

1 Opportunities and Challenges for Neuromorphic Computing  
2 Algorithms and Applications

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7 **Abstract**

8 Neuromorphic computing technologies will be important for the future of computing, but much of  
9 the work in neuromorphic computing has focused on hardware development. Here, we review recent  
10 results in neuromorphic computing algorithms and applications. We highlight characteristics of neuromorphic  
computing technologies that make them attractive for the future of computing and we discuss  
opportunities for future development of algorithms and applications on these systems.

11 **1 Promises of Neuromorphic Computing**

12 With the end of Moore’s law approaching and Dennard scaling ending, the computing community is in-  
13 creasingly looking at new technologies to enable continued performance improvements. Neuromorphic com-  
14 puters are one such new computing technology. “Neuromorphic” was coined by Carver Mead in the late  
15 1980’s [63, 64], and at that time, primarily referred to mixed analog-digital implementations of brain-inspired  
16 computing. However, as the field has continued to evolve and with the advent of large-scale funding op-  
17 portunities for brain-inspired computing systems such as the DARPA Synapse project and the European  
18 Union’s Human Brain Project, the term neuromorphic has come to encompass a wider variety of hardware  
19 implementations.

20 We define neuromorphic computers as non-von Neumann computers whose structure and function are  
21 inspired by brains and that are composed of neurons and synapses. Von Neumann computers are composed  
22 of central processing units (CPUs) and memory units, where data and instructions are stored. In a neuromorphic  
23 computer, on the other hand, both processing and memory are governed by the neurons and  
24 the synapses. Rather than explicit instructions as in a von Neumann computer, programs in neuromorphic  
25 computers are defined by the structure of the neural network and its parameters. While Von Neumann com-  
26 puters encode information as numerical values represented by binary values, neuromorphic systems receive  
27 spikes as input, where the associated time at which they occur, their magnitude, and their shape can be used  
28 to encode numerical information. Binary values can be turned into spikes and vice versa, but the precise  
29 way to perform this conversion is still an area of study in neuromorphic computing [97].

30 Given the aforementioned contrasting characteristics between the two architectures, neuromorphic com-  
31 puters present some fundamental operational differences:

32 • **Massively parallel operation:** Neuromorphic computers are inherently massively parallel, where all of  
33 the neurons and synapses can potentially be operating simultaneously. However, the computations  
34 performed by neurons and synapses are relatively simple when compared with the parallelized von  
35 Neumann systems.

36 • **Collocated processing and memory:** There is no notion of a separation of processing and memory  
37 in neuromorphic hardware. Although neurons are sometimes thought of as “processing” units and

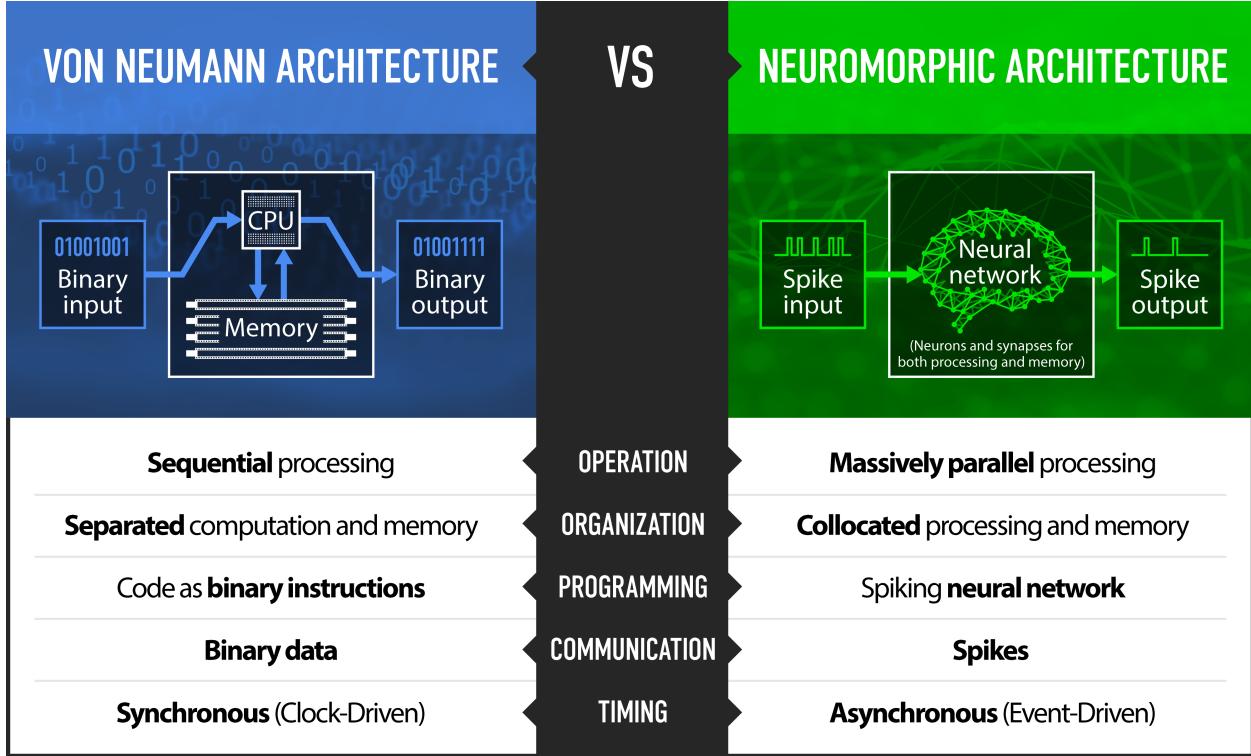


Figure 1: Comparison of the von Neumann architecture with the neuromorphic architecture.

38 synapses are sometimes thought of as “memory,” the neurons and synapses both perform processing and  
 39 store values in many implementations. The collocation of processing and memory helps mitigate the von  
 40 Neumann bottleneck i.e., the processor memory separation that causes a slowdown in the maximum  
 41 throughput that can be achieved. Additionally, the data accesses from main memory consumes a  
 42 significant amount of energy compared to the compute energy [110]) present in conventional computing  
 43 systems.

44

- 45 Inherent scalability: Neuromorphic systems are meant to be inherently scalable because adding ad-  
 46 ditional neuromorphic chips is simply increasing the number of neurons and synapses that can be  
 47 realized. It is possible to take multiple physical neuromorphic chips and treat them as a single large  
 48 neuromorphic implementation to run larger and larger networks. This has been successfully accom-  
 49 plished across a variety of large-scale neuromorphic hardware systems, including SpiNNaker [62, 31]  
 and Loihi [22].
- 50 Event-driven computation: Neuromorphic systems leverage event-driven computation (i.e., computing  
 51 only when data is available) and temporally sparse activity to allow for extremely efficient computa-  
 52 tion [70, 7]. Neurons and synapses only perform work when there are spikes to process, and typically,  
 53 spikes are relatively sparse within the operation of the network.
- 54 Stochasticity: Neuromorphic systems can include a notion of randomness, such as in the firing of  
 55 neurons, to allow for noise and enable different algorithms.

56 The features of a neuromorphic computer are well noted in the literature and are given as motivations  
 57 for implementing and using neuromorphic computers [98, 44, 109, 112, 23, 40]. One of the most attractive  
 58 features of neuromorphic systems for computation is their extremely low power operation; they can often  
 59 operate on orders of magnitude less power than traditional computing systems. This low power operation

60 is due to their event driven nature and massively parallel nature, where typically only a small portion  
61 of the entire system is active at any given time and the rest is idle. Because of the increasing energy  
62 cost of computing, as well as applications in which there are energy constraints (such as edge computing  
63 applications), energy efficiency alone is a compelling reason to investigate the use of neuromorphic systems.  
64 Additionally, because they inherently implement neural network-style computation, neuromorphic systems  
65 are a natural platform for many of today’s artificial intelligence and machine learning applications. There  
66 is also promise to leverage the inherent computational properties of neuromorphic computers to perform a  
67 wide variety of different types of computation [2].

68 Each of these features of neuromorphic computers are inspired by characteristics of the brain and have  
69 been prioritized in the implementation of neuromorphic computers in recent years. However, it is not clear  
70 whether they are the only aspects of biological brains that are important for performing computation. For  
71 example, though neurons and synapses have been chosen as the primary computational units of neuromorphic  
72 computers, there are a variety of other types of neural components that may be useful for computation,  
73 including glial cells [88, 41]. Moreover, neurons and synapses have been a convenient level of abstraction  
74 for neuromorphic systems, but whether they are the most appropriate level of abstraction is still an open  
75 question [89].

76 Unlike some of future computing technologies, many physical realizations of neuromorphic hardware are  
77 currently under development or are even available for use to the research community. Several large-scale neuromorphic  
78 systems have been developed with a variety of approaches and goals [36]. The European Union’s  
79 Human Brain Project sponsored the development of SpiNNaker [31] and BrainScaleS [94] to enable neuromorphic  
80 simulations at scale. An optimized digital neuromorphic processor called ODIN (Online-learning  
81 Digital spiking Neuromorphic) has also been proposed [35], allowing the use of slightly more complex neuron  
82 models. One of the neuromorphic platforms targeting more general computations for wider classes of  
83 applications is the Tianjic chip, a platform, that supports both neuromorphic spiking neural networks and  
84 the traditional artificial neural networks for different categories of problems [84]. Industry has also taken  
85 an interest to neuromorphic systems. Some examples include IBM’s TrueNorth [65] and Intel’s Loihi [22],  
86 and there are also a variety of academic efforts, including DYNAPs [69], Neurogrid [12], IFAT [112], and  
87 BrainScales2 [93]. The usefulness of neuromorphic hardware, such as BrainScales2, has been demonstrated  
88 in carrying out optimizations for learning to learn (i.e., where an optimization process is used to define how  
89 learning occurs) scenarios for spiking neural networks as it runs at a much accelerated timescales compared  
90 to biological timescales [15].

91 All the aforementioned large-scale neuromorphic systems are silicon-based and implemented using conventional  
92 CMOS (Complementary Metal Oxide Semiconductor) technology. However, there is a tremendous  
93 amount of research in the neuromorphic community on developing new types of materials for neuromorphic  
94 implementations, such as phase-change, ferroelectric, nonfilamentary, topological insulators, or channel-  
95 doped biomembranes [42, 75, 74]. One popular approach in the literature is using memristors as the fundamental  
96 device to have resistive memory to collocate processing and memory [45, 59], but other types of  
97 devices have also been used to implement neuromorphic systems, including optoelectronic devices [98]. Each  
98 device and material used to implement neuromorphic systems has unique operating characteristics, such as  
99 how fast they operate, their energy consumption, and the level of resemblance to biology. The diversity of  
100 devices and materials used to implement neuromorphic hardware today offers the opportunity to customize  
101 the properties required for a given application.

102 Most research in the field of neuromorphic computing today fall in the realm of the aforementioned  
103 hardware systems, devices, and materials. However, to most effectively use neuromorphic computers in the  
104 future, exploit all their unique computational characteristics, and help drive their hardware design, they must  
105 be connected to neuromorphic algorithms and applications. From this perspective, we provide an overview  
106 of the current state of the art in neuromorphic algorithms and applications and provide a forward-looking  
107 view on opportunities for the future for neuromorphic computing in computer science and computational  
108 science.

109 Additionally, it is worth noting that the term neuromorphic computing has been used for a wide array of  
110 different types of technologies. As noted previously, the original definition only encompassed mixed analog-

111 digital implementations. In this work, we consider all types of hardware implementations (digital, mixed  
112 analog-digital, analog) as neuromorphic, but we restrict our attention to *spiking* neuromorphic systems, i.e.,  
113 those that implement spiking neural networks.

## 114 2 Neuromorphic Algorithms and Applications

115 Programming a neuromorphic computer entails creating a spiking neural network (SNN) that can be deployed  
116 to that neuromorphic system (see Box 1). SNNs take an additional level of inspiration from biological neural  
117 systems in the way that they perform computation. In particular, SNNs include time in the way that they  
118 perform computation; neurons and synapses both include notions of time within most neuromorphic systems.  
119 For example, spiking neurons might leak charge over time based on a particular time constant, and neurons  
120 and/or synapses in SNNs might have an associated time delay.

121 Algorithms for neuromorphic implementations often entail how to define an SNN for a given application.  
122 There are a wide variety of algorithmic approaches for neuromorphic computing systems that fall into two  
123 broad categories: (1) algorithms for training or learning an SNN to be deployed to a neuromorphic system  
124 (Figure 2) and (2) non-machine learning algorithms in which SNNs are hand-constructed to solve a particular  
125 task. It is worth noting that here training and learning algorithms refer to the mechanism of optimizing the  
126 parameters of an SNN (typically the synaptic weights) for a particular problem.

127 This section provides an overview of some state-of-the-art approaches for neuromorphic computing algo-  
128 rithms, both on benchmarking datasets and demonstrating their potential applicability to real-world appli-  
129 cations.

## Spiking Neural Networks

Spiking neural networks (SNNs) are a particular type of artificial neural network in which the function of the neurons and the synapses in the network are more inspired by biology than other types of artificial neural networks, such as multi-layer perceptrons. The key difference between traditional artificial neural networks and SNNs is that SNNs take into account timing in their operation. Neuron models implemented in SNNs in the literature range from simpler integrate and fire models, in which charge is integrated over time until a threshold value is reached, to much more complex and biologically plausible models, such as the Hodgkin-Huxley neuron model, which approximates the functionality of specific aspects of biological neurons such as ion channels [98]. Both neurons and synapses in SNNs can include time components that affect their functionality.

Neurons in spiking neural networks accumulate charge over time from either the environment (via input information to the network) or from internal communications (usually via spikes from other neurons in the network). Neurons have an associated threshold value, and when the charge value on that neuron reaches the threshold value, it fires, sending communications along all of its outgoing synapses. Neurons may also include a notion of “leak,” where the accumulated charge that is not above the threshold dissipates as time passes. Additionally, neurons may have an associated axonal delay, in which outgoing information from the neuron is delayed before it affects its outgoing synapses. Synapses form the connections between neurons, and each synapse has a pre-synaptic neuron and a post-synaptic neuron. Synapses have an associated weight value, and that value may be positive (excitatory) or negative (inhibitory). Synapses may have an associated delay value such that communications from the pre-synaptic neuron are delayed in reaching the post-synaptic neuron. Synapses also commonly include a learning mechanism in which the weight value of the synapse changes over time based on activity in the network. Neuromorphic systems often realize a particular fabric of connectivity, but the synapses may be turned on and off to realize a network structure within that connectivity. Additionally, parameters of the neurons and synapses such as neuron thresholds, synaptic weights, axonal delays, and synaptic delays are often programmable within a neuromorphic architecture.

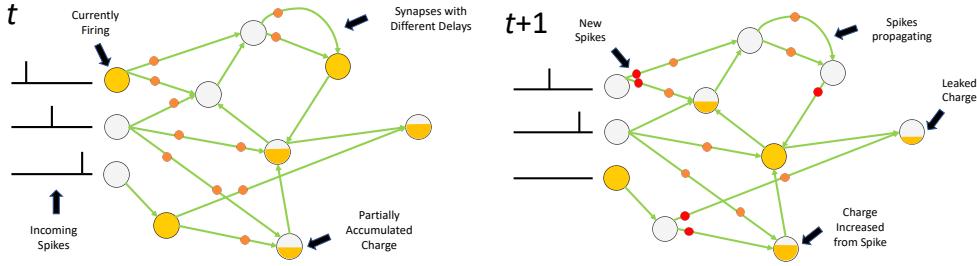
Unlike in traditional artificial neural networks where information is received at the input and then synchronously passed between layers in the network, in SNNs, even if input information is received at the same time and the SNN is organized in layers, because the delays on each synapse and neuron may be different, information is propagated asynchronously throughout the network, arriving at different times. Thus, this is beneficial while realizing SNNs on a neuromorphic hardware, which can be designed to operate in an event-driven or asynchronous manner that fits well with the temporal dynamics of spiking neurons and synapses. An example SNN and how it operates in the temporal domain is shown in the figure. In this example, synapses are shown with a time delay. Information is communicated by spikes passed throughout the network. In this example, the network’s operation at time  $t$  (left) and time  $t+1$  (right) is shown, to show how the network’s state changes with time.

130

## 2.1 Machine Learning Algorithms

### 2.1.1 Spike-Based Quasi-Back-Propagation

131 Back-propagation and stochastic gradient descent have shown impressive performance in the field of deep  
132 learning. However, these approaches do not map directly to SNNs because spiking neurons do not have  
133 differentiable activation functions (i.e., many spiking neurons use a threshold function, which is not directly  
134 differentiable). Additionally, the temporal processing component of SNNs can add another difficulty in train-  
135 ing and learning for these approaches. Algorithms that have been successful for deep learning applications  
136  
137



138 must be adapted to work with SNNs (top-left in Figure 2), and these adaptations can reduce the accuracy  
 139 of the SNN compared with a similar artificial neural network [117, 53, 8, 9].

140 Some of the approaches that adapt deep learning-style training include using a surrogate gradient and  
 141 having a smoothed activation function to compute the error gradients while performing weight adjustments  
 142 in each of the successive layers [76, 119]. There have also been a few demonstrations on computing the  
 143 spike error gradient [34, 57, 56] that have shown close to state-of-the-art classification performance on the  
 144 Modified National Institute of Standards and Technology (MNIST) handwritten digits dataset. To make use  
 145 of the inherent temporal dimension in SNNs, there have been efforts attempting to employ rules that have  
 146 been used to train recurrent neural networks, albeit with several approximations. As surveyed by Zenke  
 147 and Neftci [121], approaches such as backpropagation through time and real-time recurrent learning have  
 148 been demonstrated on neuromorphic datasets, such as the Spiking Heidelberg Digits (SHD) and the Spiking  
 149 Speech Command (SSC) dataset [20].

### 150 2.1.2 Mapping a Pre-Trained Deep Neural Network

151 Because deep neural networks (DNNs) have an established training mechanism, several efforts to deploy a  
 152 neuromorphic solution for a problem begin by training a DNN and then performing a mapping process to  
 153 convert it to an SNN for inference purposes (see top-right in Figure 2). Most of these approaches have yielded  
 154 near state-of-the-art performance with potential for significant energy reduction in computation due to the  
 155 use of only accumulate computations (AC) over multiply and accumulate computations (MAC) in DNNs on  
 156 several commonly employed datasets, such as MNIST, Canadian Institute For Advanced Research (CIFAR)-  
 157 10, and ImageNet [27, 39, 99, 101]. Most initial conversion techniques used weight normalization or activation  
 158 normalization or employed average pooling instead of max pooling [27, 90, 99]. Other approaches involve  
 159 training DNNs in a constrained manner so that the neuron's activation function iteratively starts resembling  
 160 that of a spiking neuron [39, 101]. Stockl et al., have proposed a new mapping strategy where SNNs make use  
 161 of Few Spikes neuron model (FS-neuron) that can represent complex activation functions temporally with  
 162 at most two spikes [108]. They have shown close to deep neural network accuracies on benchmark image  
 163 classification datasets with significantly fewer time-steps per inference compared to previously demonstrated  
 164 conversion strategies. Several applications demonstrated on neuromorphic hardware have employed some  
 165 of the aforementioned mapping techniques. Tasks such as keyword spotting, medical image analysis, and  
 166 object detection have been demonstrated to run efficiently on existing platforms such as Intel's Loihi and  
 167 IBM's TrueNorth [14, 32, 103].

168 It is worth noting that training a conventional DNN and then mapping to neuromorphic hardware,  
 169 especially emerging hardware systems, can see a reduction in accuracy not only because of the change from  
 170 DNNs to SNNs, but also because of the neuromorphic hardware itself. Often neuromorphic hardware systems  
 171 that are implemented with emerging hardware devices such as memristors will have reduced precision in  
 172 the synaptic weight values they can realize, and they may also have cycle-to-cycle and device variation.  
 173 When creating a mapping technique, it is important to take into account how these characteristics might  
 174 influence the inference performance of a mapped network.

175 Algorithms that use deep learning-style training to train SNNs often do not leverage all the inherent  
 176 computational capabilities of SNNs, and using those approaches limits the capabilities of SNNs to what

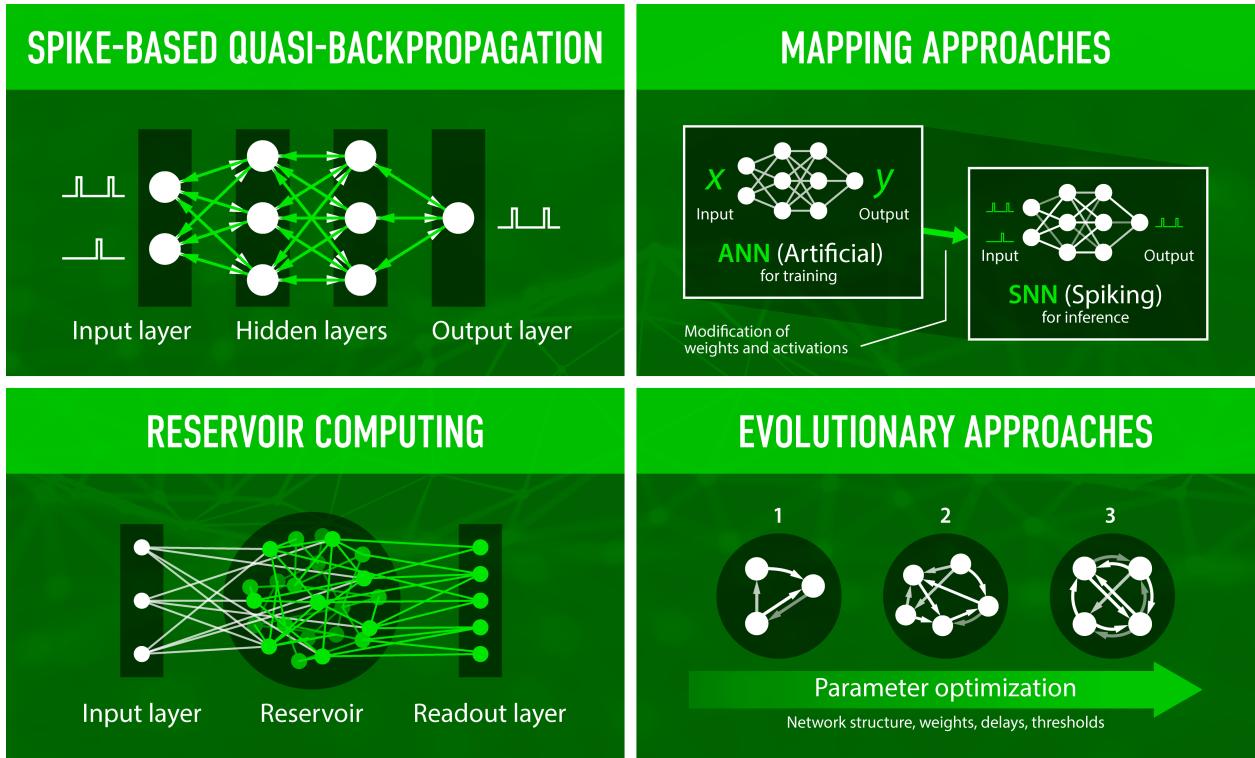


Figure 2: Common training approaches for SNNs. In the top-left panel, the structure of a network for a spike-based quasi-backpropagation is depicted. In this case, the training approach is performed directly on the SNN. The top-right panel shows the procedure for mapping approaches, where a traditional ANN is trained and then mapped into an SNN. The bottom-left panel shows the structure of a typical reservoir computing solution, including the input layer, the reservoir, and the readout layer. The bottom-right panel shows an evolutionary approach to evolving the structures and parameters of an SNN over time.

177 traditional artificial neural networks can already achieve. For example, most gradient descent style rules,  
178 including mapping approaches, do not focus on the temporal aspect of SNN computation.

### 179 **2.1.3 Reservoir Computing**

180 Another common algorithm used in SNNs is reservoir computing or liquid state machines (see bottom-left in  
181 Figure 2). In reservoir computing, a sparse recurrent SNN is defined to function as the “liquid” or reservoir.  
182 This liquid is typically randomly defined, but is required to have two properties: input separability, which  
183 requires that different inputs result in different outputs, and fading memory, which requires that signals do  
184 not continue to propagate infinitely through the reservoir and instead will eventually die out. In addition to  
185 the liquid itself, which is untrained, a reservoir computing approach also includes a readout mechanism, such  
186 as a linear regression, is trained to recognize the output of the reservoir. The key advantage of a reservoir  
187 computing approach is that it does not require any training of the SNN component. Reservoir computing  
188 in SNNs uses the sparse and recurrent connections with synaptic delays in networks of spiking neurons  
189 to cast the input to a spatially and temporally higher dimensional space [111]. Several demonstrations  
190 of spike-based reservoir computing have shown their effectiveness at processing temporally varying signals  
191 [51, 29, 116]. Variants of this computing framework have ranged from simple reservoir networks for bio-signal  
192 processing and prosthetic control applications [51] to using hierarchical layers of liquid state machines—a  
193 type of reservoir network—interconnected with layers trained in supervised mode for video [106] and audio  
194 signal processing applications [116].

### 195 **2.1.4 Evolutionary Approaches**

196 Evolutionary approaches to training or designing SNNs (see bottom-right in Figure 2) have also been used [96,  
197 92, 95]. In an evolutionary algorithm, a random collection of potential solutions is created to form an initial  
198 population. Each member of the population is evaluated and assigned a score, which are then used to perform  
199 selection (preferentially selecting better performing individuals) and reproduction (creating new individuals  
200 through recombination of old individuals and mutations) to produce a new population. In the context of  
201 SNNs for neuromorphic computing, evolutionary approaches can be used to determine parameters of the  
202 SNN, such as neuron thresholds or synaptic delays, or the structure of the network, such as the number of  
203 neurons and how they are connected to each other with synapses. These approaches are attractive because  
204 they do not require differentiability in the activation functions and do not rely on any particular network  
205 structure (i.e., feed-forward, recurrent). Additionally, they can be used to evolve the structure of the network  
206 and the parameters. However, their flexibility has a cost: evolutionary approaches can be slow to converge  
207 compared with other training approaches. Evolutionary approaches have been most successfully applied to  
208 control applications, such as video games [86] and autonomous robot navigation [92, 67].

### 209 **2.1.5 Plasticity**

210 Several neurobiological studies have reported the modulation of synaptic strength based on the activity of the  
211 connected neurons, which has been postulated as a learning mechanism for various tasks [13]. Spike timing-  
212 dependent plasticity (STDP), which operates on the underlying principle of adjusting the weights based  
213 on relative spike timings from pre- and post-synaptic neurons, is the most commonly implemented synaptic  
214 plasticity mechanism in neuromorphic literature [98]. Several different mathematical formulations of this rule  
215 have been demonstrated on the MNIST, CIFAR-10, and ImageNet datasets [102, 72, 55, 46, 11, 61]. Shrestha  
216 et al. presents a hardware-friendly modification of the exponential STDP rule, albeit the classification  
217 performance on MNIST was lower than the best results achieved so far with SNNs [102]. STDP-style rules  
218 have also been shown to approximate several machine learning approaches, such as clustering and Bayesian  
219 inference [73, 77]. STDP as a clustering mechanism has been demonstrated as a spike sorter in brain  
220 machine interface applications [73]. Combinations of spiking reservoirs and STDP have also been employed  
221 in an SNN approach called NeuCube [48]. NeuCube has been used to process electroencephalograms and

222 functional magnetic resonance imaging signals in applications such as sleep state detection and prosthetic  
223 controllers [48, 17, 54].

224 A much broader class of SNNs for modeling dynamical systems are the recurrent networks with delays  
225 and synaptic plasticity. One such class of networks are the polychronization networks [43], which have been  
226 employed for different spatio-temporal classification tasks [114]. Alemi, et al. demonstrate a local learning  
227 rule with recurrent SNNs with fewer spikes to realize non-linear dynamical systems [4]. Such recurrent SNNs  
228 have shown greater classification ability with winner-take-all models [60, 79, 47]. To leverage the temporal  
229 dimension of SNN, some learning algorithms aim at generating single or multiple spikes at desired times,  
230 which have been applied in classification tasks [36, 16, 115, 58, 120]. Most of these algorithms also depend  
231 on the spike representation used to encode the input signals. There have been several approaches to encode  
232 signals in terms of spike rates, latency, and neuron population, etc. [97, 85].

## 233 2.2 Non-Machine Learning Algorithms

234 The typical use cases for neuromorphic computing have been mainly machine learning, but neuromorphic  
235 computers have also recently been considered for non-machine learning algorithms. One common  
236 class of algorithms that have been mapped onto neuromorphic implementations comes from graph theory  
237 [38] [19][49] [5]. A graph is a model that consists of vertices, or *nodes*, along with directed pairwise  
238 relationships between nodes called *edges*. The underlying architecture of a neuromorphic system is a directed  
239 graph, so when there is a graph of interest, it can be embedded directly into a neuromorphic architecture  
240 with suitable parameter settings, and the spike raster can reveal graph properties. For example, with the  
241 correct parameter sets, a given node can be spiked, and the time at which other nodes spike corresponds  
242 exactly with the length of the shortest path from the source node [3]. During the COVID-19 pandemic,  
243 neuromorphic computing was coupled with graph theory as a tool for analyzing the spread of disease [37].

244 Random walks have also been implemented within neuromorphic systems. In a random walk, a random  
245 node is selected as a starting point, and an agent moves along an edge departing from that node selected  
246 at random. The process is repeated for several steps, and the locations visited by the random agent can  
247 reveal an important characteristic related to the underlying network. Random walk analyses frequently  
248 involve performing many random walks and then aggregating the results for analysis. Although traditional  
249 hardware performs the parallel step well, the aggregation and analysis step requires high energy usage  
250 sequential operation and does not always benefit from parallel architectures, such as GPUs. Severa et al. [100]  
251 showed that in certain settings, random walks could be studied in low-energy neuromorphic settings and that  
252 the analysis can be done in an inherently parallel fashion. Random walks are central to many problems in  
253 computational physics. Smith, et.al. [105] used neuromorphic deployments of discrete time Markov chains to  
254 approximate solutions for both particle transport problems and heat flow on complex geometries with energy  
255 efficient time scalable approaches. Given that graphs are a special class of objects called relational structures,  
256 foundational work of Cook [18] on relational structures has proven to be compatible with neuromorphic  
257 hardware, finding application to learning in cortical networks [26] and unsupervised learning tasks [25].

258 Neuromorphic computing have also been used to find approximate solutions to NP-complete problems,  
259 which are some of the most difficult problems in computation. Neuromorphic systems can perform comparably  
260 in terms of time-to-solution and accuracy of the solution to their conventional counterparts, which use  
261 CPUs and GPUs to approximately solve NP-complete problems. The energy efficiency of the neuromorphic  
262 approaches makes them amenable to be used in edge computing applications. Thus, a considerable amount  
263 of energy can potentially be saved using a neuromorphic computer to address NP-complete problems without  
264 compromising on the time-to-solution and the accuracy of the solution. Several neuromorphic approaches  
265 for approximating NP-complete problems have been proposed in the literature. Alom et al. used the IBM  
266 TrueNorth Neurosynaptic system to approximately solve the quadratic unconstrained binary optimization  
267 (QUBO) problem, which is NP-hard [6].

268 Mniszewski converted the NP-complete graph partitioning problem to the QUBO problem and used the  
269 IBM TrueNorth system to solve it approximately [68]. In some cases, their solutions are more accurate  
270 than the solutions returned by the D-Wave quantum computer, which was designed to approximately solve  
271 the QUBO problem. Yakopcic et al. leveraged Intel Loihi to approximately solve the boolean satisfiability

272 (SAT) problem [118]. They were able to obtain the solutions of many different SAT problems with varying  
273 sizes on the Loihi chip. Earlier work of Mostafa et al. developed neural network techniques for approxi-  
274 mately solving many constraint satisfaction problems [71], later on dedicated neuromorphic hardware [70].  
275 Fonseca and Furber develop a software framework for solving NP-complete constraint SAT problems on  
276 the SpiNNaker architecture [30]. Pecevski et al. [83] used neuromorphic hardware to perform inference and  
277 sampling on general graphical structures, such as Bayes' nets, which is NP complete for random variables  
278 with probabilities not bounded away from 0 [21].

### 279 **3 Closing the Gap Between Promises and Reality**

280 Though neuromorphic hardware is available in the research community and there have been a wide variety  
281 of algorithms proposed, the applications have been primarily targeted towards benchmark datasets and  
282 demonstrations. Neuromorphic systems are not currently being used in real-world applications, and there  
283 are still a wide variety of challenges that restrict or inhibit rapid growth in algorithmic and application  
284 development.

#### 285 **3.1 Limiting of Algorithmic Focus**

286 There has yet to be a machine learning algorithm/application combination for which neuromorphic com-  
287 puting substantially outperforms deep learning approaches in terms of accuracy, though there have been  
288 compelling demonstrations in which neuromorphic solutions outperform other hardware implementations  
289 such as neural hardware and edge GPUs in terms of energy efficiency [14]. This has led to the argument that  
290 neuromorphic systems are primarily interesting because of their low power computing abilities. However, we  
291 believe that there is tremendous algorithmic opportunity for neuromorphic systems as well. There has been  
292 a focus on backpropagation-based training approaches because of their state-of-the-art performance in deep  
293 learning. By limiting focus to those algorithms, however, we may also be limiting ourselves to achieving  
294 results that are only comparable with (rather than surpassing) deep learning approaches. We believe that  
295 there is more opportunity to develop approaches that utilize the inherent features of spiking neuromorphic  
296 systems, such as evolutionary algorithms or neuroscience-inspired approaches. At the same time, though  
297 these approaches have also been iterated on for decades, they similarly have not achieved state-of-the-art re-  
298 sults. Since these approaches use the native features of SNNs and thus require computing SNNs, iterating on  
299 and refining these algorithms is inherently bound by how efficiently SNNs can be computed. Neuromorphic  
300 computers have the opportunity to significantly speed up SNN evaluation and thus, provide the opportunity  
301 to accelerate development of SNN-based algorithms.

#### 302 **3.2 Limited Access to and Usability of Existing Hardware and Software**

303 One key issue that inhibits algorithmic and application development for neuromorphic systems is the lack  
304 of readily accessible and usable software and hardware systems for the entire computational and computer  
305 science communities. Several neuromorphic systems are available; however, there are a limited number of  
306 these systems and they are typically only available via cloud access to the broader community. Several  
307 open-source neuromorphic simulators have support for different hardware back ends, such as multinode  
308 CPUs, GPUs, and emerging neuromorphic hardware (e.g., SpiNNaker [50]). Although simulators such  
309 as NEST [33], Brian [35], and Nengo [10] are available, none are universally used, and they are often  
310 built for a specific purpose. For example, NEST targets primarily computational neuroscience workloads,  
311 whereas Nengo implements computation as framed by the Neural Engineering Framework [107]. Because  
312 these software systems are developed for particular communities and use cases, their broader usability and  
313 accessibility are limited outside those communities. In the future, to enable broader usability, development  
314 of neuromorphic simulators, hardware, and software should take into account the more broad applicability of  
315 these systems. Many of these simulators also have limited performance when operating at scale [52]. With the  
316 current data explosion comes the need to process data quickly enough to keep up with data generation speeds,

317 hence emphasising the need for highly performant and scalable neuromorphic simulators that can effectively  
318 leverage current high-performance computing systems to develop and evaluate neuromorphic workloads.  
319 The above mentioned limitations of simulators and also the large training times of current neuromorphic  
320 algorithms compared to non-spiking approaches have limited the usage of neuromorphic solutions to real-  
321 world applications, which actively needs to be addressed. Additionally, because the simulators are slow, it  
322 is very difficult to rapidly evaluate new algorithmic approaches, leading to slow algorithmic evolution. To  
323 enable more rapid advancement, the community needs performant hardware simulators or emulators that  
324 can be used when hardware is difficult or impossible to access.

### 325 **3.3 Incorporation of Neuromorphic Computers into Broader Computing Envi- 326 ronments**

327 In addition to the lack of availability for neuromorphic hardware and performant simulators, many future  
328 use cases of neuromorphic computers are likely to be included as part of a broader heterogeneous computing  
329 environment rather than be operated in isolation. Because of performance constraints (e.g., energy usage  
330 or processing speed) in existing hardware, more exotic hardware systems, such as neuromorphic and quan-  
331 tum computers, will increasingly be included in the computing landscape to accelerate particular types of  
332 computation. Since neuromorphic computing is likely not the best accelerator for all possible computations  
333 but is likely to be the most performant on certain applications and certain metrics (i.e., energy efficiency for  
334 neural network computation and speed for neuroscience simulations), they will potentially be used alongside  
335 other accelerators or specialized hardware for other applications. For example, a neuromorphic system could  
336 be operating as a co-processor with a CPU with other nontraditional computing systems—such as reconfig-  
337 urable, quantum, and approximate computing systems—also possibly operating as co-processors. Integrating  
338 these diverse systems into a single compute environment and developing programming models that enable  
339 the effective use of diverse heterogeneous systems is an ongoing challenge [113]. Additionally, neuromorphic  
340 systems are heavily reliant on host machines for defining the software structure that is deployed to the neu-  
341 romorphic implementation and often for communication to and from the outside world (i.e., interfacing with  
342 sensors and actuators for real-world applications). This reliance can have a significant impact on the perfor-  
343 mance benefits of using a neuromorphic computer, to the point where factoring in communication and host  
344 machine costs eliminates the benefits of using a neuromorphic computer to implement an application [24].  
345 A key challenge moving forward is how to minimize this reliance on traditional computers, as well as to  
346 optimize communication between them.

### 347 **3.4 Lack of Benchmarks and Metrics**

348 Another key challenge for neuromorphic algorithmic development is the lack of clearly established bench-  
349 marks, metrics, and challenge problems. Without common benchmarks and metrics, it is extremely difficult  
350 to evaluate which hardware system is most suitable for a given algorithm or application. Moreover, evalu-  
351 ating whether a new algorithm performs well can be extremely difficult without commonly defined metrics.  
352 Challenge problems, such as the ImageNet task for deep learning, drove significant advances in the field [66].  
353 The field of neuromorphic computing does not have a well-defined task or set of tasks that the entire com-  
354 munity is attempting to solve. Several groups have created datasets with event/spike based representation  
355 and having temporal dimension specifically for benchmarking neuromorphic training algorithms, such as the  
356 neuromorphic MNIST [78], DVS Gesture [7], and the Spiking Heidelberg audio datasets [20]. Though there  
357 are an increasing number of these neuromorphic-targeted datasets, these datasets have not yet been broadly  
358 adopted by the field at large as common benchmarks, limiting their utility at present. Datasets such as  
359 MNIST, CIFAR-10, and ImageNet dominate the benchmarks in neuromorphic, but these datasets do not  
360 require the native temporal processing capabilities present in neuromorphic systems, and as such, do not  
361 showcase the full capabilities of neuromorphic systems. Though the field needs benchmarks and challenge  
362 problems to target, it is also worth noting that creating a single challenge problem can also be dangerous  
363 because it may result in advances that target only that application, which can narrow the broader utility of  
364 the technology (an issue that affects the field of machine learning as a whole). Because of the wide variety of

365 algorithms and applications of neuromorphic systems as detailed in the previous sections, we propose that  
366 instead of a single benchmark or challenge problem, there should instead be a suite of challenge problems,  
367 drawing from both machine learning and non-machine learning use cases.

### 368 **3.5 Lack of Programming Abstractions**

369 Finally, an additional challenge specific to the development of non-machine learning algorithms for neuro-  
370 morphic deployment is the lack of programming abstractions for neuromorphic implementations. Currently,  
371 these approaches require that the programmer design the SNN for a particular task at the neuron and  
372 synapse level, defining all parameter values of those elements and how they are connected. Not only is this  
373 a fundamentally different way of thinking about how programming is performed but it is often also very  
374 time consuming and error prone to implement operations at this level. It is no coincidence that many of the  
375 non-machine learning algorithms for neuromorphic are centered on graph algorithms, because there is a very  
376 clear approach for mapping a graph into a network (i.e., nodes to neurons and edges to synapses). There  
377 have been attempts to describe programming abstractions at a higher level, such as the Neural Engineering  
378 Framework (NEF) [107] and Dynamic Neural Fields (DNF) [91]. However, these are often restricted to spe-  
379 cific use cases and algorithms, such as biologically-plausible neural models in the case of NEF and modeling  
380 embodied cognition for DNF. We believe both NEF and DNF are important abstractions for neuromorphic,  
381 but we also believe that there is still a gap in defining abstractions for using neuromorphic computers more  
382 broadly. One approach that could be taken is defining subnetworks of spiking neurons and synapses to per-  
383 form specific tasks that are familiar to programmers, such as binary operations, conditionals, and loops, in  
384 addition to those defined by NEF and DNF, as well as guidance for composing these subnetworks into larger  
385 networks capable of more complex tasks. There has been some initial work in this direction. In particular,  
386 Plank, et al. describe subnetworks that perform basic tasks such as AND, OR, and XOR using different  
387 spike encoding schemes [87]. However, there is still tremendous opportunity to influence how these subsys-  
388 tems should be defined and composed. It is clear that they can be used for more than just neural network  
389 computation; however, until clearer program abstractions are defined and/or the broader computing com-  
390 munity becomes more familiar with the computational primitives of neuromorphic computing, non-machine  
391 learning neuromorphic algorithms will be slow to develop. It is also worth noting that although it is possible  
392 to implement a variety of different types of computations on neuromorphic computers, this does not mean  
393 that every problem *should* be mapped onto a neuromorphic computer because not every problem is likely to  
394 benefit from the computational characteristics of neuromorphic systems described in Section 1. It is better  
395 to think of neuromorphic computers as specialized processors than general purpose computer. However, we  
396 do want to emphasize with this work that the scope of specialized processors is not just neuroscience or  
397 machine learning algorithms, but a wide variety of other types of computation as well.

## 398 **4 Outlook**

399 Neuromorphic processors are energy efficient and adept at performing machine learning and some non-  
400 machine learning computations. They offer tremendous potential for computing beyond Moore’s law. We  
401 envision at least three use cases for neuromorphic processors. First, because of their low power consumption,  
402 neuromorphic processors will be indispensable for edge computing applications, such as autonomous sys-  
403 tems (e.g., vehicles, drones), robotics, remote sensing, wearable technology, and IoT. Second, neuromorphic  
404 computers are well poised to become the artificial intelligence accelerators and co-processors in personal  
405 computing devices, such as smart phones, laptops, and desktops. Accelerators and specialized architectures  
406 have already been widely adopted in mobile phones, and the need for extremely energy efficient operations  
407 to improve battery life in those systems as well as laptops continues to be an important factor. Neu-  
408 romorphic systems can help realize those operations with potentially orders of magnitude less power than  
409 today’s accelerators. Finally, because of their ability to perform certain non-machine learning computations,  
410 we envision that neuromorphic computers will be added on as co-processors in next-generation extremely  
411 heterogeneous high-performance computing systems. In this scenario, neuromorphic computers would be ex-

412 pected to enable spike-based simulations [37], run graph algorithms [49, 38], solve differential equations [104],  
413 and efficiently approximate NP-complete problems [68]. It is worth noting that the different use cases of  
414 neuromorphic computers, from edge devices to accelerators and co-processors are likely to look very different  
415 in their implementations. Neuromorphic systems deployed at the edge may be specialized to operate with  
416 one particular application and have a focus on, for example, extremely low power inference performance,  
417 whereas neuromorphic systems for broader types of computations in an HPC setting will have a focus on  
418 enabling reconfigurability and training acceleration. Though neuromorphic systems are not currently present  
419 in these use cases, we do expect that they will begin to emerge in these technologies in the future, likely first  
420 in the edge computing space as specialized processors, and later into future heterogeneous computers.

421 Although several large-scale neuromorphic hardware systems are already available to the research com-  
422 munity, these systems are all being actively developed. Moreover, there is a wide variety of research efforts  
423 in developing new materials and devices to implement neuromorphic hardware. As such, there is an oppor-  
424 tunity to engage in a software-hardware codesign process in the development of neuromorphic hardware [1].  
425 Currently, most neuromorphic hardware design begins from the bottom of the compute stack (i.e., materials  
426 and devices) up to the algorithms and applications. That is, the hardware substrate is defined first, and  
427 the onus is then on the algorithms and applications developers to map them onto that particular hardware  
428 implementation. However, because the hardware is being actively developed and new materials and devices  
429 for neuromorphic computing are being actively investigated, there is tremendous opportunity to engage in  
430 codesign all across the compute stack, for example, so that the algorithms and applications can influence  
431 the underlying hardware design (Figure 3). There is an opportunity to tailor the underlying hardware im-  
432 plementation to suit a particular application’s needs or constraints. This opens up new horizons to not only  
433 focus on digital computing, but also to rethink using analog, approximate, and mixed-signal computing [28],  
434 since biological neural computation itself is inherently analog and stochastic. Among several approaches  
435 proposed in the literature on software-hardware codesign, one is using Bayesian optimization and Neural  
436 Architecture Search approaches in which several stacks of computing that range from materials and devices  
437 to algorithm and applications are codesigned to optimize overall system performance [82, 80, 81]. For ex-  
438 ample, in a memristive crossbar-based accelerator, an automatic codesign optimization approach has the  
439 opportunity to define the number and sizes of crossbars to optimize the accuracy and energy efficiency of  
440 the design for different applications or datasets. In addition to the opportunity for whole-stack co-design  
441 driven by algorithms and applications noted here and by Aimone in [1], there is also the opportunity to  
442 allow for emerging materials and devices for neuromorphic computers to inspire our algorithmic approaches,  
443 for example, in the implementation of plasticity. Today, the process of implementing synaptic plasticity  
444 on devices begins with the inspiration of plasticity in biological brains, is implemented and demonstrated  
445 on emerging devices (top-down co-design), and then the specific plasticity algorithm is adapted to however  
446 plasticity functions on that device (bottom-up co-design). However, plasticity mechanisms in biological brain  
447 evolved to use biological materials and components. We believe there may be opportunities to look at the  
448 underlying physical behaviors of other devices and materials to inform new neuromorphic algorithms.

449 The potential of neuromorphic computers in the future of computing and computational science is only  
450 beginning to be understood, and there is tremendous opportunity to leverage the inherent computational  
451 characteristics of these systems for machine learning and certain non-machine learning computations as well.  
452 Using neuromorphic computers most effectively will require a paradigm shift in how researchers think about  
453 programming. We believe that there are opportunities to achieve unprecedented algorithmic performance in  
454 terms of speed and energy efficiency on many applications with neuromorphic computers. In particular, in  
455 addition to their clear benefits for neural network-style computation, we believe that two areas that have the  
456 opportunity to see tremendous benefits from neuromorphic computers are graph algorithms and optimization  
457 tasks. Both of these types of algorithms and applications have the opportunity to benefit from the massively  
458 parallel, event-driven, and/or stochastic operation of neuromorphic systems. With the confluence of many  
459 different types of algorithms and applications in neuromorphic, along with the active development of large-  
460 scale neuromorphic hardware and emerging devices and materials, now is the time for the greater computing  
461 community to begin considering neuromorphic computers a part of the greater computing landscape.

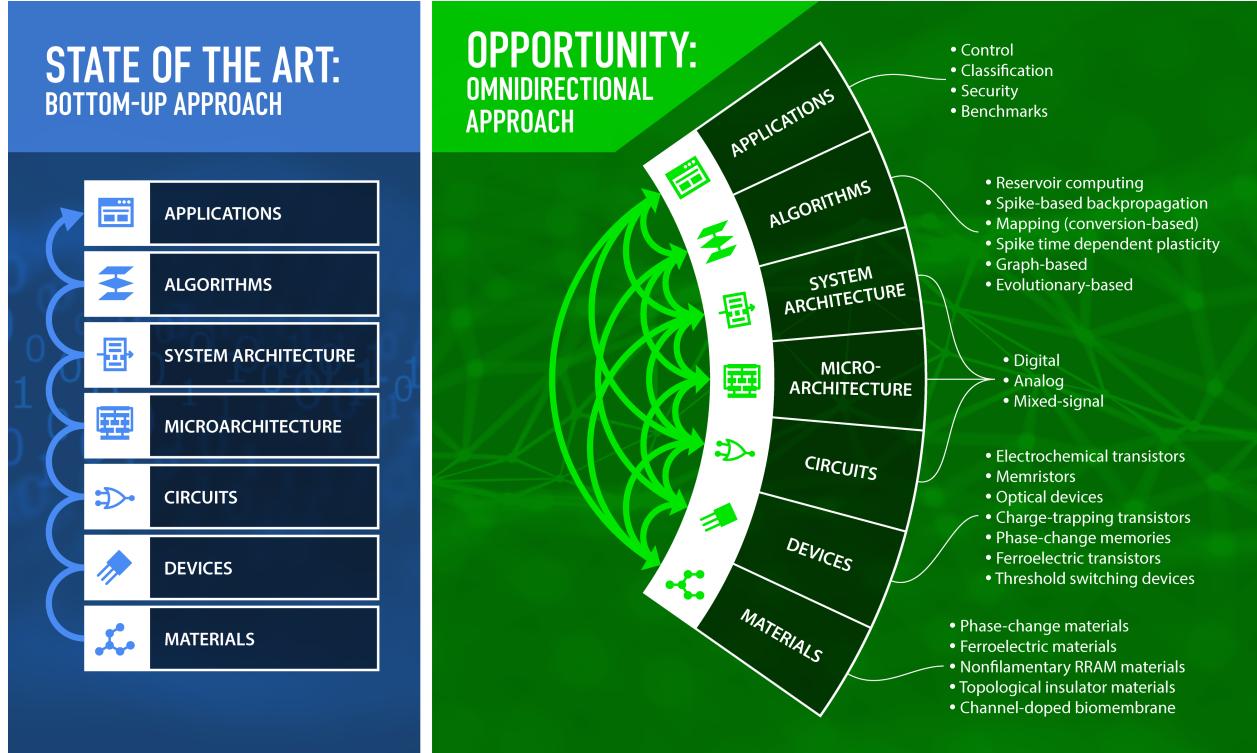


Figure 3: Opportunity for full compute stack co-design in neuromorphic systems. The current approach (shown on the left) is a bottom-up approach, where materials and devices are defined first, and those inform the architectures, algorithms, and applications sequentially. The opportunity for a future co-design approach (shown on the right) is for all aspects of the design stack to influence other components directly, e.g., for applications to directly influence the materials chosen or for the algorithms to directly influence the circuits used.

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