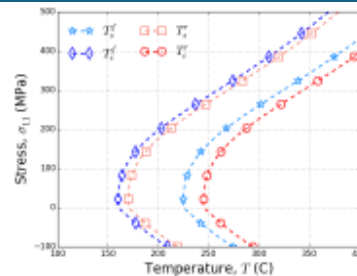
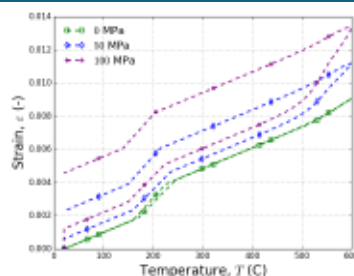
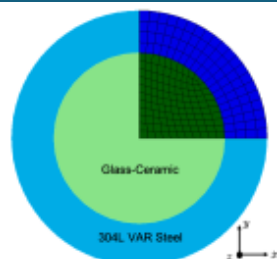
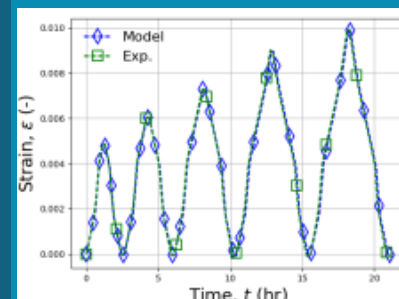




# Constitutive and Application Modeling of Glass-Ceramic Materials



Brian T. Lester  
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Sandia National Laboratories

July 28,  
2021 US National Congress on  
Computational

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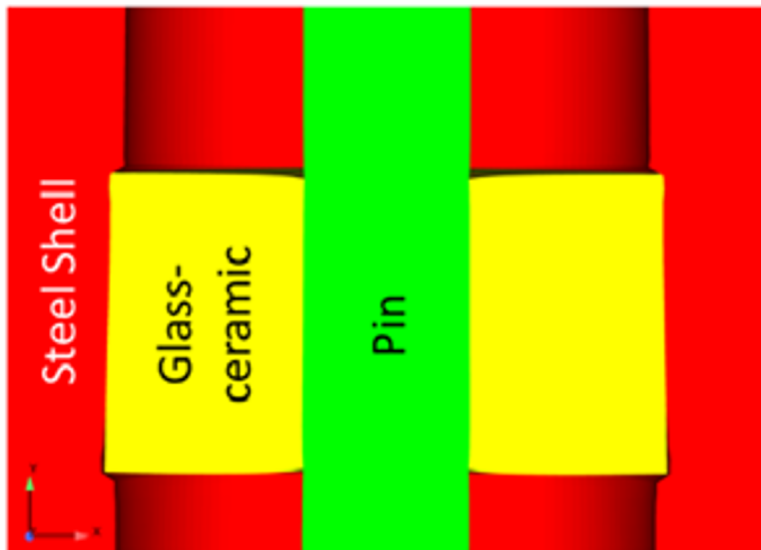
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# Glass-ceramic to Metal Seals (GcTMS)

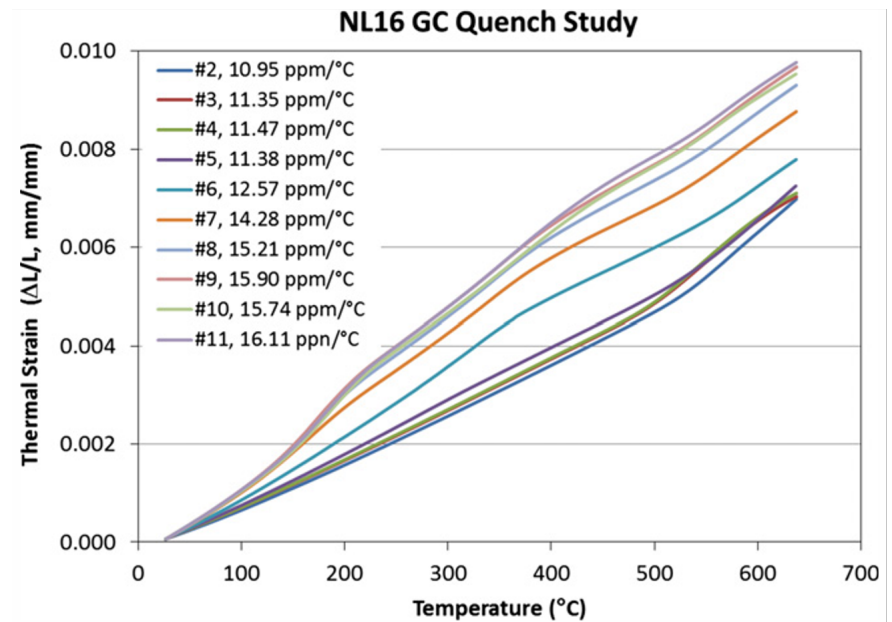


- Variety of industrial applications for glass-ceramics
  - Hermetic glass-ceramic to metal seals (GcTMS)
  - Subject to complex thermomechanical histories

Example Seal



Dai *et al.*, 2017, *J Am Ceram Soc*, 100, pp.3652-3661

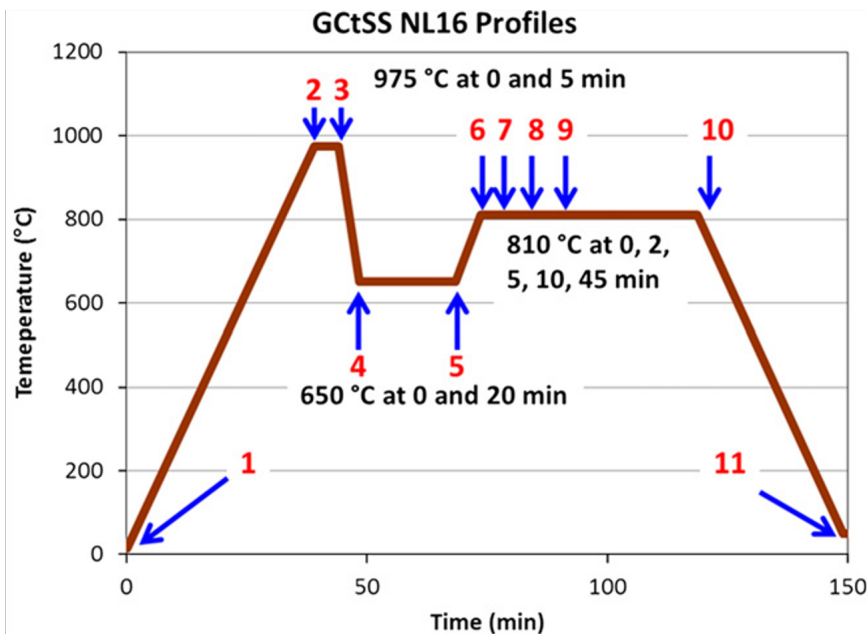


Dai *et al.*, 2016, *J Am Ceram Soc*, 99, pp.3719-3725

# Glass-Ceramics – Microstructure

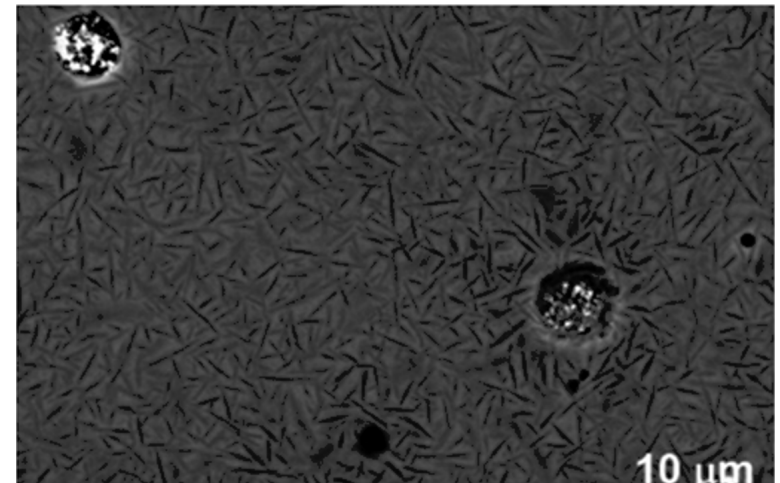


- Glass-ceramics are produced by inducing a ceramic phase(s) in an inorganic base glass
- Advantageous features arise from microstructure
  - Up to 5 constituents
  - Inelasticity from residual glass and silica polymorphs



Dai *et al.*, 2016, *J Am Ceram Soc*, 99 (11), pp.3719-3725

**NL16**



Rodriguez *et al.*, 2016, *J Am Ceram Soc*, 99 (11), pp.3726-3733

# Objective



- Need to be able to simulate sealing process
  - Predict residual stress states
  - Optimize thermal profile during sealing
- Need appropriate constitutive model for glass-ceramic
  - No existing specialized glass-ceramic constitutive models
  - Existing efforts adapt other model forms (e.g. non-linear viscoelastic)
  - Neglect complexity and details of current combination of mechanisms
- The objective of the current work is to develop a glass-ceramic constitutive model
  - Theory coupling viscoelastic and phase-transformation response
  - Robust 3D numerical implementation
  - Use model to simulate sealing process



- Seek macroscale representation of glass-ceramics via use of internal state variable/continuum thermodynamics theory
  - Thermoviscoelastic theory for response of glass
  - Utilize shape memory alloy (SMA) theory as basis (Lagoudas model) for phase transformations

$$G(\sigma_{ij}, T, t, \xi, \varepsilon_{ij}^t; \delta^i) = G^{\text{te}}(\sigma_{ij}, T, \xi; \delta^i) + G^{\text{in}}(\sigma_{ij}, T, t, \xi, \varepsilon_{ij}^t; \delta^i)$$

$\sigma_{ij}, T, t$

External State Variables

$\xi, \varepsilon_{ij}^t$

Internal State Variables

$\delta^i$

Constituent Volume Fractions

$$G^{\text{te}}(\sigma_{ij}, T, \xi; \delta^i) = \sum_{r=Q, LO, LM, A} \delta^r G^r(\sigma_{ij}, T) + \delta^C G^C(\sigma_{ij}, T, \xi)$$

$$G^{\text{in}}(\sigma_{ij}, T, t, \xi, \varepsilon_{ij}^t) = G^{\text{in-t}}(\sigma_{ij}, \xi, \varepsilon_{ij}^t) + G^{\text{neq}}(\sigma_{ij}, T, t)$$



- Coleman-Noll and 2<sup>nd</sup> Law arguments produce:
  - All constituents/phases assumed isotropic

$$\begin{aligned} \varepsilon_{ij} = & \frac{1}{2\bar{\mu}} \sigma'_{ij} + \frac{1}{9\bar{K}} \sigma_{kk} \delta_{ij} + g_\varepsilon \varepsilon_{ij}^t + \bar{\alpha} (T - T_0) \delta_{ij} - \\ & - g_v \frac{\Delta\mu}{2\mu^{\text{eq}} \mu^g} H_{ij}^2 - g_v \frac{\Delta K}{9K^{\text{eq}} K^g} H^1 \delta_{ij} + g_v \Delta\alpha H^3 \delta_{ij} \end{aligned}$$

- Transformation functions
  - Utilize  $J_2 - I_1$  model
  - Combines parts of Qidwai & Lagoudas (IJP, 2000) and Lagoudas *et al.* (IJP, 2012)

$$f(X_{ij}, p) = [\phi(X_{ij}) - p]^2 - p_0^2$$

$$\phi(X_{ij}) = \gamma_1(J_2) \sqrt{3J_2} - \gamma_2 I_1$$

# Viscoelasticity



- Hereditary integral based formulation

- Creep – not relaxation – spectra needed for use of Gibbs free energy
- Shift-factor relates “material” and “laboratory” time

$$t^* = \int_0^t \frac{ds}{a(s)}$$

- Investigate impact of two shift factors

- WLF – equilibrated shift factor

$$\log_{10} a = \frac{-C_1 (T - T_{\text{ref}})}{C_2 + (T - T_{\text{ref}})},$$

- WLF-Lag

- Incorporate some history dependence
- Sealing problem exhibits large temperature ranges of interest ( $RT \ll T_g$ )

$$\log_{10} a^{\text{WLF-Lag}} = \frac{-C_1 \left( T - T_{\text{ref}} - \int_0^t (1 - j_v(t^* - s^*, 0)) \frac{\partial T}{\partial s} ds \right)}{C_2 + \left( T - T_{\text{ref}} - \int_0^t (1 - j_v(t^* - s^*, 0)) \frac{\partial T}{\partial s} ds \right)}$$

# Numerical Implementation



- 3D numerical implementation formulated and implemented
  - Sierra/SolidMechanics FE code constitutive library (LAMÉ)
  - Fully implicit integration
  - Line-search augmented Newton-Raphson
  - Verified through a variety of loadings

## Non-Linear Solve

$$\sigma_{ij}^{n+1} = \sigma_{ij}^n + \Delta t \dot{\sigma}_{ij}^{n+1}$$

$$\varepsilon_{ij}^{t(n+1)} = \varepsilon_{ij}^{t(n)} + \Delta t \dot{\varepsilon}_{ij}^{t(n+1)}$$

$$\xi^{n+1} \rightarrow f(\sigma^{n+1}, T^{n+1}, \xi^{n+1}) = 0$$

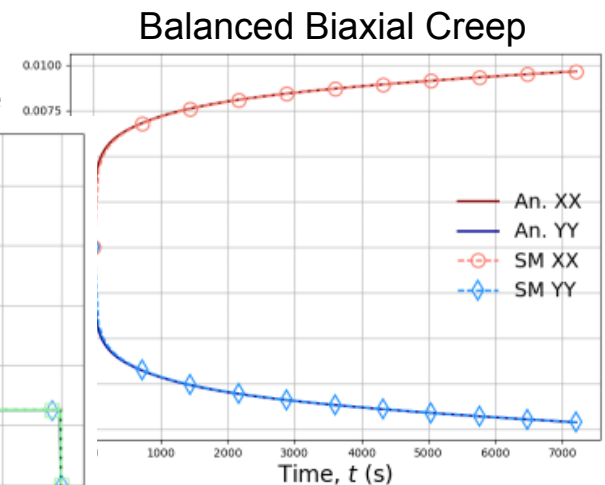
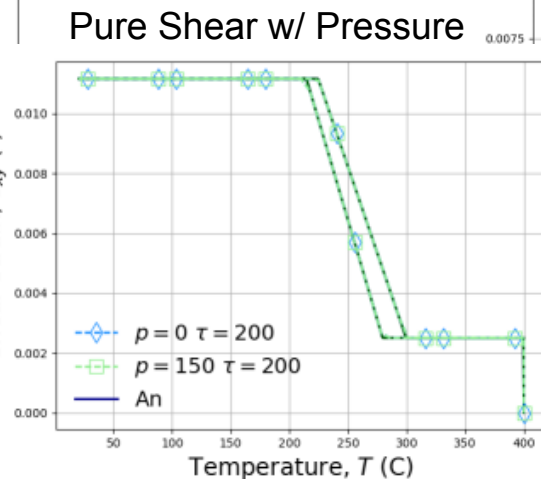
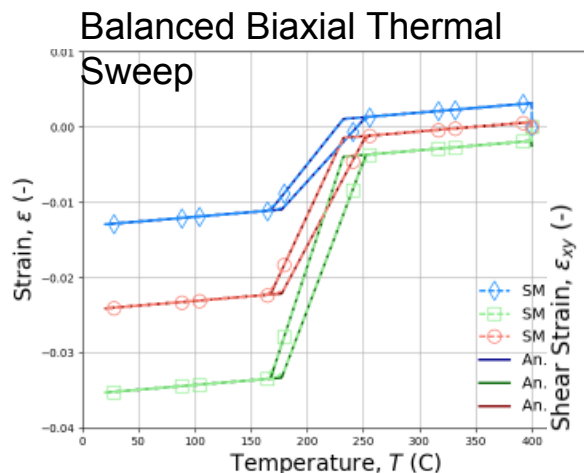
## Direct Solve

$$H_{n+1}^1 = H_n^1 + \Delta t \dot{H}_{n+1}^1$$

$$H_{ij}^{2(n+1)} = H_{ij}^{2(n)} + \Delta t \dot{H}_{ij}^{2(n+1)}$$

$$H_{n+1}^3 = H_n^3 + \Delta t \dot{H}_{n+1}^3$$

## Example Verification Tests

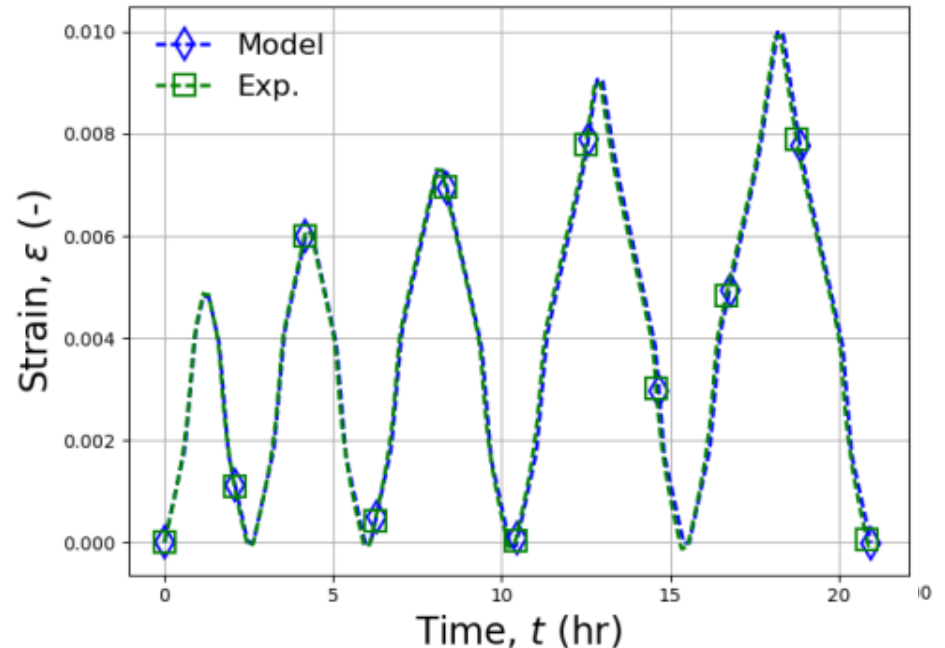
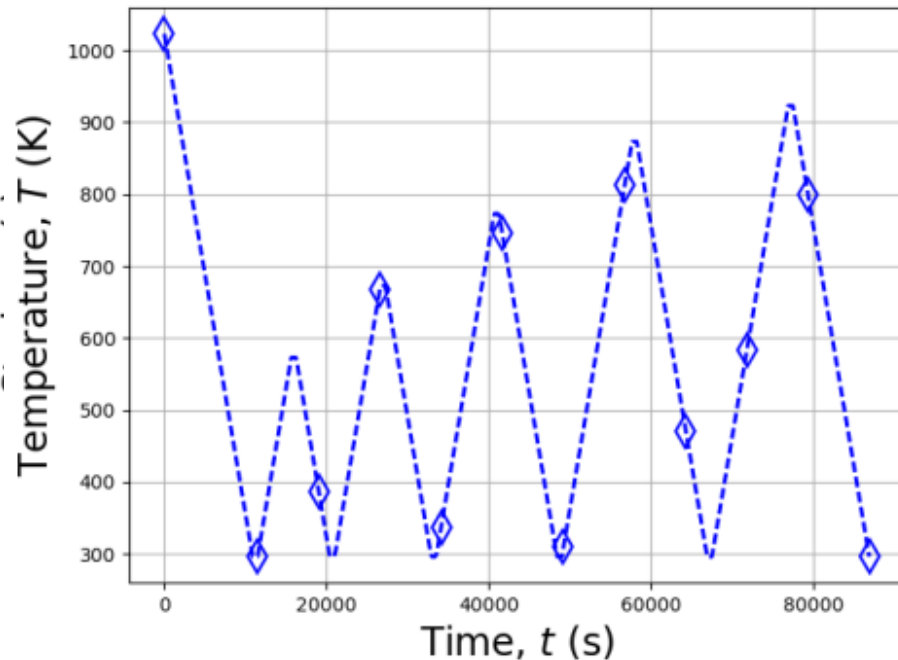




# Validation



- Consider response through “no”-load thermal sweeps
  - “Ladder”/ratcheting tests
  - Dilatometer (courtesy S. Dai, SNL)
    - Stress free thermal sweeps
    - Measures axial strain

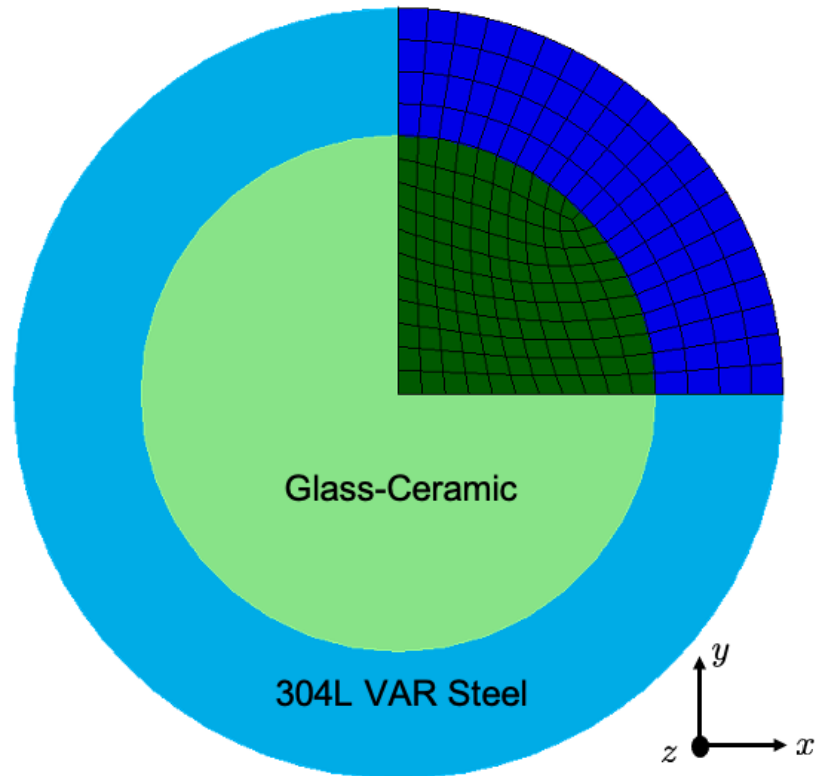
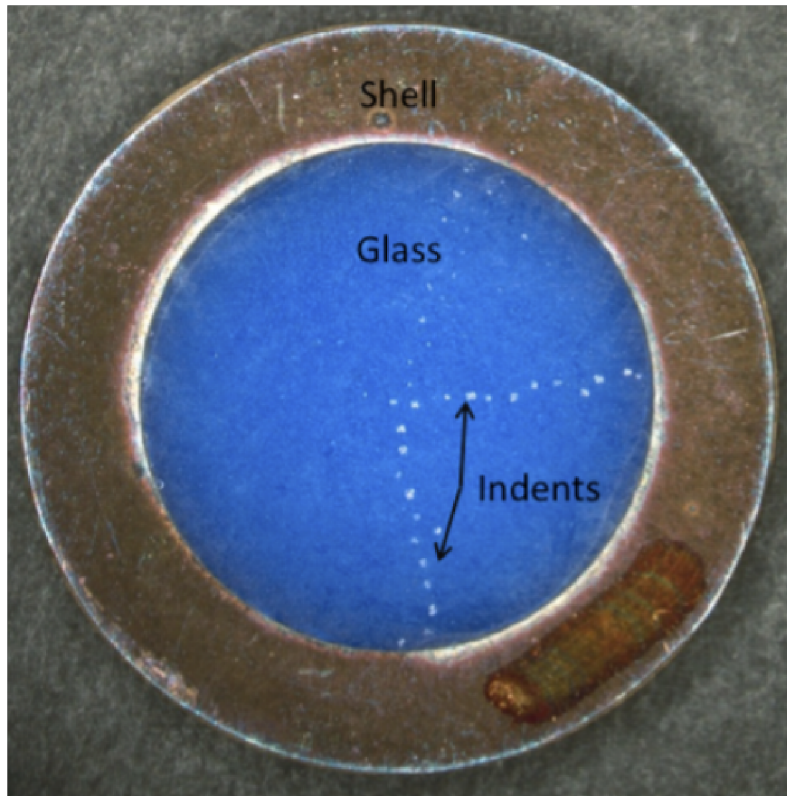


- Working on extending validation against other experiments

# Example Problem – Simple Seal



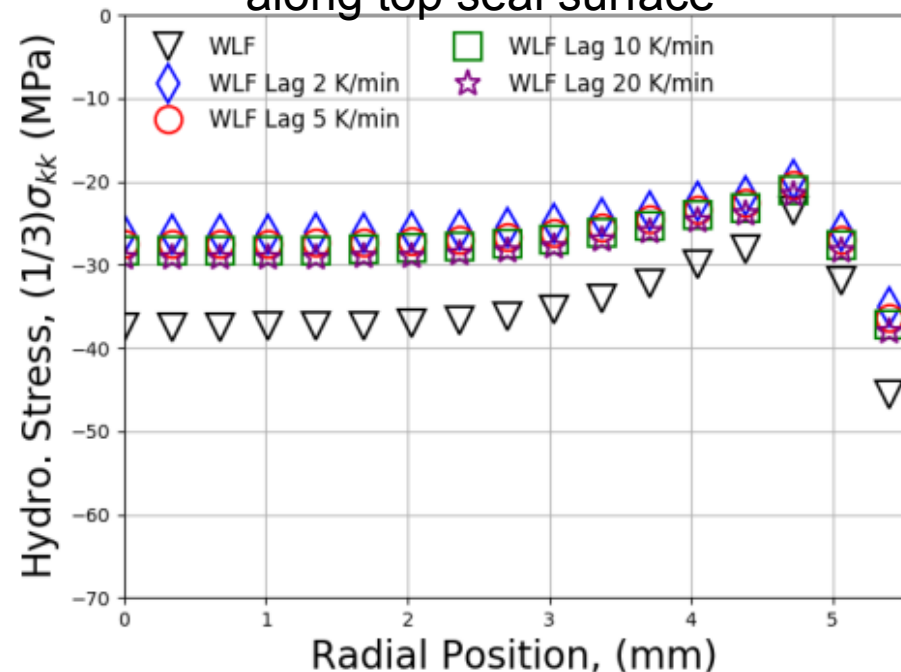
- Simple seal used as representative example problem
  - Common test for prediction and measurement of residual stress
  - GC Seal enclosed in concentric metal (stainless steel) shell
  - Cooled from above  $T_g$  to RT



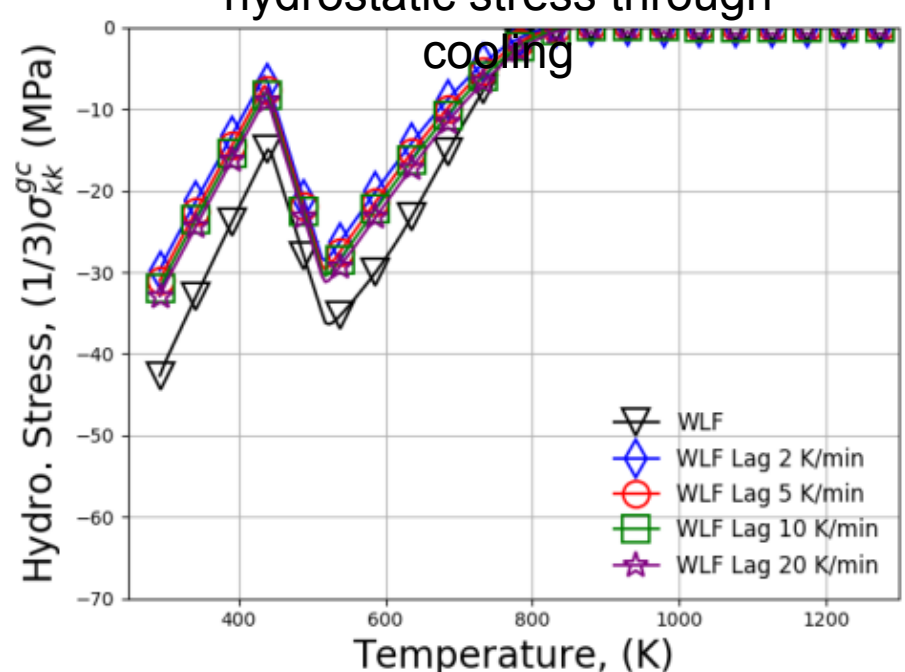
# Impact of Shift Factor

- Investigate simple seal with multiple shift factor
  - WLF-Lag at different cooling rates
  - WLF cooled at 2 K/min
  - Purely volumetric flow rule  $\gamma_1^0 = 0$ ,  $\gamma_2^0 = \bar{\gamma}$

Hydrostatic stress  
along top seal surface



Volume averaged  
hydrostatic stress through

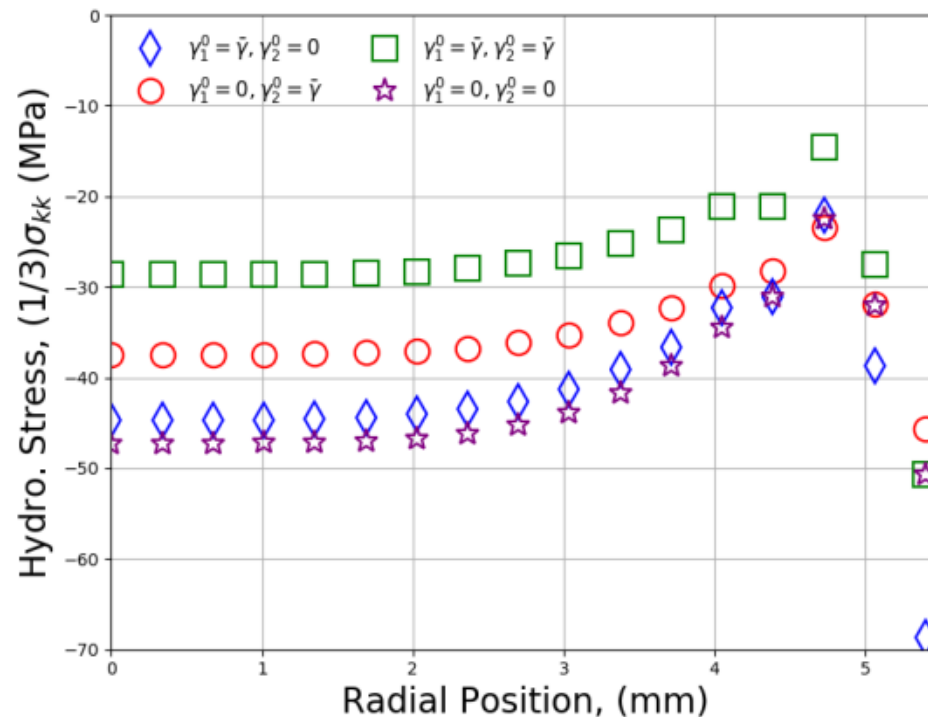


- Stress state remains compressive through loading
- Impact of both viscoelastic and transformation mechanisms may be observed

# Impact of Effective Stress



- Consider four different effective stress forms
  - Pure deviatoric,  $\gamma_1^0 = \bar{\gamma}$ ,  $\gamma_2^0 = 0$
  - Pure volumetric,  $\gamma_1^0 = 0$ ,  $\gamma_2^0 = \bar{\gamma}$
  - Both volumetric and deviatoric,  $\gamma_1^0 = \bar{\gamma}$ ,  $\gamma_2^0 = \bar{\gamma}$
  - No transformation,  $\gamma_1^0 = 0$ ,  $\gamma_2^0 = 0$

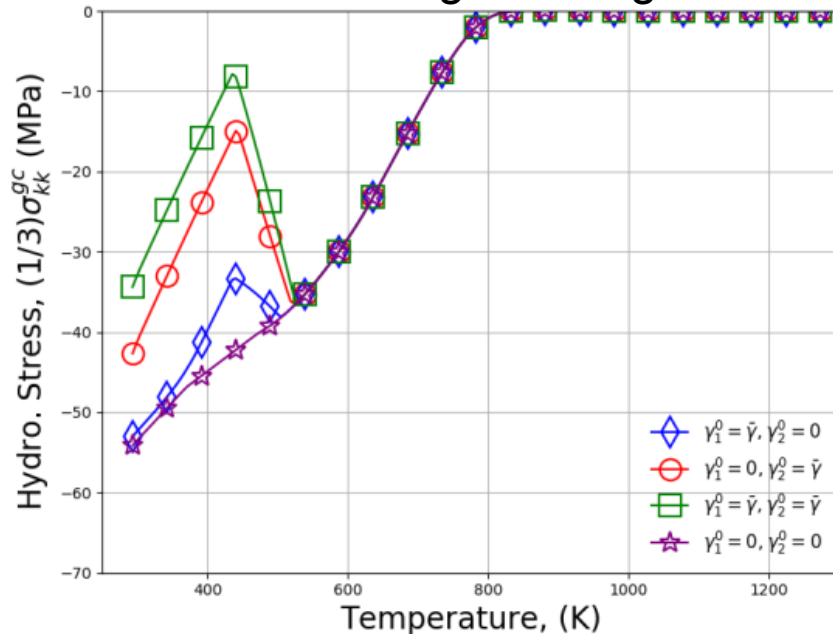


- Presence of hydrostatic strain yields lower hydrostatic stress magnitudes

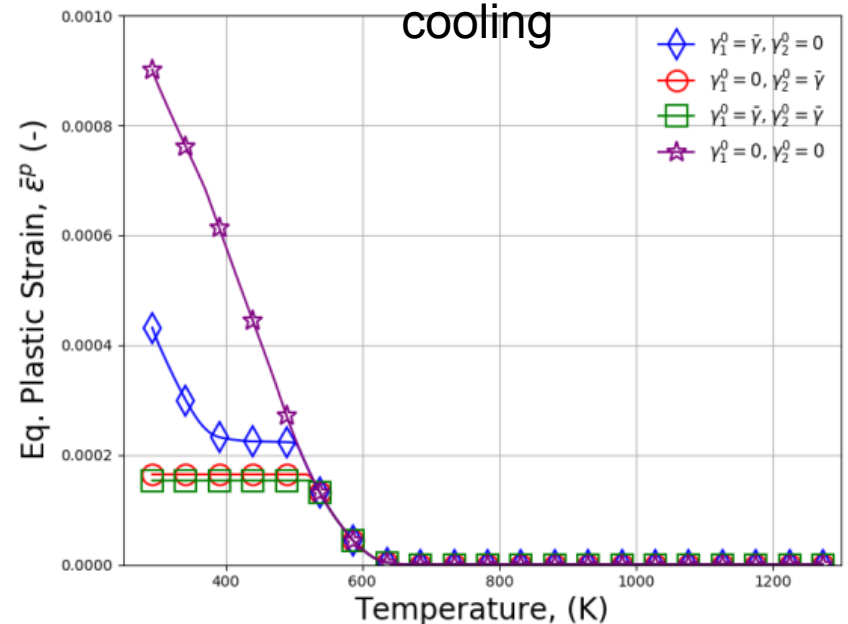
# Impact of Flow Rule



Vol. Avg. Hydrostatic stress through cooling



Vol. Avg. shell equiv. plastic strain through cooling



- Onset of transformation leads to differences in stress evolution
- Cases with hydrostatic stress decrease stress magnitudes and plastic strain evolution in shell
- Differences in shell versus seal dominated response

# Conclusion and Summary



- Developed new phenomenological constitutive model for glass-ceramic materials
  - Coupled viscoelasticity and phase transformation
  - 3D numerical implementation
- Results show promise for use in modeling seal applications
  - Validation against simple, existing experiments
  - 3D form considered for simple seal case
  - Explored impact of shift factor and flow rule
  - Interesting interaction between different mechanisms
- Future work
  - Expanded validation exercises
  - Consideration of quartz response

# Acknowledgements



- Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525. This paper describes objective technical results and analysis. Any subjective views or opinions do not necessarily represent the views of the U.S. Department of Energy or the United States Government.
- Thanks to S. Dai (SNL) for dilatometer results





Questions?

