How to use manufacturing statistics when tolerancing a lens system

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ABSTRACT

Certain companies particularly those with strong design and optical manufacturing units keep strict but private statistical records regarding optical manufacturing and also use the data for design purposes. Design houses without manufacturing sectors are at a disadvantage. However, there is a small but growing public body of knowledge regarding these statistics. In this work, we develop a process to go from gathering raw manufacturing data to using the data for lens system tolerancing. We will describe a tolerancing practice using CODE V and our existing data with the goal of improving our ability to predict manufacturing outcomes. We present reasonable parameter values for the truncated normal and other distributions for variables such as wedge. In some cases, we will link the parameter values to standard tolerance categories that many manufacturers give: commercial, precision, and high precision. The tolerancing method will use established CODE V practice as well as macros with parameter values derived from data as one of the inputs. An example of how to use these techniques on a lens design will be given. We also provide an appendix that classifies glass type by manufacturability. The data provided in the appendix can be used in the tolerancing process.

Keywords: Optical manufacturing, tolerancing, lens, CODE V, statistics, Monte Carlo, optomechanics, glass

1. INTRODUCTION

Lens design software analyzes optical tolerances by modeling a perturbed optical system to capture the effect of simultaneous changes in manufacturing and alignment errors. In this work, we use the CODE V optical design software[1] with specialized macros developed by Richard Juergens[2] to devise a final tolerance scheme that is likely to result in a system that meets performance requirements. The tolerancing process often involves making assumptions concerning the statistical distributions of the relevant parameters for Monte Carlo simulations. In some cases, the presumed distributions do not resemble the actual lens manufacturing distributions. For example, optical designers assume a center thickness tolerance that is symmetric about the nominal value, but lens manufacturing brings parts to tolerance, not nominal values. The best option would use actual manufacturing data to build a statistical distribution for Monte Carlo simulations. Understanding true manufacturing distributions will have other beneficial consequences. Insight into the fabrication process yields insight into the shop statistics; conversely, analyzing these statistics and distributions gives insight into the fabrication process. The desired result is a more cost-effective lens design and an increased probability that the fielded optical system will perform as expected. Figure 1 depicts the major steps in this workflow.

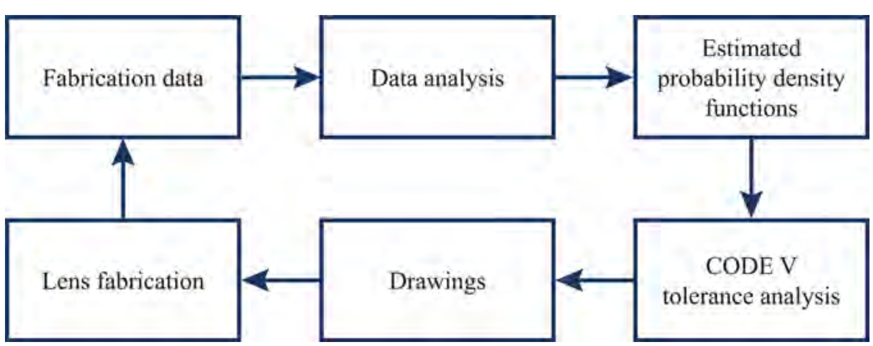

Figure 1. A workflow designed to minimize deviations from the expected result.

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We estimate probability density functions (PDFs) and use them as input to the CODE V tolerance analysis. The following four variables are of interest to us: (1) radius, (2) center thickness, (3) wedge, and (4) irregularity. We then use these distributions in the tolerancing process.

The subject of optical tolerancing is a niche within the much larger body of research on tolerancing. Of the seven categories of tolerance research identified by Hong and Chang,[3] the following three are most relevant to this study: tolerance analysis (how to estimate assembly-level results given variability of the components), tolerance synthesis (how to allocate the parts of an error budget), and tolerance transfer (how to turn tolerancing decisions into a manufacturing plan). We discuss and analyze these concepts, drawing on the larger body of knowledge about tolerancing. Our strategy is to base our analysis on actual manufacturing data, fabrication experience, and phenomenology, as we propose that such an analysis can better inform subsequent strategic decisions related to tolerance synthesis (or allocation) and tolerance transfer.

This strategy also requires careful statistical thinking. Manufacturing distributions may be truncated, bimodal, or skewed. Certain manufacturing data may be unreported or unavailable. Data outside the tolerance band, for example, may be unreported and thus excluded from the data set, which leads to truncated data and distributions. Bimodal data often suggest that there are multiple processes at work. The center thickness distribution is typically bimodal. In addition, manufacturing distributions are sometimes not centered about the nominal value. This leads to skewness, and the skewness in turn leads to nonintuitive implications for concepts like mean, median, and mode. It also implies that the expected value of a variable like radius may differ significantly from the nominal value. Tolerance analysis generally has the baseline assumption that the expected value is equal to the nominal value. This baseline assumption, however, is routinely violated in practice.

The origin of data skewness for radius or center thickness is related to what is sometimes called the “material safe” strategy. Suppliers are motivated to bring their lenses to tolerance, not to nominal. This explains why center thickness is often closer to the high side of the tolerance band and why convex surfaces are often flatter than nominal (assuming the nominal is in the center of the tolerance band). At this time, CODE V assumes that the nominal is centered within the tolerance band for radius and center thickness.

Although the normal distribution is important in describing manufacturing data, other distributions also come into play. As a consequence, it becomes important to continuously describe data with as few assumptions as possible. This is where nonparametric distributions (sometimes called smooth histograms) become extremely useful. We briefly explain the concept of nonparametric statistics as it relates to our present analysis. Problems and techniques related to nonparametric distributions have been described in detail elsewhere.[4]

The goal of this paper is to describe the process of taking raw manufacturing data and deriving statistical distributions that can be used for tolerancing analysis. We then take the resulting distributions and explain how to use them in CODE V tolerance simulations with the assistance of specialized macros. We also evaluate the usefulness of this exercise.

The benefit of this effort is twofold. Consider a case where the supplier’s capability is well within the needed tolerance. In this instance, tolerances could be made looser and costs lowered because the probability of getting close to the worst case is extremely low. In another scenario, an optical design looks good and an acceptable prototype is achieved, but as you go from prototype to production, the results are not what you expected. The process we describe in this paper is useful for understanding why unexpected results occur. Unexpected results may occur when the tolerancing process involves implicit assumptions that deviate significantly from the fabrication data.

We began pursuing this line of inquiry when we designed a 1300 mm long zoom lens for the Cygnus radiographic x-ray source.[5] We had three prototypes made. When the prototypes came in, our as-built optimization required moving some of the lenses by as much as a few centimeters. Our desire to understand how this happened is what motivated us to embark on this research.

2. STATISTICAL DATA ANALYSIS

The first major step in our process is to identify statistical distributions that realistically depict the four variables of concern during the tolerancing of a lens system: radius, center thickness, wedge, and irregularity. There are two errors that we wish to avoid. While an overly pessimistic distribution (like the uniform or end-point distribution) would result in costly designs in some cases, too much optimism would lead to unrealistic expectations for the system. There is a third concern to consider: excessive complexity in the tolerancing procedure could result in a lack of acceptance. We try to walk the middle ground, but there are no right answers. We propose solutions, but we also try to give enough information to allow the reader to make the choice that is most appropriate to him.

For this study, we use the term "batch" to denote an order of nominally identical lenses. We use the term "aggregate" to denote the combined data of many batches. The batch data distributions can have nonstandard shapes, but those shapes tend to even out when aggregated together.

2.1 Working with raw data

A commonly used standard (ISO 9001) requires manufacturers to maintain a high level of consistency in recording measured data in order to ensure data quality, reliability, and usability. Figure 2 shows an example of shop (manufacturing) data in a spreadsheet format. To extract data that are relevant to our analysis, we created an application in Mathematica, version 13.1. All of the plots also were generated with Mathematica.

ACME lens		**Part finished:**		**12**	**Qty Started:**		**15**			
Material:	N-BK7			**Side 1**	CX		SINGLE	**Side 2**	CC	
Maximum:	**6.6**	**50.1**	**0.005**	**51.676**	**0.25**			**86.701**	**0.4**	
Nominal:	**6.55**	**50**		**51.6**		**60-20**		**86.682**		**60-20**
Minimum:	**6.5**	**49.9**		**51.64**				**86.663**		
Units:	mm	mm	mm	mm	Fr			mm	Fr	
Serial #:	**CT**	**Dia**	**ETD**	**R1**	**IRR1**	**SQ1**	**METH S1**	**R2**	**IRR2**	**SQ2**
#1	6.581	50.088	0.002	51.674	0.113	✓	Pitch Polish Full Aperture	86.685	0.255	✓
#2	6.596	50.087	0.003	51.66	0.187	✓	Pitch Polish Full Aperture	86.692	0.255	✓
#3	6.59	50.08	0.002	51.671	0.197	✓	Pitch Polish Full Aperture	86.699	0.165	✓

Figure 2. An example of shop data; numbers are fictitious.

Data, however, can be messy; there can be blank fields and occasional data entry problems. Our program ignores blank fields. In some cases, we can have data that are on the edge of being acceptable, like a wedge measurement that is exactly equal to the maximum wedge specification. This is especially true when a tolerance is very stringent and can present a problem for the data analysis algorithms. In addition, algorithmic round-off error can inadvertently push a number outside the tolerance zone. We force the edge data to be slightly inside the tolerance zone for computational reasons. We did not address measurement error.

As we shall later see, the edge data are important for several reasons. For example, the data may be an indication that some lenses did not pass inspection. Reduced yield is an important cost driver, and it also has implications in the downstream tolerance analysis. In some cases, you get no data near the tolerance limits because the process is well within the capability of the supplier. Analysis of edge data helps the optical designer gain insight into the manufacturing process. The tolerance zone may be relaxed when the probability of getting a measurement near the edge is very low.

We found it advantageous to normalize the data to the maximum tolerance values so that radius and center thickness values originally on the interval $[-T, T]$ are scaled to the interval $[-1, 1]$. We normalized the irregularity and wedge data to $[0, 1]$. Because we are publishing the data, normalizing the data also renders the data anonymous. Data with different tolerance values can be combined, and this is important. Another advantage is that CODE V accepts PDFs on the interval $[-1, 1]$.

2.2 A summary of exploratory data analysis

We previously conducted what could be described as exploratory data analysis (EDA) to analyze and characterize lens manufacturing data.[4,6] We found, for example, that irregularity and edge thickness deviation (also called wedge or total indicator runout) are strongly correlated with the tolerance class. In this work, we segment the data in ways that are guided by the correlation analysis. Each segment will have a characteristic statistical distribution shape that we use when deciding which probability function to apply during tolerancing.

Radius measurements were found to be related to the qualitative shape (e.g., convex or concave) and a metric of material difficulty. This metric is related to the ratio of hardness to modulus of elasticity, but we do not have a phenomenological explanation as to why it might matter, and there may be more than one reason why a particular glass is problematic.

Hardness is a metric of inelastic deformation, whereas modulus is a metric of elastic deformation. The material factor we refer to is described elsewhere.[4] Figure 3 is a glass map with this factor identified. A list of easy and difficult glass types is also given in the appendix. Further research and input could help improve this list and explore whether there might be a better metric than our metric of material difficulty.

For many glass types, it is possible to have a prescriptive methodology for grinding and polishing. This means we can say "if this, do that." For glass with a high level of material difficulty, the prescriptive methods break down, the cost is higher, the yield is lower, and the results are less predictable.

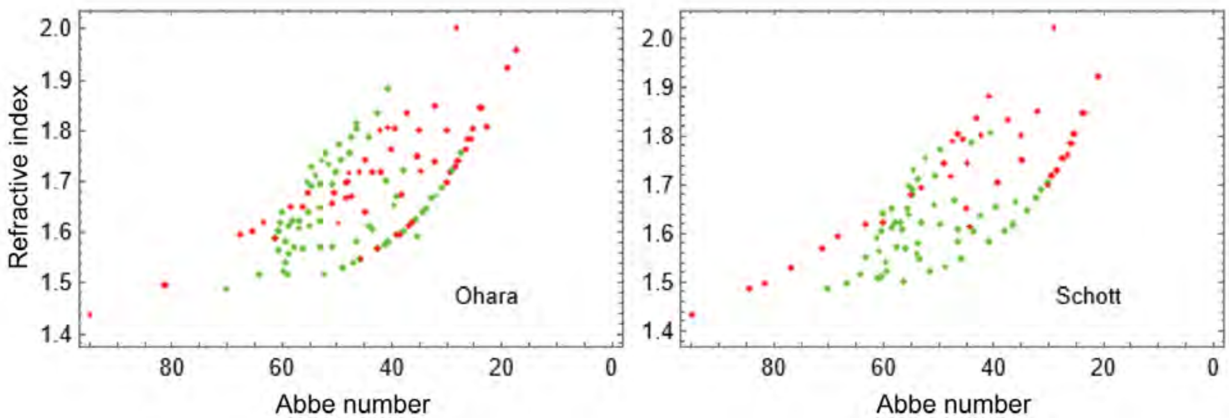

Figure 3. Glass maps with red and green denoting difficult and easy glass respectively. The Ohara plot includes i-Line but not low melt temperature glass.

The data we analyzed also included center thickness measurement, but it was not clear from the analysis what factors correlated to the mean and standard deviation. Consequently, we did not segment the center thickness data.

2.3 Statistical methods

The statistical end result we are looking for is a parametric distribution that can be used in the tolerancing process, e.g., the normal distribution. There are, however, a few issues that require careful consideration. Both the data and the resulting distributions end at the tolerance limit; this mathematical operation is called truncation. Truncated distributions are defined only inside the support interval and are zero outside that interval. The truncated distribution requires a multiplication factor such that the integral of the truncated distribution is unity.

The radius error can sometimes be modeled as a truncated normal distribution, but we found that other distributions can be helpful. It is known that the center thickness distribution is often bimodal with one large mode near +1 and a much smaller mode near −1. This phenomenon is due to lenses not passing inspection the first time and needing rework. The result is called a mixture distribution (actually a truncated mixture distribution).

The remaining variables of interest, irregularity and wedge, are nominally zero, but a certain amount of error is acceptable. We find that the PDF value near the origin is also zero, so distributions such as the truncated gamma or Rayleigh distribution can work.

In CODE V, the internally available PDFs span the interval [−1, 1], including wedge and irregularity. Thus, our normalization practice is mostly consistent with the internal CODE V practice, except for wedge and irregularity data. Irregularity is a peak-to-valley measurement. The peak is an additive defect, whereas the valley could be considered a subtractive defect, but the measurement difference is always reported as a positive number. We report a normalized distribution on the interval [0, 1] for irregularity. To translate the resulting distribution into something that CODE V can interpret would require a mixture distribution where half of the values are of the original PDF and the other half are of the reflected PDF. We discuss in more detail how this is done in section 2.5.

In CODE V, irregularity tolerance means that there may be a gap between the nominal spherical surface and manufactured surface of ±½T. The wedge distribution has issues that are similar to irregularity. The root mean square (RMS) is another metric of irregularity that is sometimes used. Opticians favor RMS because it is less sensitive to small measurement errors. We did not have enough data to report results of this metric.

Nonparametric methods also can be extremely useful. Nonparametric methods (sometimes called smooth histograms) involve a kernel function. Truncated (bounded) data present an additional challenge, but fortunately we find that this challenge can be addressed effectively with the methods developed by Silverman.[4,7] We used the reflection method to obtain most of the plots. For wedge and irregularity, we assumed that the error cannot be zero at the origin, so we used reflection on the right side and antireflection on the left side to force zero at the origin, along with a multiplier to ensure that the integral of the curve was unity.

It is helpful to compare the nonparametric results to the fitted parametric results for evaluation. Unfortunately, the nonparametric boundary estimates (assuming a truncated distribution) are known to be more error prone than the interior estimates. Nevertheless, the boundary density estimates are quite important to understand.

2.4 Strategies for segmenting data, calculation methods, and numerical results

Results from our previous effort to analyze lens manufacturing data showed strong correlations between the tolerance class and errors for both wedge (edge thickness deviation) and irregularity.[4] A tolerance class macro (TOLCLASS.seq) is included in the vender-provided macro library in CODE V, and the vendor-suggested tolerances are given in Table 1. Note that the tolerances are defined in terms of wavelength (λ), but they are measured and often listed in drawings in fringes (Fr). Note that 1λ = 2 Fr. By default, the radius tolerance is 4x the irregularity tolerance. Note also that CODE V lists wedge and irregularity as plus/minus values, because this is how it represents these variables.

Table 1. CODE V tolerance class suggestions.

	Class			
Attribute	**Regular**	**Select**	**Premium**	**Ultimate**
Delta sag	±5λ (10 Fr)	±2.5λ (5 Fr)	±1.25λ (2.5 Fr)	±0.25λ (0.5 Fr)
Center thickness	±0.13 mm	±0.05 mm	±0.025 mm	±0.0065 mm
Irregularity	±1λ (2 Fr)	±0.25λ (0.5 Fr)	±0.05λ (0.1 Fr)	±0.02λ (0.04 Fr)
Wedge	±25 µm	±13 µm	±6.5 µm	±2.5 µm

We fitted the available lens data for irregularity and wedge (edge thickness deviation) to the gamma distribution. Note that there have also been studies that have fitted data for irregularity and wedge to the Rayleigh distribution; however, we find that the gamma distribution gives us a shape that better matches the data and the nonparametric approximation. In Mathematica, the data fit is done by the "EstimatedDistribution" and "GammaDistribution" functions, where (k, θ) are the output variables. The variables (k, θ) are positive and sometimes called the shape and scale parameters, respectively.

There is an interesting relationship between the normal, Rayleigh, gamma, and Rice distributions. Suppose x and y are independent random variables with identical distributions and $z=\sqrt{x^2+y^2}$. If x and y are normally distributed with a zero mean, then z will be Rayleigh distributed. The z variable will be Rice distributed if x and y are normally distributed regardless of the mean location, but the Rice distribution is complicated to evaluate. We find from numerical experiments that z will be nearly gamma distributed as well (Figure 4). This finding leads us to believe that the gamma distribution is a reasonable choice for describing wedge and irregularity, especially when they have tight tolerances.

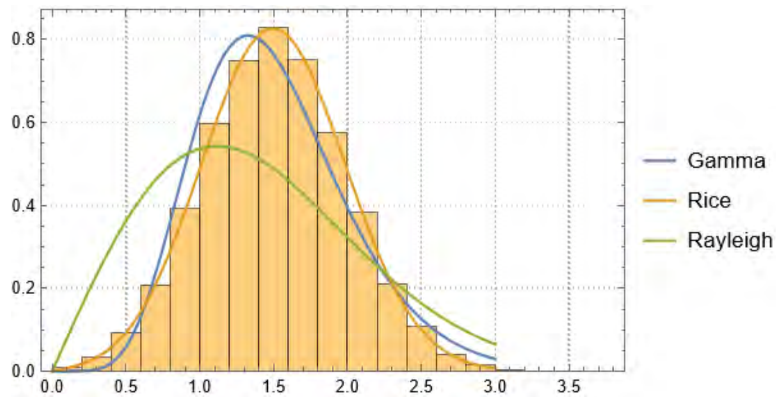

Figure 4. Numerical experiment with three distributions. In this case, x and y are both $N(1, 0.5)$. Distributions are fit by maximizing the log-likelihood function.

Mathematica allows a user to nest the target distribution (e.g., gamma or normal) inside the "TruncatedDistribution" operator. This operator sets the distribution to zero outside the truncation region. It also makes an adjustment such that the resulting PDF has an integral (on the appropriate support interval) that is equal to unity. CODE V requires the PDF for each of the tolerance variables to be truncated on the interval [−1, 1]. Mathematica also has the "MixtureDistribution" operator. This operator takes two or more distributions and adds them with weights specified for each one such that the resulting distribution will integrate to unity, and therefore is useful for bimodal data. The "EstimatedDistribution" operator in Mathematica takes data and a distribution with parameters and estimates the parameters.

It is also possible to fit the data in Mathematica by applying the "NMaximize" function to the "LogLikelihood" function. The Mathematica function "NMaximize" is a constrained global optimization function and gives the user more control over the results.

Figure 5 shows wedge distributions used in our analysis, where the solid lines represent best-fit curves (for the truncated gamma distribution) and the dashed lines nonparametric curves. Note the distributions shown in Figure 5c and also in Figure 6; we thought it important to match the nonparametric right boundary density (approximately 0.8) to avoid an overly optimistic fit from a tolerance analysis point of view. We did this by defining the output domain of k and θ (of the gamma distribution) to positive reals and adding a constraint such that the PDF[$x = 1 \mid k, \theta$] is equal to the estimated right boundary value (using "NMaximize"). Figure 6 also shows that the two-parameter gamma distribution gives the user more control than the one-parameter Rayleigh distribution. The results of our irregularity analysis are summarized in Figure 7. Table 2 lists selected irregularity and wedge categories that we used with guidance from optics manufacturer Optimax Systems. The data are segmented into three tolerance levels: commercial, precision, and high precision.

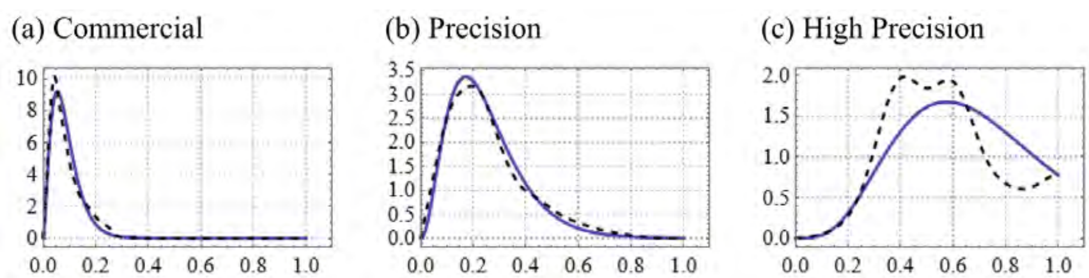

Figure 5. Wedge distributions: (a), (b), and (c) represent commercial, precision, and high precision, respectively.

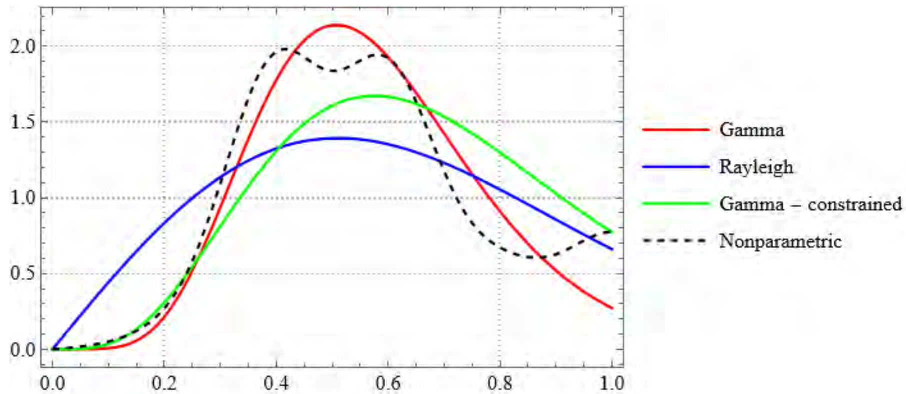

Figure 6. Distribution comparisons for high-precision wedge data.

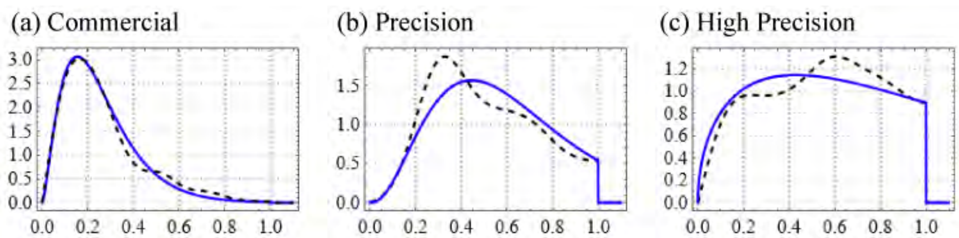

Figure 7. Irregularity distributions: (*a*), (*b*), and (*c*) represent commercial, precision, and high precision, respectively

Table 2. Irregularity and wedge tolerance and distributions with guidance from Optimax's Manufacturing Tolerance Chart.[8]

	Tolerance level		
Attribute	**Commercial**	**Precision**	**High Precision**
Irregularity			
Tolerance[a] (peak to valley)	$T \geq 1$ Fr	1 Fr > $T \geq 0.25$ Fr	$T < 0.25$ Fr
Distribution[b]	*G*(2.557, 0.099)	*G*(3.455, 0.181)	*G*(1.507, 0.846)
Wedge			
Tolerance[a]	$T \geq 0.02$ mm	0.02 mm > $T \geq 0.009$ mm	$T < 0.009$ mm
Distribution[b]	*G*(2.694, 0.032)	*G*(3.262, 0.076)	*G*(5.179, 0.138)

[a]*T* = tolerance value as measured and specified on the drawing.

[b]*G* = truncated gamma distribution.

Our previous data analysis effort also demonstrated statistically significant correlations between a metric of material difficulty and the radius error.[4] We also found subtle differences in the distributions of errors of concave and convex surfaces in our previous study.[6] We know that these differences are related to variations in the grinding process between concave and convex surfaces. With convex lenses, grinding begins at the diameter and moves inward; with concave lenses grinding begins in the center and moves outward. We segmented the data into concave or convex, and then further separated each according to the material difficulty (easy vs. difficult glass). We found striking differences between results when the data were segmented in this way (although bear in mind that the data were derived from a relatively small sample of lenses). The results are summarized in Table 3 and Figure 8. The solid lines in Figure 8 represent best-fit curves for the assumed distributions listed in Table 3 and the dashed lines nonparametric curves.

The results suggest that lenses ground from easy glass have symmetric distributions in the aggregate, although individual batches might not be symmetric. A symmetric distribution implies that the data average is equal to the nominal value as shown in the drawing. It is generally assumed that the nominal value is in the center of the tolerance band.

On the other hand, we find that lenses ground from difficult glass have asymmetric distributions that are different for convex and concave surfaces. We looked at only spherical surfaces for this study. Convex surfaces had a curvature to the right of the center of the tolerance band (i.e., flatter), whereas concave surfaces had a curvature to the left of the center of the tolerance band (i.e., rounder). This result is related to what is referred to as the "material safe" strategy, wherein the optician will attempt to meet tolerance while removing as little material as possible.

This finding suggests that for difficult glass, we should consider an asymmetric tolerance band. A possible goal is to use historical data to place the nominal value with respect to the tolerance band close to the most probable result.

Note that conservative assumptions like a uniform or end-point probability distribution are still symmetric. Consequently, even these conservative assumptions may lead to an incorrect result if the underlying assumptions of tolerance analysis do not take the possibility of asymmetry into account.

Table 3. Segmentation and distribution fit results for radius.

	Distribution[a]	
Material difficulty	**Convex**	**Concave**
Easy glass	*N*(0, 0.829), Figure 8a	*C*(0, 0.33), Figure 8b
Difficult glass	*E*(1.898), Figure 8c	*N*(−0.634, 0.730), Figure 8d

[a]*N* = normal distribution; *C* = Cauchy distribution; *E* = exponential distribution.

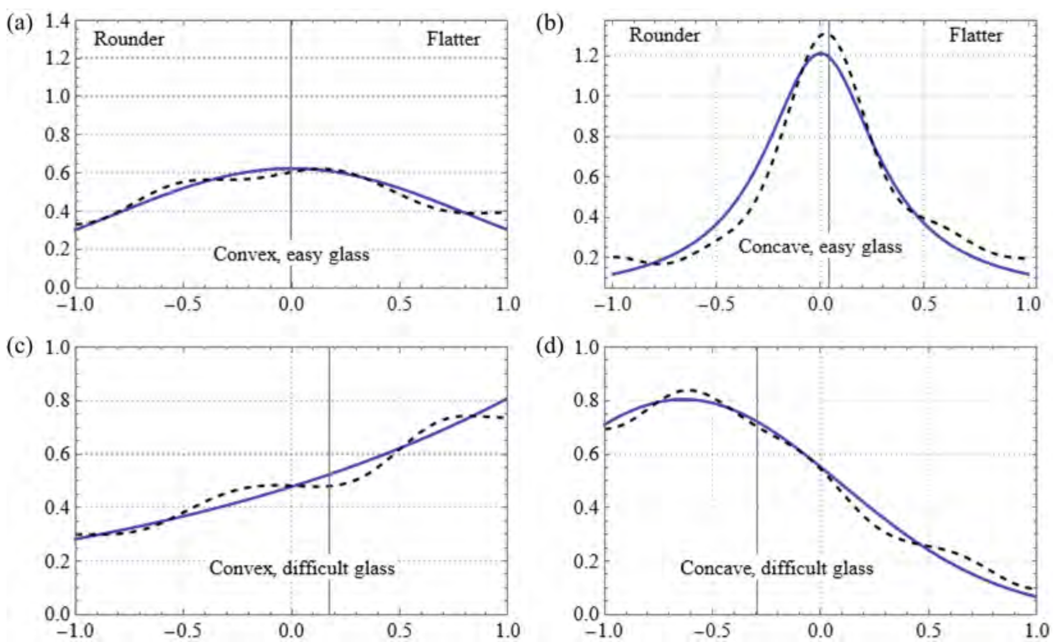

Figure 8. Radius distributions.

The distribution for lenses with concave surfaces from easy glass was a bit problematic. The data were highly symmetrical about zero and had a very strong central spike, but we also noted significant tails. The truncated Cauchy distribution was better than the normal distribution for the data fit. We looked at the Cauchy distributions *C*(0, 0.44) and *C*(0, 0.33). We found the Cauchy distribution *C*(0, 0.44) by maximizing the "LogLikelihood" function. The Cauchy distribution *C*(0, 0.33) was the compromise solution that we used; it matched the mean and standard deviation of the data, and was a little better at approximating the edge densities. Engineering judgement plays an important role in decisions of this type, and sometimes there are no objectively "best" recommendations.

Note that the distribution for the "convex, difficult glass" group also has a nonstandard shape. We approximated the fit using a version of the exponential distribution that is available in the CODE V PDF library. The exponential distribution is typically not defined for negative numbers. This version of the exponential distribution is defined over the interval [−1, 1], and has the rate parameter β. The distribution shape that we need can also be approximated with a linear or quadratic function, but is reasonably represented by the exponential distribution.

Table 4 is a data summary for the two variables needing a target value (radius [with difficult glass] and center thickness). The summary includes analysis from the data and from the fitted distributions listed in Table 3 and Figure 8. Skewness is calculated in Mathematica by dividing the third central moment by the cube of the population standard deviation. It is a metric of asymmetry for a distribution. For reference, a uniform distribution $U(-1, 1)$ has a standard deviation of 0.577 and skewness of 0.

Table 4. A summary of asymmetric data and parametric fits for radius and center thickness.

	Radius				**Center thickness**	
	Concave, difficult glass		**Convex, difficult glass**			
	Data	**$N(-0.634, 0.730)$[a]**	**Data**	**$E(1.898)$[b]**	**Data**	**$C(0.652, 0.450)$[c]**
Mean	−0.291	−0.291	0.172	0.172	0.399	0.399
Median	−0.368	−0.357	0.235	0.252	0.480	0.468
Mode[d]	−0.611	−0.634	1.000	1.000	0.652	0.652
Standard deviation	0.470	0.469	0.564	0.562	0.422	0.438
Skewness	0.577	0.548	−0.358	−0.363	−0.967	−1.020

[a]Normal distribution.

[b]Exponential distribution.

[c]Cauchy distribution.

[d]Data mode is estimated from a nonparametric fit.

We did not find strong correlations of the center thickness variable with material or geometry parameters, so the center thickness data were not segmented. The results of our analysis of center thickness data are summarized in Figure 9. We did find that the data were usually skewed toward the thicker end of the tolerance band. Approximately 84 percent of the time, we found the results between 0 and 1 assuming a normalized tolerance band [−1, 1]. This finding suggests that we can loosen center thickness tolerances, but we need to do this carefully because there is a 16 percent probability of a result falling between [−1, 0]. We would like to experiment with a center thickness nominal $+0.4T/-1.6T$; it would usually be nominal $\pm T$, but this is currently not allowed with CODE V.

The center thickness is known to have a bimodal mixture distribution with the larger mode on the right side and the smaller mode on the left side. Physically, this means that the opticians stop polishing when the lens is within tolerance, and this would be on the thicker end of the tolerance. Mathematically, this distribution has five parameters that need to be optimized for best fit with the data. We helped the optimization by constraining the left and right boundary conditions according to the nonparametric estimates, but we also constrained the mean of the fitted distribution to be equal to the data mean. The best-fit mixture distribution was $0.980 \cdot N(0.666, 0.550) + 0.020 \cdot N(-1, 0.110)$, where N indicates the truncated normal distribution. A mixture distribution is not included in the CODE V's library of functions, but the truncated Cauchy distribution.

The Cauchy distribution is somewhat infamous in financial circles; it is sometimes used for modeling "black swan" events, events that are extremely rare and mostly unpredictable but have severe consequences (e.g., a devastating banking crisis). The Cauchy distribution has heavy tails, and those heavy tails can be very useful in roughly approximating certain manufacturing distributions. We find that the bimodal center thickness can be roughly approximated with $C[0.652, 0.450]$. We matched the primary mode and the mean of this distribution; fortunately, the standard deviation and edge densities also closely matched the data is (see Figure 9).

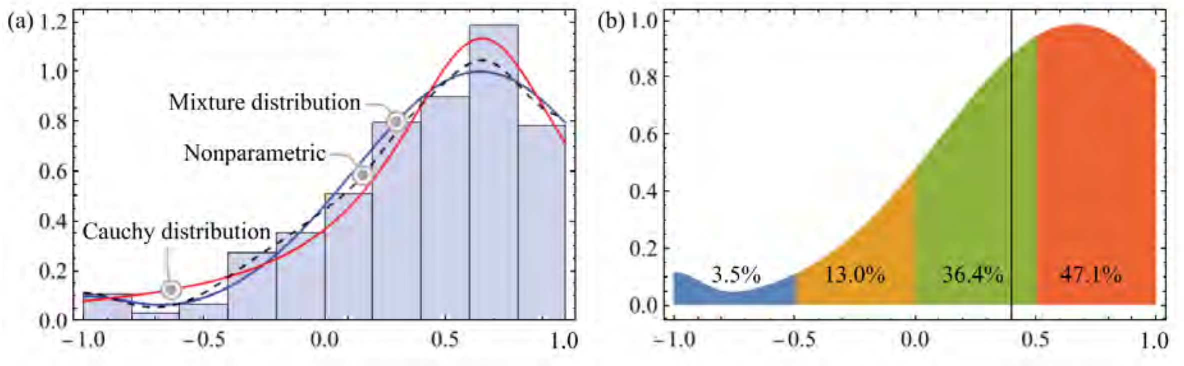

Figure 9. Center thickness data fit: (*a*) comparison between methods, and (*b*) estimated probability per quarter.

2.5 Special considerations for wedge and irregularity

Optics shops report wedge and irregularity as a positive number, but CODE V accepts a PDF on the interval [−1, 1]. We can deal with this problem by reflecting our existing PDFs about the origin. Consider, for example, the gamma distribution in Figure 10. The reflected PDF can be constructed in Mathematica with a transformation operation and a mixture operation (Figure 11). We thank Richard Juergens for incorporating the "bigamma" distribution (i.e., reflected gamma distribution as shown in Figure 11) into the TSF_CHA macro.

```
In[103]:= rule1 = {k → 3.5, θ → 0.2};
          fn1 =
            TruncatedDistribution[{0, 1},
              GammaDistribution[k, θ]] /. rule1;
          PDF[fn1, x]

Out[105]= { 103.65 e^(-5. x) x^2.5   0 < x ≤ 1
          { 0                         True
```

Figure 10. Mathematica input for displaying a gamma distribution.

```
In[99]:= fn2 = TransformedDistribution[-u, u ≈ fn1];
         fn3 =
           TruncatedDistribution[{-1, 1},
            MixtureDistribution[{.5, .5}, {fn1, fn2}]];
         Chop@PDF[fn3, x]
         GraphicsGrid[{{P1, P2}}]

Out[101]= { 0                          x == 0
          { 51.825 e^(5. x) (-x)^2.5   -1 < x < 0
          { 51.825 e^(-5. x) x^2.5     0 < x ≤ 1
          { 0                          True
```

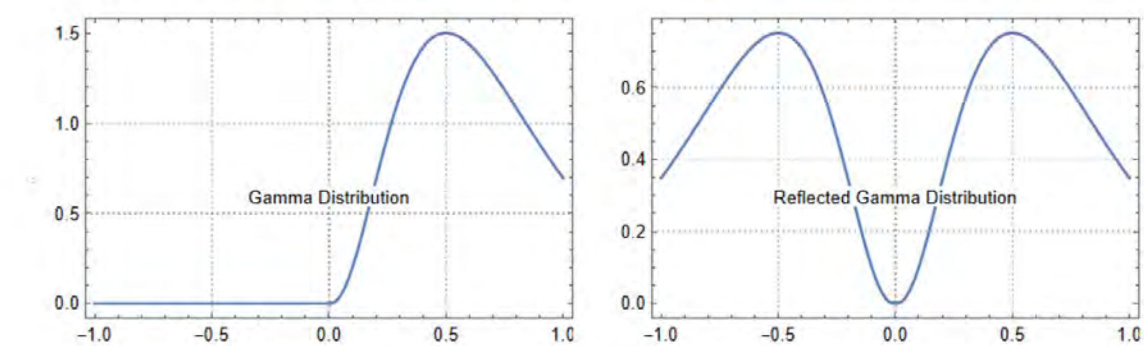

Figure 11. The Mathematica code for creating a reflected distribution.

2.6 From normalized distributions to dimensional distributions and back

All of our distribution functions were based on normalized data. The normalization procedure was done as follows: suppose V is a vector of measured values reported by our supplier with the allowable values of nominal $\pm T$ for radius or center thickness. Suppose V_{normal} is the normalized version of V: $V_{normal} = (V - \text{nominal})/T$. For wedge and irregularity, $V_{normal} = V/T$.

CODE V uses generic distributions that mostly exist on the support interval [−1, 1]. Examples of exceptions include RMS irregularity, which by definition must be greater than zero. We can infer that the software performs a scaling operation that takes the dimensionless interval [−1, 1] to the dimensional interval $[-T, T]$, where T is the tolerance value. Nevertheless, it is useful to understand what the software is doing. Suppose $f(x)$ is a PDF on the interval [−1, 1] and $g(x)$ is an equivalent PDF on the interval $[-T, T]$, where T is a positive number: $g(x) = f(x/T)/T$. This relationship also works if the interval is [0, 1] or [0, T]. Likewise, in reverse: $f(x) = g(x\cdot T)\cdot T$.

3. INCORPORATING DATA ANALYSIS INTO CODE V TOLERANCE ANALYSIS

Optical engineers using CODE V are usually familiar with tolerancing using the TOR and TOLMONTE commands. TOR uses a wavefront differential method, so while it has limitations and assumptions to consider, it is extremely fast. It is often the go-to method to understand as-built performance for intermediate design stages and can also identify sensitive tolerances. It supports the use of the following probability functions: uniform, Gaussian, second and fourth moments, and end point. It also supports the use of modulation transfer function (MTF) and RMS wavefront error performance metrics. TOLMONTE is much slower and uses methods based on Monte Carlo, but it supports a much wider range of performance metrics. TOLMONTE, however, does not allow the user to specify non-default probability functions. TOLMONTE could be used with non-default probability functions with an update to the UTOLCHNG macro. We are interested in applying special data-based probability functions to perturb error variation in variables such as lens center thickness, and to meet this requirement, we use a custom macro such as MONTE.

The MONTE and other associated macros were developed by Juergens specifically to take PDFs that are not typically used in CODE V. MONTE also allows the user to specify nonstandard performance metrics, but we did not take advantage of this feature. The scope of this study is to simply use what is already available in the existing library of functions and adapt it to our understanding of probable distributions for the four variables of interest.

MONTE calls other custom macros: TSF_CHA, TOL_MTF, and others. These macros perturb the lens (TSF_CHA), compensate for focus shifts, and evaluate specialized performance metrics if needed. The user enters the specialized PDF in a data file (PDF.dat). The currently available PDFs are described in the perturbation macro (TSF_CHA) and in a CODE V application video. This data file has a format as shown below:

n1 distribution param1 param2,

n2 distribution param1 param2,

and so forth,

where n1, n2, … are tolerance numbers. The program requires param1 and param2 even if, strictly speaking, you do not need them. The arguments are separated by spaces. You would enter zero for the nonfunctional parameter. For example, the uniform distribution does not need any parameters but still needs two arguments. An example of part of a PDF data file is shown in Figure 12.

```
8 gaussian 0 0.829          !  RADIUS, S1, E1, CX, easy glass
9 CAUCHY 0 0.33             !  RADIUS, S2, E1, CC, easy glass
10 cauchy 0 0.33            !  RADIUS, S3, E2, CC, easy glass
11 cauchy 0 0.33            !  RADIUS, S4, E2, CC, easy glass
12 exponential 1.898 0      !  RADIUS, S6, E4, CX, difficult glass (note: S5 IS THE APERTURE)
13 uniform 0 0              !  RADIUS, S7, E4-E5, doublet interface
14 gaussian 0 0.829         !  RADIUS, S8, E5, CX, easy glass
15 cauchy 0.652 0.45        !       CENTER THICKNESS, S1, E1
17 cauchy 0.652 0.45        !       CENTER THICKNESS, S3, E2
20 cauchy 0.652 0.45        !       CENTER THICKNESS, S6, E3
21 cauchy 0.652 0.45        !       CENTER THICKNESS, S7, E4
30 bigamma 3.455 0.181      ! IRR 45 deg, S1, precision
31 bigamma 3.455 0.181      ! IRR 45 deg, S2, precision
32 bigamma 3.455 0.181      ! IRR 45 deg, S3, precision
33 bigamma 3.455 0.181      ! IRR 45 deg, S4, precision
```

Figure 12. An example of a PDF.dat file.

The tolerance number listing may be seen from the CODE V GUI: Review > Tolerances. The TSF_CHA macro (as written by Juergens) has a library of PDF functions. Authorized users can access information about the distributions currently available in CODE V via the Synopsis customer support portal.

Examples of the MONTE macro usage are

in monte dbgauss_tol tol_mtf cv_macro:tolcomp 100 yynnyb1 "" 30 TAN, and

in monte dbgauss_tol tol_mtf cv_macro:tolcomp 100 yynnyb1 PDF_FILE 30 TAN,

where dbgauss_tol is the lens, 100 is the number of trials, 30 is the lp/mm for the MTF, and TAN is the orientation. An example of a PDF_FILE is shown in Figure 13, and in the first example above, the "" indicates the use of default PDFs.

Juergens uses an algorithm to generate random variates for the various distributions. The CODE V random number function “randf” generates uniformly distributed numbers between 0 and 1. The functions Juergens uses are not valid PDFs in a strict sense, but they will produce valid random variates. Valid PDFs are normalized such that an integral of the function over the support interval is unity. The algorithm used by Juergens requires that the functions be normalized such that the maximum value is always unity. An example of how Juergens generates random variates of a particular distribution is shown in Figure 13.

```
In[603]:= (* Generate 1000 random variates of a truncated
           Gaussian distribution: N[0.5, 0.1] *)
          param1 = .5;
          param2 = .1;
          n = 1000; i = 0; list = {};
          pts =
            While[i < n, rand1 = 2 * Random[] - 1; rand2 = Random[];
             fct = Exp[- (rand1 - param1)^2 / (2 * param2^2)];
             If[rand2 ≤ fct, i++; list = Append[list, rand1], Nothing];];
          Print["number of variates produced = " <> ToString@Length@list]
          Print["Recovery of original distribution:"]
          EstimatedDistribution[list,
           TruncatedDistribution[{-1, 1}, NormalDistribution[μ, σ]]]
          number of variates produced = 1000
          Recovery of original distribution:
Out[609]= TruncatedDistribution[{-1, 1}, NormalDistribution[0.507358, 0.0967223]]
```

Figure 13. Mathematica's interpretation of the Juergens algorithm for generating the Gaussian distribution variates.

4. AN EXAMPLE OF TOLERANCING WITH MONTE

We attempted tolerancing a sample double Gauss *f*/2.5 lens both with TOR and MONTE. This system has six elements as shown in Figure 14.

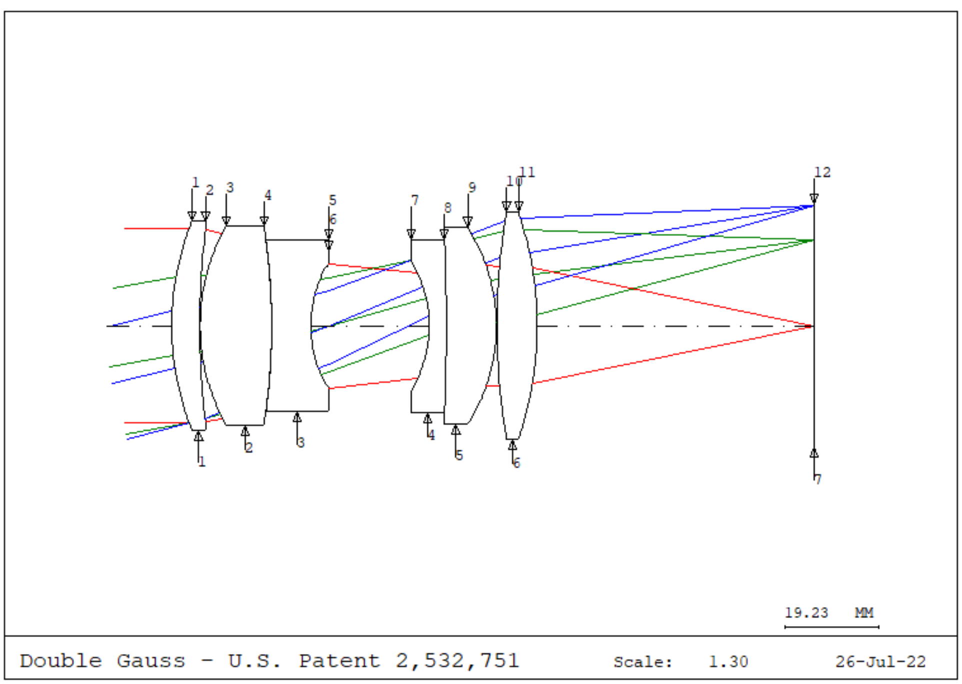

Figure 14. Double Gauss *f*/2.5 lens with six glass elements and nine surfaces; MTF evaluated at 30 lp/mm.

Table 5. Tolerancing guidance for the Gauss lens. Guidance on some glass types is a best guess.

Lens	Glass	Wedge	Surface	Radius	Irregularity
1	BSM4 (easy)	High Precision	1	Convex (Figure 8a)	Commercial
			2	Concave (Figure 8b)	Commercial
2	SK1 (difficult)	High Precision	3	Convex (Figure 8c)	Commercial
			4 (doublet interface)	Use uniform distribution	Commercial
3	F15 (easy)	High Precision	4 (doublet interface)	Use uniform distribution	Commercial
			5	Concave, uniform distribution	Commercial
4	F15 (easy)	High Precision	7	Concave (Figure 8b)	Commercial
			8 (doublet interface)	Use uniform distribution	Commercial
5	SK16 (difficult)	High Precision	8 (doublet interface)	Use uniform distribution	Commercial
			9	Convex (Figure 8c)	Commercial
6	SK16 (difficult)	Precision	10	Convex (Figure 8c)	Commercial
			11	Convex (Figure 8a)	Commercial

We were able to use the guidance on wedge and irregularity tolerances because of the addition of the reflected Gamma distribution to the TSF_CHA macro (Figure 11). The lens was designed with three fields: 0°, 10°, and 14°. As a result of recommendations from Synopsys, we added two extra fields, −10° and −14°, for comparisons with TOR. We increased the number of rays across the diameter from the default of 32 to 60 and used a simple on-axis compensation strategy with all fields evaluated in the TAN direction. Table 6 summarizes our findings for the double Gauss system.

Table 6. Tolerance trends for double Gauss lens using two methods, 400 trials with MONTE, MTF evaluated at 30 lp/mm.

Step	Description	F1, 0°	F2, 10°	F3, 14°	F4, −10°	F5, −14°
1	TOR and MONTE design values	0.674	0.518	0.619	0.521	0.619
2	Design – 2σ, TOR	0.457	0.284	0.518	0.284	0.518
3	Design – 2σ, MONTE, default PDFs	0.548	0.407	0.498	0.404	0.497
4	Design – 2σ, MONTE, custom PDFs for wedge and center thickness only	0.541	0.320	0.506	0.342	0.488
5	Design – 2σ, MONTE, custom PDFs for radius and irregularity only	0.586	0.373	0.480	0.387	0.480
6	Design – 2σ, MONTE, custom PDFs for all variables	0.560	0.353	0.509	0.346	0.513

5. DISCUSSION

We originally looked at a Tessar *f*/5.5 lens but found that the lens was relatively insensitive to variations of the tolerance distribution. When we look at the CODE V "interactive tolerancing" with "auto sort" turned on, we find that the primary contributors to MTF degradation of the Tessar lens are optomechanical (mostly barrel tilt, decenter, and lens tilt) and not glass tolerances, so they would not be affected by changes to distributions of the four variables of interest. A key point is that the distribution of errors for the four variables of interest in this study is really a very subtle tweak to the standard tolerance scheme. The standard methodology that most people use with TOR and CODE V default probability distributions is the 90 percent solution. What we do not know is when the 90 percent solution is not good enough.

We looked at the double Gauss lens at an MTF of 30 lp/mm. At this spatial frequency, the difference between TOR and MONTE with customized PDFs becomes noticeable.

We find that the design values are consistent between MONTE and TOR (Table 6, step 1). There are differences between the Design – 2σ, TOR (Table 6, step 2) and Design – 2σ, MONTE default PDFs (Table 6, step 3) values, primarily at F3. It is not clear why the F3 values in steps 2 and 3 are different, but it appears that the design is more sensitive at this field point to subtle alterations. Table 6, step 4 documents the addition of custom PDFs, but only for center thickness and wedge. The wedge tolerances mostly fall into the category of “high precision,” meaning that there is a strong probability that many of the individual lens elements have wedge measurements that are barely within tolerance. In this case, a uniform distribution is optimistic. The addition of radius and irregularity PDFs (Table 6, step 5) mostly improves the MTF. In the end (Table 6, step 6) when all variables are considered, the effect of custom PDFs is small.

It is interesting to note that the degradation of MTF is noticeable only when a special treatment for wedge and center thickness is considered. Alterations to the default CODE V PDF could make things better or worse. If you alter all four variables at once, the effects could nullify each other. While this is a good outcome, this does not provide insight into what is happening with the lens.

We were frustrated by the variation in steps 2 and 3 of Table 6: those rows should not be different. We looked at the same lens at a lower MTF (15 lp/mm) using TOR and MONTE (Table 7). This is essentially the same as a CODE V training video exercise. Steps 1 and 2 should be identical (they are very close). Steps 3 and 4 should be close (they are). We suspect that TOR and MONTE may diverge when higher spatial frequencies are considered.

Table 7. Tolerance trends for double Gauss lens using two methods, 1000 trials with MONTE, MTF evaluated at 15 lp/mm.

Step	Description	F1, 0°	F2, 10°	F3, 14°	F4, −10°	F5, −14°
1	TOR design values	0.890	0.830	0.845	0.830	0.846
2	MONTE “nominal” values	0.890	0.827	0.844	0.827	0.844
3	Design – 2σ, TOR	0.752	0.707	0.793	0.707	0.793
4	Design – 2σ, MONTE, default PDFs	0.799	0.723	0.767	0.729	0.766

It would be interesting to apply this technique on a more complicated lens. With a more complicated lens and very stringent requirements (e.g., a stepper lens), the stack-up of tolerance errors would be more egregious and the trends a little more obvious. But the compensation schemes for lenses of interest to the US Department of Energy can be quite complex, and it will take significant time and effort to develop a tolerance procedure suitable for a lens assembly of interest to us.

A basic rule of thumb for defining the assembly error is $\varepsilon_{\text{assy}} = C_f \sqrt{\sum_{i=1}^{n} \varepsilon_i^2}$, where ε_i is the component error and C_f is the correction factor. The correction factor is usually assumed to be 1. It was suggested in 1968, however, that to account for “shifts and drifts” we should use a value of 1.5.[9] This suggestion was based on experience, not analysis, but the 1.5σ shift has since been analyzed by others[10] and accepted for use by many companies. A “drift” is a gradual change in a variable. For example, a drill often makes larger holes as it wears. A “shift” could result from a setup error. For example, all the parts made on a particular day were off by 0.003 inches in the *x* direction due to a mistake in setup. The part is still within tolerance, but consistent errors add differently than random errors. It is known that opticians are incentivized to not bring parts to the nominal value in ways that are very predictable. The most obvious example is center thickness. We think that optical designers could take advantage of known trends by establishing an asymmetrical tolerance band and placing the nominal value at a location where we expect it to be consistent with historical evidence. An example might be a center thickness of 10 mm with a tolerance of +0.1 mm to −0.2 mm. Now this would be unconventional, and opticians might not like it, but they would still bid on it. We think that this technique could allow designers to lower center thickness tolerances. But things are never completely straightforward; there may be cases where consistently thicker lenses actually improve performance a little. For now, Zemax supports asymmetric tolerance bands, while CODE V does not typically support asymmetric tolerance bands (there may be special macros that enable TOLMONTE to allow asymmetric tolerance bands such as UTOLCHNG).

6. CONCLUSIONS

What is so interesting about this analysis is that it ties together four important elements of optical engineering: the material science of glass, glass grinding and polishing, optical tolerancing, and optical design. Before computers, the fabricators and designers lived under the same roof. Now with powerful computers, this is no longer needed. We gained something with the advent of computers, but we lost something too, and this is what we need to remember. To regain the insight into fabrication by optical designers now requires a leap of imagination and many phone conversations.

There are many directions this work can take in the future. For this study, we had data from 611 lenses in 24 nominally identical batches; however, a much larger study is needed. In spite of the small size of this study, there are physical reasons that explain why the distributions shown here might be representative for our industry. For the interested reader, we are simply explaining how to develop a process as described in Figure 1.

But what about engineers who have very limited access to lens manufacturing data? A wise colleague made the following comment on the data and analysis we explored as part of this inquiry: “We know three things: we have some data, we understand some of the causal relationships, and we have intuition based on decades of experience.” Data can be limited and noisy, intuition is sometimes wrong, and our causal understanding may be incomplete, but together these tell a story, and we find that the three-way conversation has been useful in refining our understanding and identifying directions for further investigation.

The data we presented come from lens grinding and polishing. There will be very different data distributions for molded systems such as cell phone lenses. Again, understanding how to segment the data will require EDA methods such as correlation analysis as well as a physical understanding of why the errors have a characteristic shape. From this, one can develop a system for segmenting the data and applying the data to an optics program.

We are certainly not the first to develop a program such as shown in Figure 1. Similar programs have been developed and implemented at Corning Tropel[11,12] and Asphericon.[13] Authors in this space are reluctant to share too much detail because of company proprietary considerations. We hope that we have opened up this branch of optical engineering so more engineers can discuss and make use of these methods.

The tolerancing methods we describe are not necessarily warranted in every case. In this work, we study the effect of a more detailed statistical treatment for four lens variables: (1) radius, (2) center thickness, (3) wedge, and (4) irregularity. In cases where the controlling tolerances are optomechanical (e.g., lens decenter, lens tilt, barrel tilt), an in-depth treatment of the four lens variables might not matter. In cases where the four variables dominate the merit function of interest, the methods described here could be beneficial. We suspect that these methods will be most beneficial for complex high-end optical systems. In cases where the direct benefits of the methods we describe here are marginal, the concepts developed here can still help inform decision-making during the optical development process.

With respect to wedge and irregularity, if the tolerance class is already within the supplier’s capability (Figures 5a, 5b, 7a, and 7b), the probability of exceeding the stated tolerance may be very low. If the tolerance is in the high-precision class, there may be a substantial probability that many of the lenses will be close to the edge of being unacceptable.

With respect to asymmetric production data (Figures 8c, 8d, and 9), be aware that the tolerance stack-up error may add differently than one might think. One may estimate the probability of a problem as follows. In the “interactive tolerancing” mode of CODE V, one may download the data into a spreadsheet, and use the “filter” feature of Excel to investigate the problem. The spreadsheet will show the effect of a positive and negative tolerance on MTF. If, for example, too many of the positive tolerances for center thickness result in in a reduction of MTF, one can assume that a consistent addition of these errors will not tend to cancel out. Alternatively, one could perturb the system such that all of the center thicknesses are $0.4T$ (for a $\pm$T tolerance) larger than nominal and then evaluate the perturbed system.

Center thickness is usually not the most important lens variable, but the possibility of loosening this tolerance is worth exploring because it could significantly improve manufacturability and reduce production cost. This is why we think designers should consider asymmetric tolerance bands for center thickness.

We were frustrated by the lack of agreement between TOR and MONTE (with default PDFs) for the double Gauss lens at an MTF of 30 lp/mm. We then found agreement for those methods at 15 lp/mm. We would like to better understand the conditions under which wavefront differential and Monte Carlo methods diverge.

IN MEMORIUM

We respectfully note the passing of the following individuals this year: Gavin Griffin, Lucy Malone, and John Greivenkamp.

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Appendix, Glass list

This appendix provides a list of easy and difficult glass types. The purpose is to provide the reader with information that can be useful in the tolerancing process. Also included here are comments provided by Brandon Light, engineer at Optimax Systems and co-investigator of this study. We hope that the reader finds his first-hand commentary on some of the glass types listed here helpful.

Difficult Schott glass types:

LAFN7, LAKN13, LASFN9, N-BAF51, N-BASF64, N-FK51, N-FK56, N-KZFS4, N-LAF2, N-LAF21, N-LAF3, N-LAF32, N-LaF35, N-LaF36, N-LAF7, N-LAK12, N-LASF31, N-LASF35, N-LASF40, N-LASF41, N-LASF44, N-LaSF45, N-PK51, N-PK52, N-PSK53, N-PSK57, N-PSK58, N-SF1, N-SF10, N-SF4, N-SF56, N-SF6, SF1, SF10, SF11, SF14, SF15, SF4, SF56A, SF57, SF6, SF66, SFL57, SK51

Difficult Ohara glass types:

BAL35Y, PBL1Y, PBL26Y, PBM18Y, PBM2Y, PBM8Y, S-BAH10, S-BAH11, S-BAM12, S-BSM25, S-BSM28, S-FPL51, S-FPL51Y, S-FPL53, S-FPM2, S-LAH52, S-LAH53, S-LAH60, S-LAH63, S-LAH71, S-LAH79, S-LAL12, S-LAL54, S-LAL56, S-LAL 7, S-LAM 2, S-LAM 3, S-LAM51, S-LAM52, S-LAM55, S-LAM58, S-LAM59, S-LAM61, S-LAM66, S-LAM 7, S-NBH51, S-NBH52, S-NBH53, S-NBH55, S-NBH 8, S-NPH 1, S-NPH 2, S-NPH 3, S-NPH53, S-PHM52, S-PHM53, S-TIH1, S-TIH10, S-TIH11, S-TIH13, S-TIH14, S-TIH23, S-TIH 3, S-TIH53, S-TIH 6, S-TIM 3, S-TIM35

Easy Schott glass types:

F2, F4, F5, K10, K7, KZFSN4, KZFSN5, LAKL12, LF5, LF5G19, LLF1, N-BAF10, N-BAF3, N-BAF4, N-BAF52, N-BAK1, N-BAK2, N-BAK4, N-BALF4, N-BALF5, N-BASF2, N-BK10, N-BK7, N-F2, N-FK5, N-K5, N-KF9, N-KzFS11, N-KZFS2, N-LAF33, N-LAF34, N-LAK10 , N-LAK14, N-LAK21, N-LAK22, N-LAK33, N-LaK34, N-LaK7, N-LAK8, N-LAK9, N-LASF43, N-LF5, N-LLF1, N-LLF6, N-PSK3, N-SF15, N-SF5, N-SF64, N-SF8, N-SK10, N-SK11, N-SK14, N-SK15, N-SK16, N-SK18, N-SK2, N-SK4, N-SK5, N-SSK2, N-SSK5, N-SSK8, N-ZK7, SF2, SF5

Easy Ohara glass types:

BAL15Y, BSL7Y, BSM51Y, PBL25Y, PBL35Y, PBL6Y, S-BAH27, S-BAH28, S-BAH32, S-BAL11, S-BAL12, S-BAL14, S-BAL 2, S-BAL 3, S-BAL35, S-BAL41, S-BAL42, S-BAM 3, S-BAM 4, S-BSL 7, S-BSM10, S-BSM14, S-BSM15, S-BSM16, S-BSM18, S-BSM 2, S-BSM22, S-BSM 4, S-BSM71, S-BSM81, S-BSM 9, S-FSL 5, S-FSL5Y, S-FTM16, S-LAH51, S-LAH55V, S-LAH58, S-LAH59, S-LAH64, S-LAH65V, S-LAH66, S-LAL10, S-LAL13, S-LAL14, S-LAL18, S-LAL58, S -LAL59, S-LAL61, S-LAL 8, S-LAL 9, S-LAM54, S-LAM60, S-NBH 5, S-NBM51, S-NSL3, S-NSL36, S-NSL 5, S-TIH18, S-TIH 4, S-TIL 1, S-TIL 2, S-TIL25, S-TIL26, S-TIL27, S-TIL6, S-TIM1, S-TIM2, S-TIM22, S-TIM25, S-TIM27, S-TIM28, S-TIM39, S-TIM 5, S-TIM 8, S-YGH51

Listed below are comments made by Brandon Light of Optimax on some of the glass types listed above. A few editorial changes are made to the comments for clarity and brevity.

On the whole, this is a good list, but as you've noticed, there are always a few glass types that stand out.

SF1 is one that seems problematic when considering the hardness and elastic modulus, but it is not difficult to work with. There are some other examples, but I think that points to how this is a guideline, and not a certitude.

N-SF6 and SF6 have the same six-digit code, but the manufacturability is quite different. N-SF6 exhibits some unfavorable thermal characteristics, while SF6 is soft and scratches easily and is marked by extreme acid susceptibility. Both of them are "difficult," but one of them, SF6, is truly worthy of the title.

N-SK16 is quite problematic. Left uncleaned for a few minutes or even merely stored in a humid environment, it will stain to the point of needing repolishing. The chemical susceptibility is that strong. It would definitely go into the "difficult" category, but it has nothing to do with the mechanical properties that drive the classification.