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# Final report on subcontract B643529

J. Brannick

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## **Final report on subcontract B643529**

September 1, 2020 – August 31, 2021

Generalized Bootstrap AMG and AIR-AMG for coupled PDE systems with a focus on space-time discretizations

PI: James Brannick Penn State University

The Pennsylvania State University (“Subcontractor”) has worked on the design of new algebraic, parallel, multilevel methods that obtain the full space and time solution of systems of PDEs. In particular, the PI and his collaborators explored semi-intrusive approaches based on algebraic multigrid (AMG). The focus of the research has been on the development of these techniques for the Euler equations in 1d and 2d. The overall research focused on the development of adaptive AIR (approximate ideal restriction) AMG solvers for these problems. The PI also explored the use of smoothed aggregation and root-node energy-based AMG solvers for these problems.

The high-level accomplishments of this project are shown below.

1. Development of (nonsymmetric) AIR AMG solvers amenable to systems of PDEs, with a focus on space-time discretizations of hyperbolic systems of PDEs and systems in a mixed hyperbolic-diffusive regime.
2. Development of smoothed aggregation and root-node energy minimization techniques for space-time PDEs.
3. Applications of these techniques to the 1d and 2d Euler equations discretized using finite difference and finite element methods.

Overall, all methods were shown to be effective for cases where the advection was constant. However, in the case of non-constant advection or in the mixed hyperbolic-diffusive regime only the AIR-based AMG solver has shown promising results. Future work will focus on further development of these techniques for non-constant advection.