09

Voltage % Error:	2.00%

Timing Error 50% Example: Voltage Error Contribution

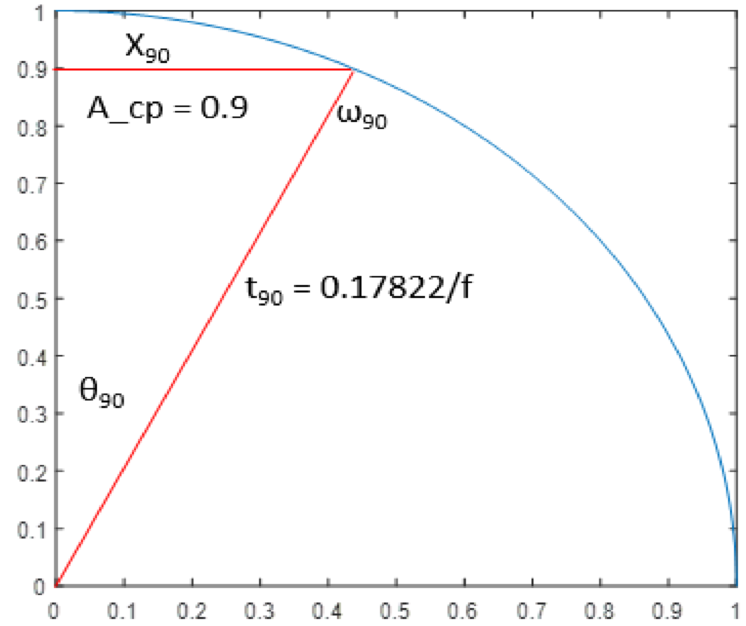


$Er_v(t)50\% \approx \pm 12\, nsec$ (±1.1% of RT)

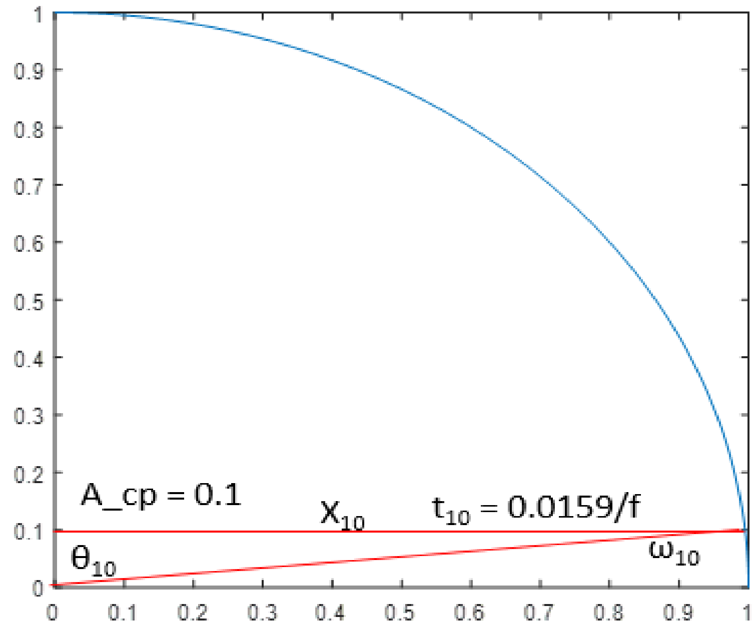
$Er_v(t)_{50\%} \approx$ ½ (Voltage % Error) * RT (nsec)

Trigonometric unit circle to estimate waveform rise time uncertainty using only frequency

- y(t)=Asin(ωt)
  - y(t) = periodic signal as a function of time
  - A = constant that establishes the peak values (+/-)
  - ω = angular frequency, also expressed as 2πf
  - f = appropriately signal-matched sine wave frequency in units of Hertz



- For any location of interest, a trig unit circle is used to determine the t(s) at any point where the frequency is known
  - $x^2 - y^2 = 1$
  - $y = A_{cp}$(amplitude of cardinal point of interest normalized for unit circle)
  - $x = (1 - A_{cp})^{\frac{1}{2}}$
  - $\theta = \arctan\left(\frac{x}{A_{cp}}\right)$
  - $\omega = \frac{\pi}{2} - \theta$
  - $\frac{\omega}{s} = 2\pi f$
  - $t = \frac{\omega}{\frac{\omega}{s}}$



Sine wave time positions are ONLY a function of frequency

- All time positions such as Rise Time, are independent of digitizer sample rate
- The unit circle shows that the time locations of cardinal points have the same theoretical time equations, regardless of digitizer sampling rate

Cardinal Point / Location	Time value equation as a function of frequency, f in Hz
t_10%	0.01594 / f
t_50%	0.83333 / f
t_90%	0.17822 / f
t_peak	0.25000 / f

Theoretical time equations, as a function of frequency, for the 10%, 50%, 90%, and peak cardinal points for a general, simplified sine wave.

Rise Time of Ideal Sine Wave	Frequency, f (Hz)	Number of points in first quarter cycle at 250MSa/s
50 ns	3.25M	19
75 ns	2.165M	29
150 ns	1.082M	58
250 ns	649k	96
500 ns	324.5k	193
1000 ns	162.25k	385
2000 ns	81k	772
4000 ns	40.575k	1540

Theoretical time values, with the associated frequencies and number of points in the first quarter cycle using a 250MSa/s example digitizer.

Methods of defining cardinal point(s) time occurrence

- Next-Point-After
  - Determines the time value for any cardinal point(s) at the data point immediately after the occurrence of the desired amplitude level if that level is not available in the data set
  - $f_s^{NPA}{}_{err\ single\ point}[sec] \approx \frac{0.5}{f_s}$
  - Use when the measurement satisfies our TUR requirement of 4-to-1 (k=2) or greater
    - The number of data points within the RT, FT interval are ≈ 400 or ≈ 800 for FWHM interval

- Interpolation
  - Uses linear interpolation to define the time at which cardinal point(s) occur
  - $f^I_{s\,err\ single\ point}[sec] \approx \frac{0.125}{f_s}$
  - Use when the measurement does not meet our TUR requirement of 4-to-1 (k=2)
  - The Interpolation method will typically provide a lower uncertainty…however depends upon "noise quality" and "signal processing condition" of raw capture waveform

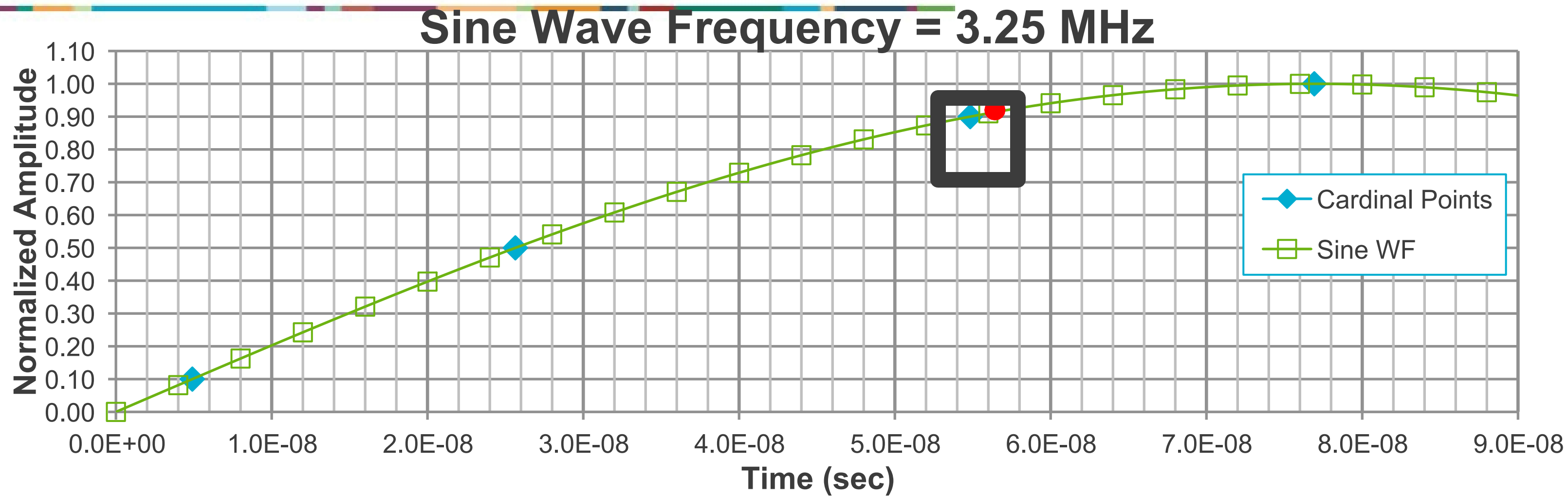
Next-Point-After example

- Trig unit circle determined t_{90} to be 55 ns at A_{90} or, A=0.9
- The user would select the time at which the closest, larger value of when A_{90} occurs
- Apply same logic to the 10% point
- The overall error of each single point is 0.1 step, which is round to 0.5 step, or 2 ns.
- The overall rise time error would be 1 step, or 4 ns.

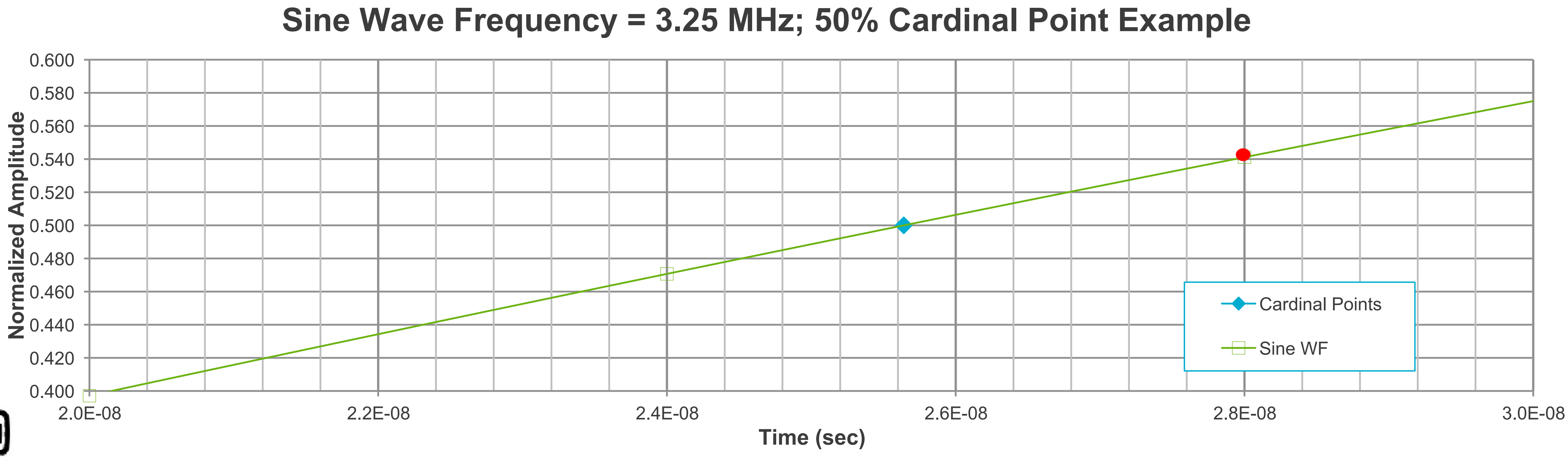
t(ns)	A(t)
0	0
4	0.082
8	**0.163**
12	0.243
16	0.321
20	0.397
24	0.471
28	0.541
32	0.608
36	0.671
40	0.729
44	0.782
48	0.831
52	0.873
56	**0.910**
60	0.941
64	0.965
68	0.983
72	0.995
76	1.000

Example data set defining time at which the 90% t_r and 10% t_r occur for a 250 M Sa/s digitizer

Next-Point-After example



The overall error of each single point is 0.1 step, which is round to 0.5 step, or ≈**2 ns**.



Interpolation example

- Example shows a digitally captured ideal sine waveform (first quarter cycle) data set normalized by the a peak value of 1
- Trig unit circle determined t_{90} to be 55 ns at A_{90} or, A=0.9
- The user would interpolate two closest, larger and smaller values that surround A_cp = 0.9
- $t_{0.9} = \frac{[A_{0.9}*(t_2 - t_1) - (A_1 * t_2) + (A_2 * t_1)]}{A_2 - A_1} = 54.9\, ns$
- where $A_{0.9}$ = 0.9, (t_1, A_1) = (52,0.873), and (t_2,A_2) = (56,0.910)
- The error for a single cardinal point is 0.01 step, rounded to .125 step, or 0.5 ns
- The overall error is 1 ns

t(ns)	A(t)
0	0
4	**0.082**
8	**0.163**
12	0.243
16	0.321
20	0.397
24	0.471
28	0.541
32	0.608
36	0.671
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76	1.000

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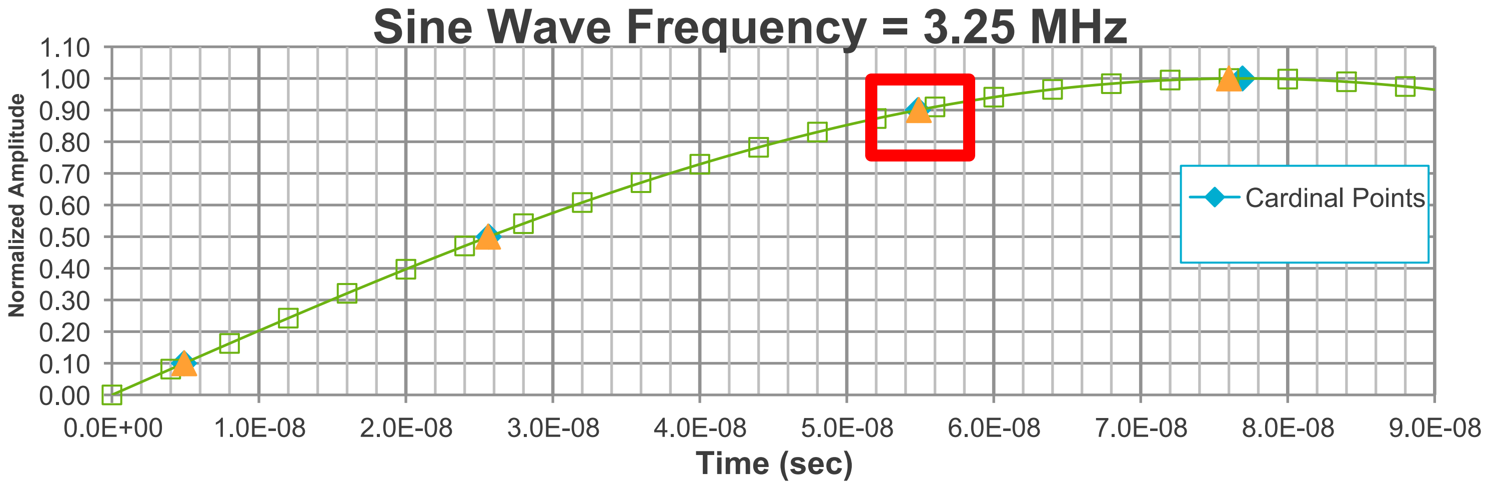
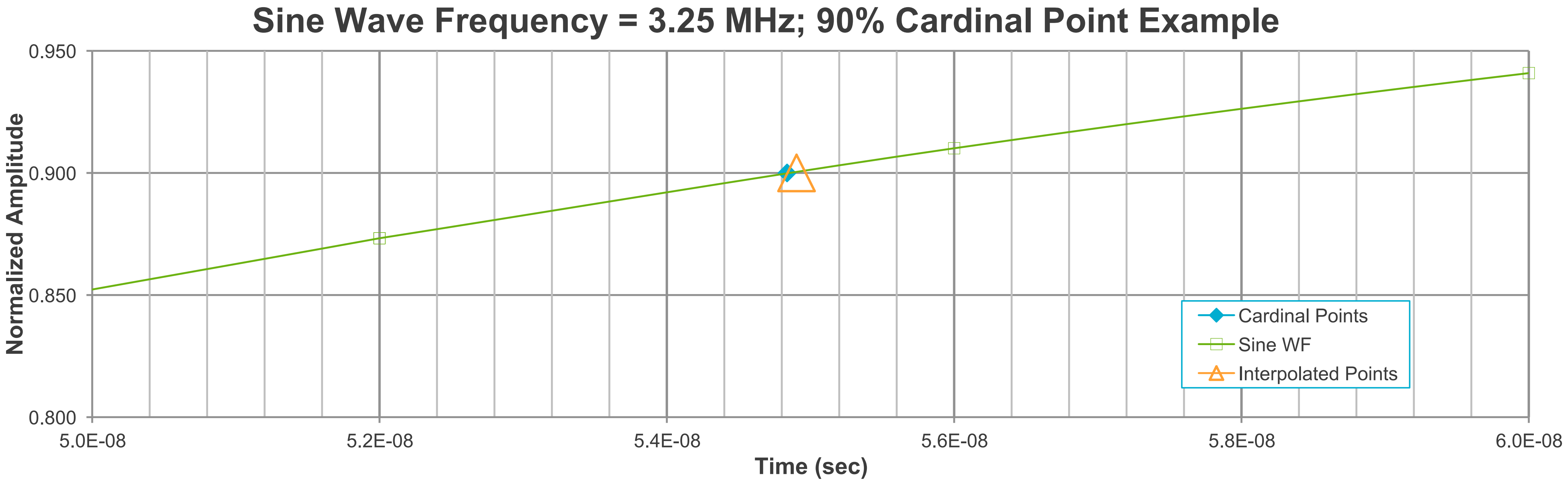
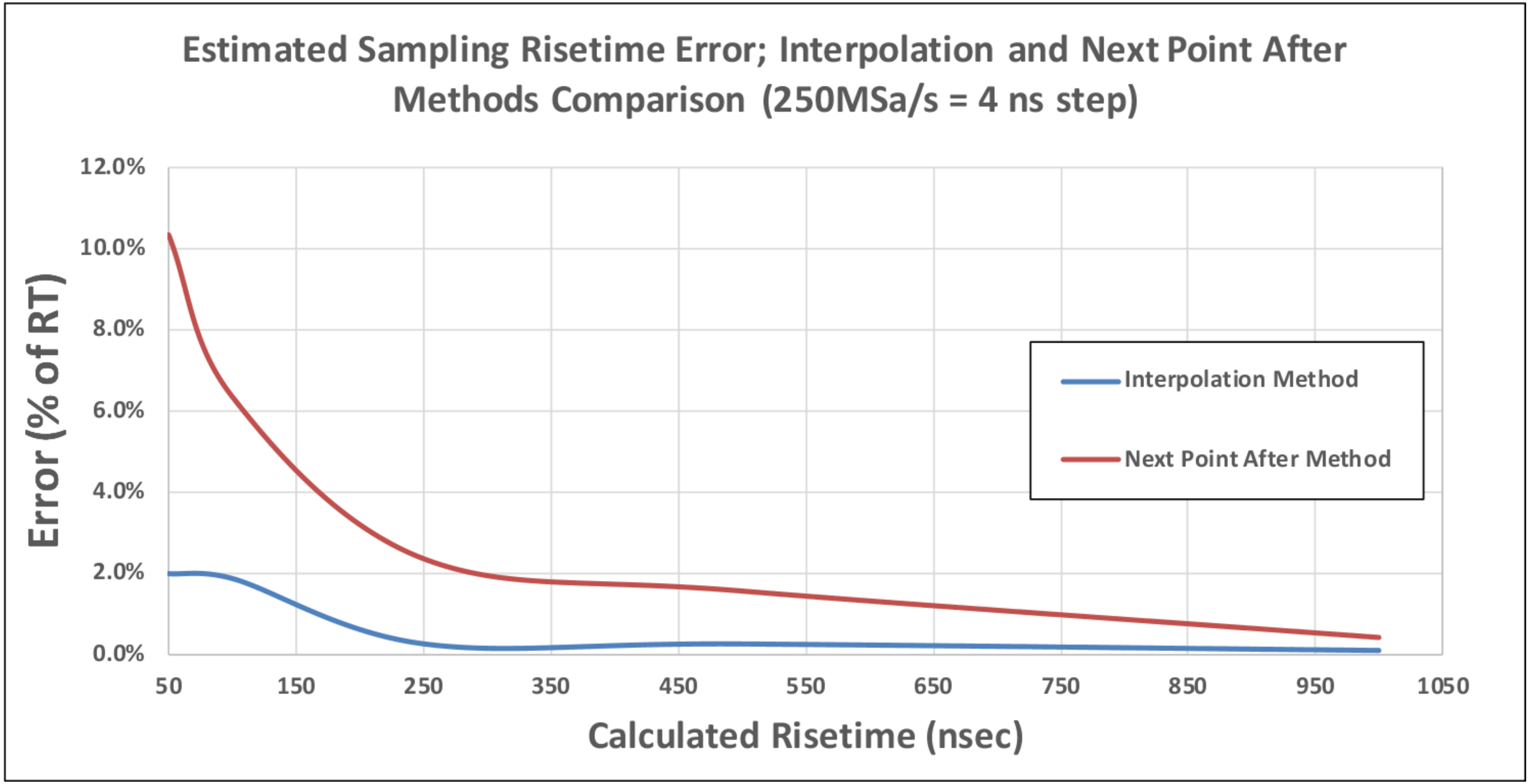
Interpolation example



The error for a single cardinal point is 0.01 step, rounded to .125 step, or ≈**0.5 ns**



Next-Point-After and Interpolation Comparison



Example problem

- **Tester Requirement:** The PT1234 will collect a burst-to-pressure delay of a 35 nanosecond risetime ± 1 nanosecond.
- **Tester Information:** Tester digitizer samples at 1 Giga-sample per second.
- **To estimate error:** We first consult Table VIIA, which is the summary table for a 1GSa/s digitizer. We see that for an approximate RT of 35 ns, a 1GSa/s digitizer is capable of capturing a 35 ns RT and can adequately estimate the timing error using the interpolation method.

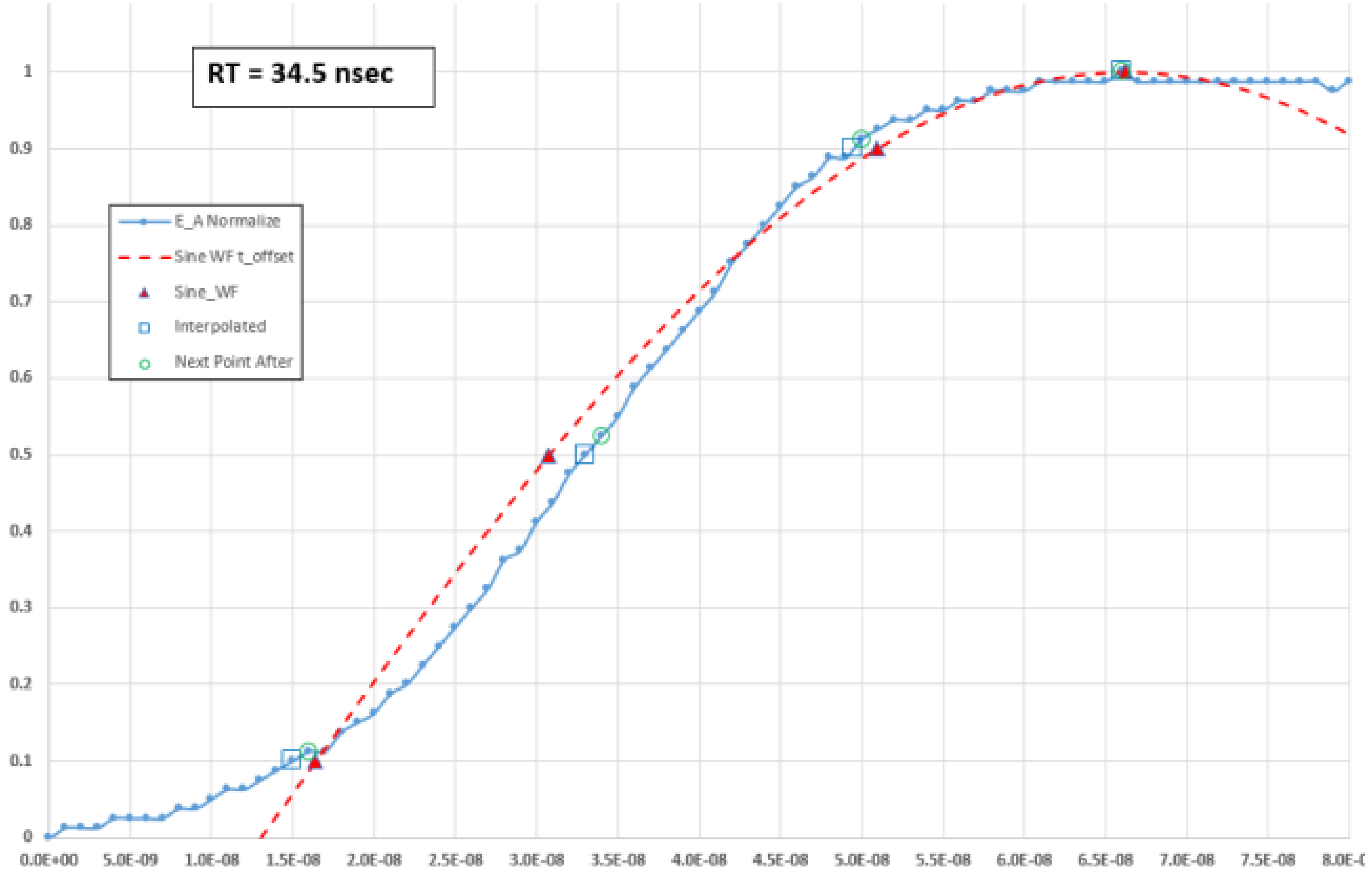
f(Hz)	f(kHz)	Calc RT (nsec)	RT Error (number of sampling steps)	Any Cardinal Point Error (number of sampling steps)	Cardinal Point Time Data Selection Method
40575	40.575	4000	1	0.5	Next Point After
81000	81	2000	1	0.5	Next Point After
162250	162.25	1000	1	0.5	Next Point After
324500	324.5	500	1	0.5	Next Point After
649000	649	250	1	0.5	Next Point After
1082000	1082	150	1	0.5	Next Point After
2165000	2165	75	1	0.5	Next Point After
3250000	3250	50	0.5	0.25	Interpolation
5000000	5000	32	0.5	0.25	Interpolation
10000000	10000	16	0.5	0.25	Interpolation

Table VIIIA. Summary of critical uncertainty parameters for a 1GSa/s (0.5 ns) digitizer.

$$f_{s\ err\ t_r}^{I}[sec] \approx \frac{.25}{f_s} \approx 0.25\ ns.$$

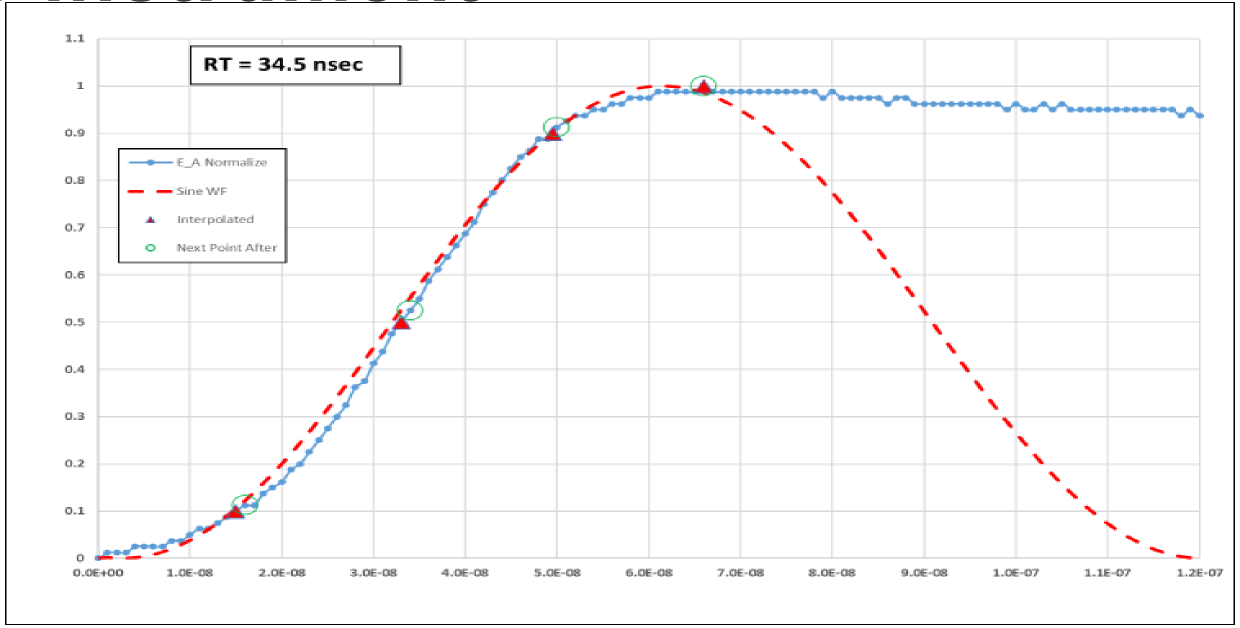
Sine wave approximation to estimate the uncertainty in a signal captured with a 1GSa/s instrument

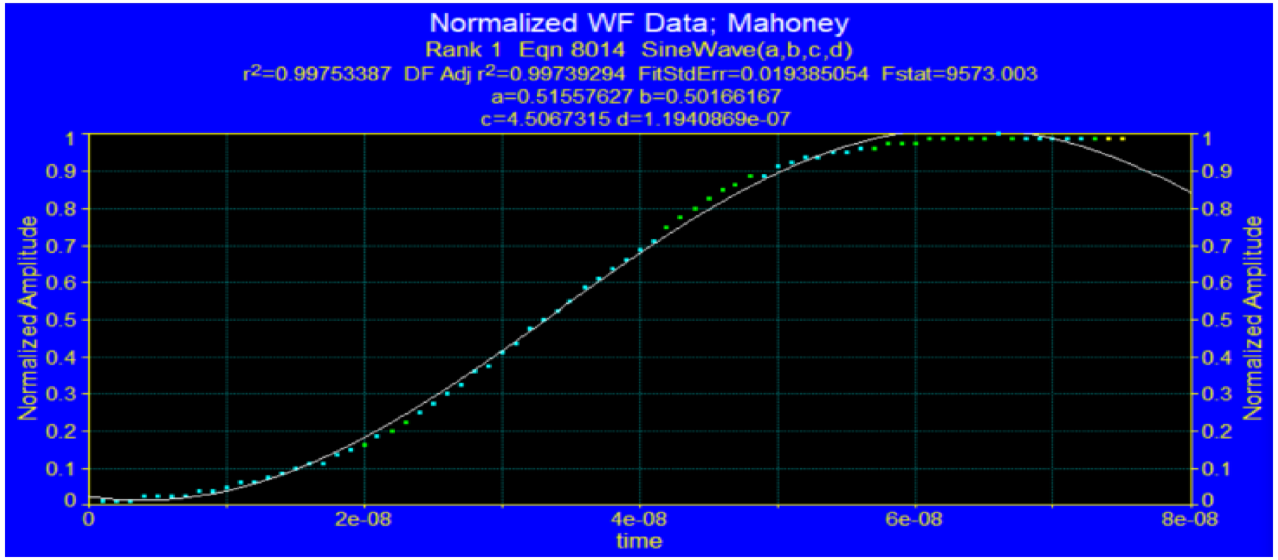
- Approximation agrees favorably
- Provides an appropriate baseline to estimate "deviation" from
- This "deviation" is our signal error



Sine wave approximation to estimate the uncertainty in a signal captured with a 1GSa/s instrument

- Using a more tailored sine approximation will result in a lower estimated uncertainty
- Use a more tailored approach if a lower uncertainty is desired





Conclusions

- We have demonstrated that a quartile sine wave approximation using the trigonometric unit circle adequately provides a baseline to estimate the timing uncertainty in a signal
- ${f_s^{NPA}}_{err\ single\ point}[sec] \approx \frac{0.5}{f_s}$
  - for RT, FT, and FWHM intervals error is 2X ${f_s^{NPA}}_{err\ single\ point}[sec]$
- Use “Next-Point-After” when
  - The measurement satisfies our TUR requirement of 4-to-1 (k=2) or greater
  - The number of data points within the RT, FT interval are ≈ 400 or ≈ 800 for FWHM interval
- $f^I_{s_{err\ single\ point}}[sec] \approx \frac{0.125}{f_s}$
  - for RT, FT, and FWHM intervals error is 2X $f^I_{s_{err\ single\ point}}[sec]$
- Use Interpolation when
  - The measurement does <u>not</u> meet our TUR requirement of 4-to-1 (k=2)
  - The Interpolation method will typically provide a lower uncertainty…however depends upon “noise quality” and “signal processing condition” of raw capture waveform