

# An Eigenmodel for Dynamic Multilayer Networks

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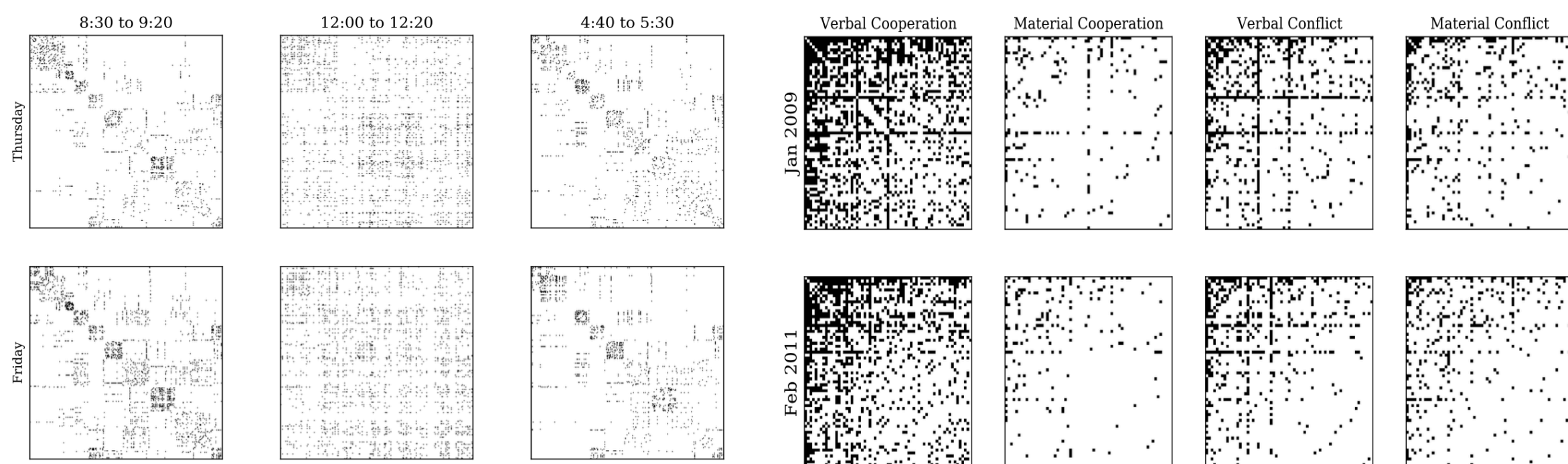
## Introduction / Motivation

### Dynamic Multilayer Networks

Many complex systems found in nature are dynamic and consist of multiple correlated relations:

- Social networks, social media, international relations, etc.

**Data:**  $K$  time-indexed adjacency matrices,  $\mathbf{Y}_t^1, \dots, \mathbf{Y}_t^K$ , where  $\mathbf{Y}_t^k \in \{0,1\}^{n \times n}$  for  $1 \leq t \leq T$  and  $1 \leq k \leq K$ , e.g.,



Face-to-face contacts at a primary school

International relations between nations

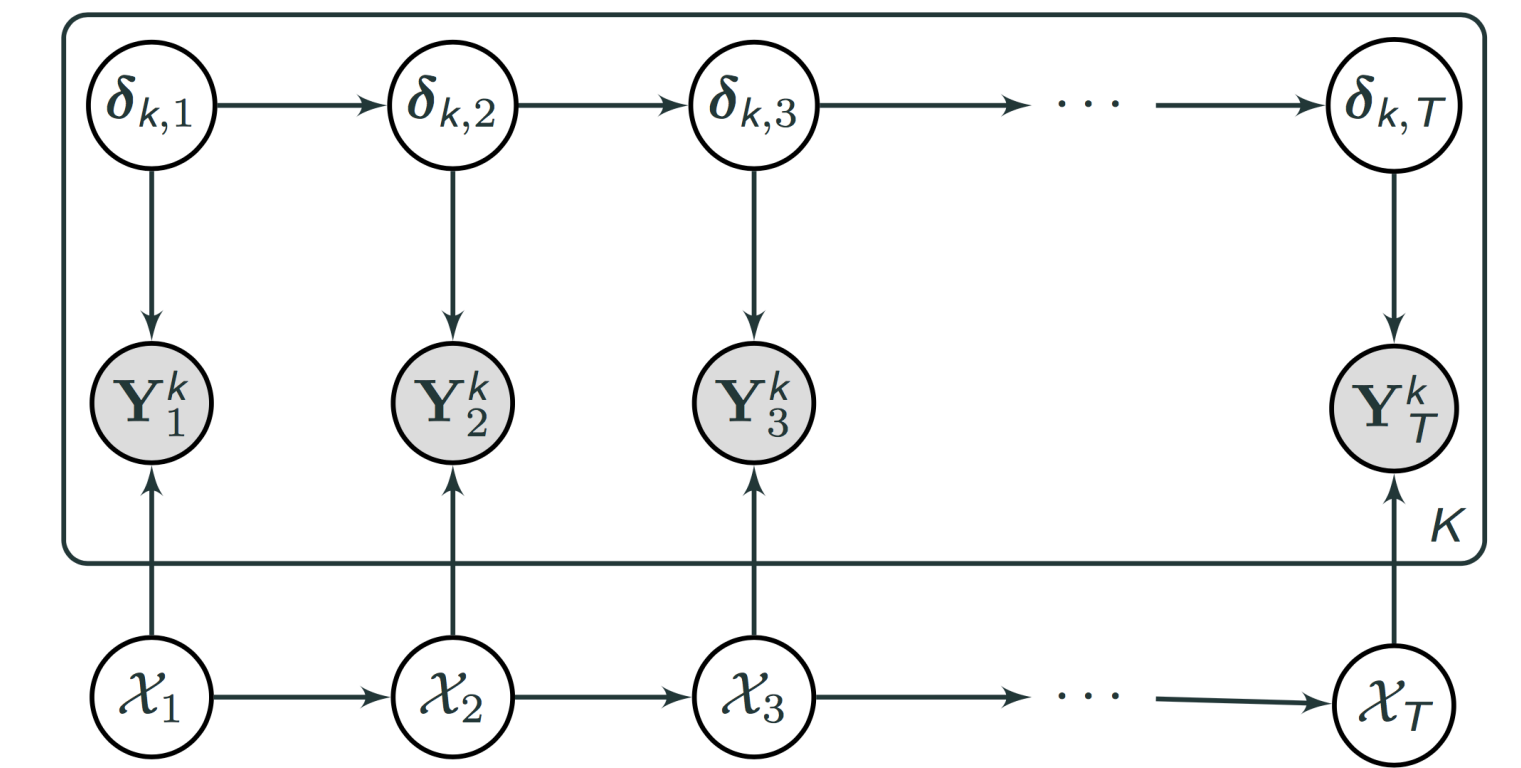
## Approach

### The Model

$$Y_{ijt}^k \stackrel{\text{ind}}{\sim} \text{Bernoulli}(P_{ijt}^k)$$

$$\text{logit}(P_{ijt}^k) = \delta_{k,t}^i + \delta_{k,t}^j + \mathbf{X}_t^i \Lambda_k \mathbf{X}_t^j$$

- $\delta_{k,t}^i \in \mathbb{R}$  : sociality, varies by time and layer.
- $\mathbf{X}_t^i \in \mathbb{R}^d$  : latent vector, shared by all layers but varies over time.
- $\Lambda_k = \text{diag}(\lambda_k) \in \mathbb{R}^{d \times d}$  : diagonal homophily matrix, varies across layers.



Latent variables evolve via a Markov process

### Variational Inference

Introduce a new structured mean-field approximation to the posterior that maintains temporal dependencies:

$$q(\theta) = \left[ \prod_{h=1}^d q(\lambda_{1h}) \right] \left[ \prod_{k=2}^K q(\lambda_k) \right] \left[ \prod_{k=1}^K \prod_{i=1}^n q(\delta_{k,1:T}^i) \right] \left[ \prod_{i=1}^n q(\mathbf{X}_{1:T}^i) \right] q(\tau^2) q(\sigma^2) q(\tau_\delta^2) q(\sigma_\delta^2).$$

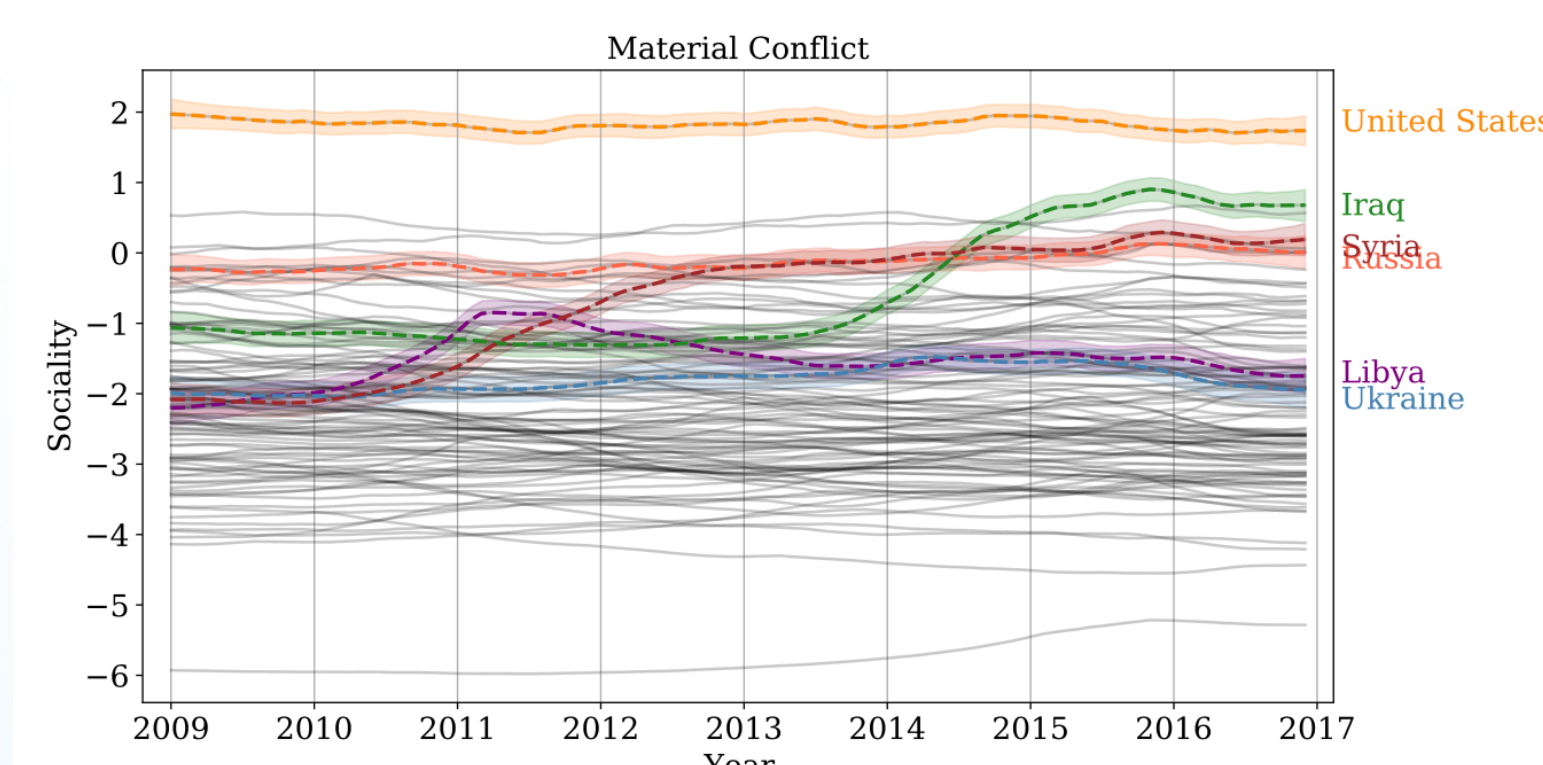
Derived a closed form coordinate ascent variational inference algorithm (CAVI) leading to efficient inference.

## Results

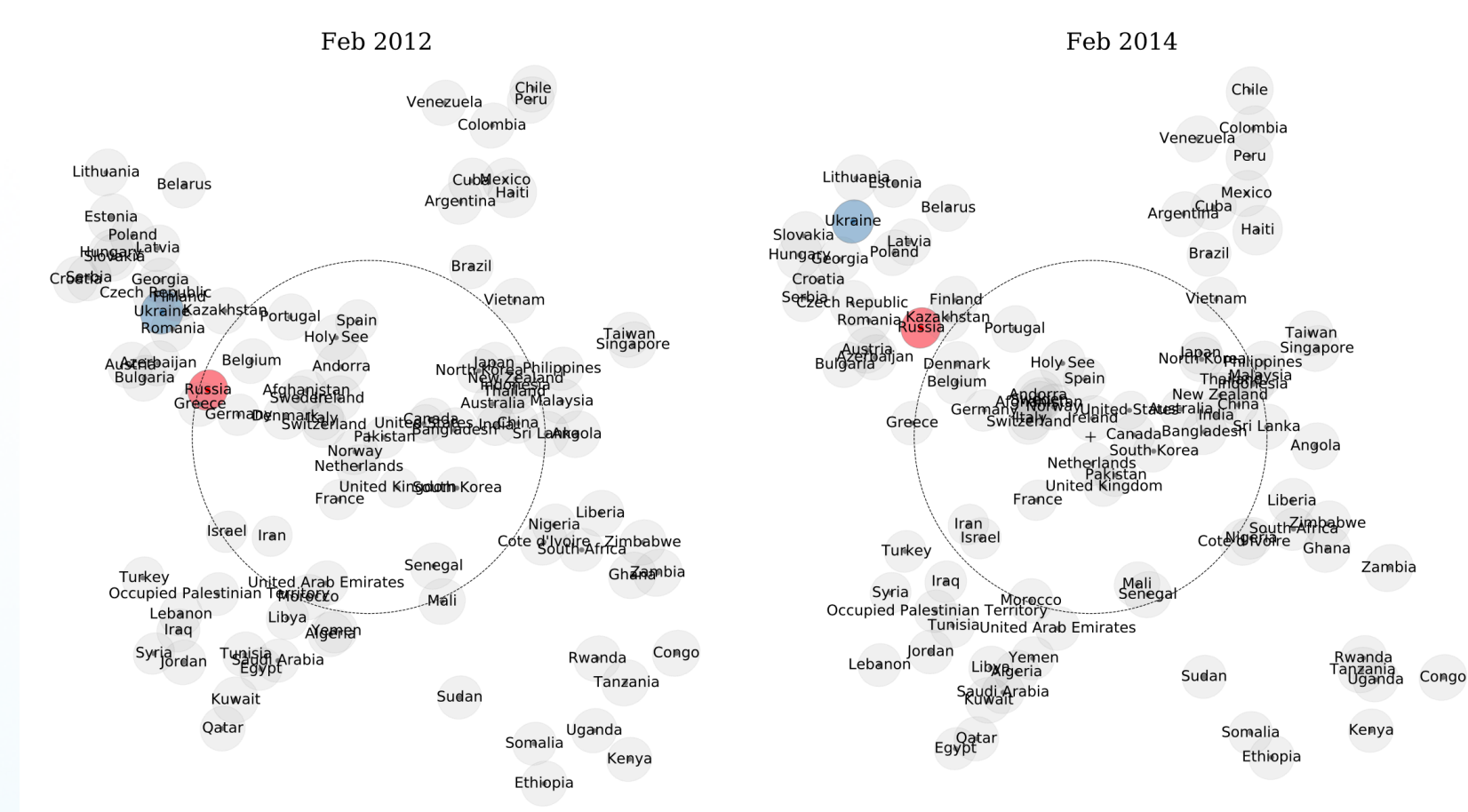
### Analysis of ICEWS Networks

Eight years during the Obama administration of monthly relational data taken from ICEWS.

- $Y_{ijt}^k = 1$  : country  $i$  and country  $j$  had a {**verbal cooperation**, **material cooperation**, **verbal conflict**, **material conflict**} on the  $t$ th month.



The nations' socialities reveal global conflicts: Arab Spring in Libya (2011), Rise of ISIL in Syria (2013 – present), The American-led intervention in Iraq (2014).

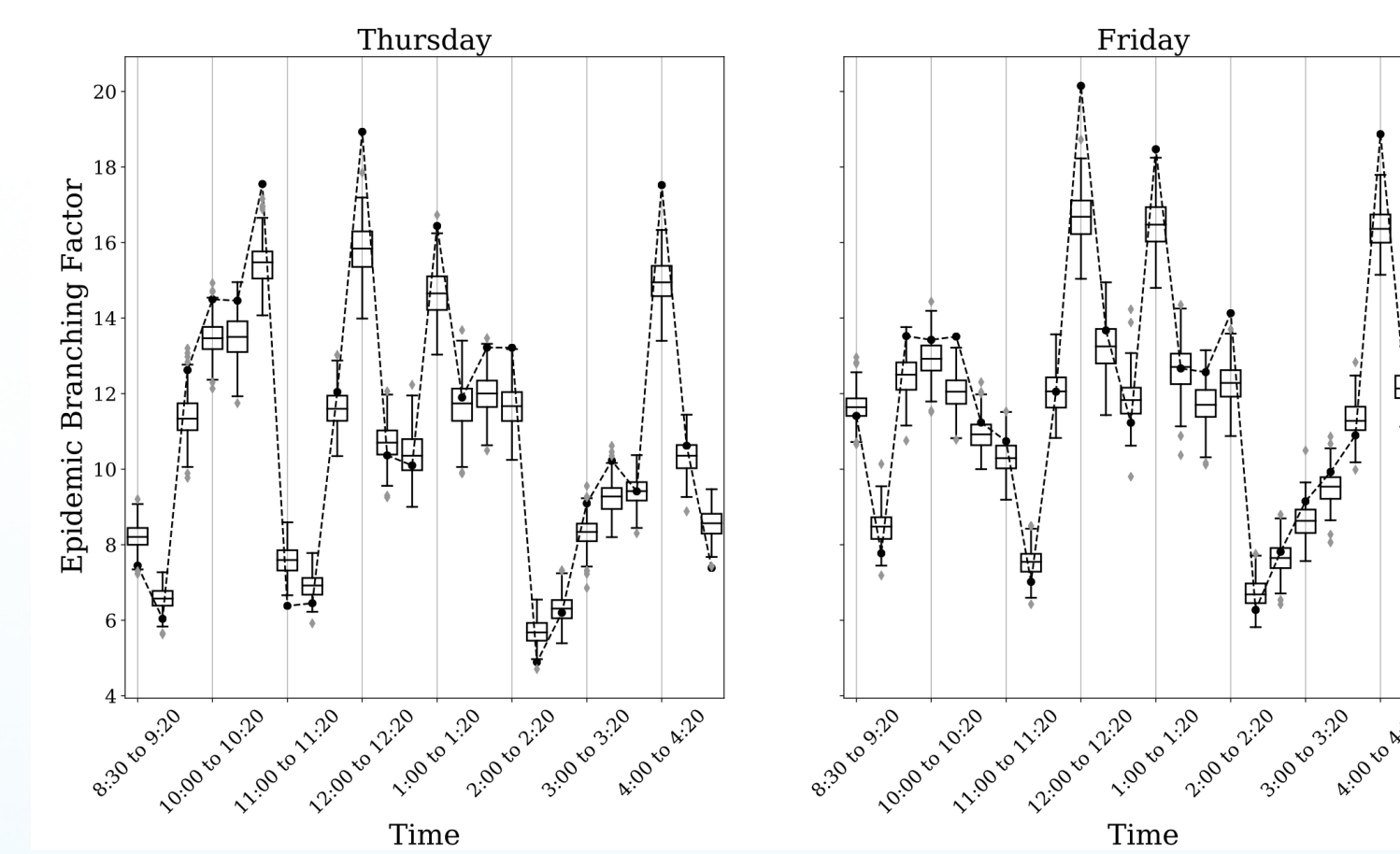


Latent positions reflect the nations' geographic location. The latent space reflects regional events such as the Crimean Crisis between Russia and Ukraine (2014).

## Challenges

### Quantifying $R_0$ in Face-to-Face Contact Networks

$Y_{ijt}^k = 1$  : individual  $i$  and individual  $j$  within 5 ft for more than 20 seconds during the  $t$ th 20 minute interval on {Thursday, Friday}.



Uncertainty quantification of the epidemic branching factor  $R_t$ . The network model underestimates sudden changes in network structure.

## Next Steps/ Future Work

- Allow inference on larger networks through stochastic optimization.
- Explore non-stationary state-space models to better model non-smooth changes.
- Extend the VB algorithm beyond binary or real-valued dyads, e.g., count data.



Scan me to download the full paper.