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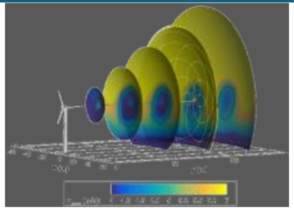
Sandia National Laboratories

Investigation of Spatial Smoothing Techniques Applied to Design Variables in Structural Optimization Problems



PRESENTED BY

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U.S. DEPARTMENT OF ENERGY NNSA

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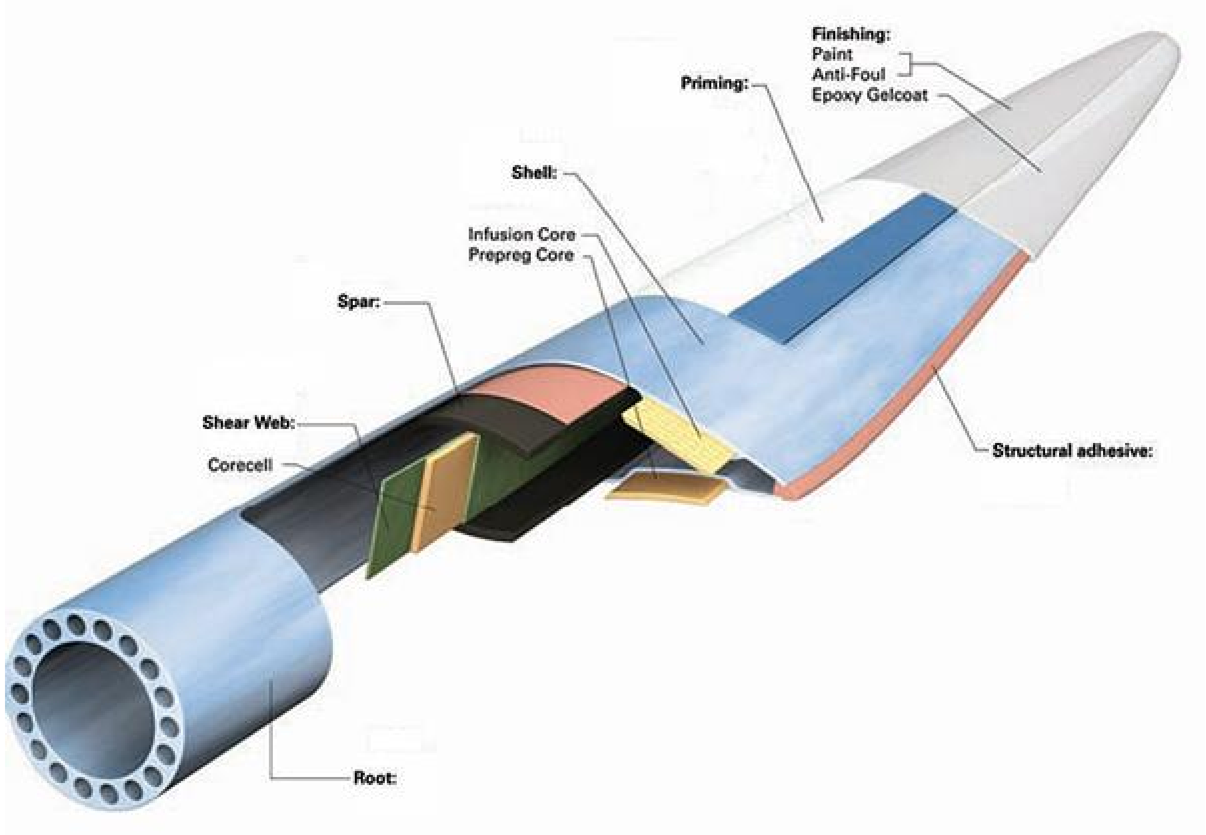
Background

Detailed structural analysis and design optimization is complex, open-ended

Critical considerations:

- Performance requirements
- Objective/constraints
- Optimization methodology
- Design space definition

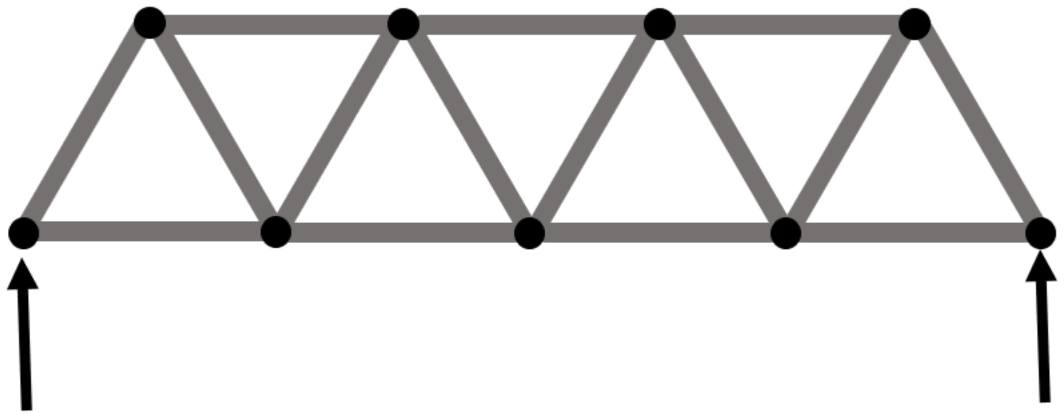


Common Challenges in Structural Optimization

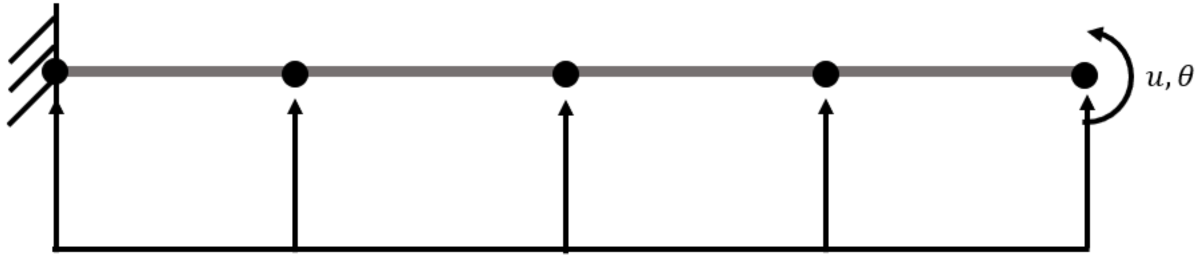
Many common methods, such as genetic algorithms and particle swarm, require many evaluations of objective function (high computational cost for high fidelity contexts).

Robust, global search but number of design variables can be limited by cost. Good for low-fidelity applications.

Low-fidelity structural truss analysis



Finite element beam modeling

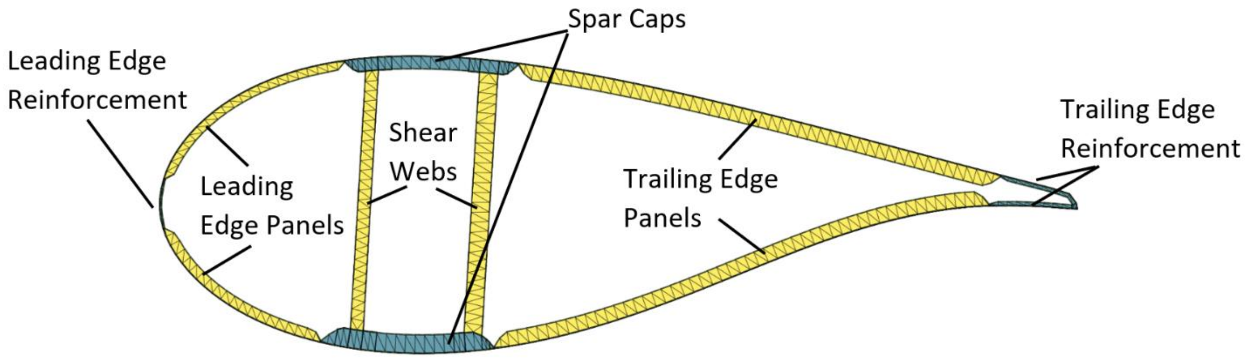


Common Challenges in Structural Optimization

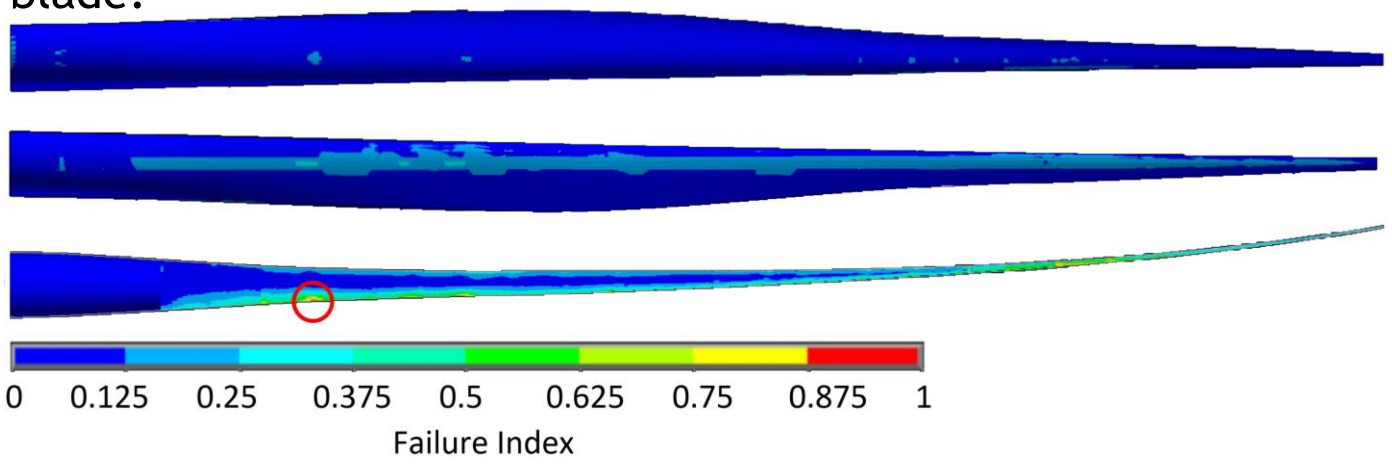
Gradient-based methods are efficient for problems with many design variables, can be favorable for high-fidelity optimization [ref 1].

If design space not defined carefully, optimizer can stall or produce solutions that are not feasible to fabricate.

Detailed 2D cross-sectional model of a wind blade.



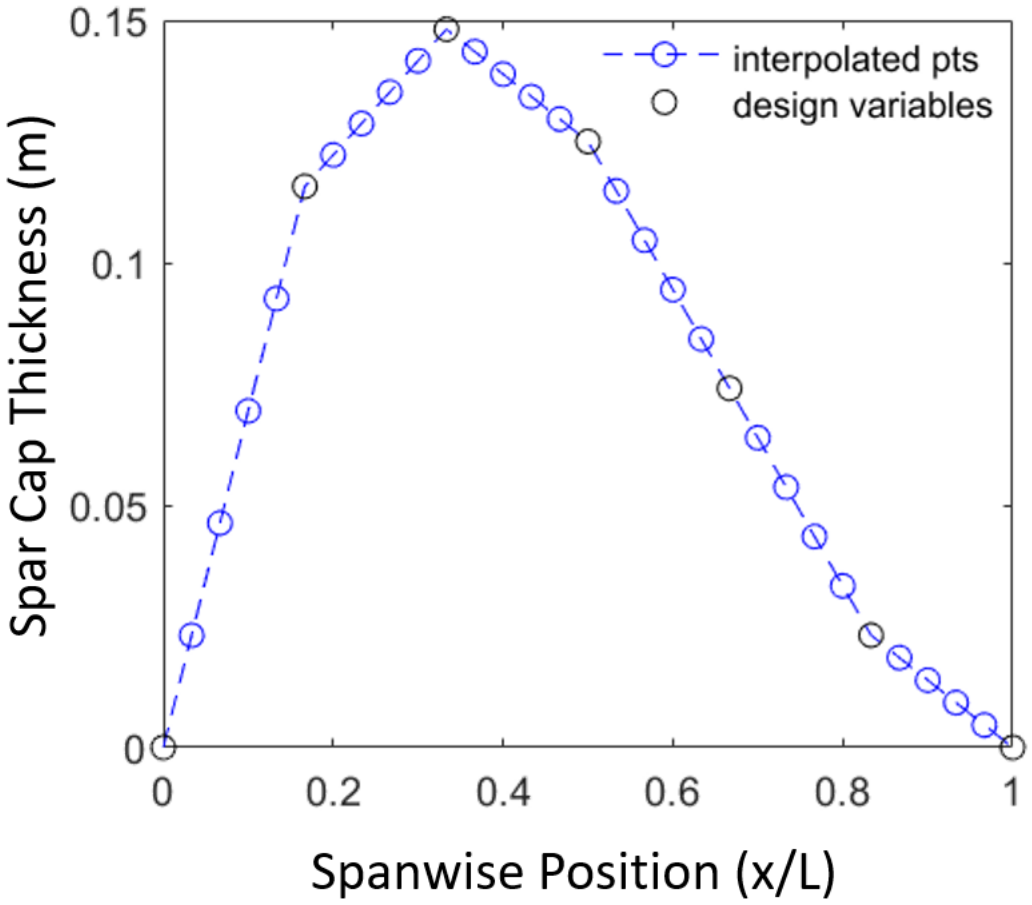
Detailed 3D finite element shell model of a wind blade.



Solutions to Challenges in Gradient-Based Optimization

Interpolation between pre-determined points a common method for defining design space [ref. 2,3].

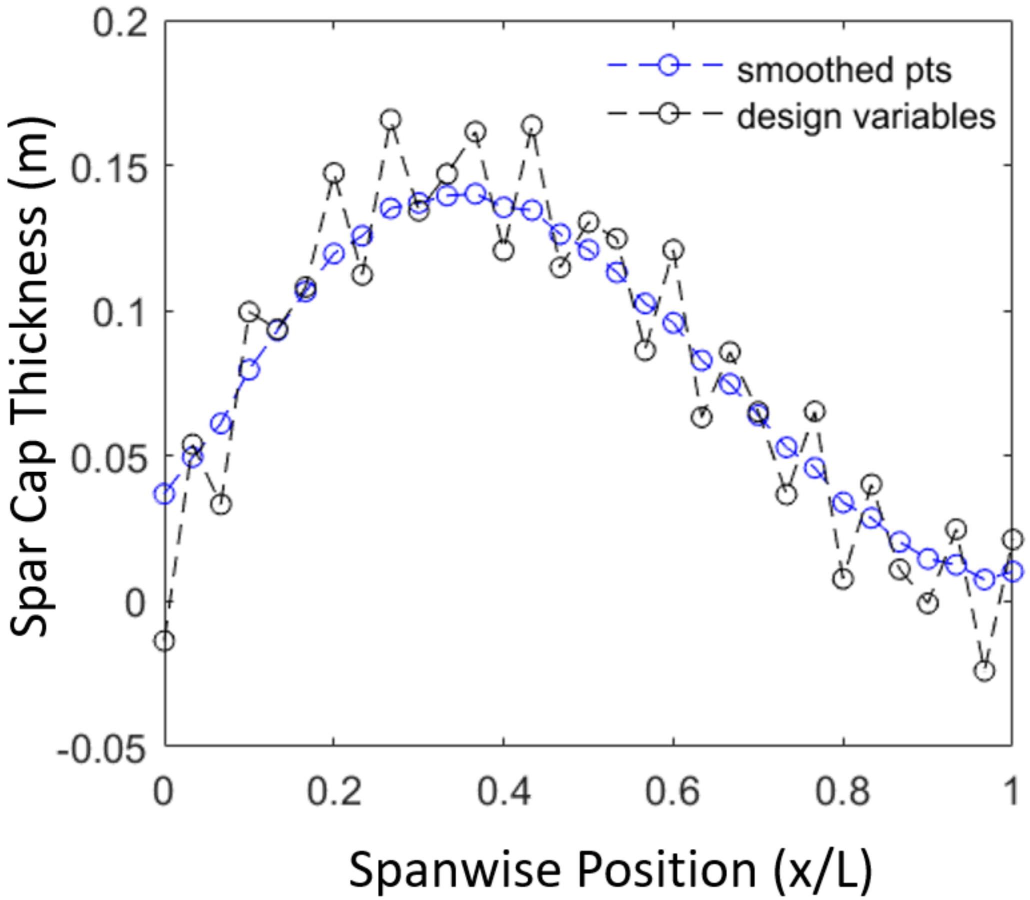
Though it may often be effective, interpolation can lack the flexibility to adapt to “hot spots” or small problematic areas that drive the design.



Solutions to Challenges in Gradient-Based Optimization

An alternative to conventional interpolation is to let all spanwise variables be active and apply a smoothing operator to preserve continuity

Least-squares homogenization is one method of smoothing



Least-Squares Homogenization

Let $\boldsymbol{x}$ be a vector of length n with a given set of values. A modified set of variables $\boldsymbol{x}'$ with greater continuity and smoothness can be formed by minimizing the sum of squared residual of the following two sets of equations:

$$\boldsymbol{x}' = \boldsymbol{x}$$

$$x_i' = x_{i+1}'$$

$$\Longrightarrow \mu(x_i' - x_{i+1}') = 0 \;\; for\ i = 1 \ldots n-1$$

Where μ is a scalar *smoothing factor*. This factor controls the strength of the smoothing effect, and provides a way to control the balance between adaptability and smoothness.

Analytical Example

For demonstration of concept, consider the closed-form analytical objective function:

$$L(\boldsymbol{x}) = \sum_{i=1}^{n} c_i x_i + k_i \left(\frac{1}{x_i}\right)^2$$

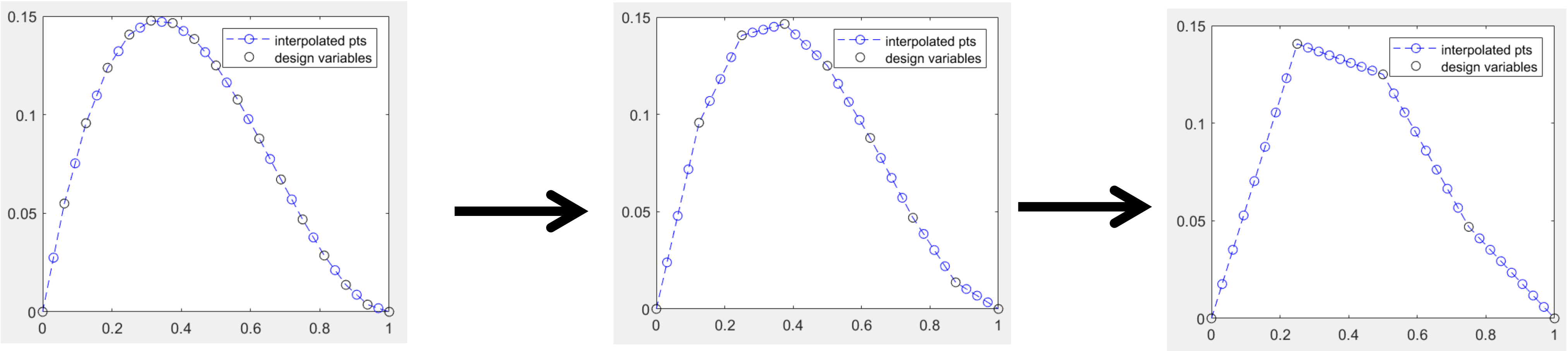
The above bears a simplified resemblance to the total mass of a structure, with a penalty term on stress with x_i representing the thickness of the i^{th} spanwise section.

Seek to minimize $L(\boldsymbol{x})$ with respect to $\boldsymbol{x}$.

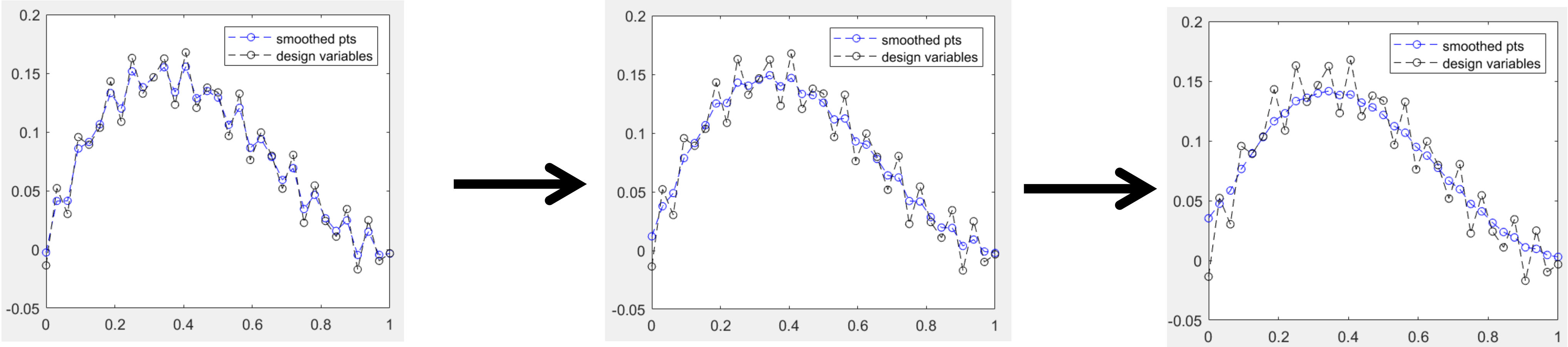
Analytical Example

For 100 randomly generated variations of $\boldsymbol{k}$ in the objective function, perform the following 2 tests:

Test 1: Minimize $L(\boldsymbol{x})$ with $\boldsymbol{x}$ interpolated between a decreasing number of selected points



Test 2: Minimize $L(\boldsymbol{x})$ with all $\boldsymbol{x}$ active, with an increasing smoothing factor



Analytical Example

Results for cases with $\boldsymbol{x}$ interpolated between key points, averaged across all 100 variations of the objective.

	Interpolation Point Spacing				
	2	4	8	16	32
Objective Error	1.55E-02	2.26E-02	2.60E-02	2.75E-02	2.84E-02
Iterations to Convergence	42	49	54	36	25

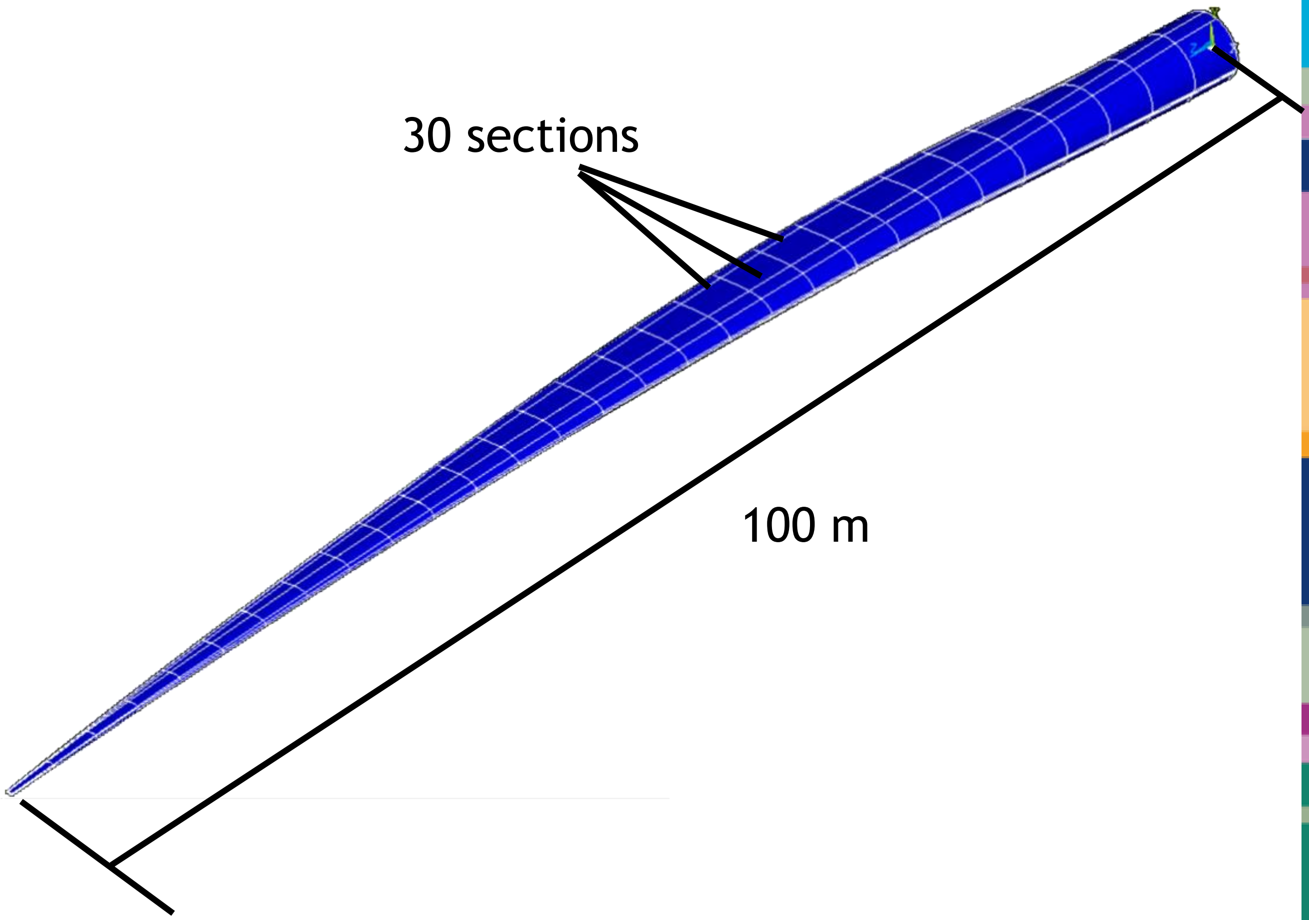
Results for cases with smoothing factor applied on $\boldsymbol{x}$ averaged across all 100 variations of the objective

	Least-Squares Smoothing Factor				
	0	0.5	1	2	4
Objective Error	1.73E-15	4.45E-15	5.02E-04	7.51E-03	1.63E-02
Iterations to Convergence	32	26	66	187	464

Example: Finite Element Wind Blade Model

Model specifications:

- Length: 100 meters
- Construction: Fiber glass composite for spar caps, reinforcements, skins, medium density foam for panel and shear web filler
- Pre-determined outer mold line/airfoil distribution, held constant for present analysis.
- Thickness of spar caps, core panels, leading and trailing edge reinforcements, shear webs and shell skin defined as design variables at 30 sections along the blade span.



Example: Finite Element Wind Blade Model

Loading: Generated from results of aeroelastic simulation using OpenFAST [ref. 4], a tool out of National Renewable Energy Laboratory (NREL).

Analysis: Finite element analysis and objective sensitivity run with AStrO, an in-house open-source code.

Optimizer: Gradient-based optimizer in MATLAB, fmincon.

Test 1: Minimize mass of blade with constraint on maximum stress by changing section thicknesses, with thicknesses interpolated between an increasing number of pre-selected spanwise sections.

Test 2: Minimize mass of blade with constraint on maximum stress by changing section thicknesses, with all sections active using an increasing smoothing factor.

Example: Finite Element Wind Blade Model

Results for cases with spanwise component thicknesses interpolated between key points

	Interpolation Point Spacing			
	2	4	8	16
Normalized Objective (scaled to initial value)	0.420	0.403	0.362	0.536
Iterations to Convergence	26	12	18	54

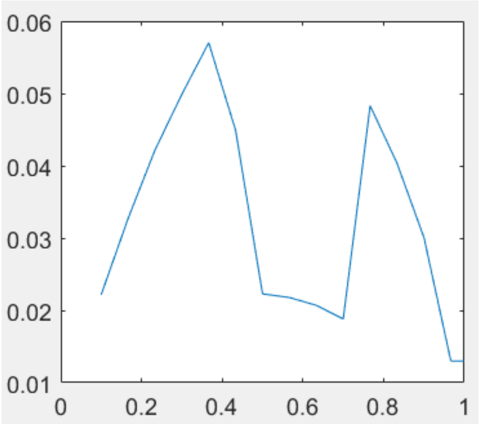
Results for cases with smoothing operator applied to the spanwise thickness

	Least-Squares Smoothing Factor			
	0.5	1	2	4
Normalized Objective (scaled to initial value)	0.403	0.390	0.411	0.346
Iterations to Convergence	51	85	39	22

Example Comparison of Final Design, Leading Edge Suction Side Filler

Thickness interpolated at spacing of 2

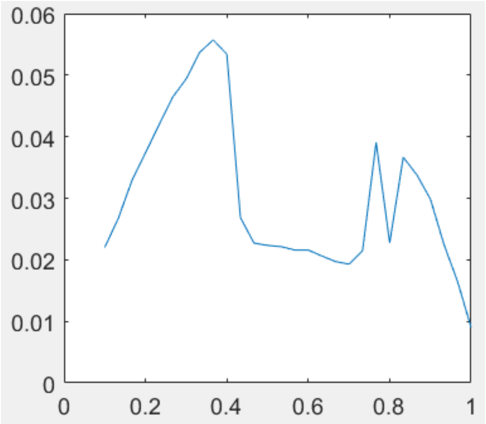
Thickness (m)



Spanwise Position (x/L)

Thickness smoothed at factor of 0.5

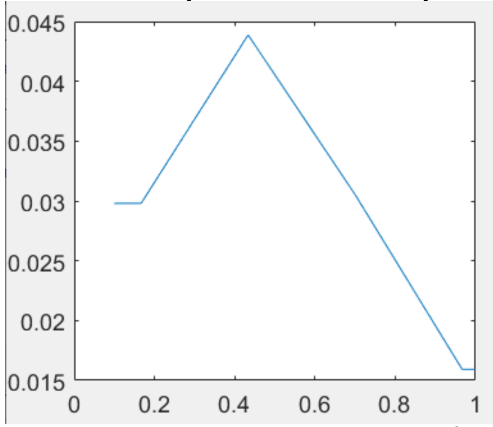
Thickness (m)



Spanwise Position (x/L)

Thickness interpolated at spacing of 8

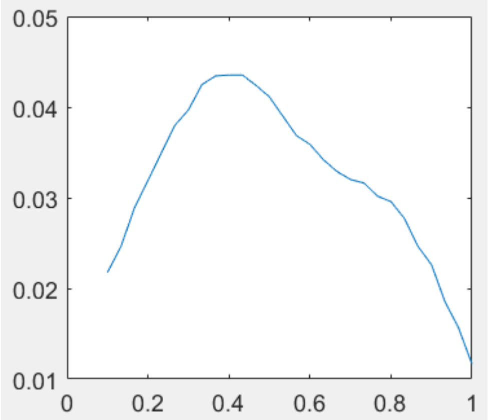
Thickness (m)



Spanwise Position (x/L)

Thickness smoothed at factor of 4

Thickness (m)



Spanwise Position (x/L)

Discussion

In analytical case, trends were seen in improved solution quality with smoothing operator compared to interpolation, but poorer quality and slower convergence for very high μ.

Trends are less clear and conclusive for wind blade optimization, but most overall favorable result was with a smoothing factor of 4. Optimizations using very low interpolation spacing or very small smoothing factors are prone to stall in convergence or produce infeasible solutions.

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4. NREL: OpenFAST, v2.5.0, https://github.com/OpenFAST/openfast, 2021a.

Least Squares Homogenization

$$\begin{bmatrix}[I]\\ [\mathrm{M}]\end{bmatrix}\{\boldsymbol{x}'\} = \begin{Bmatrix}\{\boldsymbol{x}\}\\ \{0\}\end{Bmatrix}$$

In the above, $[I]$ is the $n \times n$ identity matrix and $[\mathrm{M}]$ is an $(n-1) \times n$ matrix with μ on all diagonal terms and $-\mu$ on the first band above diagonal,

$$[\mathrm{M}] = \begin{bmatrix} \mu & -\mu & 0 & 0 & \dots & 0 & 0 \\ 0 & \mu & -\mu & 0 & \dots & 0 & 0 \\ 0 & 0 & \mu & -\mu & \dots & 0 & 0 \\ 0 & 0 & 0 & \mu & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu & -\mu \end{bmatrix}$$

Least Squares Homogenization

The least squares solution can be found with QR factorization:

$$\begin{bmatrix} [I] \\ [\mathrm{M}] \end{bmatrix} \{\boldsymbol{x}'\} = [Q][R]\{\boldsymbol{x}'\} = \begin{Bmatrix} \{\boldsymbol{x}\} \\ \{0\} \end{Bmatrix}$$

$$\Longrightarrow \{\boldsymbol{x}'\} = [R]^{-1}[Q]^T \begin{Bmatrix} \{\boldsymbol{x}\} \\ \{0\} \end{Bmatrix}$$

Since the lower portion of the vector on the right-hand-side is zero, we can define our smoothing operator $[T]$ as $[R]^{-1}$ times the first n columns of $[Q]^T$, leaving:

$$\{\boldsymbol{x}'\} = [T]\{\boldsymbol{x}\}$$