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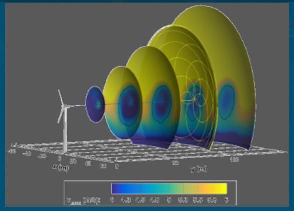
A Continuous Representation of Actuator Disk Forcing



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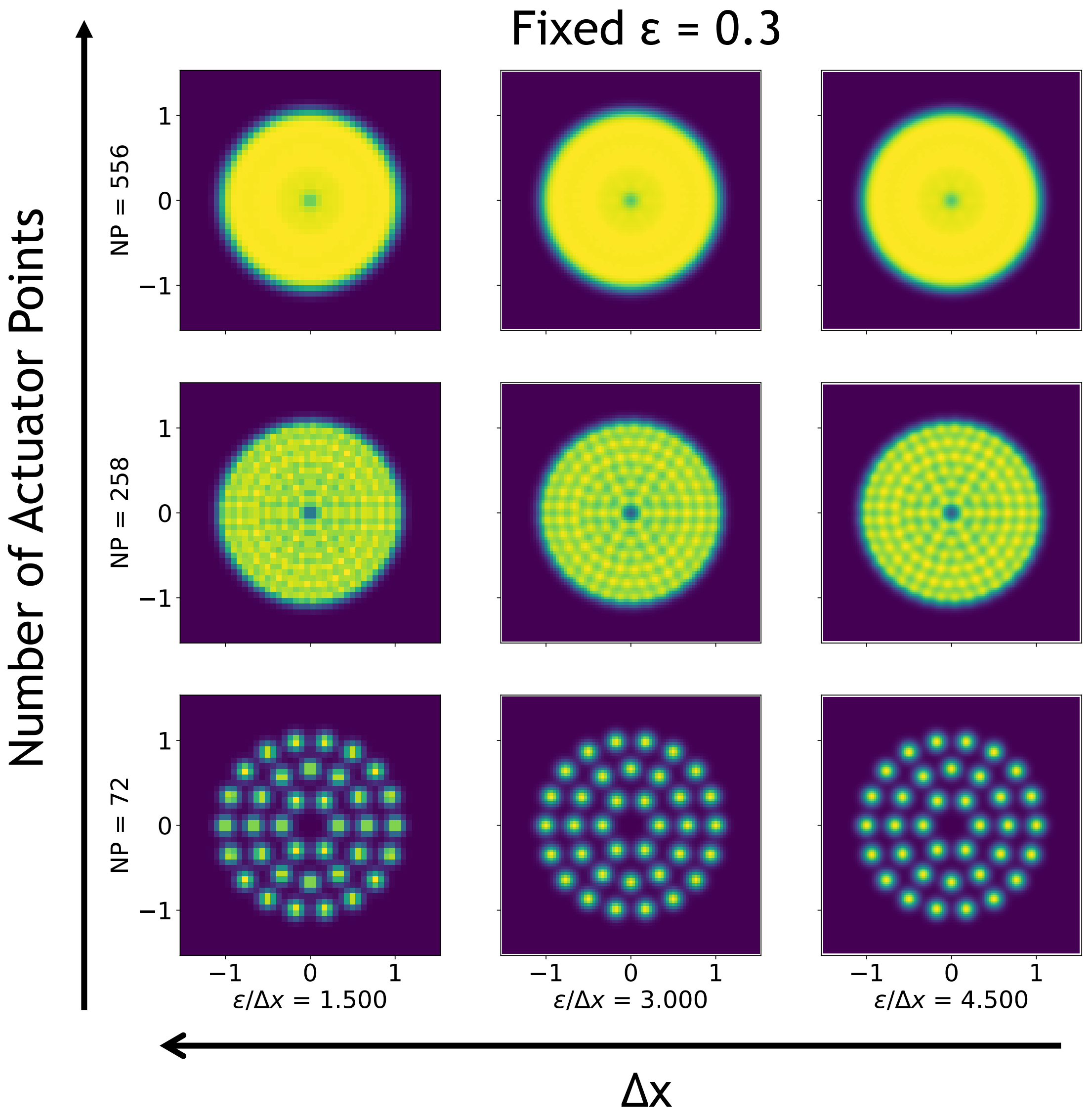
Introduction

Actuator methods (line and disk) have 3 major components

- Interpolate fluid properties
- Aerodynamic model
- Spreading function – typically Gaussians

Focus on spreading function for disk application

- Two primary purposes:
  - Transfer the force from the aerodynamic model
  - Create a geometric representation which determines wake development
- Point sampling with Gaussians can get expensive
- Dependency on Gaussian width can be confusing
- Explore alternative to Gaussian



Spreading Function:

Actuator forcing term:

$$\boldsymbol{F}(\boldsymbol{x}) = \boldsymbol{f}_j \xi_j(\boldsymbol{x})$$

Spreading function defines the geometric representation of the actuator

$$\xi_j(\boldsymbol{x}) = \frac{1}{\left(\sqrt{\pi}\epsilon\right)^3} e^{-\frac{\left|x - x_j\right|^2}{\epsilon^2}}$$

Can also be thought of as a projection onto a sub-space where the actuator geometry is defined (i.e. sub-space of $\mathbb{R}^3$)

$$\vartheta(\boldsymbol{x}) = \sum_j^N \xi_j(\boldsymbol{x})$$

$$\boldsymbol{x} \in (-\infty, \infty), \vartheta(\boldsymbol{x}) \in \mathbb{R}^3$$

Foundations for Looking Beyond the Gaussian

Gaussian is not the only option for spanning the actuator sub-space

Requirements for the projection function:

- Normalized so point force is conserved i.e. $\int \xi_j^*(\boldsymbol{x})d\boldsymbol{x} = 1$
- Ideally it will be smooth and strictly positive as well

Metrics for comparing Gaussian with alternative projection functions

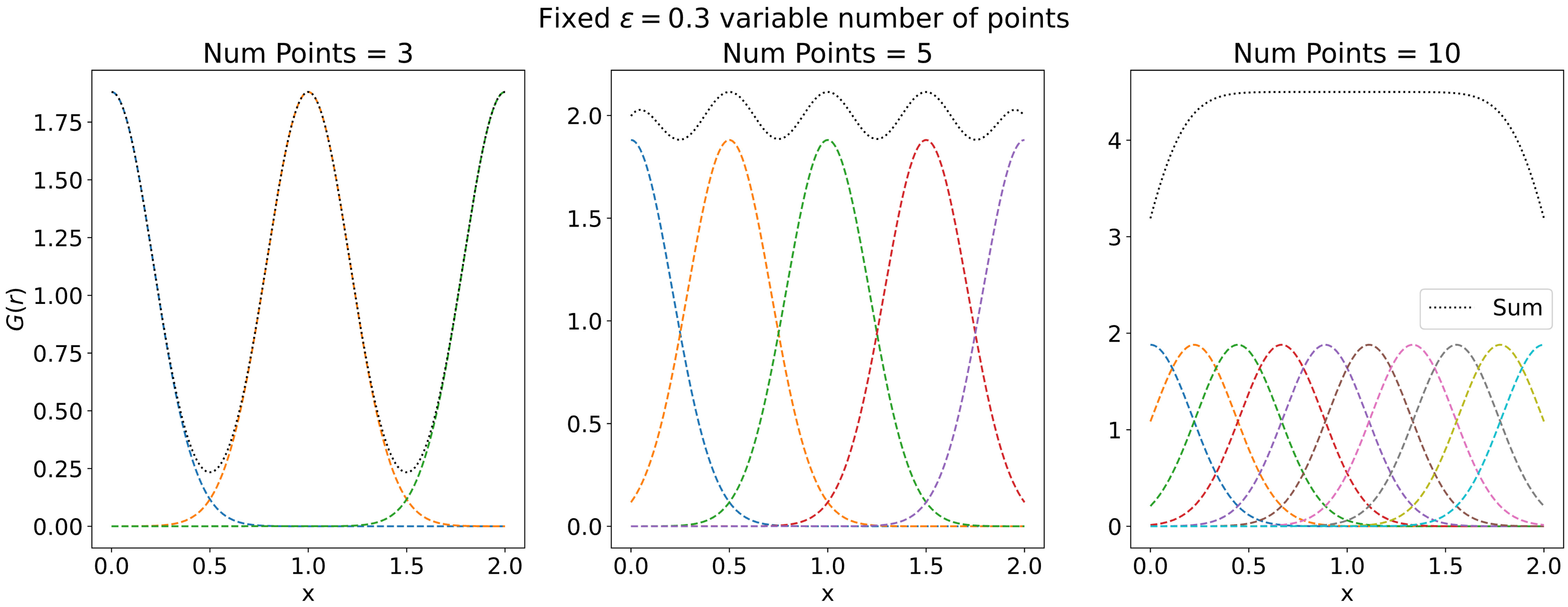
- Accuracy:
  - Measure the value of the normalized projection function on the background mesh

$$R = 1.0 - \frac{\Delta x^2}{N} \sum_j^N \xi_i^*(\boldsymbol{x})$$

- Performance:
  - Count the number of projection function calls required to compute the actuator sub-space

$$\sum_i^M \sum_j^N \xi_j^*(\boldsymbol{x}_i)$$

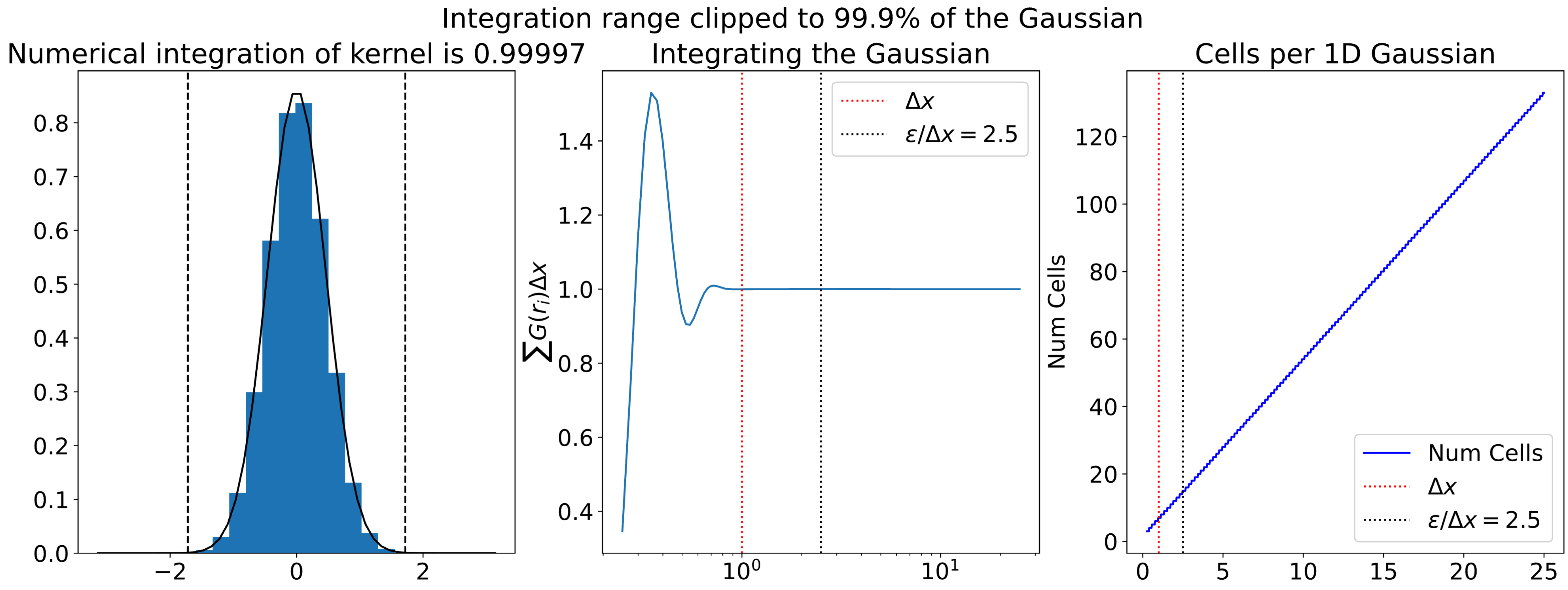
Traditional Method: Gaussians



Forcing must become diffuse to get a smooth sub-space (geometry)

- Lots of overlap between forcing from each actuator point

Accuracy and Performance: Gaussian



Accuracy is quite good, even with mid-point integration

Requires large foot print on the background mesh (10 cells > in each direction) based on common metric (Sorensen and Shen, 2002; Martinez-Tossas, Churchfield and Leonardi, 2014)

Another Word on Epsilon

Forcing sub-space geometry is dependent on combination of number of actuator points and epsilon

- Must adequately span the distance between actuator points (avoid porosity)
- Must meet requirements of background grid for integration accuracy

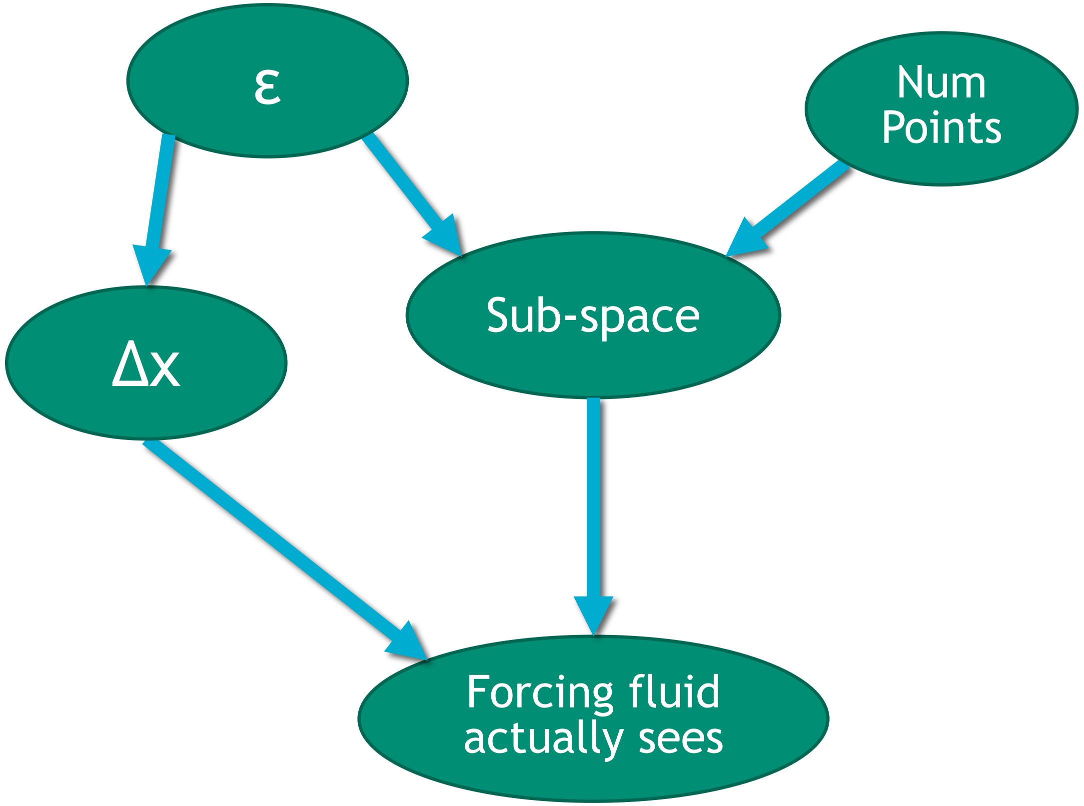
One solution is to use more points

- Acceptable for ALM but costly for ADM
- 1D vs 2D geometry

Another is to increase epsilon

- Changes geometry (makes disk bigger at the edge of the domain)
- Diffuses force over larger area
- Computational cost increase on a given grid due to larger foot print

<u>Goal to remove coupling between actuator geometry and integration requirements</u>



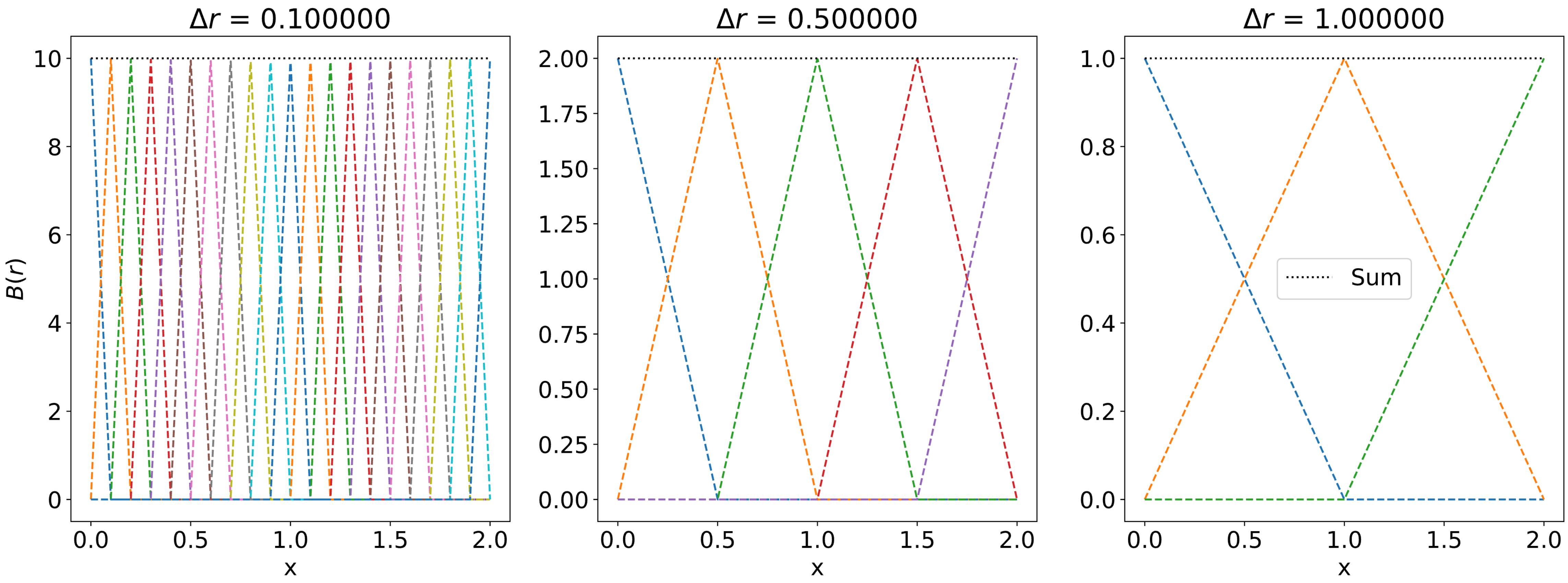
Alternative Functions

Replace Gaussians with functions that

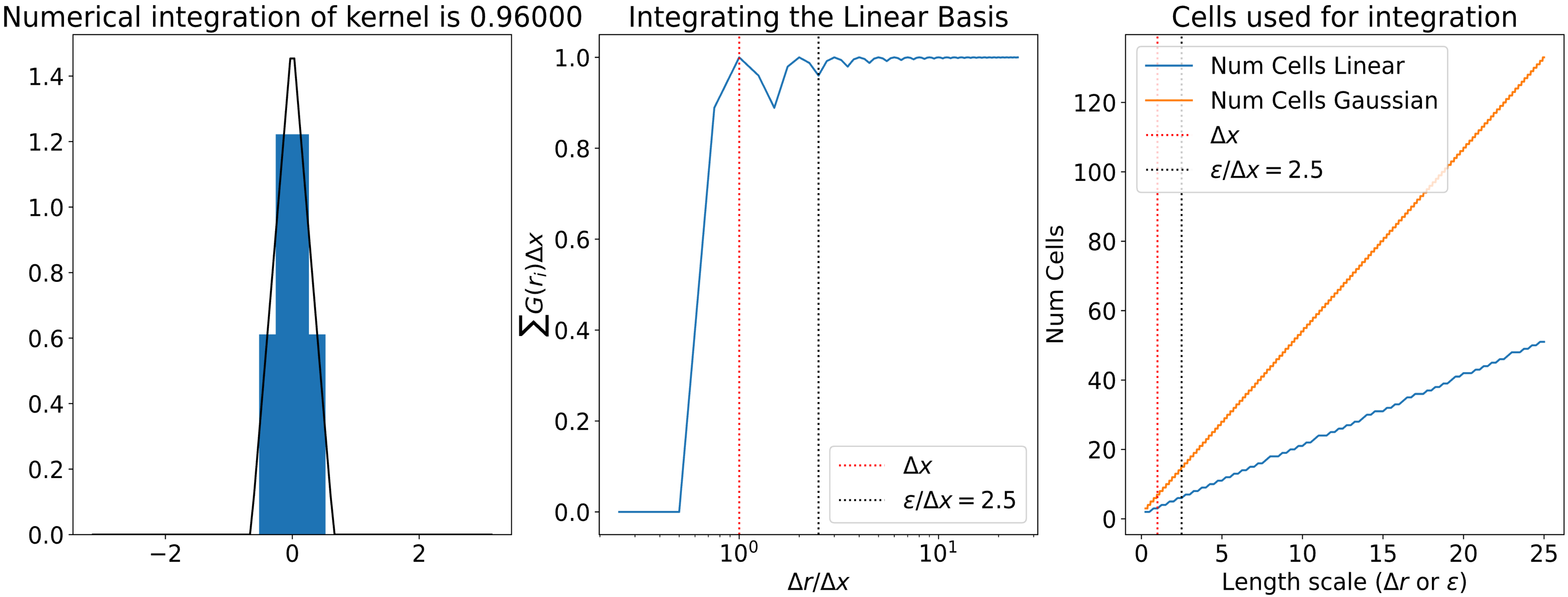
- Have compact support i.e. $x \in [-\Delta r, \Delta r],\ \vartheta^*(x) \in \mathbb{R}^1$
- Are partitions of unity: $\sum_j^N \xi_j^*(x) = 1$

Example is a normalized linear basis: $\xi_j^*(x) = \dfrac{\max\left(0, 1 - \frac{|x - x_j|}{\Delta r}\right)}{\Delta r}$

Note the peak value changes due to compact support combined with normalization. Same thing occurs with Gaussians as ε is changed.



Accuracy and Performance: Linear Basis Function

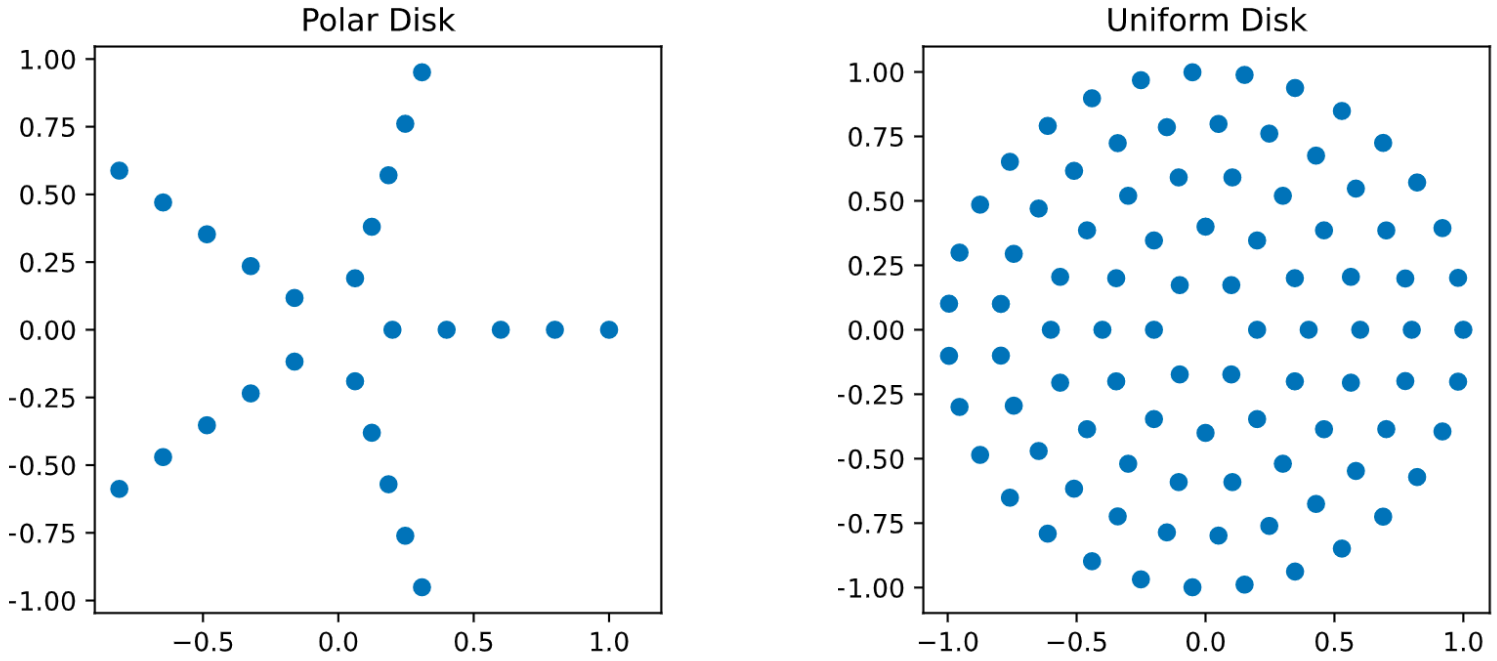


Integration foot print is smaller (restricts support to distance between actuator points)

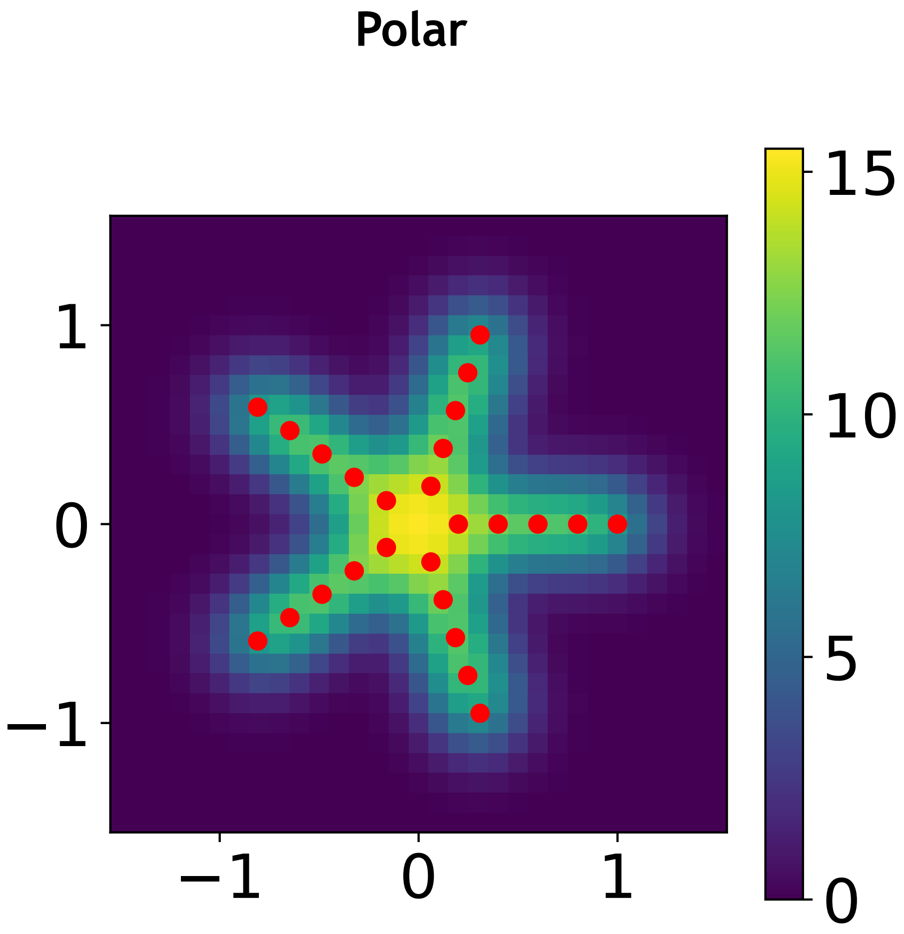
Accuracy is comparable to the Gaussian for the same foot print with mid-point rule

Comparison on a Fixed Grid

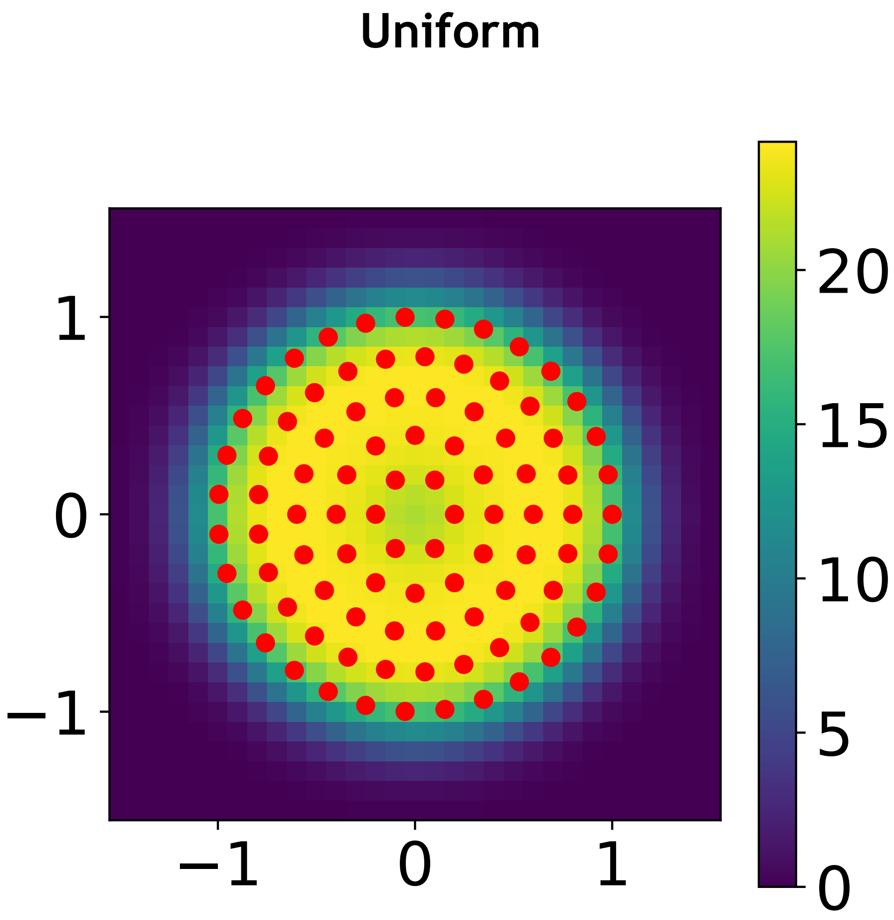
- Discretize the unit circle with actuator points
- Use a background mesh with uniform grid-spacing of 0.1 from -1.5 to 1.5



Performance and Accuracy: Gaussian



Num Function Calls: 3378
Integration Accuracy: 0.99896587

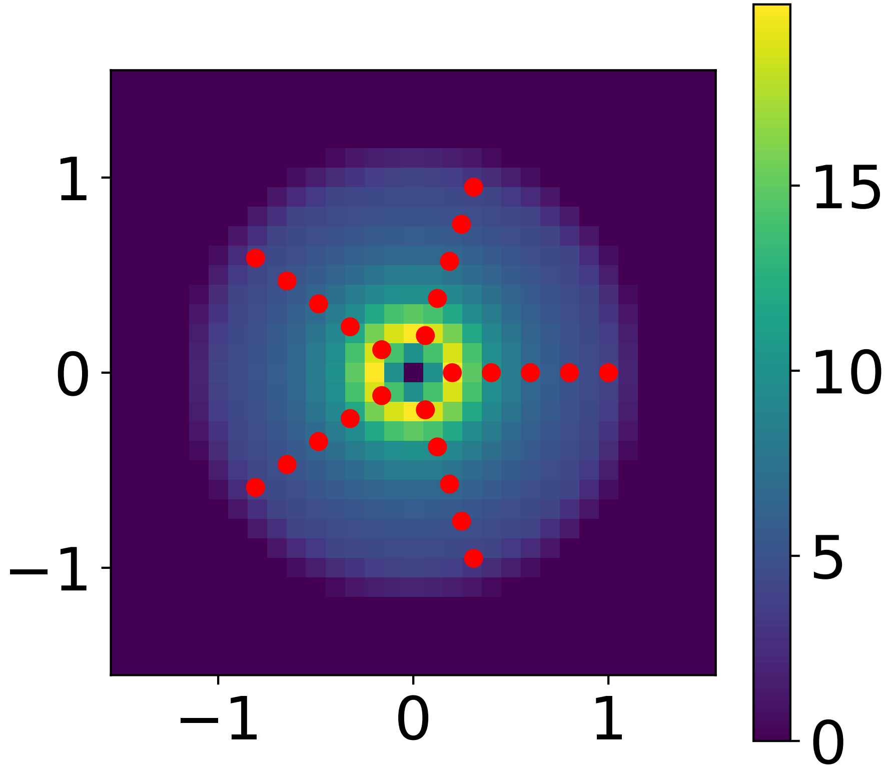


Num Function Calls: 12443
Integration Accuracy: 0.99896058

Performance and Accuracy: Linear Basis

Polar (5x5)

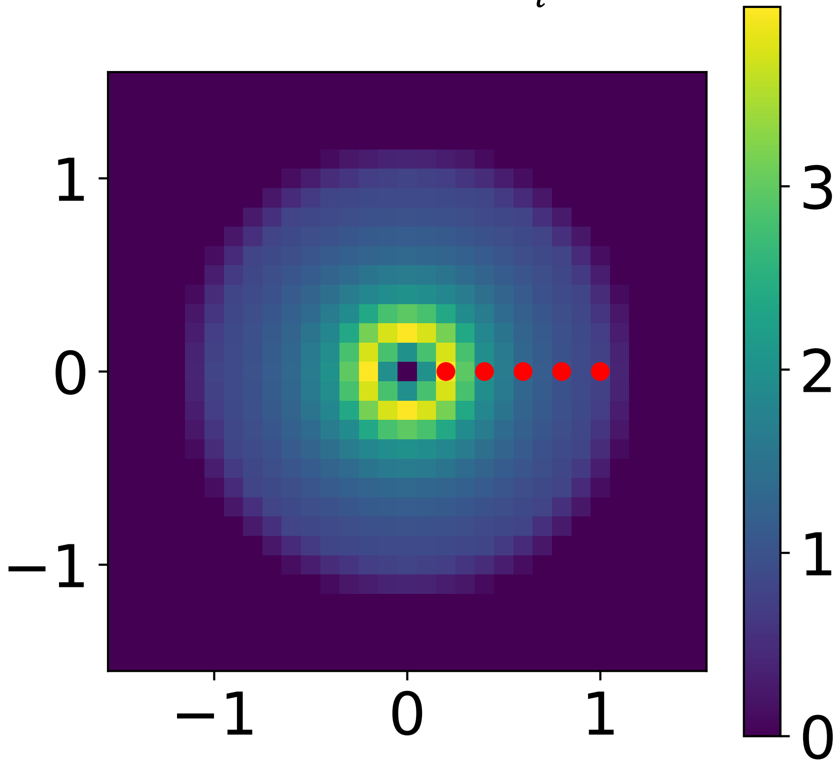
$$\eta_{ij}(r,\theta) = \xi_i^*(r, \Delta r = \Delta R) \times \xi_j^*(r\theta, \Delta r = \mathrm{r_i}\Delta\theta)$$



Num Function Calls: 1474
Integration Accuracy: 1.00008085

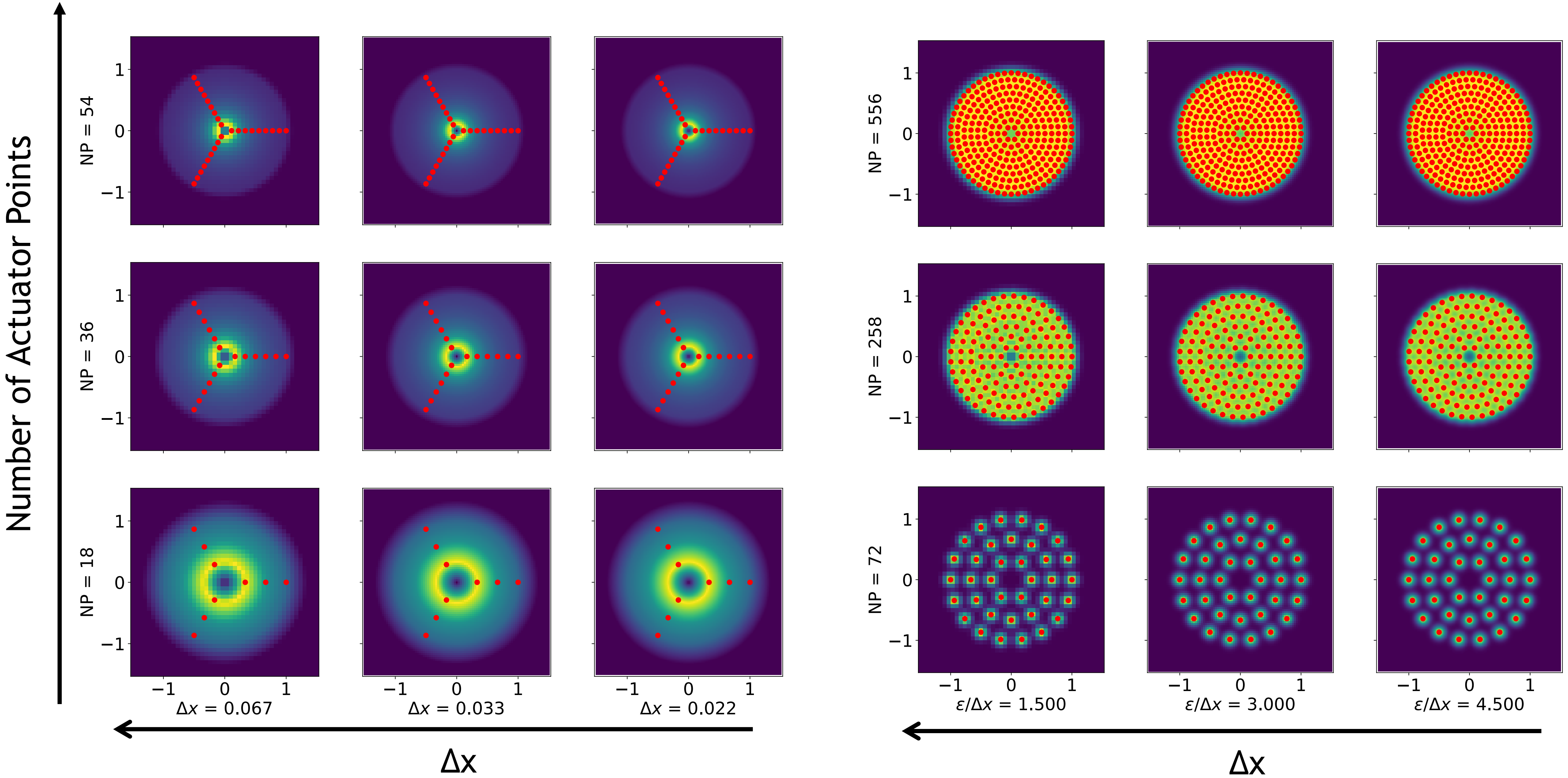
Polar (5x1)

$$\eta_i(r) = \frac{\xi_i^*(r, \Delta r = \Delta R)}{2\pi r_i}$$



Num Function Calls: 737
Integration Accuracy: 1.00008085

Point Sample Invariance



- Geometric representation of the disk is really only varies at the edges
  - Due to dependence on radial point spacing
- Practically invariant when compared to the Gaussian
- Significant reduction in points required to capture the disk

Final Comments and Conclusions

What about normal direction, disk edge?

- Can use a Gaussian, or any other normalized basis function
- Edges of sub-space can be the same as the standard model by using Gaussians

Conclusions:

- Changing the Gaussian kernels to functions with 1) compact support that are 2) partitions of unity offers several advantages
- Effectively makes geometric resolution independent from other factors (background mesh integration, aerodynamic model, etc.)
  - Geometric representation of the actuator disk can be maintained with minimal actuator points
  - Accuracy of projection to the fluid domain is not noticeably impacted even at coarser actuator point resolutions
- Less diffuse forcing with proportional decrease in computational cost due to reduction in kernel overlap

Next Steps:

- Evaluate other basis functions and fine tuning
- Extend theory to actuator line applications

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